Center Symmetry Realization in Trace Deformed Yang-Mills Theory

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Motivations

Yang-Mills (YM) theory defined on $\mathbb{R}^3 \times S^1$ has a global symmetry called center symmetry which is spontaneously broken in the limit of vanishing S^1 radius $\Rightarrow$ deconfinement.

Non perturbative properties (confinement, topology) change across the deconfinement.

The deformed theory: a tool to investigate the relation between the realization of center symmetry and the non perturbative properties of YM.

Lattice approach + Monte-Carlo methods.

Yang-Mills theory

Yang-Mills (YM) theory is a non-abelian gauge theory based on the gauge group $SU(N)$ which describes the interactions between bosonic particles called gluons.

YM is one of the key ingredient of QCD, the current theory of strong interactions.

YM shows two different regimes:

High energy: weakly coupled $\rightarrow$ perturbative methods.

Low energy: strongly coupled $\rightarrow$ Monte-Carlo methods.

Yang-Mills theory with a compactified dimension

Consider $SU(N)$ Yang-Mills (YM) theory defined on a space with a compactified direction of length L and periodic boundary conditions (PBC).

Using the path integral formulation of the theory we can interpret the compactified length L as a temperature T.

$$\boxed{T = \frac{1}{L}}$$

The high-T and low-T regime show different properties and are separated by the **deconfinement phase transition**.

Deconfinement phase transition

Compactified direction + PBC $\Rightarrow$ Finite temperature

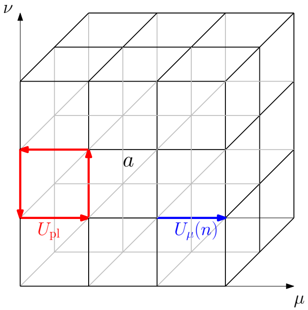
When the system is studied at high temperature (small L) it undergoes the deconfinement phase transition separating two limits:

Low- T
Quarks are confined in bound states, the hadrons.

High- T
Quarks are no more confined $\Rightarrow$ Quark Gluon Plasma (QGP).

Due to the phase transition these two regimes are not analytically connected. The weak coupling methods at high-T can not give information on low-T properties.

Lattice formulation



$$U_\mu(n) = e^{igA_\mu(n)}$$

$$U_{\rm pl}(n) = e^{iga^2F_{\mu\nu}(n)}$$

$$S_W = \sum_{\rm pl} \beta \left[1 - {\rm ReTr}(U_{\rm pl})\right]$$

Finite temperature on the lattice can be otbained using different lattice extensions:

$$L_x = L_y = L_z > L_t$$

Center symmetry (on the lattice)

YM theory defined on a space with a compactified dimension is invariant under a center symmetry transformation:

Multiply all the temporal link variables at a given time slice U_t by an element of the center of the gauge group C.

$$U_t(\vec{n}, n_t) \longrightarrow zU_t(\vec{n}, n_t) \qquad z \in C \subset SU(3)$$

The center of a given group is the subset of elements which commute with all the other elements of the group. The center of $SU(3)$ is Z_3, i.e. the cube root of the identity.

$$z \in \left(\mathbb{1}_{3\times 3},\ e^{\frac{2\pi i}{3}} \mathbb{1}_{3\times 3},\ e^{\frac{4\pi i}{3}} \mathbb{1}_{3\times 3} \right)$$

Spontaneous breaking of center symmetry

Deconfinement $\Rightarrow$ Center symmetry is broken

The Polyakov loop P is the order parameter of the deconfinement phase transition.

$$P = \prod_{t=1}^{N_t} U_4(\vec{n}, t)$$

$$\langle \mathrm{TrP} \rangle \quad \rightarrow \quad z \langle \mathrm{TrP} \rangle$$

Low-T (Confinement)	High-T (Deconfinement)
$\langle \mathrm{Tr} P \rangle = 0$	$\langle \mathrm{Tr} P \rangle \neq 0$

Volume Independence

It has been shown that YM theory is volume independent in the large N limit as long as center symmetry is not broken. (T. Eguchi and H. Kawai: PRL, **48**, (1982)).

It could be possible to study infinte volume YM performing simulations on smaller lattices, even on a single site one!

Large N limit + Center symmetry

$\Rightarrow$ Volume independence.

Question: Is it possible to preserve center symmetry also for small compactification radii without changing the physical properties?

Answer: The double trace deformation.

The double trace deformation

$$S^{\mathrm{def}} = S_W + h \underbrace{\sum_{\vec{n}} \left|\mathrm{Tr} P(\vec{n})\right|^2}_{\text{deformation}}$$

The deformation was proposed as a tool to preserve center symmetry and allow weak coupling methods in the high-T regime.

(M. Unsal and G. Yaffe: PRD, **78**, (2008)).

In the updating procedure the configurations are drawn with a statistical weight $e^{-S^{\mathrm{def}}}$, thus **configurations with $\mathrm{Tr} P \neq 0$ are suppressed.**

We choose the parameter h in order to restore centre symmetry.

NUMERICAL RESULTS

Topology and θ-term

$$\mathcal{L}_\theta = \mathcal{L}_{\rm YM} - i\theta Q(x)$$

$$F(\theta, T) = -\frac{1}{V_4}\ln \int [dA] \exp\left\{-\int_0^{\frac{1}{T}} dt \int d^3x \mathcal{L}_\theta\right\}$$

The **free energy** $F(\theta, T)$ can be parametrized as follows:

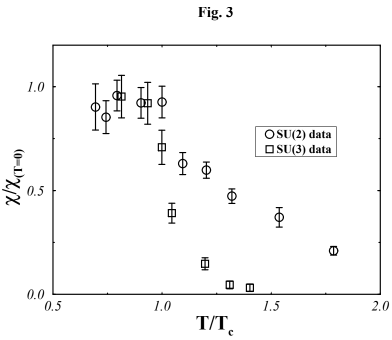
$$F(\theta, T) - F(0, T) = \frac{1}{2}\chi(T)\theta^2 \left(1 + b_2(T)\theta^2 + b_4(T)\theta^4 + \cdots\right)$$

and it is easy to see that

$$\chi = \frac{\langle Q^2 \rangle_{\theta=0}}{V_4}$$

$$b_2 = -\frac{1}{12\langle Q^2 \rangle_{\theta=0}}\left[\langle Q^4 \rangle_{\theta=0} - 3\langle Q^2 \rangle^2_{\theta=0}\right]$$

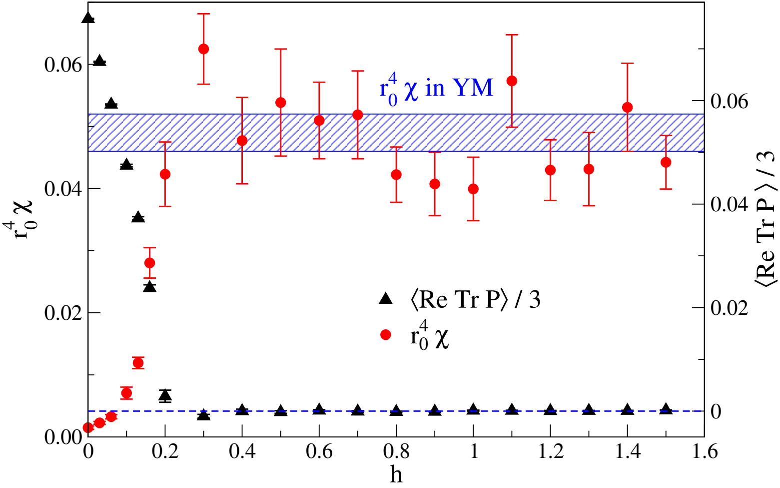
Topology and Finite Temperature: MC Results



- Figure: (Allès, D'Elia, Di Giacomo: PLB 412 1997)
- **Topological suscceptibility changes drastically from the low-T to the high-T regime.**

What happens in the deformed theory?

Deformed $SU(3)$ results. 8×32^3 $\beta = 6.4$



- More detail $\rightarrow$ (C. Bonati, MC and M. D'Elia: PRD, **98**, (2018)).
- The blue band is the $T = 0$ result

  $\rightarrow$ (C. Bonati, M. D'Elia and A. Scapellato: PRD, **93**, (2016)).

The $SU(4)$ case

$SU(4) \longrightarrow$ Center Simmetry has two breaking patterns:

$$\boxed{\mathbb{Z}_4 \quad \rightarrow \quad \mathrm{Id}} \qquad \boxed{\mathbb{Z}_4 \quad \rightarrow \quad \mathbb{Z}_2}$$

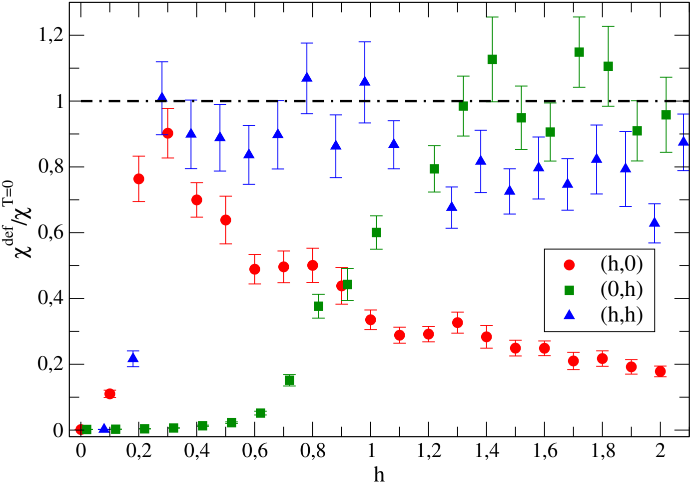
The order parameter are

$$\boxed{\langle \mathrm{Tr} P \rangle} \qquad \boxed{\langle \mathrm{Tr} P^2 \rangle}$$

In order to recover the full center symmetry we must consider two deformations:

$$\boxed{\mathrm{S}^{\mathrm{def}} = \mathrm{S}_{\mathrm{W}} + h_1 \sum_{\vec{n}} \left| \mathrm{Tr} P(\vec{n}) \right|^2 + h_2 \sum_{\vec{n}} \left| \mathrm{Tr} P^2(\vec{n}) \right|^2}$$

Deformed $SU(4)$ results. 6×32^3 $\beta = 11.40$



- Detail $\to$ (C. Bonati, MC, M. D'Elia, F. Mazziotti: PRD, **101**, (2020)).
- $T = 0 \to$ (C. Bonati, M. D'Elia, P. Rossi, E. Vicari: PRD, **94**, (2016)).

Glueball spectrum

YM theory has a non trivial spectrum made of bound states built from the gluon fields.

The states can be labelled using theri Angular momentum (J), parity (P) and charge (C).

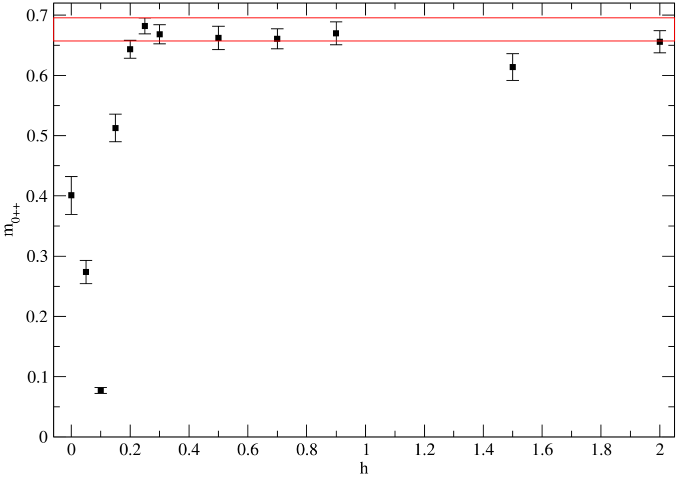
It is possible to extract the mass from lattice simulation using the relation

$$\langle O^*(x_0)O(x_0 = 0)\rangle = \sum_n |\langle n|\,\hat{O}\,|\Omega\rangle|^2 \mathrm{e}^{-E_n x_0}$$

where O is an operator built with link variables and $|\Omega\rangle$ is the ground state.

$x_0 \to \infty \quad \Rightarrow$ we can compute E_0 looking at the decay.

$SU(3)$ 0++ mass. 6×30^3 $\beta = 6.0$



- (In preparation)

Summary

Why trace deformation?

- A tool to understand how the confining properties are related to the realization of center symmetry.
- Exploit volume independence.

Lattice results:

- Both topological properties and gluebal masses computed in the center-stabilized regime are in agreement with their values in standard non-deformed confined phase.

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THANK YOU!