α'-corrections to integrable deformations

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based on 2003.05867 in collaboration with

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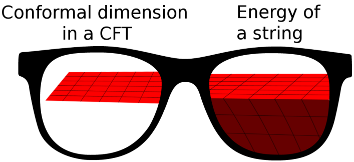
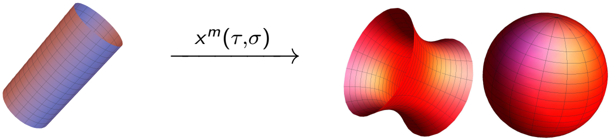
Introduction

[Maldacena 97] **AdS/CFT** or **holographic duality**

$\mathcal{N} = 4$ SYM



$AdS_5 \times S^5$



Integrability (classical) of the string

2d σ-model equations of motion $\iff$ flatness of **Lax connection**

$$\partial_\tau L_\sigma - \partial_\sigma L_\tau + [L_\tau, L_\sigma] = 0$$

Exact spectrum in large-color limit of 't Hooft

See review [Beisert et al. 09]

Low energy limit of the string $\longrightarrow$ **supergravity**

Metric G_{mn}, **Kalb-Ramond** B_{mn}, **dilaton** Φ (here bosonic string)

$$0 = R_{mn} - \tfrac{1}{4} H^2_{mn} + 2\nabla_m \nabla_n \Phi + \alpha'(\tfrac{1}{2} R_{mpqr} R_n{}^{pqr} + \ldots) + \mathcal{O}(\alpha'^2),$$
$$0 = \nabla^p H_{mnp} + \ldots + \alpha'(\ldots) + \mathcal{O}(\alpha'^2), \qquad H = dB$$
$$0 = \Box\Phi + \ldots + \alpha'(\ldots) + \mathcal{O}(\alpha'^2)$$

Higher-derivative corrections in the **inverse tension** α'

Low energy limit of the string $\longrightarrow$ **supergravity**

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$$
\begin{aligned}
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0 &= \nabla^p H_{mnp} + \ldots + \alpha'(\ldots) + \mathcal{O}(\alpha'^2), \qquad\qquad H = dB\\
0 &= \Box\Phi + \ldots + \alpha'(\ldots) + \mathcal{O}(\alpha'^2)
\end{aligned}
$$

Higher-derivative corrections in the **inverse tension** α'

Low energy limit of the string $\longrightarrow$ **supergravity**

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\begin{aligned}
0 &= R_{mn} - \tfrac{1}{4}H^2_{mn} + 2\nabla_m\nabla_n\Phi + \alpha'\left(\tfrac{1}{2}R_{mpqr}R_n{}^{pqr} + \dots\right) + \mathcal{O}(\alpha'^2),\\
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\end{aligned}
$$

Higher-derivative corrections in the **inverse tension** α'

The Yang-Baxter deformation

[Klimčik] . . . [many more]

Start with solution G, B, Φ with **isometries**
k_i^m Killing vectors $[k_i, k_j] = -f_{ij}{}^k k_k$

$$\boxed{\widetilde{G} - \widetilde{B} = (G - B)[1 + \Theta(G - B)]^{-1}, \qquad \Theta^{mn} = \eta\; k_i^m k_j^n R^{ij}}$$

$$\boxed{e^{-2\widetilde{\Phi}}\sqrt{\det \widetilde{G}} = e^{-2\Phi}\sqrt{\det G}}$$

- **Antisymmetry** $R^{ij} = -R^{ji}$
- **Classical Yang-Baxter equation** $R^{l[i} R^{|m|j} f_{lm}{}^{k]} = 0$
- **Unimodularity condition** $R^{ij} f_{ij}{}^k = 0$ (sufficient)

[Klimčik] ... [many more]

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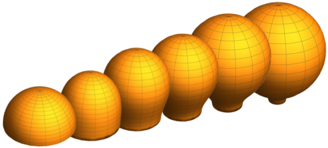
E.g. def. of $S^3 \times S^1$ ($U(2)$ PCM)



Double Field Theory

[Siegel] ... [many more]

$X^M = (\tilde{x}_m, x^m)$ where $m = 1, \ldots, D$

Generalized metric

$$\mathcal{H}^{MN} = \begin{pmatrix} (G - BG^{-1}B)_{mn} & (BG^{-1})_m{}^n \\ -(G^{-1}B)^m{}_n & G^{mn} \end{pmatrix}$$

Capital indices are raised and lowered with the $O(D, D)$ **metric**

$$\eta_{MN} = \begin{pmatrix} \mathbf{0} & \delta^m{}_n \\ \delta_m{}^n & \mathbf{0} \end{pmatrix}$$

$$e^{-2\hat{d}} = e^{-2\Phi}\sqrt{\det G}$$

T-duality (Buscher's rules) as simple $O(D, D)$ transformation

Yang-Baxter deformation in doubled language

$$\widetilde{\mathcal{H}} = \mathcal{O}^T \mathcal{H} \mathcal{O}\,, \qquad \mathcal{O}_M{}^N = \delta_M{}^N + \Theta_M{}^N, \quad \Theta_M{}^N = \begin{pmatrix} \mathbf{0} & \Theta^{mn} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}$$

$$\mathcal{H}^{MN} = E_A{}^M \mathcal{H}^{AB} E_B{}^N, \qquad \mathcal{H}^{AB} = \begin{pmatrix} \bar{\eta}_{ab} & \mathbf{0} \\ \mathbf{0} & \bar{\eta}^{ab} \end{pmatrix}$$

with $\bar{\eta}_{ab}$ Minkowski metric

$$\widetilde{E}_A{}^M = E_A{}^N \mathcal{O}_N{}^M$$

$$\eta^{MN} = E_A{}^M \eta^{AB} E_B{}^N, \qquad \eta^{AB} = \begin{pmatrix} \bar{\eta}_{ab} & \mathbf{0} \\ \mathbf{0} & -\bar{\eta}^{ab} \end{pmatrix}$$

DFT equations of motion written in terms of **generalised fluxes** $\mathcal{F}$ and their **derivatives** $\partial_A \mathcal{F}$ [Geissbühler 11]

Weitzenböck **connection** $\Omega_{ABC} = E_A{}^M \partial_M E_B{}^N E_{CN}$

Fluxes $\mathcal{F}_{ABC} = 3\Omega_{[ABC]}\,, \qquad \mathcal{F}_A = \Omega^B{}_{BA} + 2E_A{}^M \partial_M \hat{d}$

DFT equations of motion are equivalent to **supergravity** equations when reducing to D dimension and imposing only x^m-dependence of fields

[RB, Wulff 20]

$$\widetilde{E} = E\mathcal{O}, \qquad \widetilde{\Omega}_{ABC} = \widetilde{E}_A{}^M \partial_M \widetilde{E}_B{}^N \widetilde{E}_{CN}$$

$$\widetilde{\mathcal{F}}_{ABC} = 3\widetilde{\Omega}_{[ABC]} = \mathcal{F}_{ABC} + 3E_{[A}{}^M E_B{}^N E_{C]}{}^P \; \Theta_M{}^Q \partial_Q \Theta_{NP}$$

$$\Theta_{[M}{}^Q \partial_Q \Theta_{NP]} = 0 \iff \textbf{CYBE for } R$$

$$\widetilde{\mathcal{F}}_A = \widetilde{\Omega}^B{}_{BA} + 2\widetilde{E}_A{}^M \partial_M \hat{d} = \mathcal{F}_A + E_A{}^M \Delta\mathcal{F}_M$$

$$\Delta\mathcal{F}_M = \begin{pmatrix} -2\nabla_n \Theta^{mn} \\ 0 \end{pmatrix}$$

$$\nabla_n \Theta^{mn} = 0 \iff \textbf{unimodularity condition for } R$$

α'-corrections

Double Lorentz $SO(1,D-1)\times SO(D-1,1)$

$$\Lambda_A{}^B=\delta_A{}^B+\lambda_A{}^B+\mathcal{O}(\lambda^2),\qquad \lambda_A{}^B=\begin{pmatrix}\lambda^{[+]a}{}_b & \mathbf{0}\\ \mathbf{0} & \lambda_a^{[-]b}\end{pmatrix}$$

Anomalous double Lorentz at α' order [Marqués, Núñez 15]

$$\begin{aligned}\hat\delta G^{(\mathrm{DFT})}_{mn} &= -\frac{1}{2}\partial_{(m}\lambda^{[+]ab}\omega^{(+)}_{n)ab}-\frac{1}{2}\partial_{(m}\lambda^{[-]ab}\omega^{(-)}_{n)cd}\,,\\ \hat\delta B^{(\mathrm{DFT})}_{mn} &= \frac{1}{2}\partial_{[m}\lambda^{[+]ab}\omega^{(+)}_{n]ab}-\frac{1}{2}\partial_{[m}\lambda^{[-]ab}\omega^{(-)}_{n]ab}\end{aligned}$$

$\omega^{(\pm)}=\omega\pm\frac{1}{2}H$

[RB, Wulff 20]

Finite first-order α'-corrections of YB deformations

$$\Delta(\widetilde{G}-\widetilde{B})^{(\text{DFT})}_{mn}=\tfrac{1}{2}\widetilde{\omega}^{(-)}_{mab}\left(\partial_n\Lambda\Lambda^{-1}\right)^{ab}+\tfrac{1}{4}\partial_m\Lambda^{ab}\partial_n\Lambda_{ab}-B^{\text{WZW}}_{mn}$$

$$\Lambda_a{}^b=\left(O^{[+]}O^{[-]-1}\right)_a{}^b,\qquad O^{[\pm]}=\mathbf{1}\pm(G\mp B)\Theta$$

$$dB^{\text{WZW}}=-\tfrac{1}{12}\,\text{Tr}(\Lambda^T d\Lambda)^3$$

Correction of **dilaton** from invariance of $e^{-2\hat{d}}=e^{-2\Phi}\sqrt{\det G}$

Tests in covariant schemes

Conclusions

- Deformations of 2d σ-models preserving classical **integrability**
- Applications to **AdS/CFT** correspondence
- Formulation in **Double Field Theory** and α'-**corrections**
- Explore other **solution-generating techniques** in supergravity: *non-abelian T-duality, Poisson-Lie T-duality,* η/λ *deformations*
- Investigate **higher** α'-**corrections**
- **Develop integrability methods** for deformed models
- ...

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Thank you!

Back-up slides

$$E_A{}^M = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{[+]an}(G-B)_{nm} & e^{[+]am} \\ -e_a^{[-]n}(G+B)_{nm} & e_a^{[-]m} \end{pmatrix}$$

$e_m^{[\pm]a} e_n^{[\pm]b} \bar{\eta}_{ab} = G_{mn}$

Diagonal gauge $e^{[+]} = e^{[-]}$ when going to sugra in D dimensions

$$\widetilde{e}_a^{[\pm]m} = e_a^{[\pm]n} O_n{}^{[\pm]m}, \qquad O^{[\pm]} = \mathbf{1} \pm (G \mp B)\Theta$$

Need **compensating Lorentz transformation**, e.g.

$$\Lambda_A{}^B = \begin{pmatrix} \delta^a{}_b & \mathbf{0} \\ \mathbf{0} & \Lambda_a{}^b \end{pmatrix}, \qquad \Lambda_a{}^b = \left(O^{[+]} O^{[-]-1}\right)_a{}^b$$

[RB, Wulff 20]

Finite form of anomalous Lorentz transformation

- $\hat{\delta}\widetilde{G}^{(\text{DFT})}_{mn} = -\frac{1}{2}\partial_{(m}\lambda^{ab}\widetilde{\omega}^{(-)}_{n)ab} + \mathcal{O}(\lambda^2)$

$$\implies \widetilde{G}^{(\text{DFT})}_{mn} + \frac{\alpha'}{4}\widetilde{\omega}^{(-)cd}_{m}\widetilde{\omega}^{(-)}_{ncd} \text{ is invariant}$$

- $\hat{\delta}\widetilde{B}^{(\text{DFT})}_{mn} = \hat{\delta}_1\widetilde{B}^{(\text{DFT})}_{mn} + \hat{\delta}_2\widetilde{B}^{(\text{DFT})}_{mn}$

$$\hat{\delta}_1\widetilde{B}^{(\text{DFT})}_{mn} = -\frac{1}{2}\partial_{[m}\lambda^{ab}\widetilde{\omega}_{n]ab} + \mathcal{O}(\lambda^2)$$
$$\hat{\delta}_2\widetilde{B}^{(\text{DFT})}_{mn} = \frac{1}{2}\partial_{[m}\lambda^{ab}\frac{1}{2}\widetilde{H}_{n]ab} + \mathcal{O}(\lambda^2)$$

$$\implies \widetilde{B}^{(\text{DFT})}_{mn} - \frac{\alpha'}{4}\widetilde{\omega}^{cd}_{[m}\widetilde{H}_{n]cd} \text{ is invariant under } \hat{\delta}_2$$

$$\implies \hat{\delta}_1\widetilde{H}^{(\text{DFT})} = -\frac{1}{4}\hat{\delta}CS(\widetilde{\omega}), \quad CS(\widetilde{\omega}) = \text{Tr}\left(\widetilde{\omega}d\widetilde{\omega} + \frac{2}{3}\widetilde{\omega}\widetilde{\omega}\widetilde{\omega}\right)$$