Classicalization, Scrambling and Thermalization in QCD at high energies

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Outline of lectures

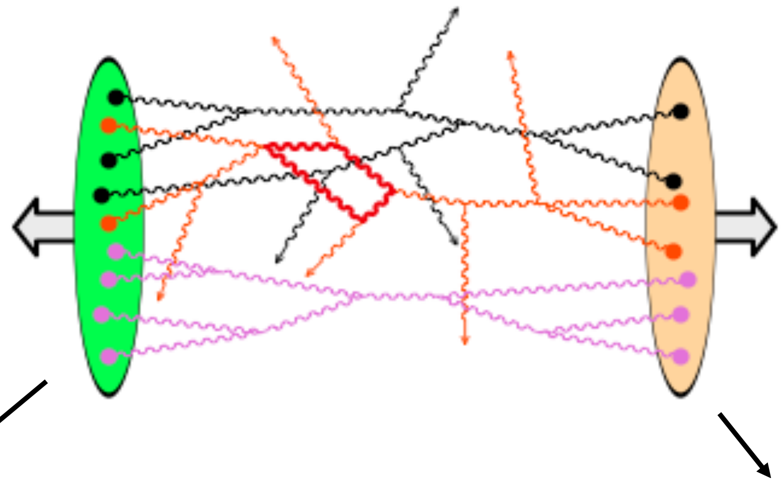
Lecture I: Classicalization: The hadron wavefunction at high energies as a Color Glass Condensate

Lecture II: The CGC Effective Field Theory

Lecture III: From CGC to the Glasma, key features of the Glasma

Lecture IV: Thermalization and interdisciplinary connections

Particle production in presence of strong time-dependent sources

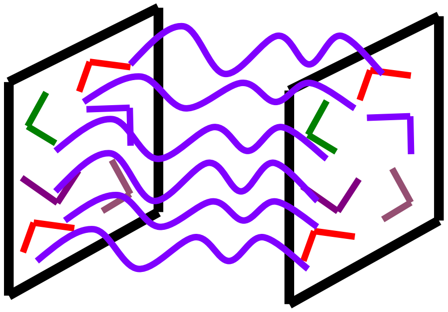


$\rho_A \sim \frac{1}{g} >> 1$

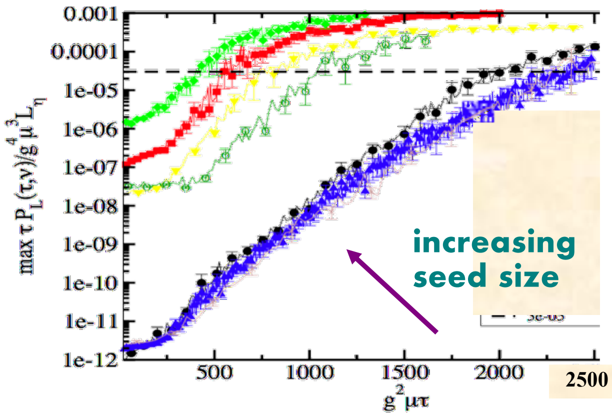
$\rho_B \sim \frac{1}{g} >> 1$

P_n obtained from cut vacuum graphs in field theories
with strong time dependent sources

From Glasma to Plasma



Quant. fluctuations grow exponentially after collision



$$\langle\langle T^{\mu\nu}\rangle\rangle_{\mathrm{LLx+Linst.}} = \int [D\rho_1][D\rho_2]\, W_{\mathrm{Y_{beam}-Y}}[\rho_1]\, W_{\mathrm{Y_{beam}+Y}}[\rho_2]$$
$$\times \int [da(u)]\; F_{\mathrm{init}}[a]\; T^{\mu\nu}_{\mathrm{LO}}[A_{\mathrm{cl}}(\rho_1,\rho_2)+a]$$

Path integral over multiple initializations of classical trajectories in one event can lead to quasi-ergodic "eigenstate thermalization" Berry; Srednicki; Rigol et al.; …

This *scrambling of information* is seen in many systems in nature and can be understood To lead to decoherence of the primordial classical fields

Initial conditions in the overpopulated Glasma

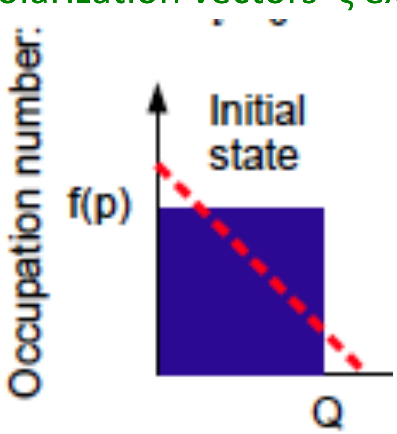
Choose for the Gaussian random gauge fields for the initial conditions

$$A^a_\mu(\tau_0, \mathbf{x}_\perp, \eta) = \sum_{\lambda=1,2} \int \frac{\mathrm{d}^2\mathbf{k}_\perp}{(2\pi)^2} \frac{\mathrm{d}\nu}{2\pi} \sqrt{f(\mathbf{k}_\perp, \nu, \tau_0)} \times \left[c^{\mathbf{k}_\perp \nu}_{\lambda,a}\, \xi^{(\lambda)\mathbf{k}_\perp \nu +}_\mu(\tau_0)\, e^{i\mathbf{k}_\perp \mathbf{x}_\perp}\, e^{i\nu\eta} + c.c. \right]$$

Stochastic random variables

$$\begin{aligned}
\langle c^{(\lambda)\boldsymbol{k}_\perp \nu} c^{(\lambda')\boldsymbol{k}'_\perp \nu'} \rangle &= 0\,, \\
\langle c^{(\lambda)\boldsymbol{k}_\perp \nu} c^{*(\lambda')\boldsymbol{k}'_\perp \nu'} \rangle &= (2\pi)^3 \delta^{\lambda\lambda'} \delta(\boldsymbol{k} - \boldsymbol{k}')\delta(\nu - \nu') \\
\langle c^{*(\lambda)\boldsymbol{k}_\perp \nu} c^{*(\lambda')\boldsymbol{k}'_\perp \nu'} \rangle &= 0.
\end{aligned}$$

Polarization vectors ξ expressed in terms of Hankel functions in Fock-Schwinger gauge $A^\tau = 0$



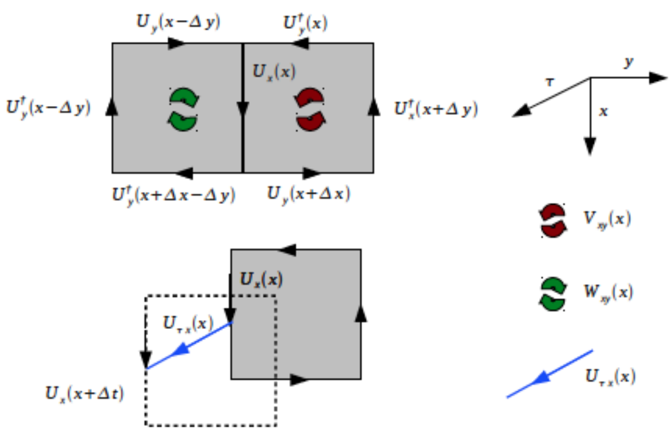
$$f(p_\perp, p_z, t_0) = \frac{n_0}{\alpha_S}\, \Theta\left(Q - \sqrt{p_\perp^2 + (\xi_0 p_z)^2} \right)$$

Controls “prolateness” or “oblateness” of initial momentum distribution

Temporal evolution in the overpopulated QGP

Solve Hamilton's equation for 3+1-D SU(2) gauge theory in Fock-Schwinger gauge

Berges,Boguslavski,Schlichting,Venugopalan
arXiv: 1303.5650, 1311.3005



Fix residual gauge freedom imposing Coloumb gauge at each readout time

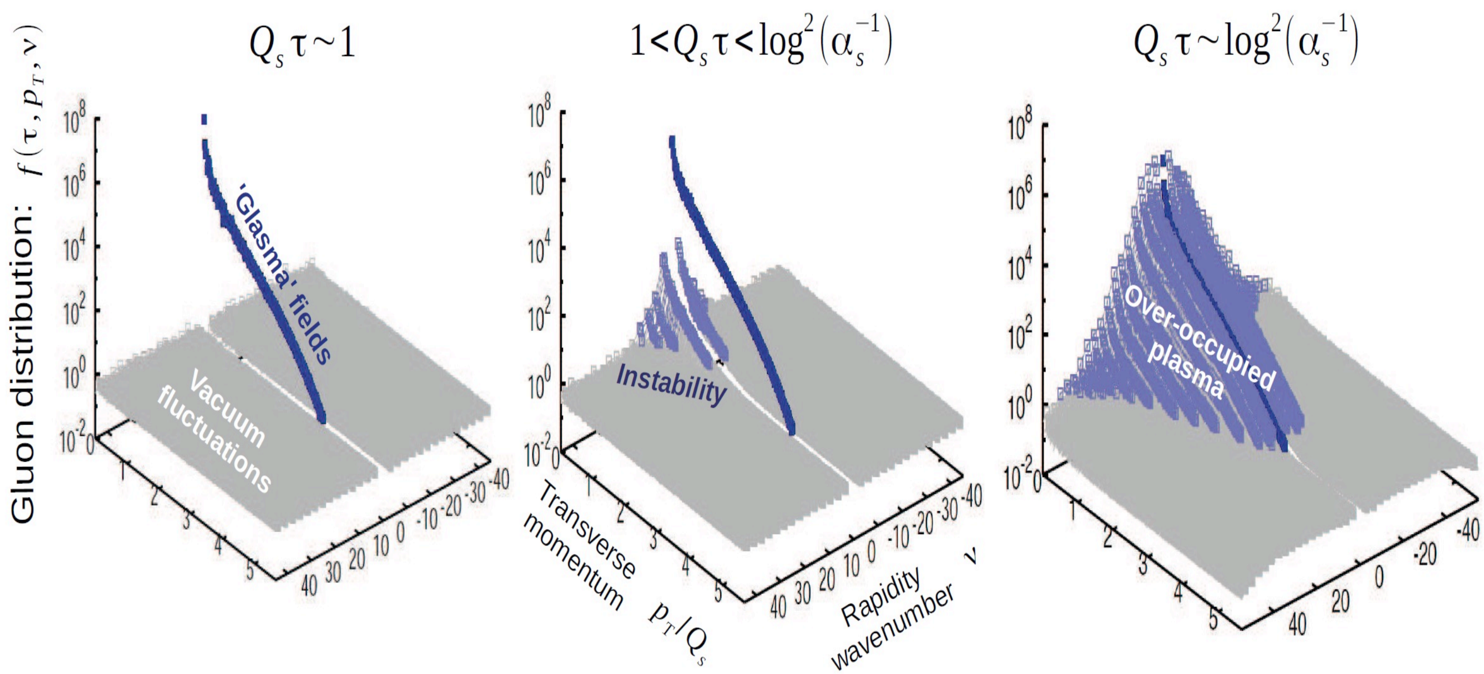
$$\partial_i A_i + t^{-2}\partial_\eta A_\eta = 0$$

Largest classical-statistical numerical simulations of expanding Yang-Mills to date:
$256^2 \times 4096$ lattices

From Glasma to Quark Gluon Plasma

Glasma fields produced in the shock wave collision are unstable to quantum fluctuations…

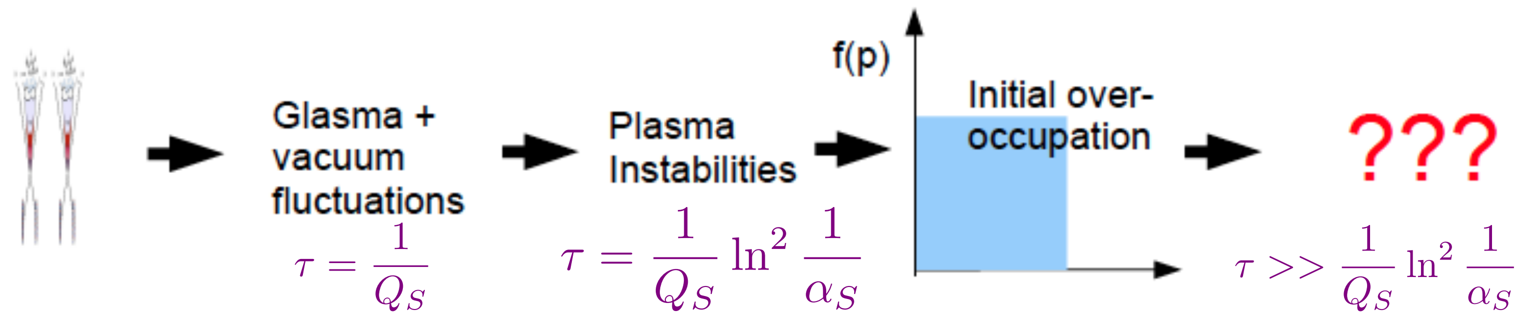
This instability leads to rapid overpopulation of all momentum modes



Classical-statistical QFT numerical lattice simulations of gluon fields exploding into the vacuum

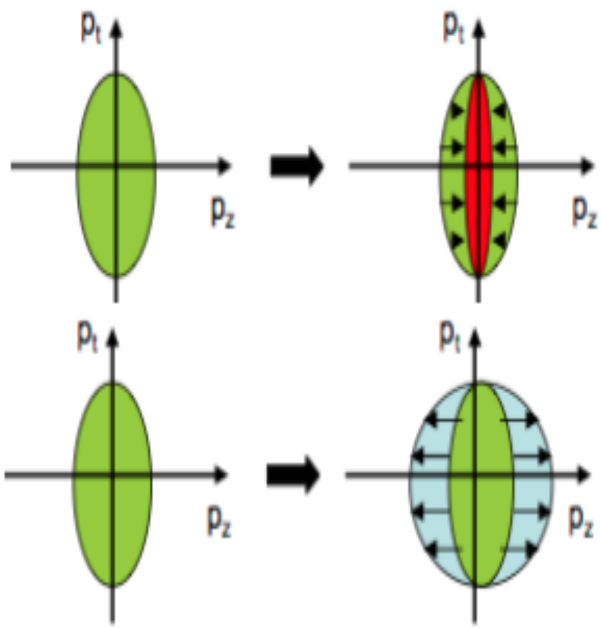
Berges,Schenke,Schlichting,RV, NPA 931 (2014) 348

From Glasma to Quark Gluon Plasma



From Glasma to Quark Gluon Plasma

- There is a natural ***competition*** between ***interactions*** and the ***longitudinal expansion*** which renders the system ***anisotropic*** on large time scales



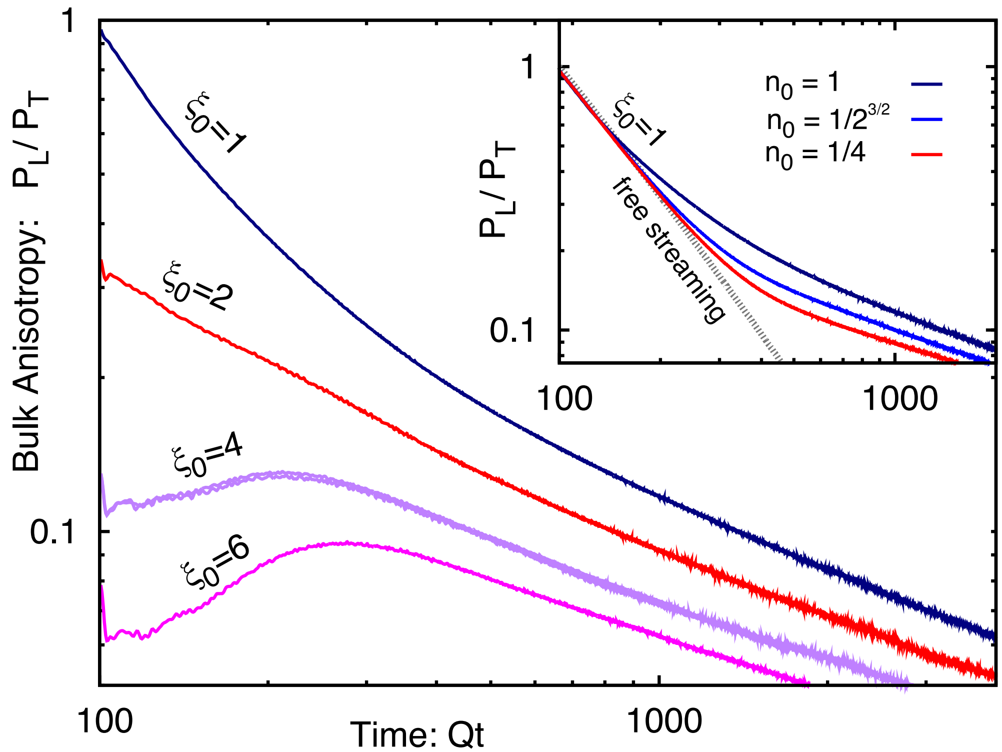
Longitudinal Expansion:

- Red-shift of longitudinal momenta p_z
  $\rightarrow$ increase of anisotropy
- Dilution of the system

Interactions:

- Isotropize the system

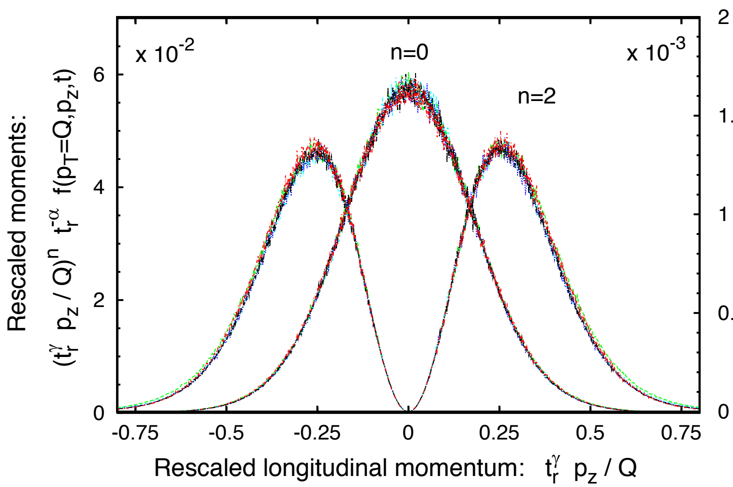
Pressure becomes increasingly anisotropic

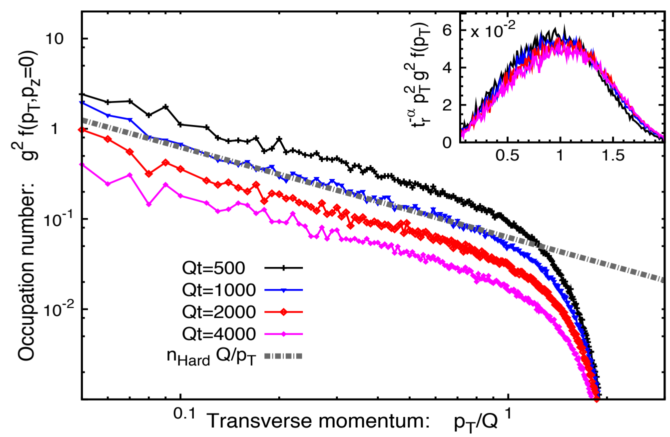


P_L/P_T approaches a universal $\tau^{-2/3}$ behavior

Result: a universal non-thermal fixed point

Conjecture: $f(p_\perp, p_z, t) = t^\alpha \; f_S(t^\beta p_T, t^\gamma p_z)$





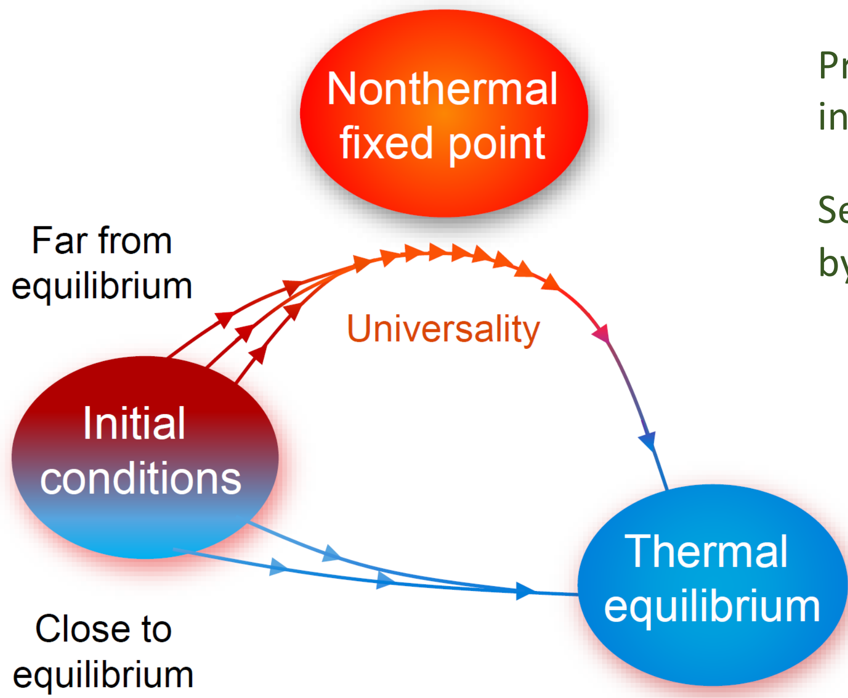
Moments of distribution extracted over range of time slices lie on universal curves

Distribution as function of p_T displays 2-D thermal behavior

Kinetic theory in the overoccupied regime

For $1 < f < 1/\alpha_S$ a dual description is feasible either in terms of kinetic theory or classical-statistical dynamics …

Mueller,Son (2002)
Jeon (2005)



Properties independent of initial conditions

Self-similar evolution characterized by universal scaling exponents

Kinetic theory in the overoccupied regime

Different scenarios:

- Elastic multiple scattering dominates in the Glasma

BMSS: Baier,Mueller,Schiff,Son

- Rescattering influenced by plasma (Weibel) instabilities

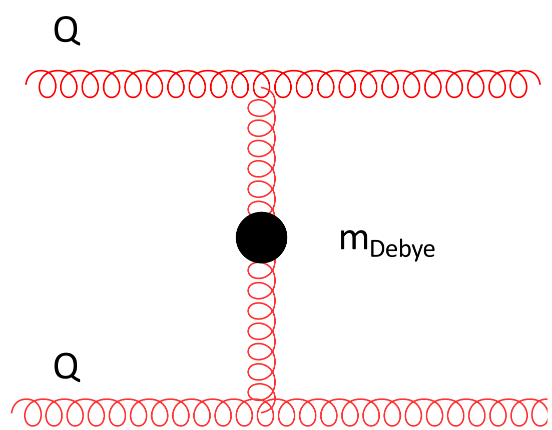
DB: Bodeker
KM: Kurkela, Moore

- Transient Bose condensation+multiple scattering

BGLMV: Blaizot,Gelis,Liao,McLerran,Venugopalan

Gell-Mann's totalitarian principle: Anything that is not forbidden is allowed

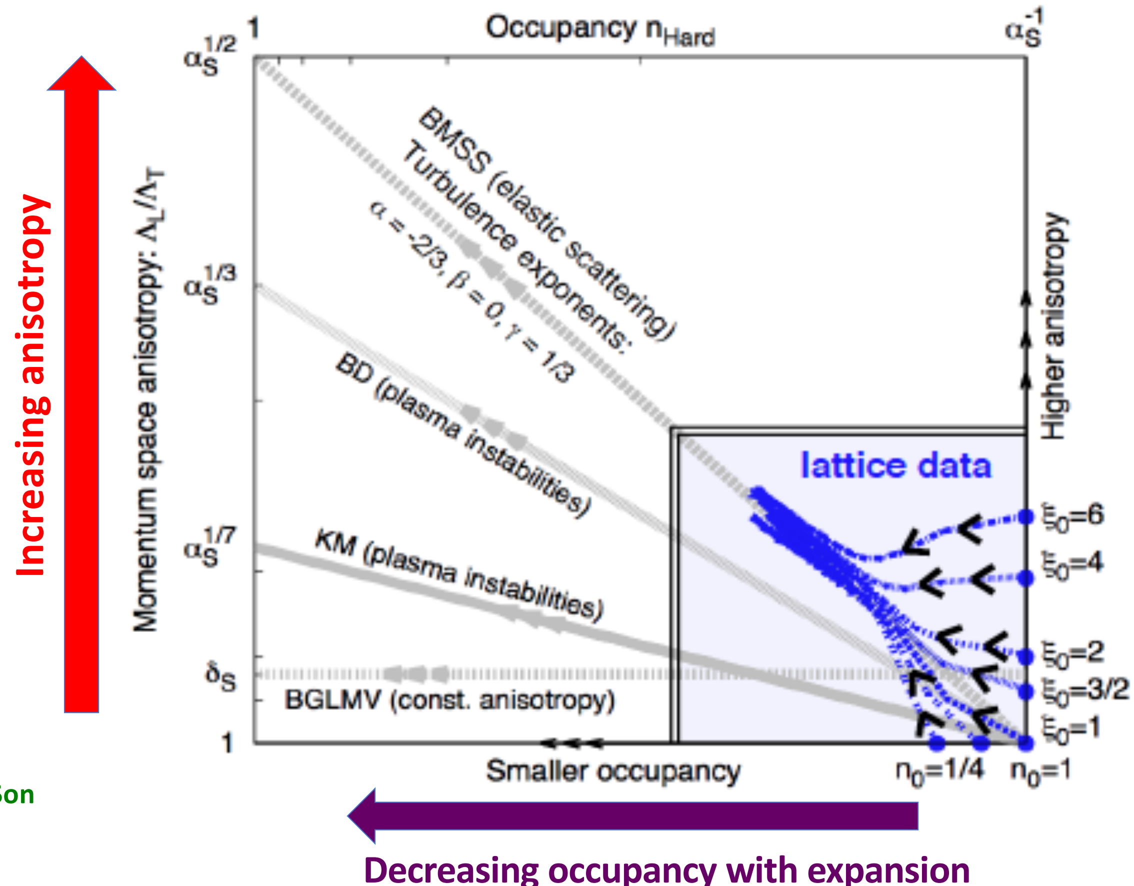
Kinetic theory in the overoccupied regime



Differences arise due to assumptions of non-perturbative behavior at $p \approx m_{Debye}$

Non-thermal fixed point in overpopulated QGP

Berges,Boguslavski,Schlichting,Venugopalan. PRD89 (2014) 114007



BMSS: Baier,Mueller,Schiff,Son
BD: Bodeker
KM: Kurkela, Moore
BGLMV: Blaizot,Gelis,Liao,McLerran,Venugopalan

Kinetic interpretation of self-similar behavior

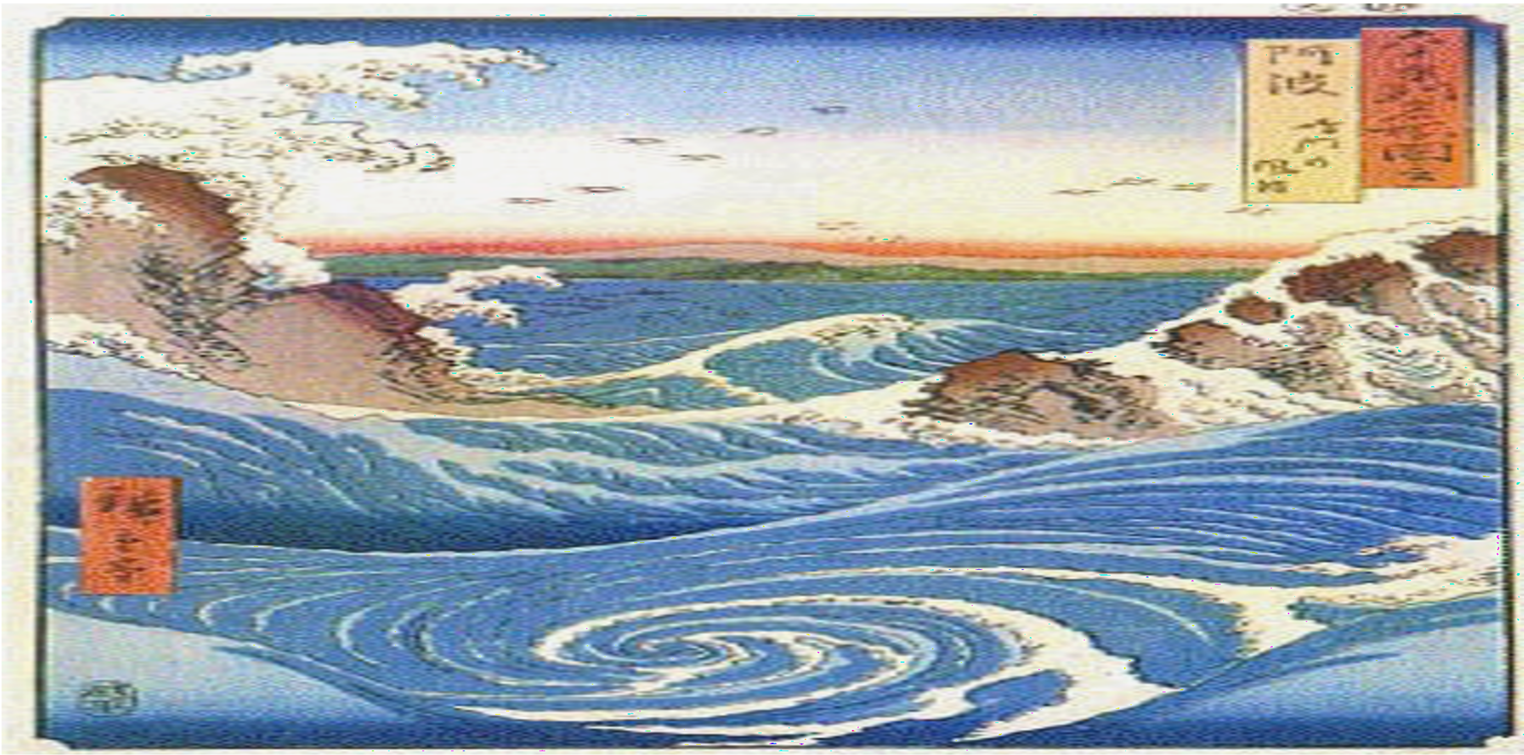
For self-similar scaling solution, a) small angle elastic scattering
b) energy conservation
c) number conservation

give unique results: $\alpha = -2/3$, $\beta = 0$, $\gamma = 1/3$

- These are the same exponents (within errors) extracted from our numerical simulations !

- The same exponents appear in the "bottom-up" thermalization scenario of Baier, Mueller, Schiff, Son (BMSS)

Universal non-thermal attractor in QCD

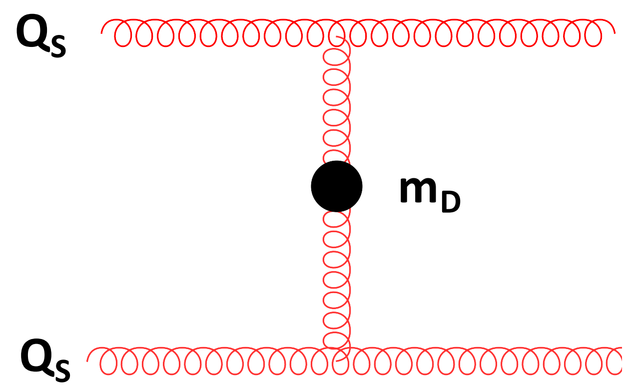


"Big whorls have little whorls, which feed on their velocity,
And little whorls have lesser whorls, and so on to viscosity."

Quo vadis, thermal QGP?

Baier,Mueller,Schiff,Son (2001)

The "bottom-up" scenario:



Scale for scattering of produced gluons ($t > 1/Q_S$) set by

$$m_D^2 \propto g^2 \int_{\vec{p}} \frac{f_{\rm hard}}{p}$$

Build up p_z (which fights the red shift of $p_z \sim 1/t$) with multiple scattering

$$p_z^2 = N_{\rm coll.}\, m_D^2 \qquad \frac{dN_{\rm col}}{d\tau} \sim \sigma N_h (1 + f_h) \sim \frac{\alpha N_h}{m_{\rm D}^2 p_z \tau} \qquad f_h = N_h/(Q_s^2 p_z)$$

$$\boxed{p_z \sim \frac{Q_s}{(Q_s \tau)^{1/3}}}$$

Our simulations support this picture– no significant role of late time instabilities (imaginary component in m_D)

Quo vadis, thermal QGP?

Occupation # $f = \frac{1}{p_z Q_s^2} \frac{Q_s^3}{\alpha_S(Q_s \tau)}$

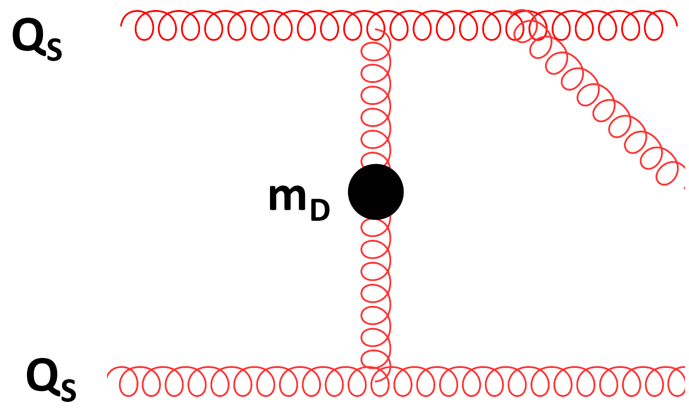
$$\boxed{f < 1 \text{ for } \tau > \frac{1}{\alpha_S^{3/2}} \frac{1}{Q_s}}$$

Classical statistical simulations break down
in this regime… have to switch to a quantum kinetic description

Quo vadis, thermal QGP?

In the quantum regime, thermalization proceeds through number changing inelastic processes:

i) Soft gluons first thermalize
ii) hard gluons at scale Q_S, that carry most of the energy, are quenched and lose energy to the bath



The final stage in "bottom up" thermalization
– is identical to the "jet quenching" formalism that Prof. Blaizot discussed in his lectures

Thermalized soft bath of gluons for $\tau > \frac{1}{\alpha_S^{5/2}} \frac{1}{Q_S}$

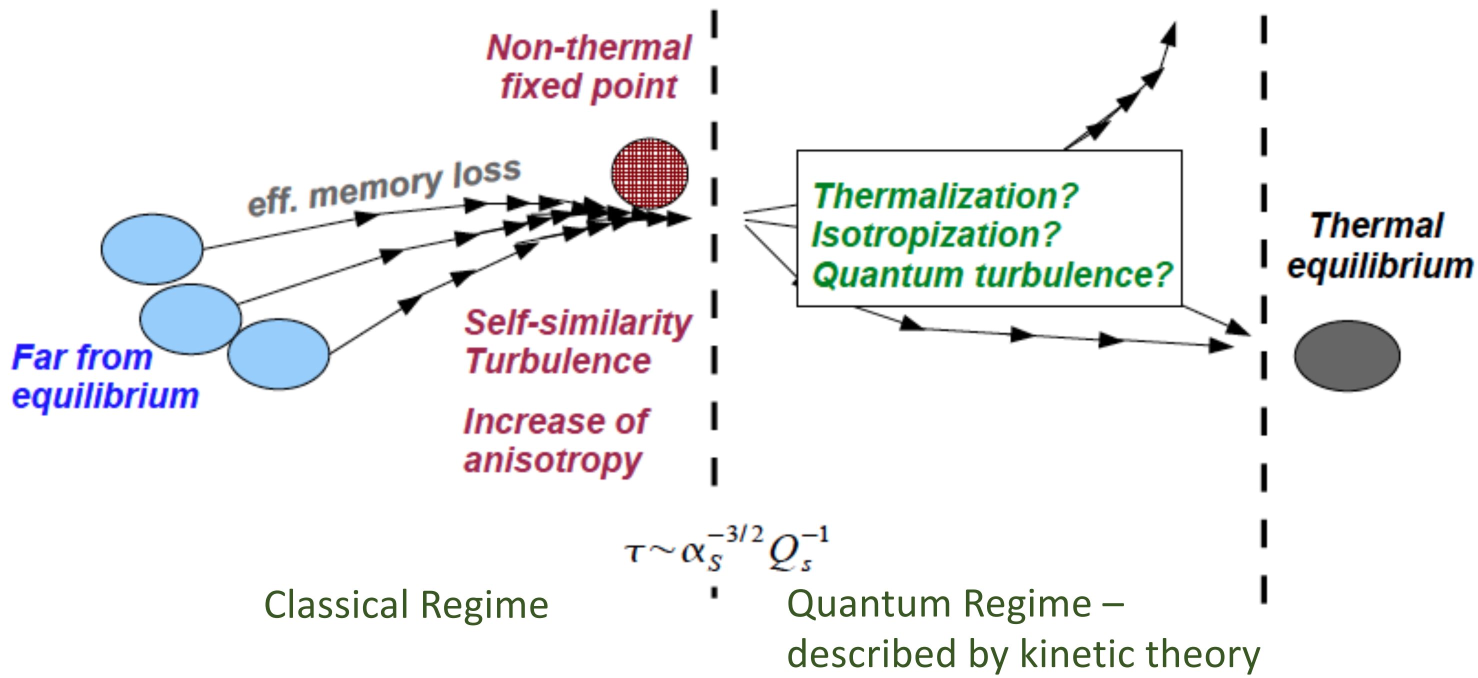
Thermalization temperature of $T_i = \alpha_S^{2/5} Q_S$

Thermalization in the Regge limit

A final consequence is that in the Regge limit: $\alpha_S \to 0, f\ \alpha_S$ = 1

thermalization occurs almost instantaneously in QCD compared to the lifetime of the system given by the size R ….

Quo vadis, thermal QGP?

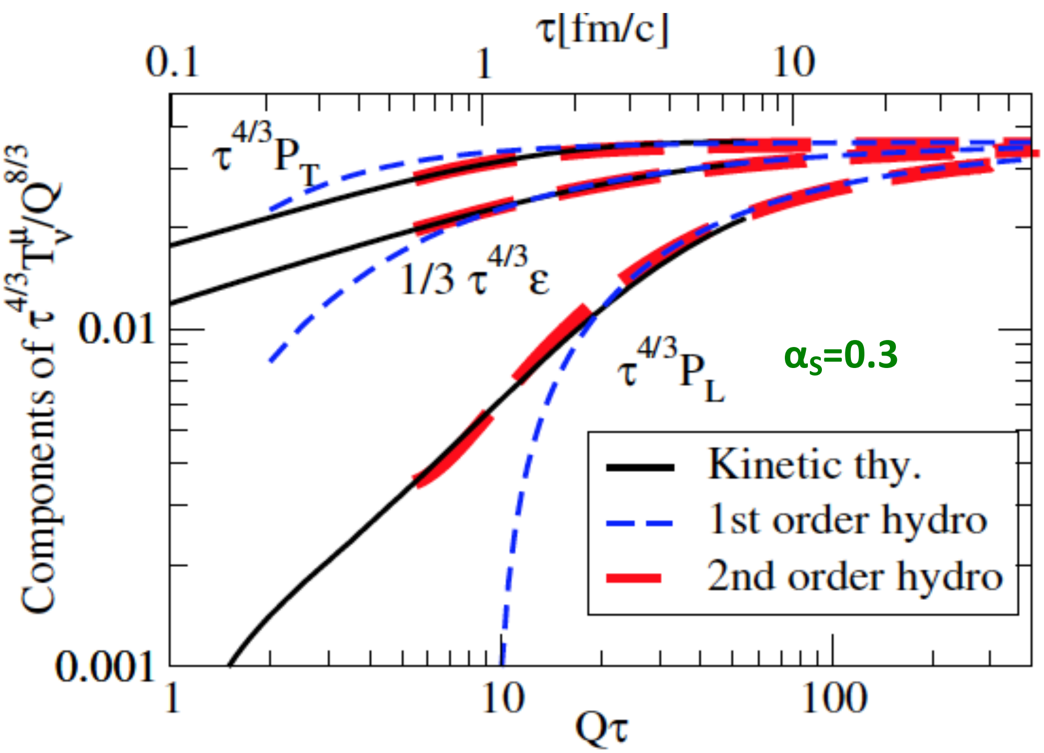


Thermalized soft bath of gluons for $\tau > \dfrac{1}{\alpha_S^{5/2}} \dfrac{1}{Q_S}$

Thermalization temperature of $T_i = \alpha_S^{2/5} Q_S$

Matching the Glasma to viscous hydrodynamics

Kurkela, Zhu, arXiv: 1506.06647



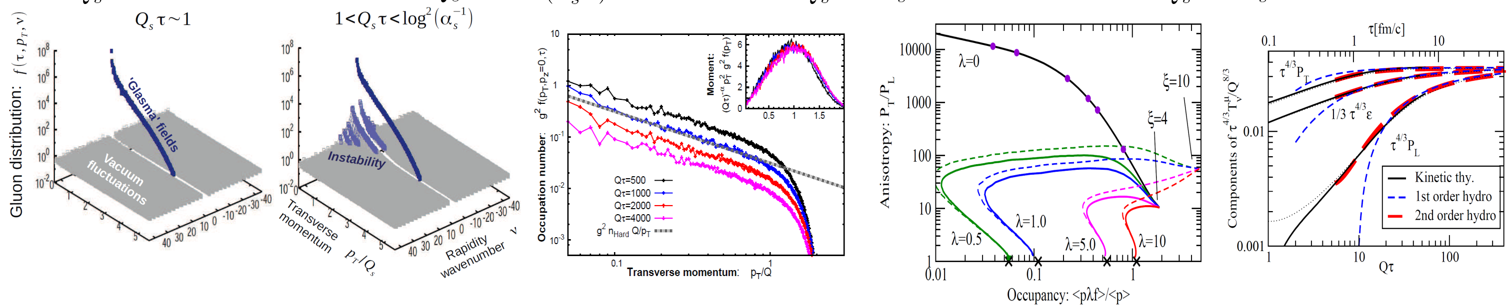
Good matching of quantitative implementation of kinetic theory to hydrodynamics at times ~ 1 fm

… when extrapolated to realistic couplings (many caveats remain)

Glasma to Plasma: from nuts to soup

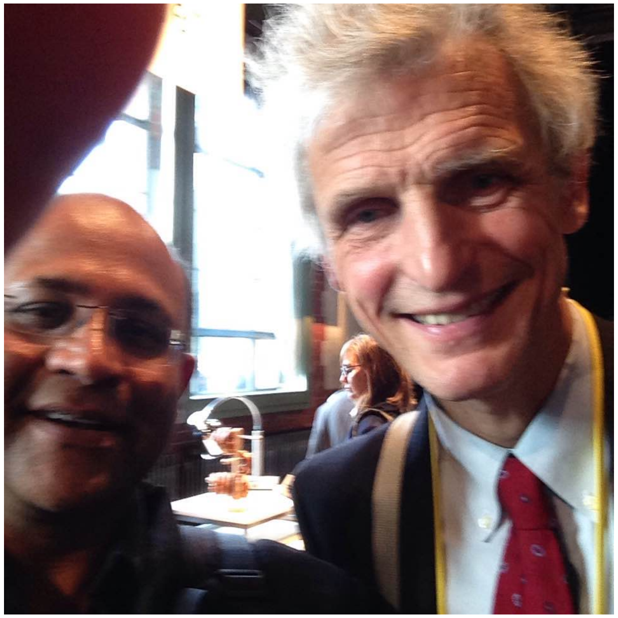
K. Boguslavski

$Q_s\tau = 0^+$ $\quad$ $Q_s\tau = \ln^2(\alpha_s^{-1})$ $\quad$ $Q_s\tau = \alpha_s^{-3/2}$ $\quad$ $Q_s\tau = \alpha_s^{-13/5}$



Glasma; **Instabilities;** **Classical NTFP;** **Constant anisotropy, Radiational breakup;** **Hydrodynamics, Thermalization**

Universality: hotness is also cool



Wolfgang Ketterle, Nobel Prize (2001)

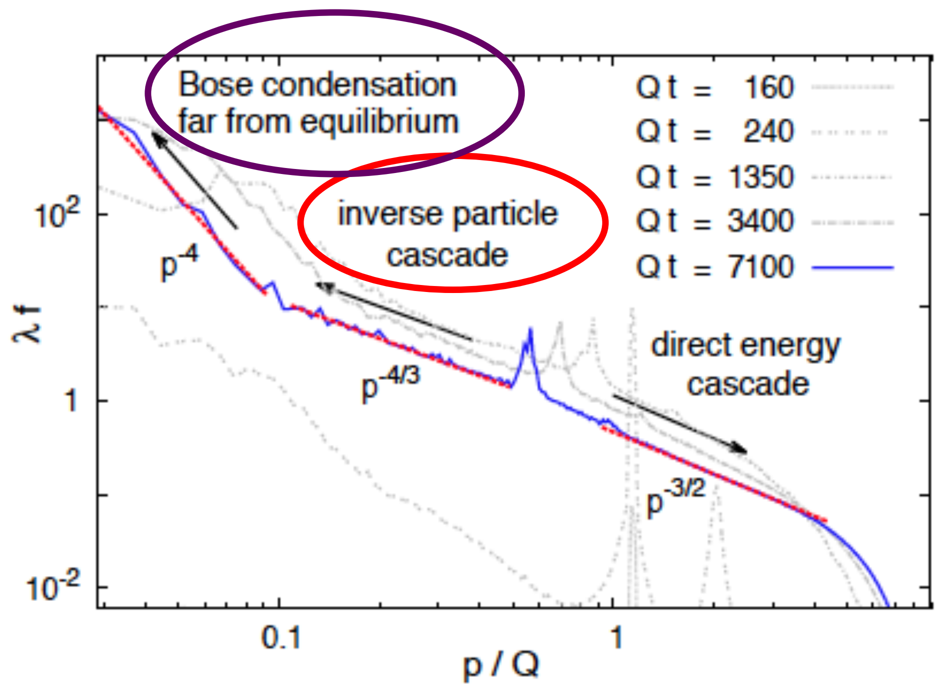
For the achievement of Bose-Einstein condensation in dilute gases of alkali atoms, and for early fundamental studies of the properties of the condensates

Non-Equilibrium dynamics of overoccupied scalar fields

$$S = \int d\tau d^2x_T d\eta\ \tau \left(\frac{g^{\mu\nu}}{2} (\partial_\mu \varphi_a)(\partial_\nu \varphi_a) - \frac{\lambda}{4!N} (\varphi_a \varphi_a)^2 \right)$$

In a non-relativistic limit, models cold atomic gases

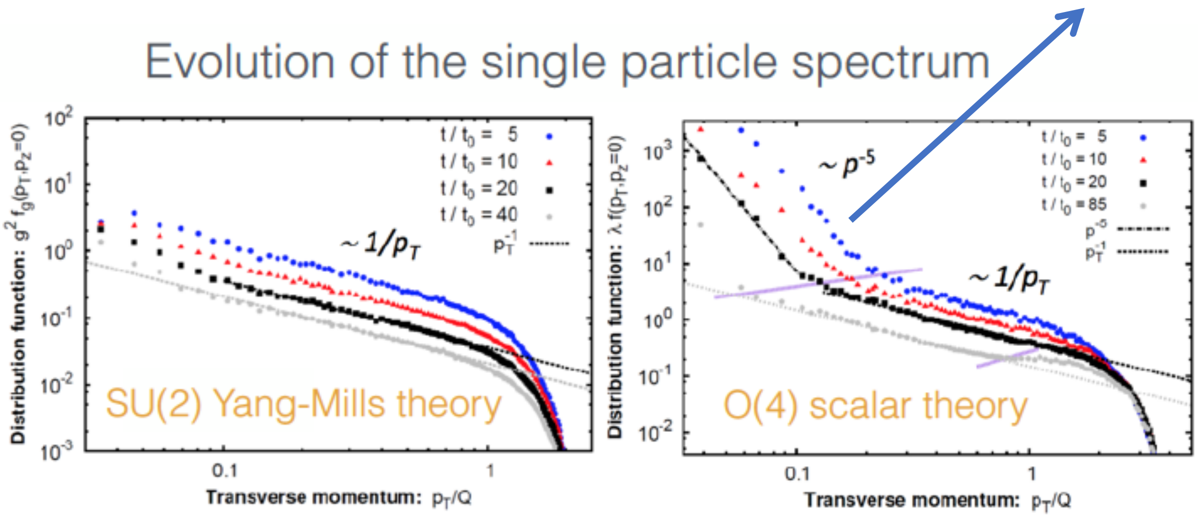
Scheppach,Berges,Gasenzer, PRA 81 (2010) 033611
Nowak, Schole, Sexty, Gasenzer, PRA85 (2012) 043627
Nowak et al., arXiv 1302.1448



Berges, Sexty PRL 108 (2012) 161601
Berges,Boguslavski,Orioli, PRD 92, 025041 (2015)
Berges,Boguslavskii,Schlichting,Venugopalan, JHEP 1405 (2014) 054

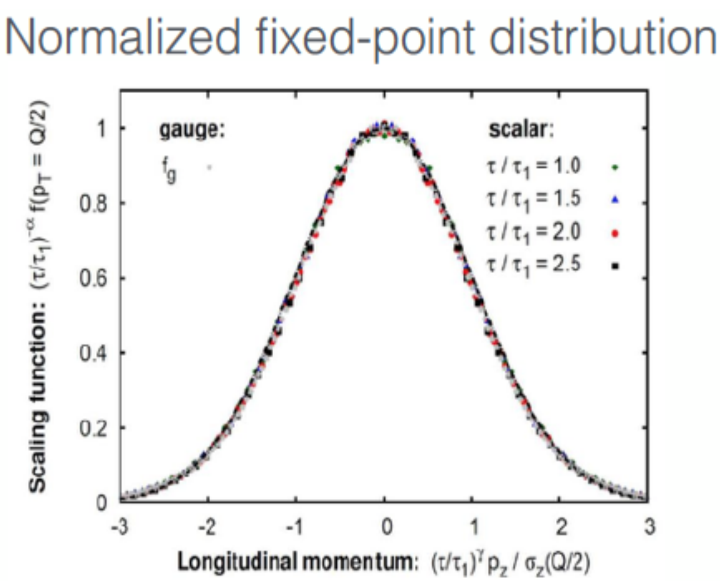
Remarkable universality between world's hottest and coolest fluids

Leads to Bose-Einstein condensation



Berges,Boguslavski,Schlichting,Venugopalan, PRL 114 (2015) 061601, Editor's suggestion & PRD92 (2015) 096 006

Remarkable universality between world's hottest and coolest fluids



In a wide inertial range, scalars and gauge fields have identical scaling exponents and scaling functions

$$f(p_T, p_z, \tau) = \tau^{\alpha} f_S(\tau^{\beta} p_T, \tau^{\gamma} p_z)$$

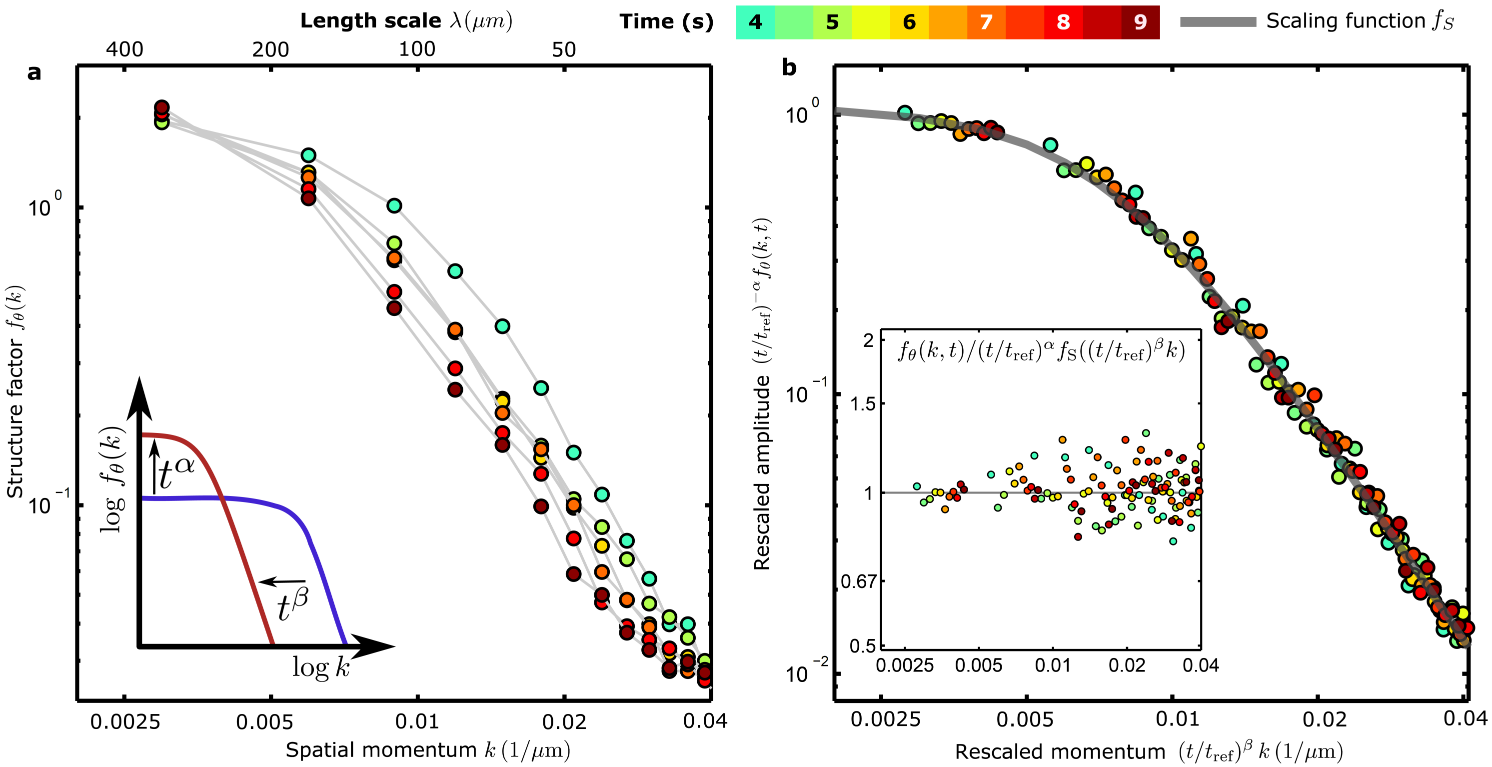
Very surprising from a kinetic theory perspective -- may reflect infrared dynamics consistent with a BEC

Berges,Boguslavski,Schlichting, Venugopalan, PRD92 (2015) 096 006
Tanji, Venugopalan, PRD (2017)

Turbulence is everywhere yet baffles deep thinkers

I am an old man now, and when I die and go to heaven there are two matters on which I hope for enlightenment. One is quantum electrodynamics, and the other is the turbulent motion of fluids. And about the former I am rather optimistic.   - Horace Lamb

Non-thermal fixed points in quantum gases



$$f_\theta(k,t) = t^\alpha f_S(t^\beta k)$$

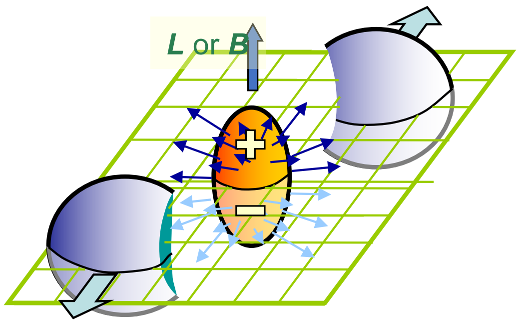
^{87}Rb BEC in a quasi 1D optical trap
only isotropic geometry thus far

$\alpha = 0.33 \pm 0.08 \quad \beta = 0.54 \pm 0.06$

Oberthaler BEC Labs
Prüfer et al, arXiv:1805.11881, *Nature* (2018)

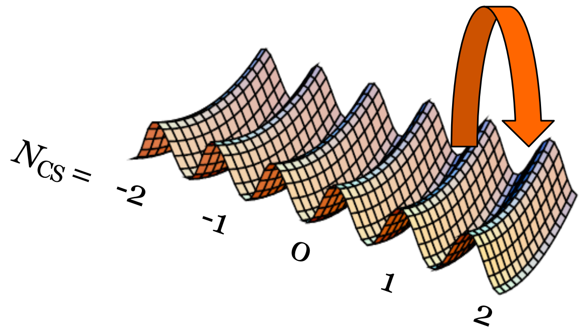
Topology in heavy-ion collisions:The Chiral Magnetic Effect

Kharzeev,McLerran,Warringa (2007)



External (QED) magnetic field
- As strong as 10^{18} Gauss !

+



Over the barrier topological (sphaleron) transitions … analogous to proposed mechanism for Electroweak Baryogenesis

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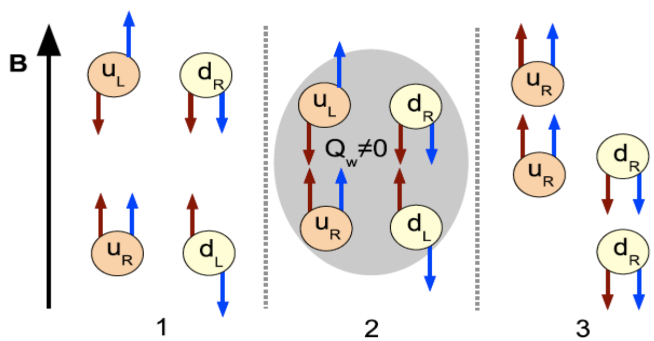
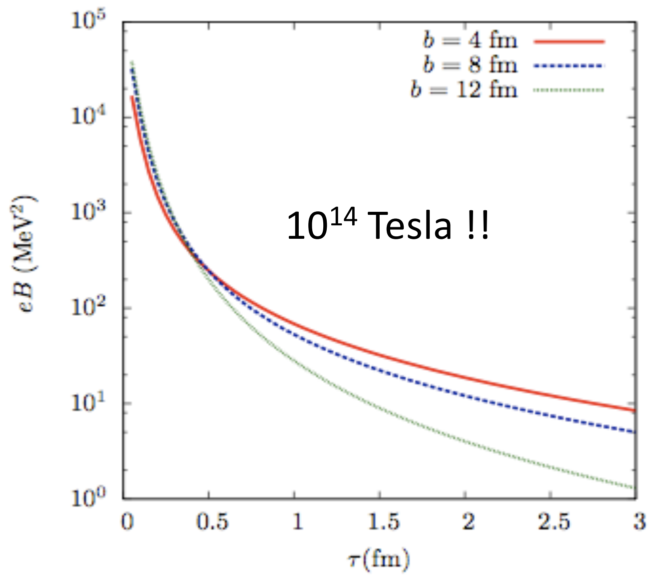


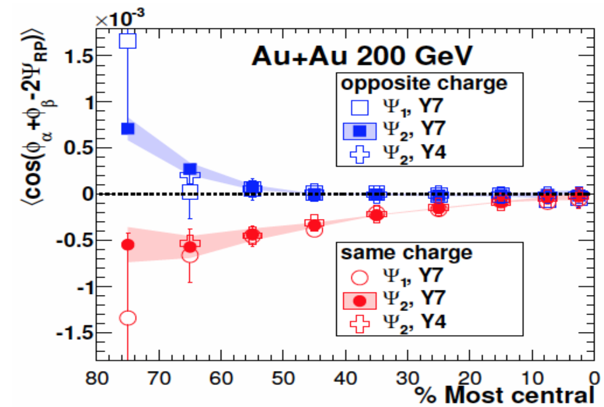
$$\vec{J}_{\text{CME}} = \frac{e^2}{2\pi^2}\,\mu_5\,\vec{B}$$

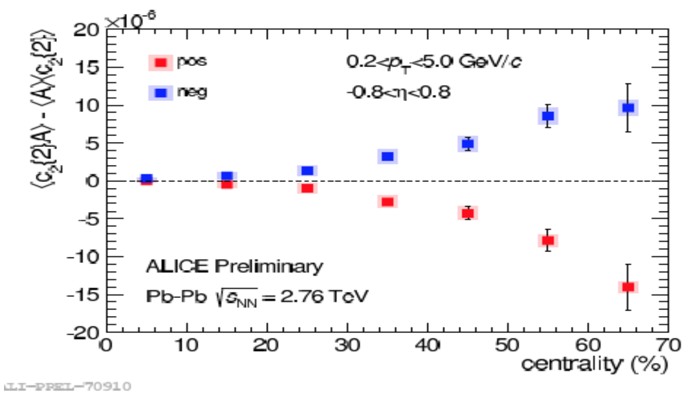
Chiral magnetic effect

Topology in heavy-ion collisions:The Chiral Magnetic Effect



External B field dies rapidly…
Effect most significant,
for transitions at early times





Consistent (caveat emptor!) with results from STAR and ALICE…searches underway

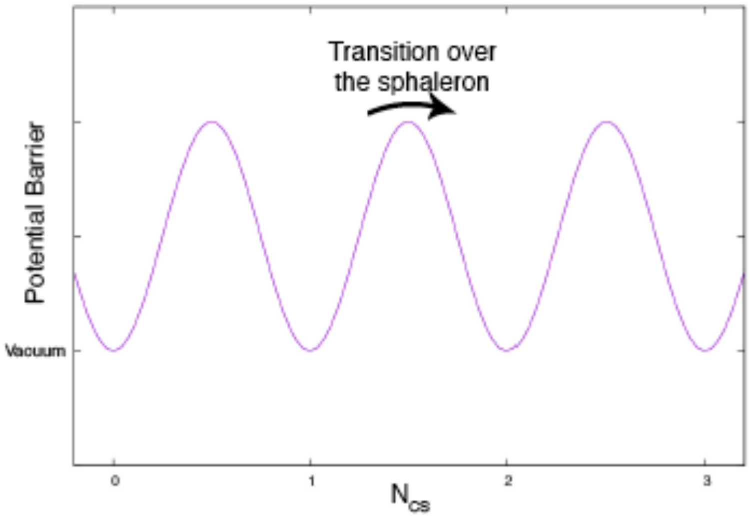
Chiral magnetic effect seen in condensed matter systems

Q. Li et al., Nature Physics (2015)

Sphaleron transitions in QCD

Sphaleron: spatially localized, unstable finite energy classical solutions
(σφαλερος -``ready to fall”)

EW theory: Klinkhamer, Manton, PRD30 (1984) 2212
QCD: McLerran,Shaposhnikov,Turok,Voloshin, PLB256 (1991) 451



Chiral Anomaly:

$$\partial_\mu J^\mu_{5,f} = 2m_f \bar{q}\gamma_5 q - \frac{g^2}{16\pi^2} F^a_{\mu\nu}\tilde{F}^{\mu\nu,a}$$

Chern-Simons current:

$$K^\mu = \frac{g^2}{32\pi^2}\epsilon^{\mu\nu\rho\sigma}\left(F^a_{\nu\rho}A^a_\sigma - \frac{g}{3}f_{abc}A^b_\nu A^c_\rho A^c_\sigma\right)$$

Chern-Simons #:

$$N_{CS}(t) = \int d^3x\, K^0(t,\mathbf{x})$$

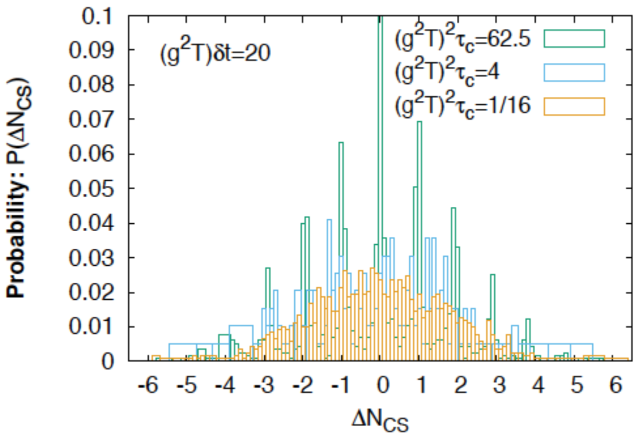
Rate of change of CS #

$$\frac{dN_{CS}(t)}{dt} = \frac{g^2}{8\pi^2}\int d^3x\, E^a_i(\mathbf{x})B^a_i(\mathbf{x})$$

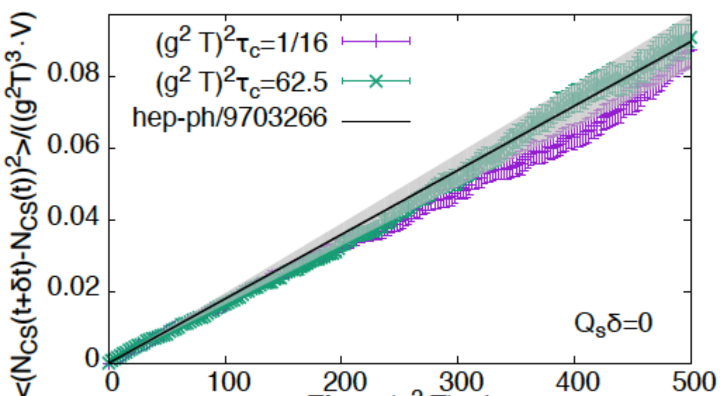
Key quantity: Sphaleron transition rate

$$\Gamma^{eq} = \lim_{\delta t -> \infty} \frac{\langle (N_{\rm CS}(t+\delta t) - N_{\rm CS}(t))^2\rangle_{\rm eq}}{V\delta t}$$

Thermal sphaleron transition rate



Cooled configurations—containing increasingly soft infrared modes cluster about integer values of CS #



Reproduce seminal results of Guy D. Moore

Moore, Tassler, arXiv:1011.1167

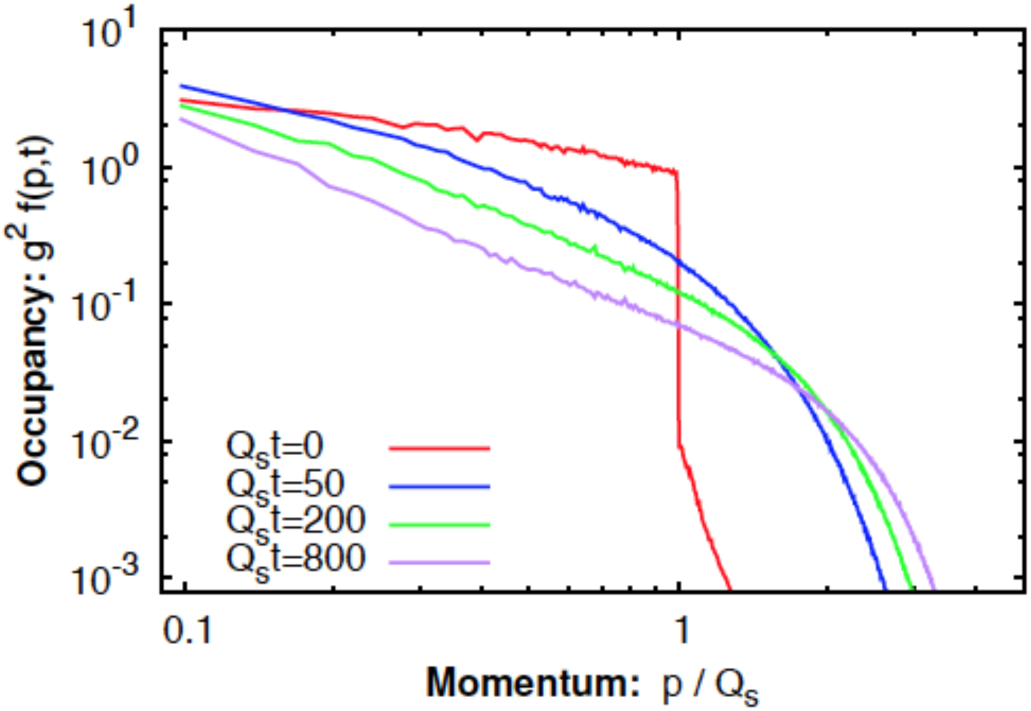
$$\Gamma_{\rm sphal} = (132 \pm 4)\left(\frac{g^2T^2}{m_{\rm D}^2}\right)\left(\ln\frac{m_{\rm D}}{\gamma} + 3.0410\right)\alpha_{\rm s}^5 T^4 ,$$

$$\gamma = \frac{N_{\rm c} g^2 T}{4\pi}\left(\ln\frac{m_{\rm D}}{\gamma} + 3.041\right) ,$$

$$m_{\rm D}^2 = \frac{2N_{\rm c}+N_{\rm f}}{6}g^2T^2 = \frac{6+N_{\rm f}}{6}g^2T^2 .$$

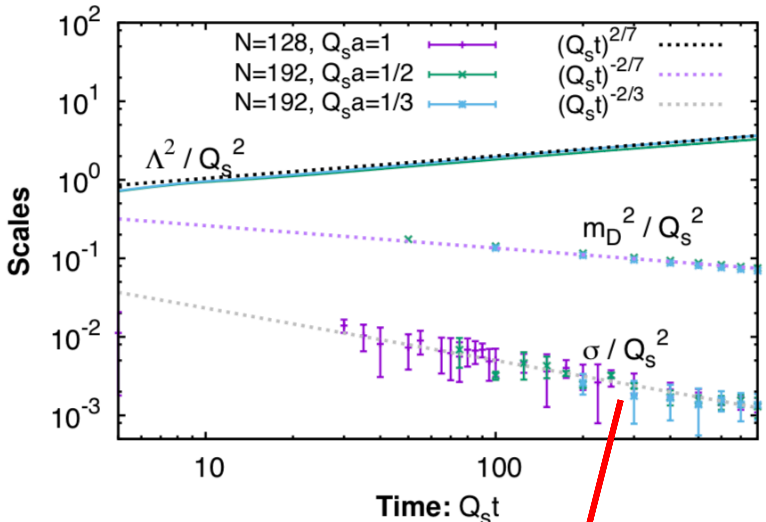
Parametric form predicted by Arnold, Son, Yaffe

Sphaleron transitions in the Glasma



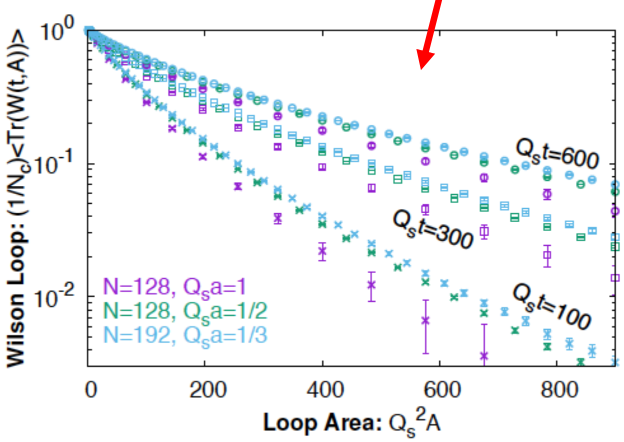
For simplicity, only consider Glasma in a box, and SU(2) gauge theory

Emergence of infrared structure in the Glasma



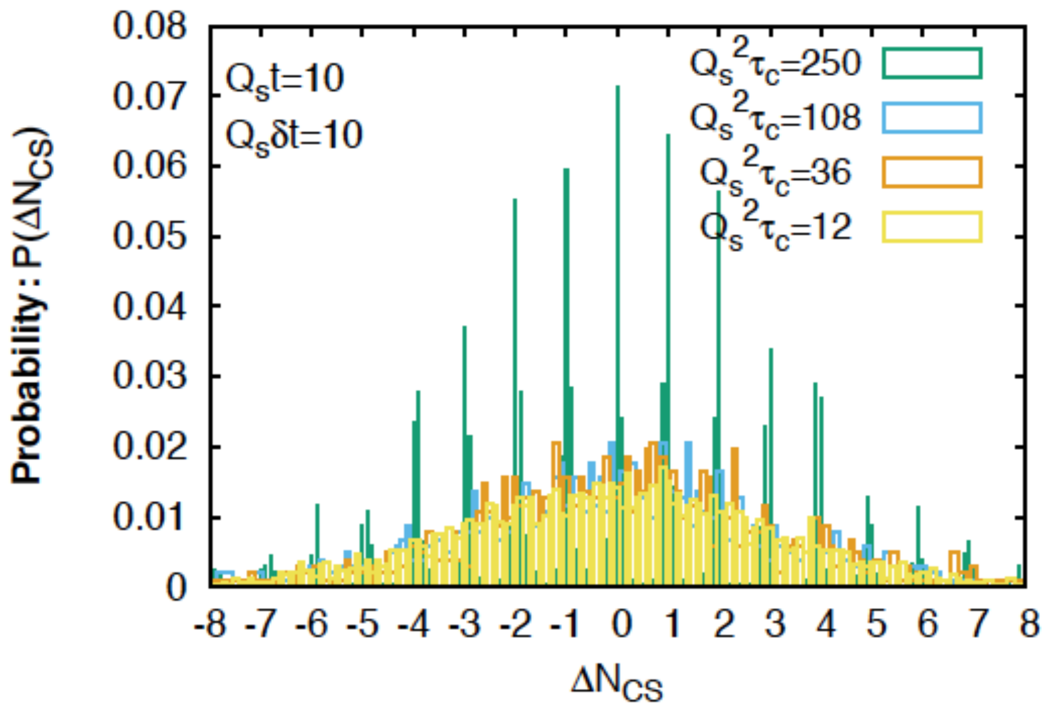
A (parametrically) one scale Glasma evolves towards the separation of length scales characteristic of a QGP:

$T >> gT >> g^2T$



Trace of spatial Wilson loop obeys area law – string tension σ sensitive to infrared dynamics on magnetic scale

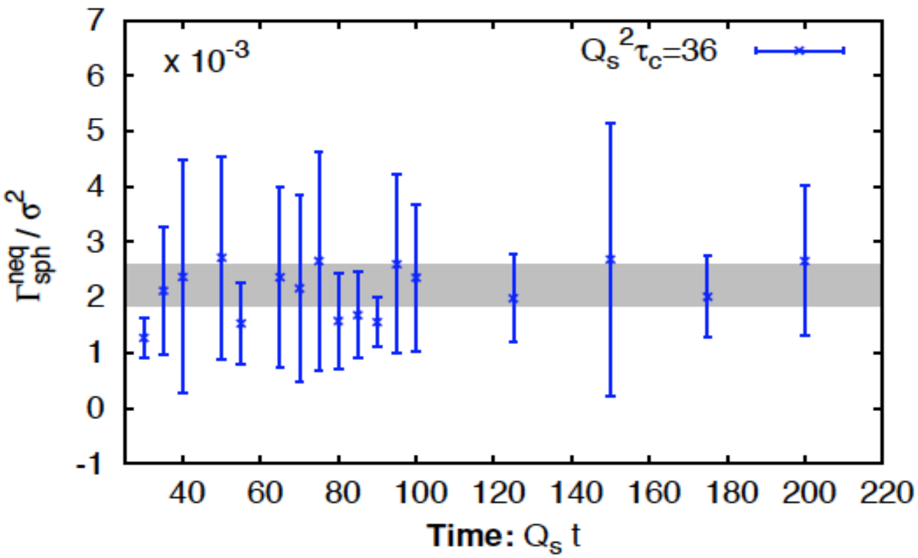
Topological transitions in the Glasma

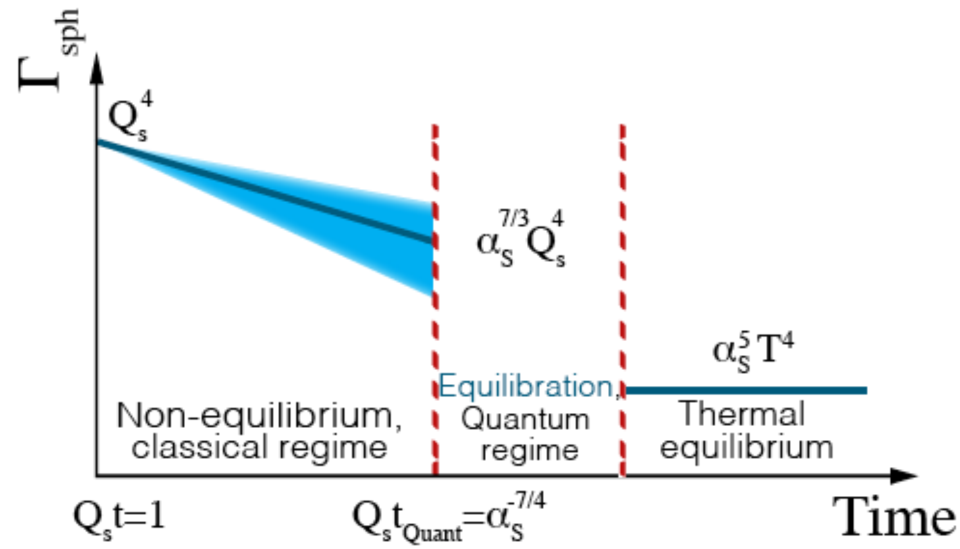


“Cooled” soft Glue configurations in the Glasma are topological!

Mace,Schlichting,Venugopalan, PRD (2016)

Topological transitions in the Glasma





Sphaleron transition rate scales with the string tension in the Glasma
- rate is large at early times !

Mace,Schlichting,Venugopalan,PRD (2016)

Thanks for listening !

Please feel free to contact me if you have any questions regarding these lectures.