Exact results with defects

based on: 1805.04111, 1910.06332, 1911.05082, 2004.07849
with M. Lemos, M. Meineri; M. Billó, F. Galvagno, A. Lerda; G. Bliard, V. Forini, L. Griguolo, D. Seminara

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May 28^{th}, 2020.
Cortona Young

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  2. Boundaries and interfaces
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- They preserve part of the original (super)symmetry, leading to constraints on physical observables.

CFT

- Set of conformal primary operators $\mathcal{O}_{\Delta,\ell}(x)$ plus descendants $\partial_{\mu_1}\dots\partial_{\mu_n}\mathcal{O}_{\Delta,\ell}$.
- Operator product expansion (OPE)

$$\mathcal{O}_i(x)\mathcal{O}_j(0) = \sum_{k\in\text{prim.}} c_{ijk}|x|^{\Delta_k-\Delta_i-\Delta_j}\left(\mathcal{O}_k(0)\underbrace{+x^\mu\partial_\mu\mathcal{O}_k(0)+\dots}_{\text{all fixed}}\right)$$

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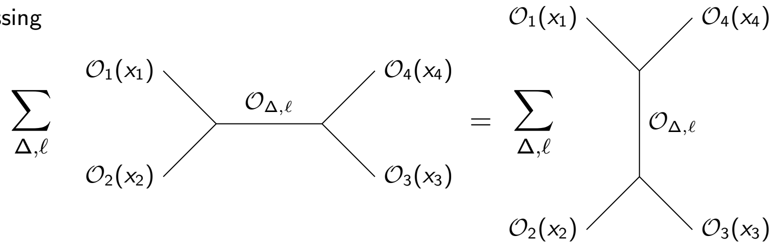

DCFT [Billò, Goncalves, Lauria, Meineri, 2016]

- Defect operators $\hat{\mathcal{O}}_{\hat{\Delta},\hat{\ell},s}$ with parallel ($\hat{\ell}$) and orthogonal spin (s) and descendants $\partial_{a_1}\ldots\partial_{a_n}\hat{\mathcal{O}}_{\hat{\Delta},\hat{\ell},s}$.

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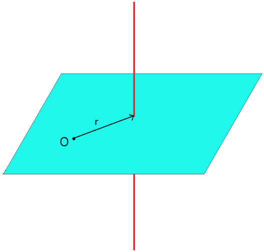

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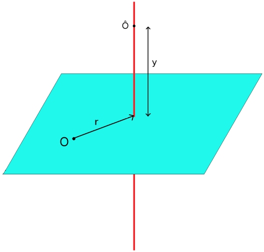

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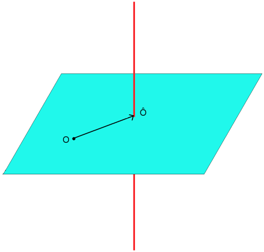

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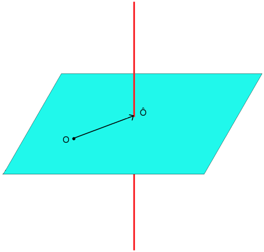

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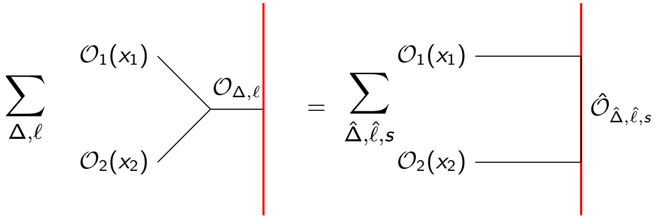
- The naive set of defect CFT data is $\{a_{\mathcal{O}}, b_{\mathcal{O}\hat{\mathcal{O}}}, \hat{\Delta}_{\hat{\mathcal{O}}}, \hat{c}_{\hat{\mathcal{O}}_1\hat{\mathcal{O}}_2\hat{\mathcal{O}}_3}\}$.

DCFT

- Defect crossing



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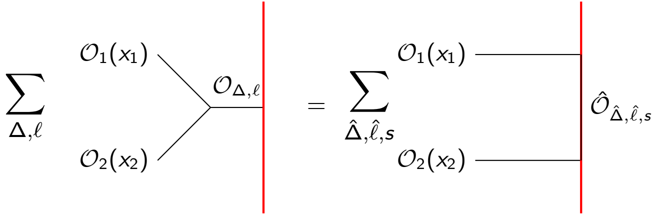

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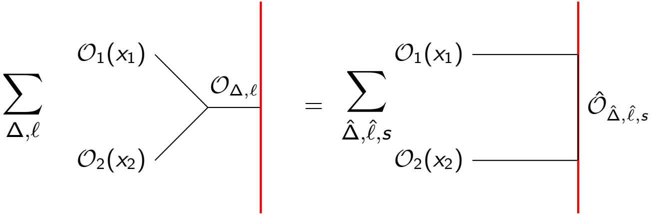
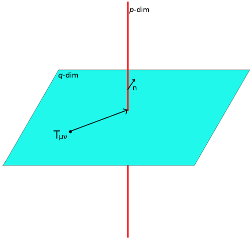


- Subset of defect CFT data:
  1. Physically (or geometrically) relevant
  2. Universal (present in any defect CFT)
- Stress-tensor one-point function

$$\langle T^{ab}\rangle_W = -h\frac{(q-1)\delta^{ab}}{|x_\perp|^d}$$

$$\langle T^{ij}\rangle_W = h\frac{(p+1)\delta^{ij} - d\, n^i n^j}{|x_\perp|^d}$$



Displacement operator

- A defect breaks translation invariance

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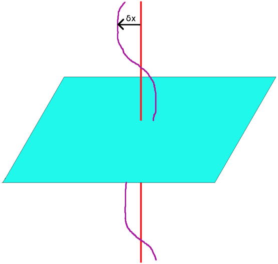

- $\mathbb{D}^i(x_\parallel)$ is the displacement operator
- It implements small modifications of the defect

$$\delta \langle X \rangle_W = -\int d^p x_\parallel\, \delta x_i(x_\parallel) \langle \mathbb{D}^i(x_\parallel) X \rangle_W$$

- Its two-point function is fixed by conformal symmetry

$$\langle \mathbb{D}^i(x_\parallel)\, \mathbb{D}^j(0) \rangle_W = C_D\, \frac{\delta^{ij}}{|x_\parallel|^{2(p+1)}}\,.$$

Normalization fixed by Ward identity, C_D is physical. Like $\langle T_{\mu\nu} T_{\rho\sigma} \rangle \sim c$.

Relation between C_D and h

For superconformal defects [LB, Lemos, 2019]

$$C_D = \frac{2^{p+1}(q+p-1)(p+2)}{q-1} \frac{\Gamma(\frac{p+1}{2})}{\pi^{\frac{p+1}{2}}} \frac{\pi^{\frac{q}{2}}}{\Gamma(\frac{q}{2})} h \,.$$

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- The relation is theory independent, but C_D and h are non-trivial functions of the parameters (e.g. λ, N).
- Conjectured for Wilson lines in $\mathcal{N}=4$ SYM and ABJM theory [Lewkowycz, Maldacena, 2014].
- Proven for $d=4$ and any $q>1$ (any SUSY) [LB, Lemos, Meineri, 2018; LB, Lemos, 2019].
- Proof is general, no conceptual difficulty in its generalization.

Examples

$$\begin{aligned} p=1,\ q=3 &\qquad C_D = 36\ h \\ p=1,\ q=2 &\qquad C_D = 24\ h \\ p=2,\ q=2 &\qquad C_D = 48\ h \end{aligned}$$

Wilson lines

$$\mathcal{W} = \mathrm{Tr}\mathcal{P}e^{\mathrm{i}\oint A_\mu dx^\mu}$$

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Supersymmetric Wilson line for $\mathcal{N} \geq 2$ SYM

$$\mathcal{W} = \mathrm{Tr}\mathcal{P}e^{\mathrm{i}\oint A_\mu dx^\mu + \oint |dx|\phi}$$

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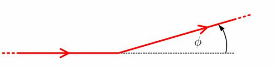
Displacement operator

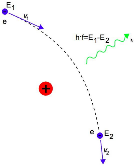
$$\mathbb{D}^i = F^{ti} + \mathrm{i}D^i\phi$$

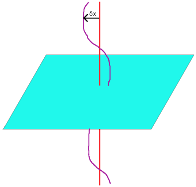
- Correlators

$$\langle \mathbb{D}^i(t_1)\mathbb{D}^j(t_2)\rangle_{\mathcal{W}} = \langle \mathrm{Tr}(\mathcal{W}_{-\infty,t_1}\mathbb{D}^i(t_1)\mathcal{W}_{t_1,t_2}\mathbb{D}^j(t_2)\mathcal{W}_{t_2,\infty})\rangle_W$$

- The displacement two-point function is related to the energy emitted by an accelerated heavy probe, the Bremsstrahlung function.







$$\Gamma_{\text{cusp}}(\phi) \sim -B\,\phi^2 \qquad \Delta E \sim 2\pi\,B \int dt\,\dot{v}^2 \qquad \langle \mathbb{D}^i(\tau)\,\mathbb{D}^j(0)\rangle_W = \frac{12\,B\,\delta^{ij}}{|\tau|^4}$$

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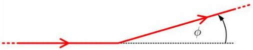

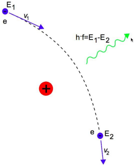

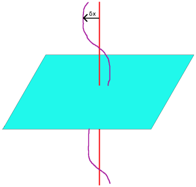

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Exact Bremsstrahlung function in $\mathcal{N}=4$ SYM [Correa, Henn, Maldacena, Sever, 2012]

$$B = \frac{1}{2\pi^2}\lambda\partial_\lambda \log \langle W_{\text{circle}}\rangle$$

- $\langle W_{\text{circle}}\rangle$ is known exactly [Erickson,Semenoff,Zarembo,2000; Drukker,Gross,2000; Pestun,2007].

Two-point function in $\mathcal{N} = 2$ SCFTs

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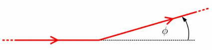

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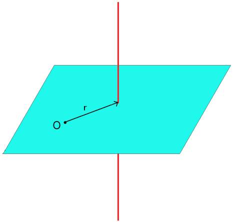

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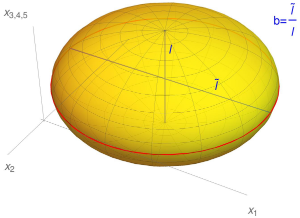

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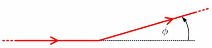

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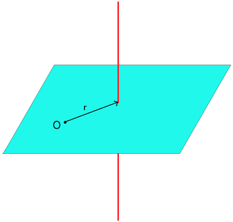

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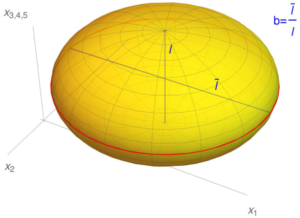

- It was initially conjectured [Fiol, Gerchkowitz, Komargodski, 2015] based on consistent perturbative evidence [Fiol, Gerchkowitz, Komargodski, 2015; Gomez, Mauri, Penati, 2018].
- The first equality is equivalent to $C_D = 36h$ [LB,Lemos, Meineri, 2018].
- The second equality was proven using defect CFT techniques [LB, Billò, Galvagno, Lerda, 2019].

$$\partial_b \ln \langle W_b \rangle \Big|_{b=1} = \int_{S^4} \frac{1}{2} \langle T_{\mu\nu} \rangle_W \partial_b g^{\mu\nu} \Big|_{b=1} + \langle O_2 \rangle_W \partial_b M \Big|_{b=1} + \ldots$$

Four-point function

- Four-point functions in 1d CFT are functions of a single cross ratio

$$\langle \phi(t_1)\phi(t_2)\phi(t_3)\phi(t_4)\rangle_{\mathcal{W}} = \frac{1}{t_{13}^{2\Delta}t_{24}^{2\Delta}}g(\chi) \qquad \chi = \frac{t_{12}t_{34}}{t_{13}t_{24}}$$

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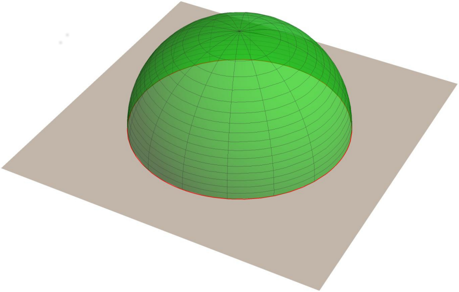

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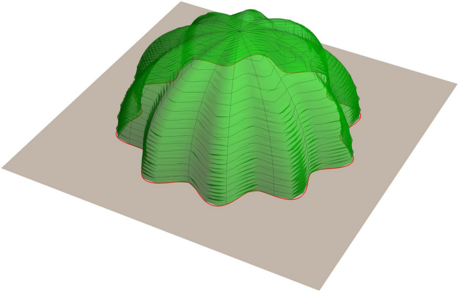

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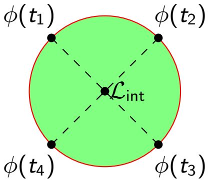
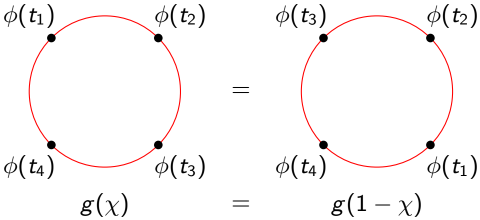
Witten diagrams [Giombi, Roiban, Tseytlin, 2017; LB, Bliard, Forini, Griguolo, Seminara, 2020]



Analytic bootstrap [Liendo, Meneghelli, Mitev, 2018; LB, Bliard, Forini, Griguolo, Seminara, 2020]



Surface defects and chiral algebras [LB, Lemos, 2019]

- Surface operators in superconformal theories are realized [Gukov, Witten, 2006; Alday, Gaiotto, Gukov, Tachikawa, Verlinde, 2010] by prescribing a singular behaviour to ambient 4d fields at the 2d submanifold or by coupling 2d and 4d matter [Gomis, Le Floch, 2016]

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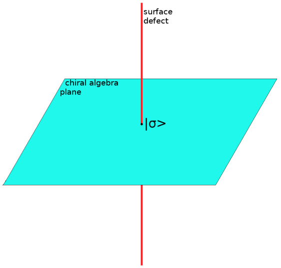

- The defect identity introduces in chiral algebra a non-vacuum module $|\sigma\rangle$ [Gaiotto, Cordova, Shao, 2017]
- We worked out several non-trivial properties of this module, in particular [LB, Lemos, 2019]

$$\langle\sigma|T(z)|\sigma\rangle = -3\pi^2 h/z^2$$

- Identifying the module associated to the defect identity provides a way of computing h.
- The construction gives access to an infinite number of defect CFT data upon the identification of 4d and 2d operators.

Conclusions and open questions

- Defect correlators in CFTs are related to physically interesting observables.
- The development of exact techniques for these quantities allows us to explore the non-perturbative regime of CFTs.

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  1. What is the landscape of conformal defects in a given bulk CFT?
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THANK YOU

$\mathcal{N} = 6$ Chern-Simons theory with matter. SCFT in 3d

Type *IIA* superstring in $AdS_4 \times \mathbb{CP}^3$

$$\lambda = \frac{N}{k}$$

ABJM theory [Aharony, Bergman, Jafferis, Maldacena, 2008]

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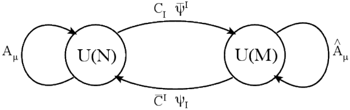
- Gauge group $U(N)_k \times U(M)_{-k}$, but here $M = N$.



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INTEGRABILITY $(N \to \infty)$

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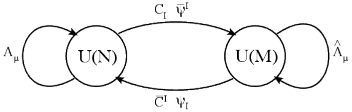

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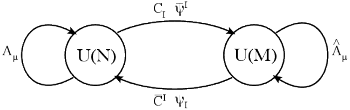

- Integrability results depend on a coupling h, in principle non-trivially related to λ.

$\mathcal{N}=4$ SYM $\quad\to\quad \lambda=(4\pi h)^2$ COMPUTED

ABJM $\quad\to\quad \lambda=\dfrac{\sinh^2 2\pi h}{2\pi}\,{}_3F_2\left(\dfrac{1}{2},\dfrac{1}{2},\dfrac{1}{2};1,\dfrac{3}{2};-\sinh^2 2\pi h\right)$ CONJECTURE

- There are checks of the conjecture [Gromov, Sizov, 2014] at weak [Leoni, Mauri, Minahan, Ohlsson Sax, Santambrogio, Sieg, 2010] and strong [LB, M.S.Bianchi, Bres, Forini, Vescovi,2014] coupling

Wilson loops in ABJM theory

- The landscape of conformal Wilson loops in ABJM is richer than $\mathcal{N}=4$ SYM.

Scalar coupling [Drukker, Plefka, Young; Chen, Wu; Rey, Suyama, Yamaguchi, 2008]

$$W = \mathrm{Tr}\mathcal{P}e^{-i\oint A\cdot x+\oint |dx|\mathcal{M}_J{}^I C_I \bar{C}^J} \qquad I,J=1,...,4$$

With the maximally supersymmetric coupling $\mathcal{M}_I{}^J = \mathrm{diag}(-1,-1,1,1)$, W is $\frac{1}{6}$BPS.

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- The $\frac{1}{2}$BPS Wilson loop is more complicated [Drukker, Trancanelli, 2010]

$$\mathcal{W} = \frac{1}{2N}\mathrm{Tr}\left[P\exp\left(-i\int dt\mathcal{L}(t)\right)\right]$$

- In this case $\mathcal{L}(t)$ is a $U(N|N)$ supermatrix

$$\mathcal{L} = \begin{pmatrix} A_\mu \dot{x}^\mu - i\mathcal{M}_J{}^I C_I \bar{C}^J & -i\eta_I\bar{\psi}^I \\ -i\psi_I\bar{\eta}^I & \hat{A}_\mu\dot{x}^\mu - i\mathcal{M}_J{}^I \bar{C}^J C_I \end{pmatrix}$$

- There is a one-parameter family of $\frac{1}{6}$BPS intermediate cases $\rightarrow$ defect conformal manifold [Cooke, Drukker, Trancanelli, 2015; Correa, Giraldo-Rivera, Silva, 2019]

Displacement multiplet [LB, Griguolo, Preti, Seminara 2017; LB, Bliard, Forini, Griguolo, Seminara, 2020]

	ABJM	$\frac{1}{2}$ BPS WL
Supergroup	$OSP(6\|4)$	$SU(1,1\|3)$
Bosonic subgroup	$SO(1,4) \times SU(4)_R$	$SO(1,2) \times U(1) \times SU(3)_R$

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Breaking	$SU(4)_R \to SU(3)_R$
Defect operators	$\mathbb{O}^a(t), \bar{\mathbb{O}}_a(t) \quad {}_{a=1,2,3}$

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- Multiplets

$$
SU(1,1|3)\ \text{chiral mult.} \left\{ \begin{array}{ccc} \mathbb{F}(t) & & \bar{\mathbb{F}}(t) \\ \downarrow \mathcal{Q}^a & & \downarrow \bar{\mathcal{Q}}_a \\ \mathbb{O}^a(t) & & \bar{\mathbb{O}}_a(t) \\ \downarrow & & \downarrow \\ \mathbb{\Lambda}_a(t) & & \bar{\mathbb{\Lambda}}^a(t) \\ \downarrow & & \downarrow \\ \mathbb{D}(t) & & \bar{\mathbb{D}}(t) \end{array} \right\} SU(1,1|3)\ \text{antichiral mult.}
$$

String in $AdS_4 \times \mathbb{C}P^3$

- The $\frac{1}{2}$ BPS Wilson line in ABJM is dual to the fundamental string solution ending on the defect at the boundary.
- Introduce static gauge [Drukker, Gross, Tseytlin, 2000]

$$ds^2_{AdS_4} = \frac{dx^\mu dx_\mu + dz^2}{z^2} \qquad x^0 = \tau \qquad z = \sigma$$

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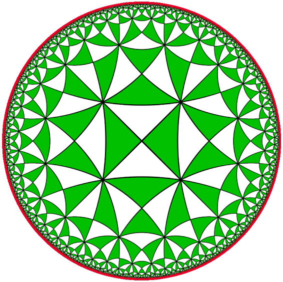
- Induced AdS_2 worldsheet metric

$$ds^2 = \frac{d\tau^2 + d\sigma^2}{\sigma^2}$$

- Fluctuation modes of the worldsheet are naturally associated to contour deformations.
- AdS dual of the displacement multiplet

GRADING	OPERATOR	Δ	m^2
Fermion	$\mathbb{F}(t)$	$\frac{1}{2}$	0
Boson	$\mathbb{O}^a(t)$	1	0
Fermion	$\mathbb{\Lambda}_a(t)$	$\frac{3}{2}$	1
Boson	$\mathbb{D}(t)$	2	2



Two-point function

- Chiral superfield $(y = t + \theta^a \bar{\theta}_a)$

$$\Phi(y,\theta) = \mathbb{F}(y) + \theta_a \mathbb{O}^a(y) - \frac{1}{2}\theta_a\theta_b\,\epsilon^{abc}\,\mathbb{\Lambda}_c(y) + \frac{1}{3}\theta_a\theta_b\theta_c\,\epsilon^{abc}\,\mathbb{D}(y)$$

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$$\langle \Phi(y_1,\theta_1)\bar{\Phi}(y_2,\bar{\theta}_2)\rangle = \frac{C_\Phi}{\langle 1\bar{2}\rangle} \qquad \langle 1\bar{2}\rangle = y_1 - y_2 - 2\theta_{a1}\bar{\theta}^a{}_2$$

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- C_Φ, a.k.a. the Bremsstrahlung function is known exactly in a closed form [LB, Preti, Vescovi, 2018] after a long effort [Lewkowycz, Maldacena, 2013; M.S.Bianchi, Griguolo, Leoni, Penati, Seminara, 2014; Aguilera-Damia, Correa, Silva, 2014; LB, Griguolo, Preti, Seminara, 2017; M.S.Bianchi, Griguolo, Mauri, Penati, Seminara, 2018]

$$\frac{1}{2}C_\Phi = B_{1/2} = \frac{\kappa}{64\pi}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};2;-\frac{\kappa^2}{16}\right) \qquad \lambda = \frac{\kappa}{8\pi}\,{}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};1,\frac{3}{2};-\frac{\kappa^2}{16}\right)$$

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- The effective coupling κ is simply related to the conjectured expression for $h(\lambda)$

$$\kappa = 4\sinh 2\pi h$$

- We expect a relatively simple result in terms of the integrability coupling h.
- Unfortunately, an integrability result is still lacking.

Four-point function [LB, Bliard, Forini, Griguolo, Seminara, 2020]

- Single superconformal invariant compatible with chirality (no nilpotent invariants)

$$\langle \Phi(y_1,\theta_1)\bar\Phi(y_2,\bar\theta_2)\Phi(y_3,\theta_3)\bar\Phi(y_4,\bar\theta_4)\rangle = \frac{C_\Phi^2}{\langle 1\bar2\rangle\langle 3\bar4\rangle} f(\mathcal{Z}) \qquad \mathcal{Z} = \frac{\langle 1\bar2\rangle\langle 3\bar4\rangle}{\langle 1\bar4\rangle\langle 3\bar2\rangle}$$

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- The superprimary correlator contains all the information

$$\langle \mathbb{F}(t_1)\bar{\mathbb{F}}(t_2)\mathbb{F}(t_3)\bar{\mathbb{F}}(t_4)\rangle = \frac{C_\Phi^2}{t_{12}t_{34}} f(z) \qquad z = \frac{t_{12}t_{34}}{t_{13}t_{24}}$$

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- We consider the perturbation

$$f(z) = f^{(0)}(z) + \epsilon\, f^{(1)}(z)$$

- The first order analytic bootstrap analysis gives infinitely many solution.
- We selected the "minimal" one according to a criterium on the asymptotic behaviour of anomalous dimensions established in [Liendo, Meneghelli, Mitev, 2018]

$$f^{(1)}(z) = -\frac{(1-z)^3}{z}\log(1-z) + z(3-z)\log(-z) + z - 1$$

- The result can be confirmed by the computation of Witten diagrams in AdS_2

$$S_B \equiv T \int d^2\sigma \sqrt{g}\; L_B \,, \qquad L_B = \; L_2 + L_{4X} + L_{2X,2w} + L_{4w} + \ldots$$

$$L_2 = g^{\alpha\beta}\partial_\alpha X \partial_\beta \bar{X} + 2|X|^2 + g^{\alpha\beta}\partial_\alpha w^a \partial_\beta \bar{w}_a$$

$$\begin{aligned} L_{4w} = \; & -\frac{1}{2}(w^a \bar{w}_a)(g^{\alpha\beta}\partial_\alpha w^b \partial_\beta \bar{w}_b) - \frac{1}{2}(w^a \bar{w}_b)(g^{\alpha\beta}\partial_\alpha w^b \partial_\beta \bar{w}_a) + \frac{1}{2}\,(g^{\alpha\beta}\partial_\alpha w^a \partial_\beta \bar{w}_a)^2 \\ & - \frac{1}{2}(g^{\alpha\beta}\partial_\alpha w^a \partial_\beta \bar{w}_b)\,(g^{\gamma\delta}\partial_\gamma \bar{w}_a \partial_\delta w^b) - \frac{1}{2}(g^{\alpha\beta}\partial_\alpha w^a \partial_\beta w^b)\,(g^{\gamma\delta}\partial_\gamma \bar{w}_a \partial_\delta \bar{w}_b) \end{aligned}$$

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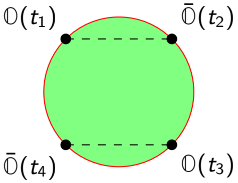
- Leading order



Witten diagrams [LB, Bliard, Forini, Griguolo, Seminara, 2020]

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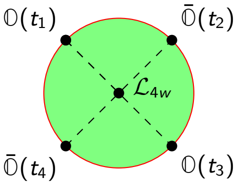
$$L_{4w} = \ -\frac{1}{2}(w^a\bar{w}_a)(g^{\alpha\beta}\partial_\alpha w^b \partial_\beta \bar{w}_b) - \frac{1}{2}(w^a\bar{w}_b)(g^{\alpha\beta}\partial_\alpha w^b \partial_\beta \bar{w}_a) + \frac{1}{2}\,(g^{\alpha\beta}\partial_\alpha w^a \partial_\beta \bar{w}_a)^2$$
$$- \frac{1}{2}(g^{\alpha\beta}\partial_\alpha w^a \partial_\beta \bar{w}_b)\,(g^{\gamma\delta}\partial_\gamma \bar{w}_a \partial_\delta w^b) - \frac{1}{2}(g^{\alpha\beta}\partial_\alpha w^a \partial_\beta w^b)\,(g^{\gamma\delta}\partial_\gamma \bar{w}_a \partial_\delta \bar{w}_b)$$

$$w^a \xrightarrow[\text{bdy}]{} \mathbb{O}^a \qquad\qquad X \xrightarrow[\text{bdy}]{} \mathbb{D}$$

- Next-to-leading order: perfect agreement with the bootstrap for $\epsilon = \frac{1}{4\pi T}$



A detour: Weyl anomaly

- For homogeneous 4d CFT in curved space

$$\langle T_\mu{}^\mu \rangle = a\ E_4 + c\ I_4$$

$$E_4 = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2 \qquad I_4 = C^{\mu\nu\rho\sigma} C_{\mu\nu\rho\sigma}$$

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For $\mathcal{N} \geq 3$ supersymmetry

$$a = c$$

- For a 2d defect in a 4d CFT [Graham, Witten, 1999; Schwimmer, Theisen, 2008]

$$\langle T_\mu{}^\mu \rangle_\Sigma = -\frac{\delta^2(x_\perp)}{2\pi}\left(b R_\Sigma + d_1 \tilde{K}^i_{ab}\tilde{K}^{ab}_i - d_2 \gamma^{ab}\gamma^{cd} W_{acbd} \right) ,$$

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- The type B coefficients d_1 and d_2 are related to defect correlators [Lewkowycz, Perlmutter, 2014;LB, Meineri, Myers, Smolkin, 2015]

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With our relation [LB, Lemos, 2019]

$$C_D = 48h \Rightarrow d_1 = d_2$$

$\mathcal{N} = (2,2)$ surfaces in 4d $\mathcal{N} = 2$

	$\mathcal{N} = 2$	$\mathcal{N} = (2,2)$ surface
Supergroup	$SU(2,2\|2)$	$SU(1,1\|1)_L \times SU(1,1\|1)_R \times U(1)_\mathcal{C}$
Bosonic subgroup	$SO(1,5) \times SU(2)_R \times U(1)_r$	$SO(1,3) \times U(1)_\perp \times U(1)_r \times U(1)_R$

- Displacement multiplets [Gaiotto, Gukov, Seiberg, 2013]

(antichiral,chiral)

$$\begin{tikzcd} & \mathbb{O}_\uparrow \arrow[dl, "\bar{G}^-_{-1/2}"'] \arrow[dr, "G^+_{-1/2}"] & \\ \Lambda^-_\uparrow \arrow[dr] & & \Lambda^+_\uparrow \arrow[dl] \\ & \mathbb{D}_\uparrow & \end{tikzcd}$$

(chiral,antichiral)

$$\begin{tikzcd} & \mathbb{O}_\downarrow \arrow[dl, "G^-_{-1/2}"'] \arrow[dr, "\bar{G}^+_{-1/2}"] & \\ \Lambda^-_\downarrow \arrow[dr] & & \Lambda^+_\downarrow \arrow[dl] \\ & \mathbb{D}_\downarrow & \end{tikzcd}$$