Classicalization, Scrambling and Thermalization in QCD at high energies

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Outline of lectures

Lecture I: Classicalization: The hadron wavefunction at high energies as a Color Glass Condensate

Lecture II: CGC continued ? Multi-particle production and scrambling in strong fields: the Glasma

Lecture III: Novel features of the Glasma: universal non-thermal fixed points, the Chiral magnetic effect

Lecture IV: Thermalization and interdisciplinary connections

The Regge-Gribov Limit

$$x_{\rm Bj} \to 0\,;\; s \to \infty\,;\; Q^2 (>> \Lambda^2_{\rm QCD}) = \text{fixed}$$

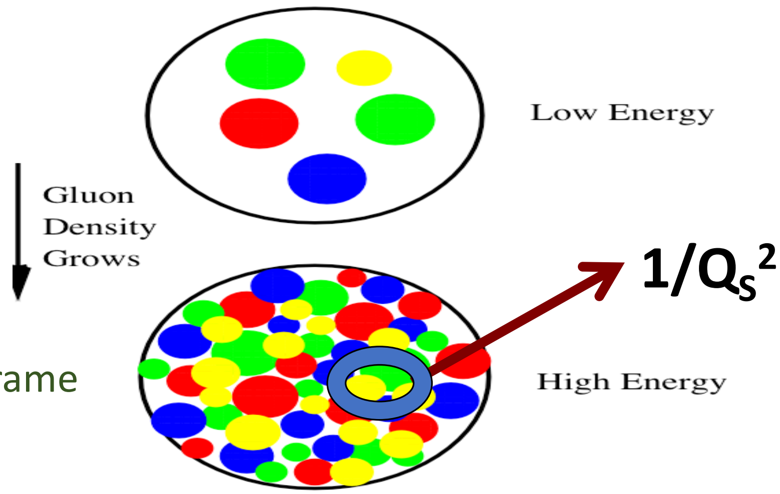
Physics of multi-particle production and strong fields in QCD

Novel universal properties of QCD ?

The boosted proton: gluon saturation

Gribov,Levin,Ryskin (1983)
Mueller, Qiu (1986)

Decoupling of longitudinal
and transverse dynamics
In the hadron infinite momentum frame



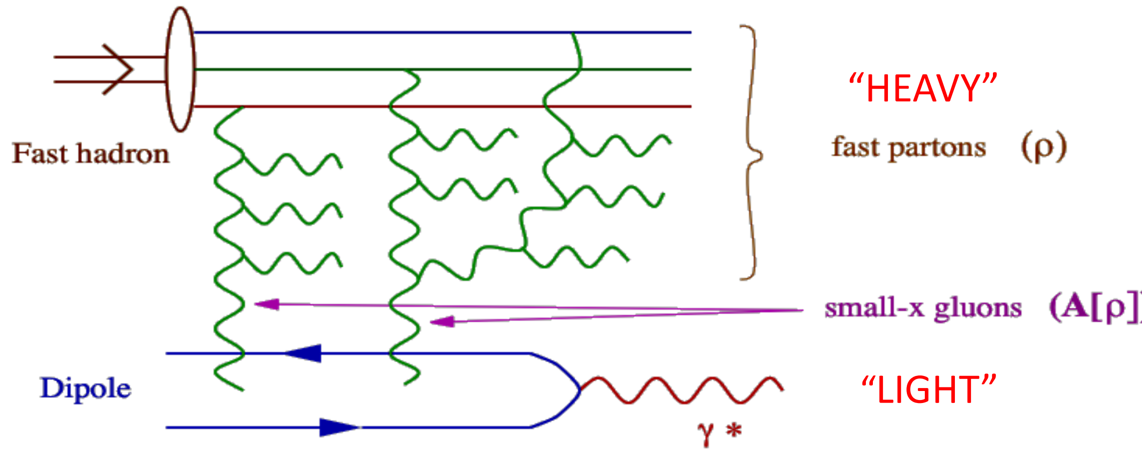
Gluons at maximal phase space occupancy $n \sim 1/\alpha_S$, resist close packing by recombining and screening their color charges -- gluon saturation

Emergent dynamical saturation scale $Q_S (x) >> \Lambda_{QCD}$

Asymptotic freedom! $\alpha_S (Q_S) << 1$ *provides non-pert weak coupling window into infrared*

Classicalization in the Regge limit: the Color Glass Condensate EFT

Born-Oppenheimer separation between fast and slow modes

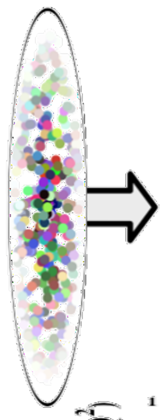


CGC: Effective Field Theory of classical static quark/gluon sources and dynamical gluon fields

Remarkably, physics of extreme quantum fluctuations becomes classical because of high gluon occupancy…

McLerran, RV (1994)

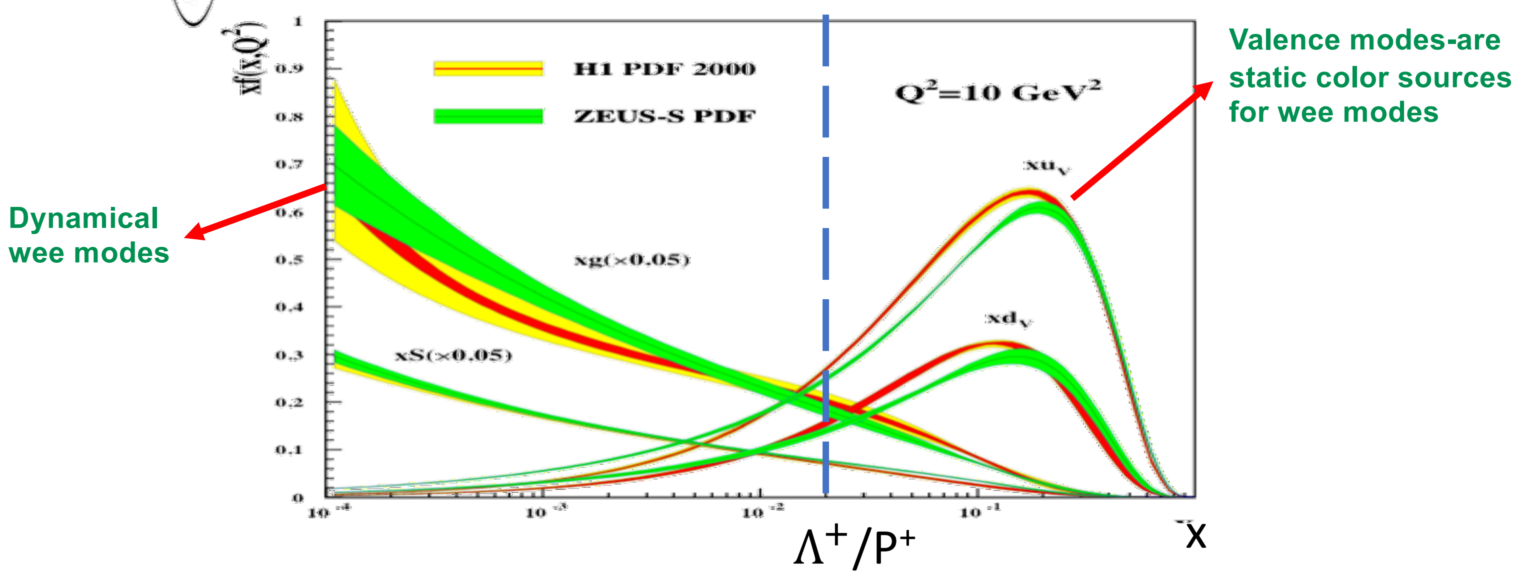
CGC EFT for gluon saturation



Nuclear wavefunction at high energies

$$|A\rangle = |qqq\ldots q\rangle + \ldots + |qqq\ldots q\,\mathbf{gg\ldots gg}\rangle$$

EFT for high Fock states in light-front wavefunction



Effective Field Theory on Light Front

Poincare group on LF ⟷ Galilean sub-group of 2D Quantum Mechanics

isomorphism

Weinberg (1966)
Susskind; Bardacki-Halpern (1968)

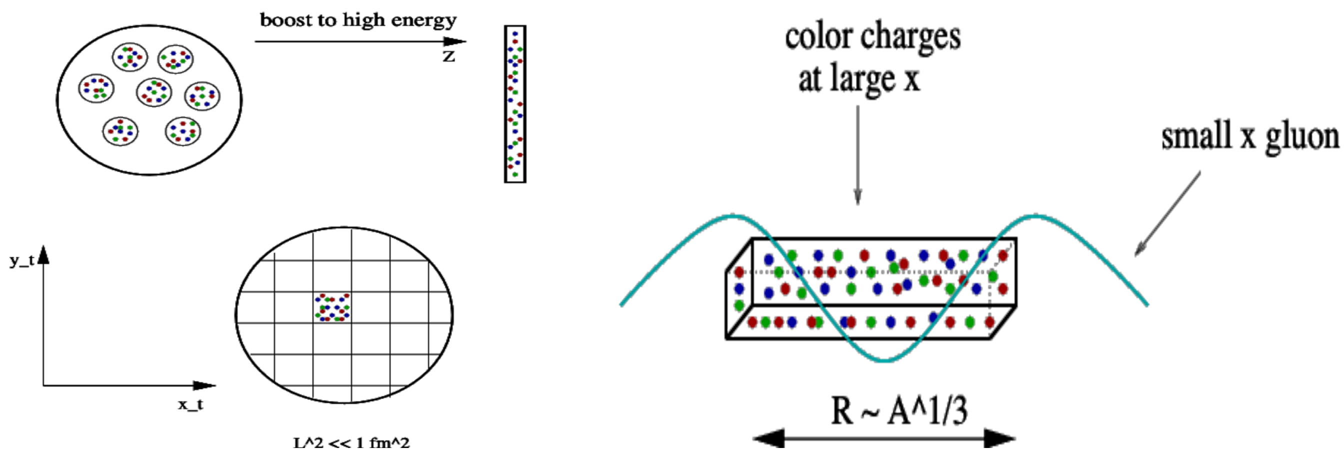
Eg., LF dispersion relation of relativistic free particle

$$P^- = \frac{P_\perp^2}{2P^+}$$

P^- → Energy; $P_\perp^2$ → Momentum; $2P^+$ → Mass

Large x (P^+) modes: static LF (color) sources ρ^a
Small x ($k^+ << P^+$) modes: dynamical fields A^a_μ

What do static color sources look like in the IMF ?



$$\lambda_{\rm wee} \approx \frac{1}{k^+} \equiv \frac{1}{xP^+} >> \lambda_{\rm val.} \equiv \frac{Rm_p}{P^+} => x << A^{-1/3}$$

In the infinite momentum frame (IMF), wee partons "see" a large density of color sources at small transverse resolutions

Effective Field Theory on Light Front

Explicit construction classical EFT in the Regge limit for large nuclei:

Gaussian stochastic distribution of k static color sources coherently coupled to gauge fields

$$\mathcal{N}\int dm\, dn\, d_{mn}\, N^{(k)}_{m,n} \;:\; \xrightarrow{\text{For SU(3) high dim. reps.}} \int [d\rho]\exp\left(-\int d^2x_\perp\left[\frac{\rho^a\rho^a}{2\mu_A^2}-\frac{d_{abc}\rho^a\rho^b\rho^c}{\kappa_A}\right]\right)$$

$$\mathcal{Z}[j] = \int [d\rho] W_{\Lambda^+}[\rho]\left\{\frac{\int^{\Lambda^+}[dA]\delta(A^+)e^{iS_{\Lambda^+}[A,\rho]-\int j\cdot A}}{\int^{\Lambda^+}[dA]\delta(A^+)e^{iS_{\Lambda^+}[A,\rho]}}\right\}$$

$W_{\Lambda^+}[\rho]$ Non-pert. gauge invariant "density matrix" defined at initial scale Λ^+

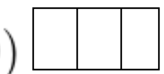
For a large nucleus, $Q_S^2 \propto \mu_A^2 \sim A^{1/3}\,\Lambda_{QCD}^2$; $\alpha_S(Q_S^2) \ll 1$ weak coupling EFT !

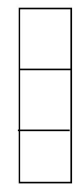
Simple understanding of "Pomeron" and "Odderon" configurations …

Coda: Path integral representation for static color sources

$$\mathcal{Z} = < P| e^{ix^+ P^-_{\rm QCD}} |P >= \lim_{x^+ \to i\infty} \sum_{N,Q} < N, Q| e^{ix^+ P^-_{\rm QCD}} |N, Q >$$

Random walk in SU(3): recursion relation from Young tableaux

$3 = (1, 0)$ 

$\bar{3} = (0, 1)$ 

Multiplicity of an (m,n) representation after k random walks

$$N^{(k+1)}_{m,n} = N^{(k)}_{m-1,n} + N^{(k)}_{m+1,n-1} + N^{(k)}_{m,n-1}$$

For large k, use Stirling's formula: $N^{(k)}_{m,n} \approx \frac{27mn(m+n)}{k^3} \frac{3^{3/2+k}}{2k\pi} \exp\left(-3\, D_2^{m,n}\right) \left(1 + 3\, D_3^{m,n}/k^2\right)$

Quadratic Casimir: $D_2^{m,n} = \frac{(m^2 + mn + n^2)}{3} + (m+n)$

Cubic Casimir $D_3^{m,n} = \frac{1}{18}(m+2n+3)(n+2m+3)(m-n)$

$$\mathcal{N} \int dm\, dn\, d_{mn}\, N^{(k)}_{m,n} \approx \left(\frac{N_c}{k\pi}\right)^4 \int d^8\mathbf{Q}\, e^{-N_c \mathbf{Q}^2/k + 3\, D_3(\mathbf{Q})/k^2}$$

Dim. of rep. $d_{mn} \approx \frac{mn(m+n)}{2}$

$$d^8Q = d\phi_1\, d\phi_2\, d\phi_3\, d\pi_1\, d\pi_2\, d\pi_3\, dm\, dn \left(m\, n\, (m+n)\, \frac{\sqrt{3}}{48}\right)$$

Canonically conjugate Darboux variables

Jeon, RV: hep-ph/0406169

2-D classical EFT

Soln. of Yang-Mills eqns in IMF (P+ $\rightarrow \infty$): pure gauges separated by shockwave discontinuity

$A_i = 0$ | $A_i = -\frac{-1}{ig} U \partial_i U^\dagger$

$x^- = 0$

Gauge choice $A^+ = 0$

Classical soln: $A- = 0$

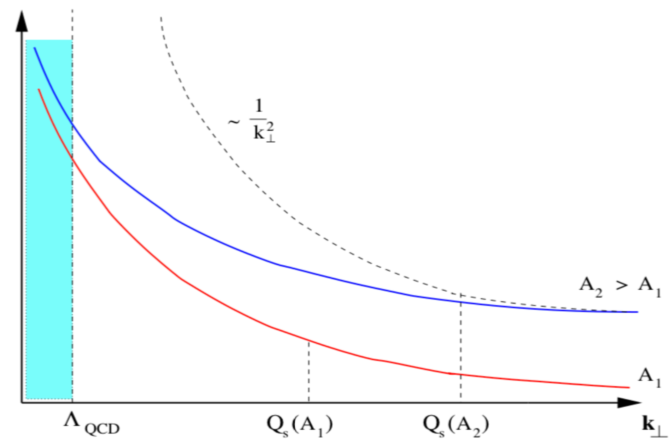
$$D_i \frac{dA^{i,a}}{dy} = g\rho^a(x_t, y)$$ with the solution $U = P \exp\left(i\int_y^\infty dy' \frac{\rho(x_t, y')}{\nabla_t^2} \right)$

rapidity $y = \ln(x^-/x_0^-)$

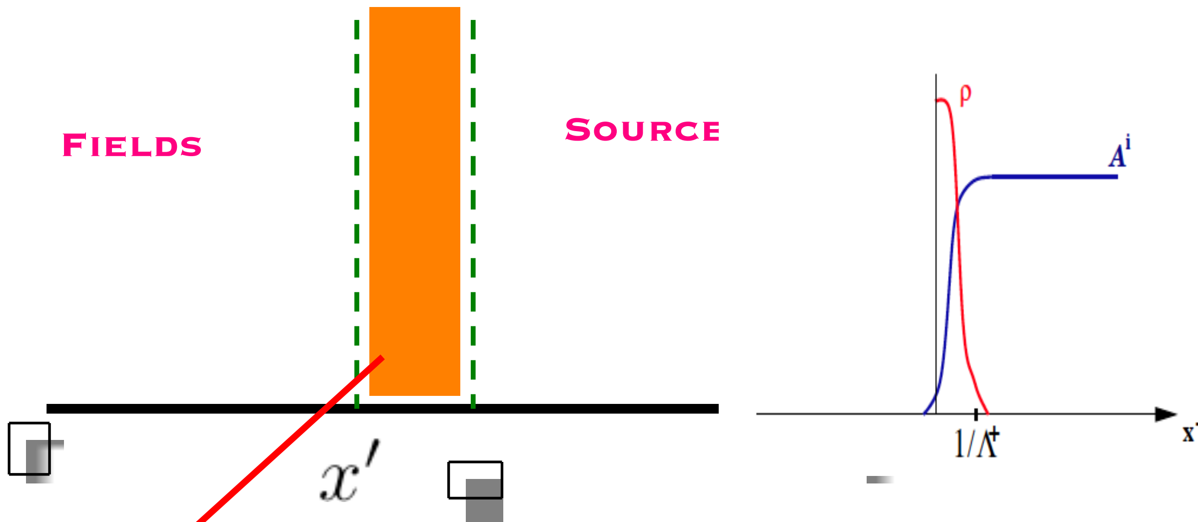
$$\langle P|\mathcal{O}|P\rangle \rightarrow \int [d\rho] W_{\Lambda^+}[\rho]\, \mathcal{O}(A_{\rm cl.}[\rho])$$

For A >>1 (Gaussian W), compute n-point correlators

Gluon distribution in nucleus:
non-Abelian Weizacker-Williams dist. $\frac{(2\pi)^3}{2(N_c^2-1)} \frac{dN}{\pi R_A^2 d^2k_\perp}$



Quantum evolution of classical theory: Wilson RG

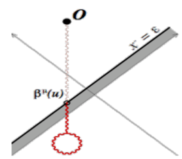
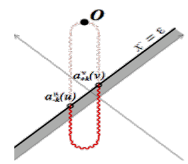


Integrate out small fluctuations => Increase color charge of sources
- Extends validity of the classical EFT to finite nuclei…

Wilsonian RG equations describe evolution of all N-point correlation functions with energy

JIMWLK Jalilian-Marian, Iancu, McLerran,Weigert, Leonidov,Kovner

JIMWLK RG evolution for a single nucleus

$$\mathcal{O}_{\rm NLO} = \Big($$

$$+$$

$$\chi(x_\perp, y_\perp)$$
$$\Big)\, \mathcal{O}_{\rm LO}$$

$$= \ln\left(\frac{\Lambda^+}{p^+}\right) \mathcal{H}\, \mathcal{O}_{\rm LO}$$ (keeping leading log divergences)

$$\langle \mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO} \rangle = \int [d\tilde{\rho}] W[\tilde{\rho}] \left[\mathcal{O}_{\rm LO} + \mathcal{O}_{\rm NLO}\right]$$
$$= \int [d\tilde{\rho}] \left\{ \left[1 + \ln\left(\frac{\Lambda^+}{p^+}\right)\mathcal{H}\right] W_{\Lambda^+} \right\} \mathcal{O}_{\rm LO}$$

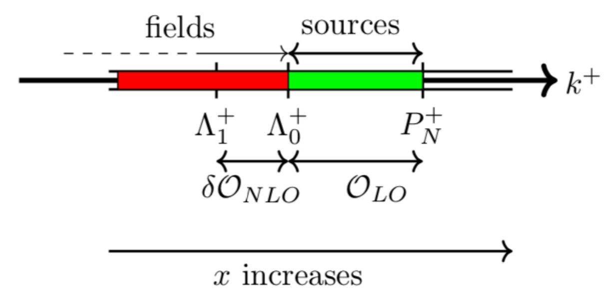
LHS independent of Λ^+ =>
$$\frac{\partial W[\tilde{\rho}]}{\partial Y} = \mathcal{H}\, W[\tilde{\rho}]$$

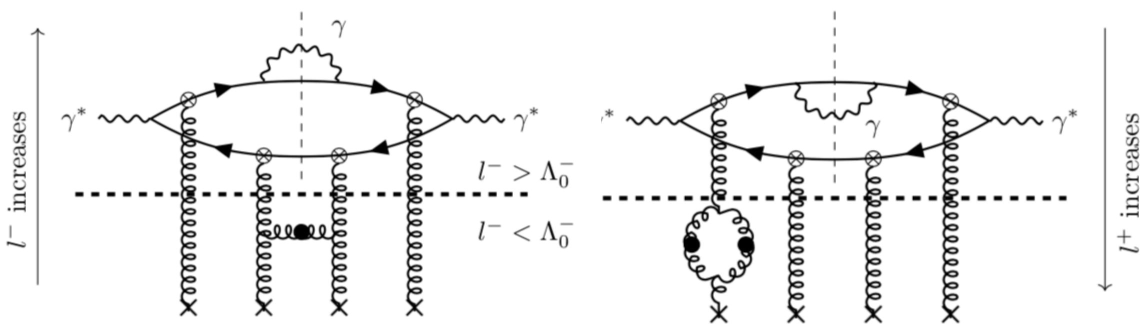
- JIMWLK Hamiltonian now computed to NLLx accuracy

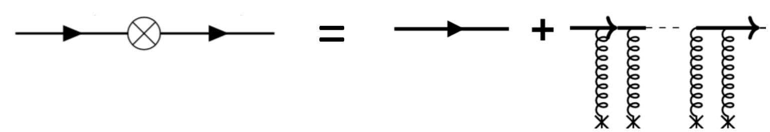
Balitsky, Chirilli; Kovchegov, Weigert; Grabovsky;
Kovner, Lublinsky, Mulian; Caron-Huot

JIMWLK RG evolution in DIS

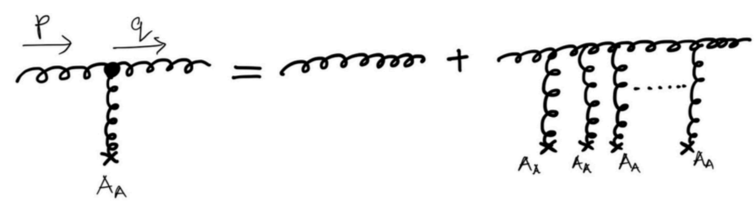
Wilsonian RG describes evolution $W_{\Lambda_0^+}[\rho] \rightarrow W_{\Lambda_1^+}[\rho']$
with scale separation between static sources and fields







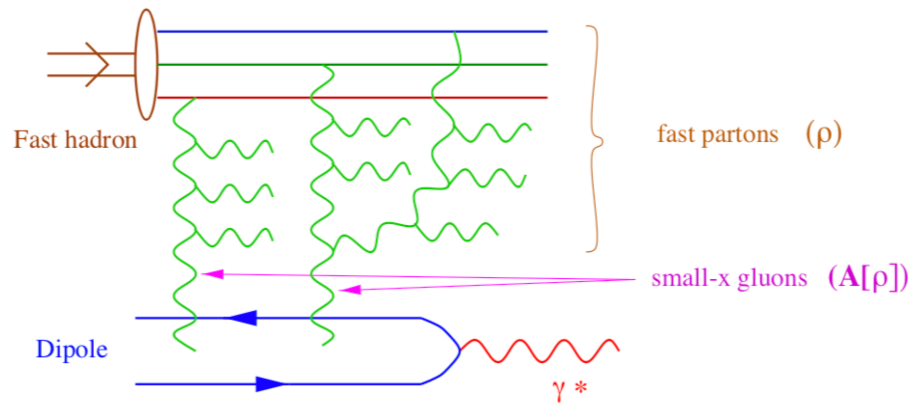
"Shockwave" propagators in strong background fields
in "wrong" light-cone gauge (A^-=0)



Effective vertices identical to quark-quark-reggeon and gluon-gluon-reggeon vertices in Lipatov's Reggeon EFT

Bondarenko,Lipatov,Pozdynyakov, Prygarin, arXiv:1708.05183
Hentschinski, arXiv:1802.06755

B-JIMWLK hierarchy of many-body correlators in QCD



$$\frac{\partial}{\partial Y}\langle\mathcal{O}[\rho]\rangle_Y = \frac{1}{2}\langle\int_{x,y}\frac{\partial}{\partial\rho^a(x)}\chi^{ab}_{x,y}\frac{\partial}{\partial\rho^b(y)}\mathcal{O}[\rho]\rangle_Y$$

"time" "diffusion coefficient"

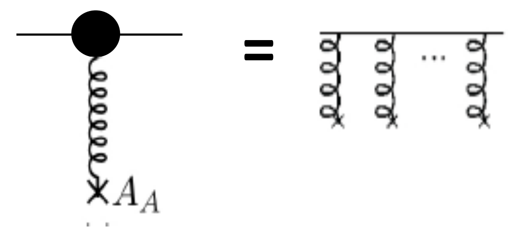
Diffusion of fuzz of "wee" partons in the functional space of colored fields

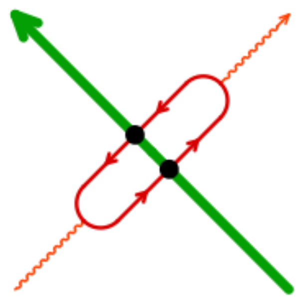
Can be represented as a Langevin equation that can be solved numerically to "leading logs in x" accuracy to compute n-point Wilson line correlators

BFKL: Balitsky-Fadin-Kuraev-Lipatov (1976-1978)
JIMWLK :Jalilian-Marian,Kovner,Leonidov,Weigert (1997); Iancu,Leonidov,McLerran (2001); Independent and equivalent formulation: Balitsky (1996)

Inclusive DIS: dipole evolution

Dipole scattering off the nuclear shock wave





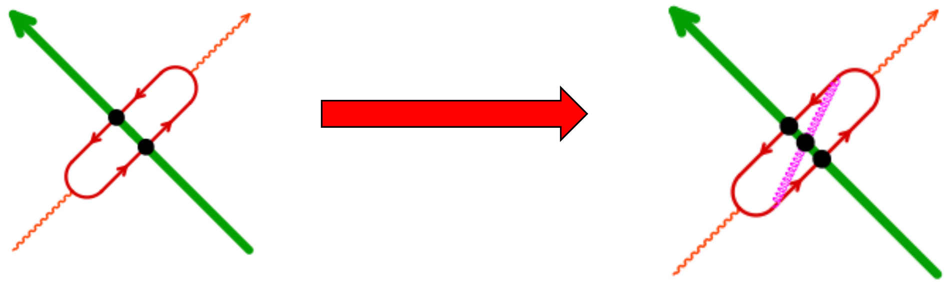
$$\sigma_{\gamma^* T} = \int_0^1 dz \int d^2 r_\perp \; |\psi(z, r_\perp)|^2 \sigma_{\text{dipole}}(x, r_\perp)$$

$$\sigma_{\text{dipole}}(x, r_\perp) = 2 \int d^2 b \int [D\rho] \; W_{\Lambda^+}[\rho] \; T(b + \frac{r_\perp}{2}, b - \frac{r_\perp}{2})$$

$$1 - \frac{1}{N_c} \text{Tr} \left(V \left(b + \frac{r_\perp}{2} \right) V^\dagger \left(b - \frac{r_\perp}{2} \right) \right)$$

$$V = P \; exp(ig \frac{\rho}{\nabla_T{}^2})$$

Inclusive DIS: dipole evolution



B-JIMWLK eqn. for dipole correlator:

$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\,\langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\,\mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

Dipole factorization:

Y = Ln(1/x)

$$\langle \mathrm{Tr}(V_x V_z^\dagger)\,\mathrm{Tr}(V_z V_y^\dagger)\rangle_Y \longrightarrow \langle \mathrm{Tr}(V_x V_z^\dagger)\rangle_Y \langle \mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

$A >> 1$, $N_c \rightarrow \infty$

Resulting closed form eqn. for a large nucleus is the Balitsky-Kovchegov (BK) eqn. widely used in phenomenological applications –

The BFKL equation is the low density $V \approx 1 - ig\rho/\nabla t^2$ limit of the BK equation…

Analytical approximations to the BK equation

The 2-point correlator $\mathcal{N}_Y = 1 - \frac{1}{N_c}\mathrm{Tr}\left(V\left(b+\frac{r_\perp}{2}\right)V^\dagger\left(b-\frac{r_\perp}{2}\right)\right)$

for $N_c \to \infty$ and $A >> 1$

$$\frac{\partial \mathcal{N}_Y(x,y)}{\partial Y} = \bar{\alpha}_s \int_z \frac{(x-y)^2}{(x-z)^2(y-z)^2}\left\{\underbrace{\mathcal{N}_Y(x,z)+\mathcal{N}_Y(z,y)-\mathcal{N}_Y(x,y)}_{\text{BFKL}} - \underbrace{\mathcal{N}_Y(x,z)\mathcal{N}_Y(z,y)}_{\text{Non-linear}}\right\}$$

For small dipole, $(r << 1/Q_s(Y)) => \text{BFKL eqn.}$

$$\mathcal{N}_Y(r) \approx (r^2 Q_0^2)^{1/2} e^{\omega \bar{\alpha}_s Y} \exp\left(-\frac{\ln^2(1/r^2 Q_0^2)}{2\beta\bar{\alpha}_s Y}\right)$$

Imposng a saturation condition,

$$\mathcal{N} = 1/2 \text{ when } r \sim 1/Q_s(Y) \Rightarrow \boxed{Q_s^2(Y) \approx Q_0^2\, e^{\lambda Y} \text{ with } \lambda \sim 4.8\,\alpha_s}$$

For a large dipole, $(r >> 1/Q_s(Y))$

Levin,Tuchin;
Iancu,McLerran;Mueller

$$\mathcal{N}_Y(r) \approx 1 - \kappa \exp\left(-\frac{1}{4c}\ln^2(r^2 Q_s^2(Y))\right) \qquad c \approx 4.8$$

Geometrical scaling

Iancu,Itakura, McLerran;
Mueller,Triantafyllopolous

Can write the solution of BFKL as:

$$\mathcal{N}_Y(r_\perp) \approx \exp\left(\omega \bar{\alpha}_s Y - \frac{\rho}{2} - \frac{\rho^2}{2\beta\bar{\alpha}_s Y}\right) \text{ with } \rho = \ln\frac{1}{r^2 Q_0^2}$$

ρ_S soln. where argument vanishes

$$=> Q_s^2 = Q_0^2\, e^{c\bar{\alpha}_s Y}, \text{with } c = 4.84$$

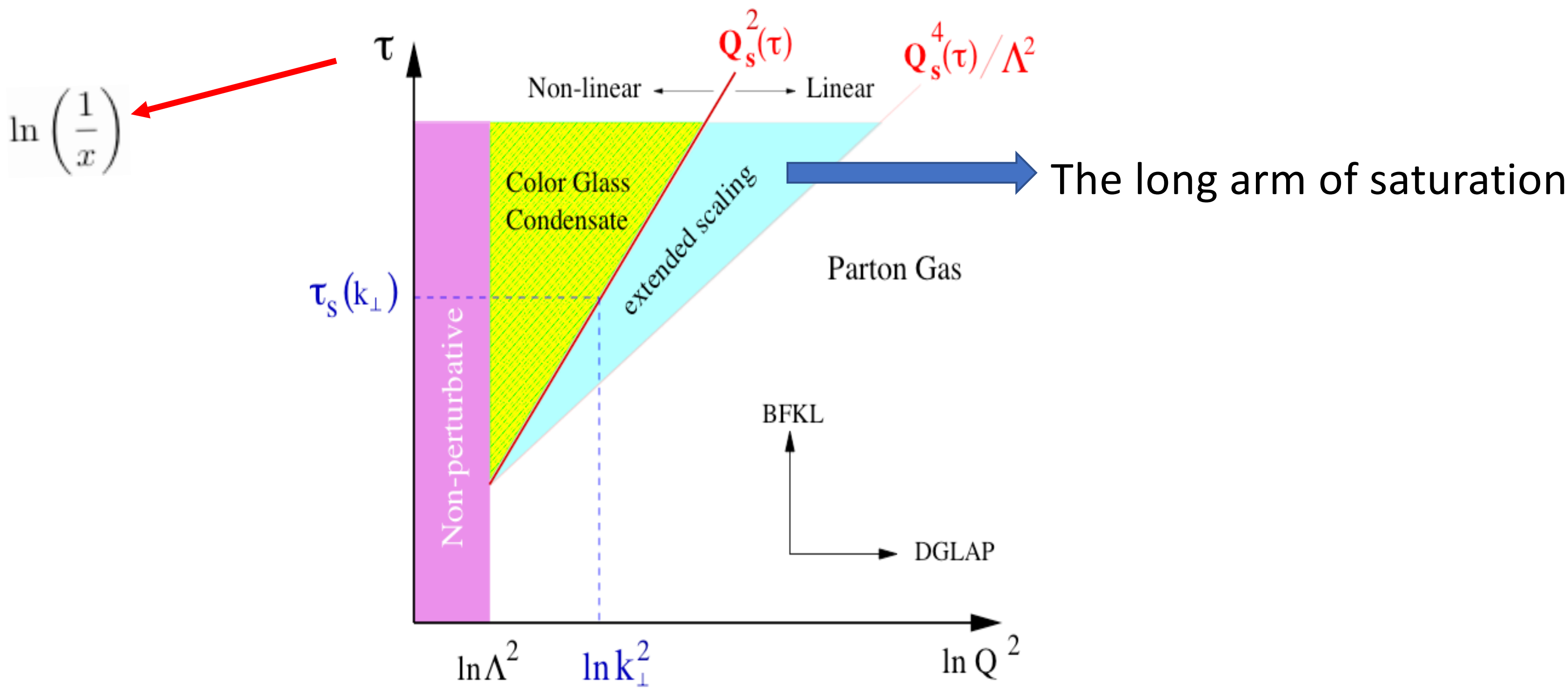
For $r_\perp < 1/Q_s$ (but close!), can write $\rho = \rho_S(Y) + \ln\frac{1}{r_\perp^2 Q_s^2} \equiv \rho_S + \delta\rho$

Plugging into $\mathcal{N}_Y$, can show simply

$$\mathcal{N}_Y \approx \left(r_\perp^2 Q_s^2(Y)\right)^\gamma \text{ for } Q_s^2 << Q^2 << \frac{Q_s^4}{Q_0^2}$$

$\gamma \sim 0.64$ is larger than BFKL anomalous dimension = 1/2

Geometrical scaling window in QCD at high energies



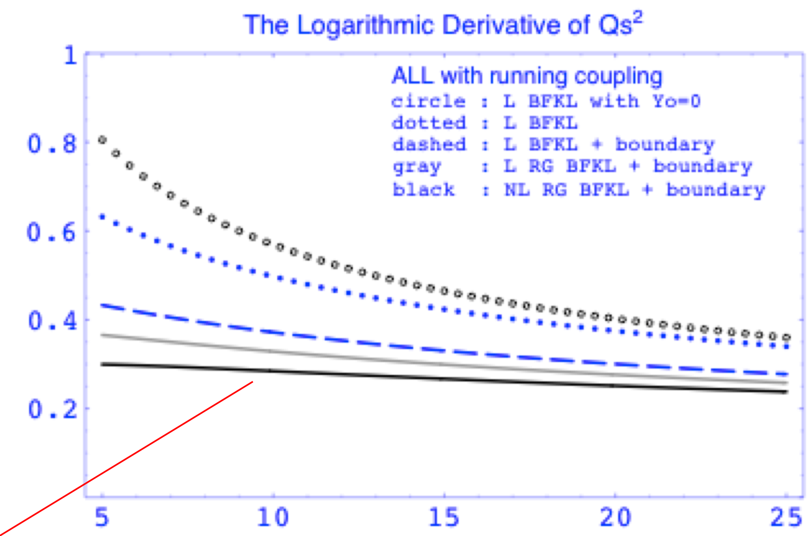
How does Q_S behave as function of Y?

Fixed coupling LO BFKL: $$Q_s^2 = Q_0^2 e^{c\,\bar{\alpha}_s Y}$$

LO BFKL+ running coupling: $$Q_s^2 = \Lambda_{\rm QCD}^2 e^{\sqrt{2b_0 c(Y+Y_0)}}$$

Triantafyllopolous

$$\lambda \equiv \frac{d \ln Q_s^2}{dY}$$

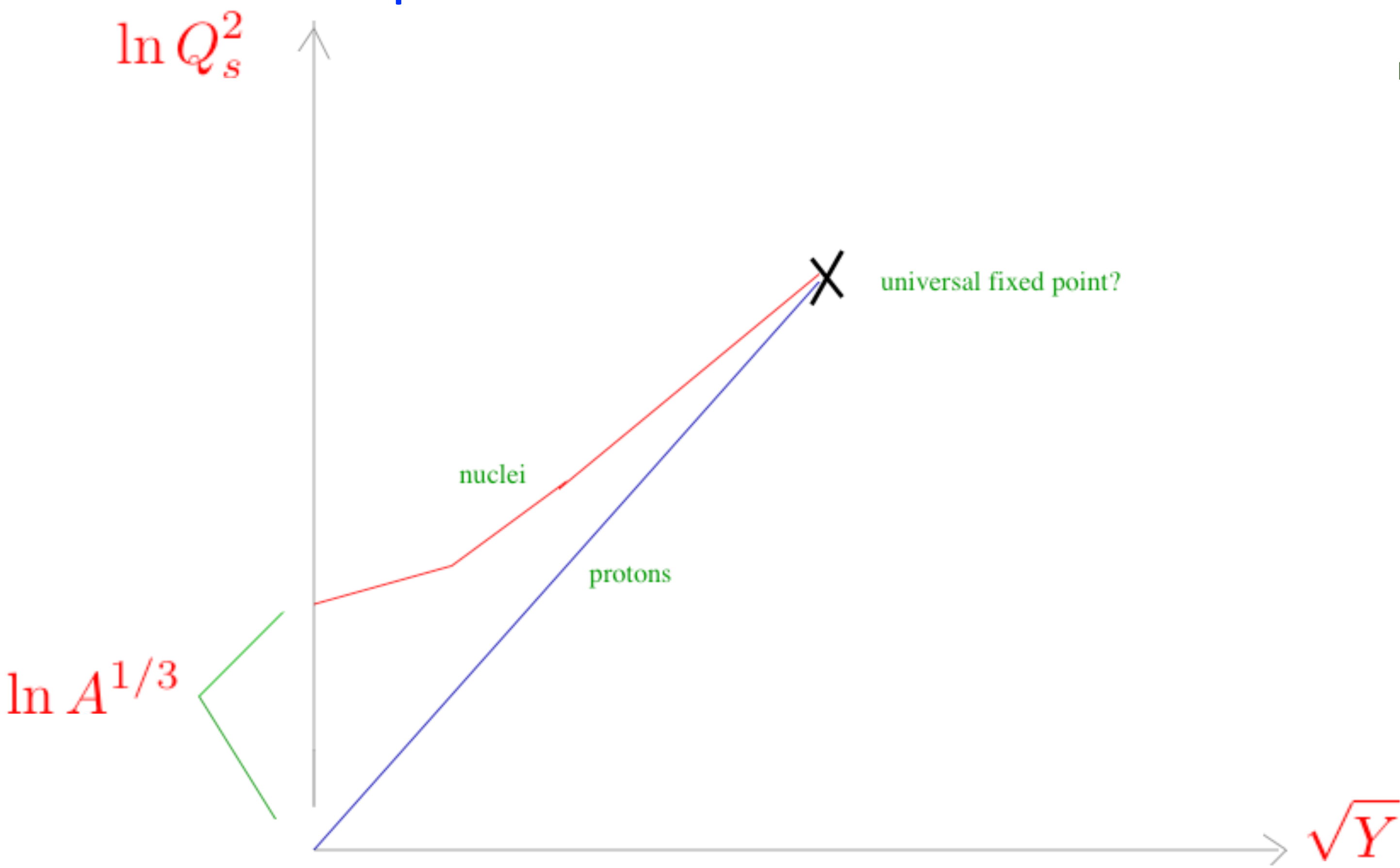


Very close to
HERA result!

Y

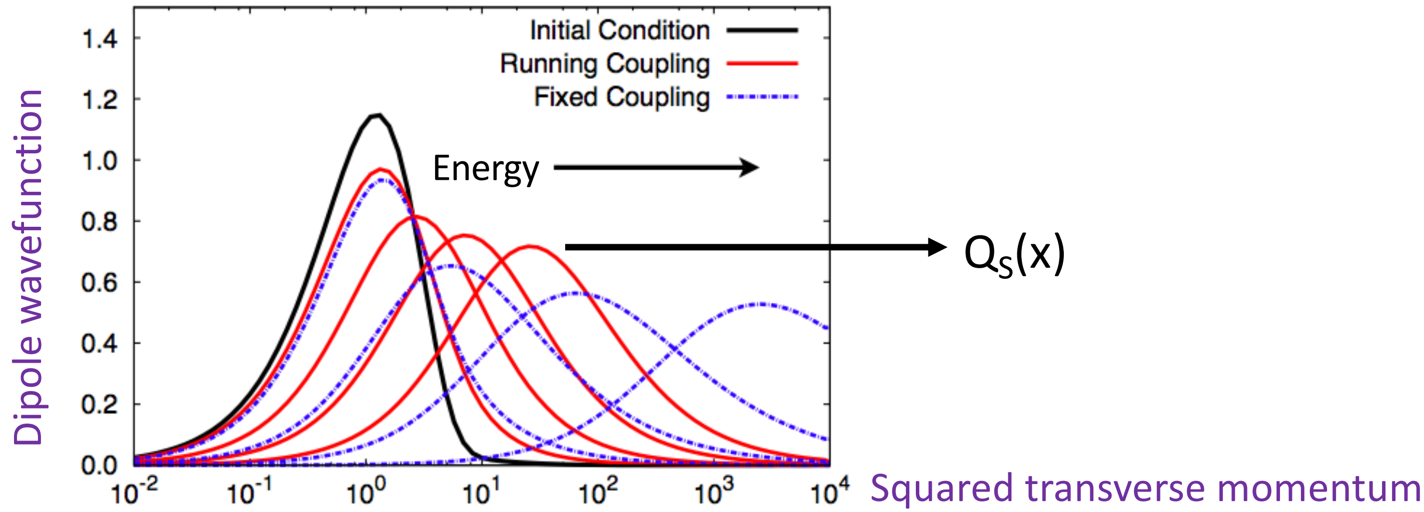
A dependence of the saturation scale

Mueller (2003)



Such interesting systematics may be tested at the EIC !

Dipole evolution in the Color Glass Condensate EFT



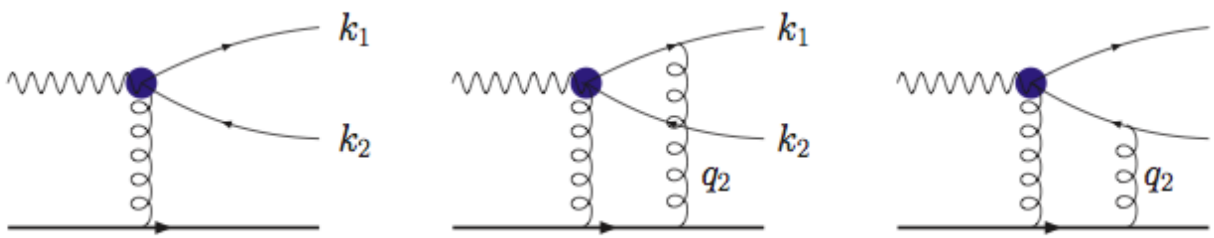
The BK equation describes how $q\bar{q}$ "dipole" probe evolves with energy
– *providing* a *clean demonstration of unitarization in strong fields*

Its dynamics can be mapped* to that of the Fischer-Kolmogorov (FKPP) eqn. describing the evolution of non-linear wave fronts. Rich synergy with stat. mech.

Munier, Peschanski (2003)

* small caveat

Semi-inclusive DIS: quadrupole evolution



Dominguez,Marquet,Xiao,Yuan (2011)

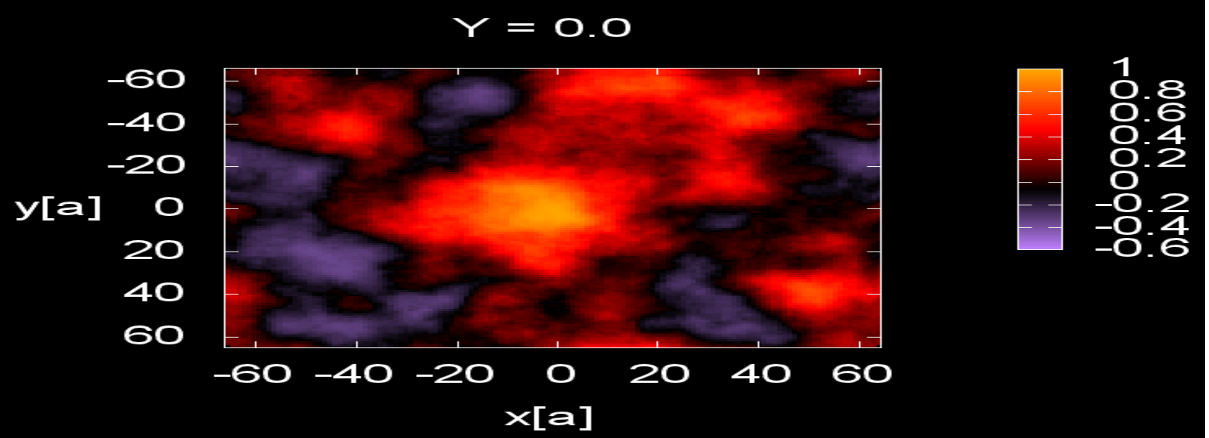
$$\frac{d\sigma^{\gamma^*_{\mathrm{T,L}}A\to q\bar{q}X}}{d^3k_1 d^3k_2} \propto \int_{x,y,\bar{x}\bar{y}} e^{ik_{1\perp}\cdot(x-\bar{x})} e^{ik_{2\perp}\cdot(y-\bar{y})} \left[1 + Q(x,y;\bar{y},\bar{x}) - D(x,y) - D(\bar{y},\bar{x})\right]$$

Functional Langevin solutions of JIMWLK hierarchy

Rummukainen,Weigert (2003)

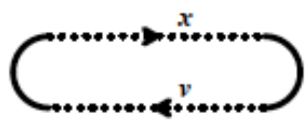
Dumitru,Jalilian-Marian,Lappi,Schenke,RV, PLB706 (2011)219

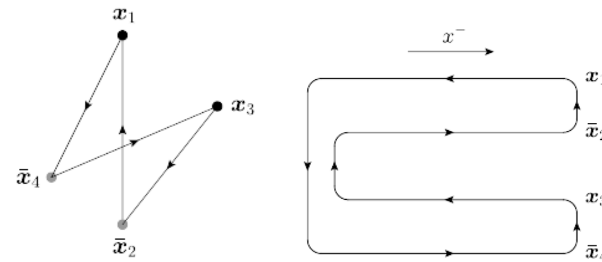
We are now able to compute all n-point correlations of a theory of strongly correlated gluons and study their evolution with energy!



Correlator of Light-like Wilson lines Tr(V(0,0)V^dagger (x,y))

Semi-inclusive DIS: quadrupole evolution



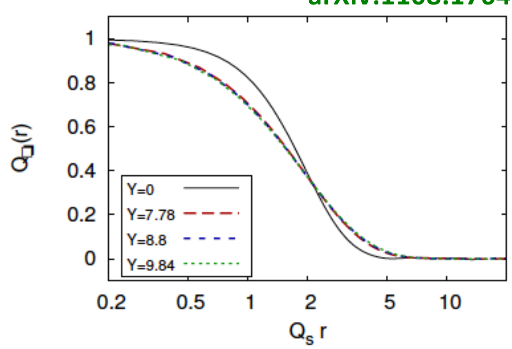


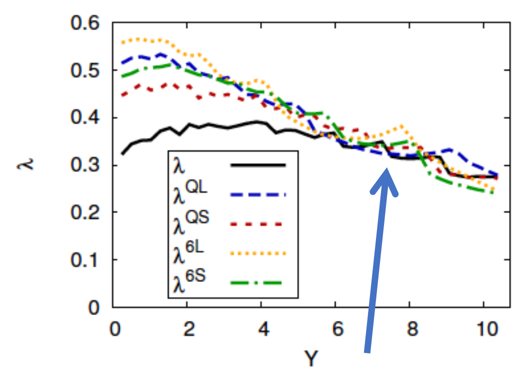
$$D(x,y) = \frac{1}{N_c}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y \qquad Q(x,y;\bar{y},\bar{x}) = \frac{1}{N_c}\langle \mathrm{Tr}(V_x V_{\bar{x}}^\dagger V_{\bar{y}} V_y^\dagger)\rangle_Y$$

RG evolution provides fresh insight into multi-parton correlations

Dumitru,Jalilian-Marian,Lappi,Schenke,RV: arXiv:1108.1764

Quadrupoles, like Dipoles, exhibit geometrical scaling





Iancu,Triantafyllopolous,arXiv:1112.1104

Rate of energy evolution of dipole and quadrupole saturation scales