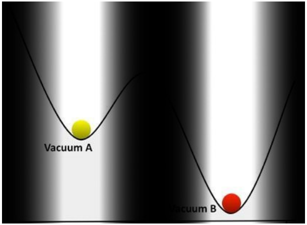
On Vacuum Stability without Supersymmetry

Brane dynamics, bubbles and holography

Ivano Basile | SNS, Pisa | *Cortona Young, May 2020*

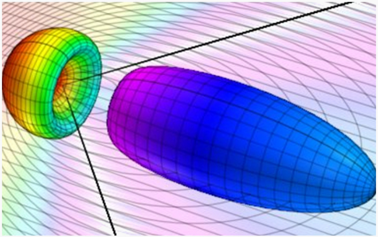
based on:

- hep-th/1811.11448
  with J. Mourad and A. Sagnotti
- hep-th/1908.04352
  with R. Antonelli
- hep-th/1806.02289
  with R. Antonelli and A. Bombini

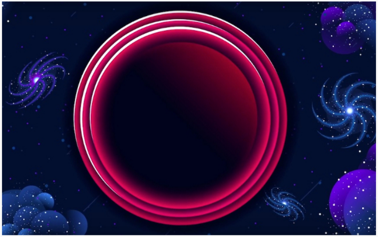


Swampland

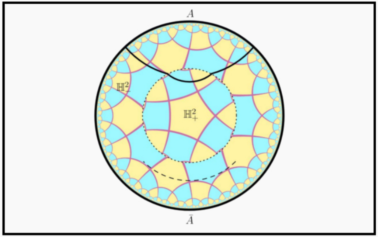
Instability



Vacuum Bubbles



Holography



[credits to Kurzgesagt ↑]

Non-SUSY 10d strings: vantage point?

1. **Heterotic** $SO(16) \times SO(16)$: **NO SUSY** (Alvarez–Gaume, Ginsparg, Moore, Vafa, 1986)
2. $U(32)$: Type 0'B **closed** **+** **open**: **NO SUSY** (Sagnotti, 1995)
3. $USp(32)$: **closed** **+** **open** (Sugimoto, 1999)
   "Brane SUSY Breaking" (Antoniadis, Dudas, Sagnotti, 1999)
   10D couplings (Dudas, Mourad; Pradisi, Riccioni, 2000)

Features

- no tachyons
- branes + orientifolds → **residual tension**

$$V(\phi) = V_0 \, e^{\gamma \, \phi} \qquad \longrightarrow \qquad \textbf{NO flat vacuum!}$$

- $\exists$ AdS $\times S$ **flux backgrounds** (Mourad, Sagnotti, 2016)

are they stable?

Low-energy description

$$S_{\text{eff}} = \int d^{10}x\sqrt{-g}\left(\mathcal{R} - \frac{1}{2}(\partial\phi)^2 - V(\phi) - \frac{e^{\alpha\phi}}{12}\, H_3^2 + \dots\right)$$

AdS flux compactifications:

- Constant dilaton
- $AdS_3 \times S^7$ (BSB $\simeq$ 0'B), $AdS_7 \times S^3$ (heterotic)
- electric vs magnetic flux N of $H_3 = dB_2$

$$\boxed{N \gg 1 \qquad \longrightarrow \qquad e^{\phi}\,,\, (\alpha'\mathcal{R}) \ll 1}$$

In orientifold models: $\mathrm{AdS}_3 \times S^7$

Parameters: $V = T\, e^{\frac{3}{2}\phi}$, coupling $\alpha = 1$ to R-R 3-form

from disk amplitude: $T = 2k_{10}^2 \times \begin{cases} 64\; T_{\mathrm{D9}} & (\mathrm{BSB}) \\ 32\; T_{\mathrm{D9}} & (0'\mathrm{B}) \end{cases}$

- electric flux

$$N = \int_{S^7} \star e^{\phi}\, H_3$$

- (super)gravity regime

$$L_3\,,\, R_7 \propto N^{3/16} \qquad e^{\phi} \propto N^{-1/4}$$

- ratio of radii

$$\frac{L_3^2}{R_7^2} = \frac{1}{6}$$

In the heteroticr model: $\mathrm{AdS}_7 \times S^3$

Parameters: $V = \Lambda\, e^{\frac{5}{2}\phi}$, coupling $\alpha = -1$ to NS-NS 3-form

from 1-loop torus amplitude: $\Lambda = (\text{modular integral}) = \frac{\mathcal{O}(1)}{\alpha'}$

- magnetic flux

$$N = \int_{S^3} H_3$$

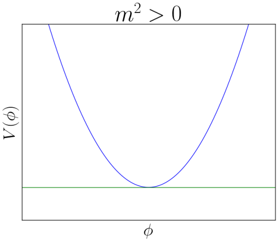
- (super)gravity regime

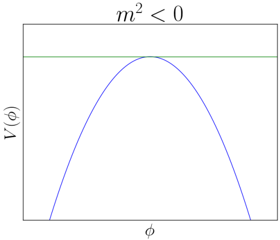
$$L_7\,,\, R_3 \propto N^{5/8} \qquad e^{\phi} \propto N^{-1/2}$$

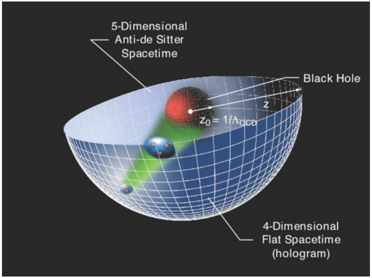
- ratio of radii

$$\frac{L_7^2}{R_3^2} = 12$$

Perturbative instabilities: Minkowski vs AdS







$m^2 < 0 \;\rightarrow\;$ modes can **grow**

AdS: *BF bound* (Breitenlohner, Freedman, 1982)

$$m^2_{\text{scalar}} \geq -\frac{(d-1)^2}{4L^2_{AdS}}$$

- **some tachyons allowed!**
- **extends** to general fluctuations

Perturbative instabilities: results (IB, Mourad, Sagnotti, 2018)

AdS vacua: unstable scalar KK modes

- BSB & 0'B $\longrightarrow$ $\ell = 2\,,\,3\,,\,4$ $\quad \mathrm{AdS}_3 \times S^7$
- Heterotic $\longrightarrow$ $\ell = 1$ $\quad \mathrm{AdS}_7 \times S^3$

Dudas-Mourad vacua (Dudas, Mourad, 2000): pert. stable...

- 9d static: ...but large corrections
- 10d cosmology: ...except isotropy?

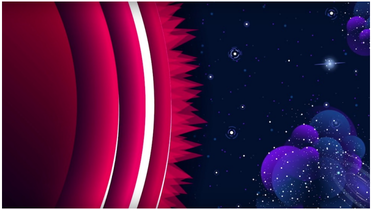
$$\delta g_{ij}(\mathbf{k} = 0) \sim A_{ij} + B_{ij} \, \log \eta$$

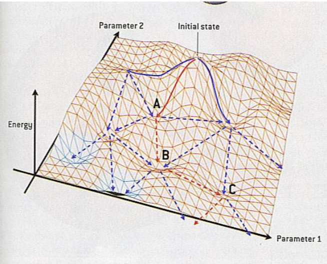
Non-perturbative instabilities

expect instantons...

$$\Gamma_{\text{decay}} \sim (\det) \times e^{-\overbrace{(S_{\text{inst.}} - S_0)}^{B}}$$

(Coleman, Callan, 1977), (Coleman, De Luccia, 1980)





BUT: no **global** knowledge
no SUSY! → any criteria?

Non-perturbative instabilities: brane picture (Antonelli, IB, 2019)

AdS vacua $\rightarrow$ *flux tunneling* (Brown, Teitelboim, 1987-1988), (Blanco-Pillado, Schwartz-Perlov, Vilenkin, 2009)

$$\mathcal{E}_{\text{vac}} \propto -N^{-3} \quad \text{or} \quad -N^{-2}$$

$$N \longrightarrow N - \delta N \quad : \quad \text{out of EFT}$$

- Instantons $\leftrightarrow$ branes (D1 or NS5)? right charge & dim.
- AdS $\rightarrow$ **near-horizon** of brane stack...

$\longrightarrow$ brane-antibrane nucleation

$$S^E_{\text{brane}} = \left[\tau_p \,\text{Area} - \frac{N\mu_p}{e^{\alpha\phi}\,R^q}\,\text{Vol}\right]_{\text{extremum}} = B_{\text{CdL}}$$

Consistency: the right branes

$$S^E_{\text{brane}} = \tau_p \, \Omega_{p+1} \, L^{p+1} \left[\frac{1}{(\beta^2-1)^{\frac{p+1}{2}}} - \frac{p+1}{2} \beta \int_0^{\frac{1}{\beta^2-1}} \frac{u^{\frac{p}{2}}}{\sqrt{1+u}} \, du \right]$$

Consistency:

- *existence:* nucl. parameter $\beta \equiv v_0 \left(\frac{\mu_p}{\tau_p} \right) g_s^{-\frac{\alpha}{2}} > 1$
- *semi-classical:* $\beta = \mathcal{O}(N^0) \longrightarrow \tau_p = T_p \, g_s^{-\frac{\alpha}{2}}$

$\longrightarrow$ **relation for fundamental (and exotic) branes**! (Bergshoeff, Riccioni et al.)

$$\tau_p^{\text{string}} = \frac{T_p}{g_s^{\sigma}} \,, \qquad \boxed{\sigma = 1 + \frac{1}{2} \alpha^{\text{string}}_{\text{electric}}}$$

After tunneling: Lorentzian evolution

probe $p/\bar{p}$-brane in (Poincaré) AdS throat at pos. Z:

$$V_{\text{probe}} = \tau_p \left(\frac{L}{Z}\right)^{p+1} \left(1 \pm v_0 \frac{\mu_p}{T_p}\right)$$

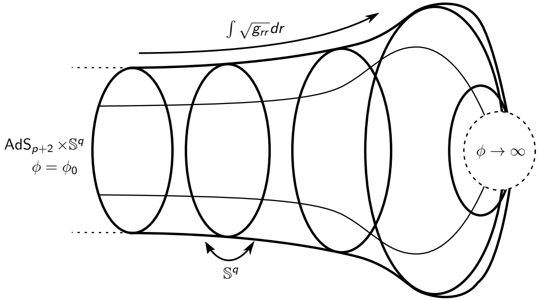
for our string models: $v_0 > 1$!

$$\textbf{WGC}: \nearrow \left(\frac{\text{charge}}{\text{tension}}\right)_{\text{eff}} \nearrow$$

$\longrightarrow$ these non-SUSY branes **feel the right forces**

Back-reaction: pinch-off at finite distance

$SO(1,p) \times SO(q)$ geometry



geodesic length: $\int \sqrt{g_{rr}}\, dr < \infty$

$p = 8$: recover 9d Dudas-Mourad

2 asymptotic free parameters: extremal tuning?

(Top-down) holography?

Dual "CFT": (IR of) world-volume gauge theory?

$$c_{\text{D1}} \propto N^{3/2}$$

Toward small N: bubble RG?

Recent developments: **"de Sitter on a brane"?**
(Banerjee, Danielsson, Dibitetto, Giri, Schillo, 2018)

$$\textit{e.g.}\ \ N \text{ NS5-branes in } SO(16) \times SO(16) \longrightarrow \boxed{\Lambda_{6d} \propto \frac{1}{N^2}}$$

AdS vacua:

- weak coupling
- discrete ($N \to g_s$, $\mathcal{R}$)
- **non-SUSY**

Flux tunneling:

- vacuum bubbles
- branes
- toward UV

Brane picture

- AdS $\longleftrightarrow$ IR world-volume theory?
- tunneling $\longleftrightarrow$ **renormalization group flows?**
- **de Sitter on a brane?**

Take-home message

SUSY breaking

↓

spontaneous dynamics

Backup slides

Perturbations and mixings

Linearized analysis: AdS tensors + angular momenta ℓ (IB, Mourad, Sagnotti, 2018)

- **Tensors:** no mixing $\longrightarrow$ stable ✓
- **Vectors:** mixing, still stable ✓

$$\delta g_{\mu i} \qquad \delta B_{\mu i}$$

- **Scalars:** Einstein eqs. $\longrightarrow$ 2 constraints!

$$\delta\phi \qquad \delta B_2 = \star_3 \, dB$$

$$\delta g_{\mu\nu} = A \, g^{(0)}_{\mu\nu}$$

$$\delta g_{ij} = C \, g^{(0)}_{ij}$$

$$\delta g_{\mu i} = \nabla_\mu \nabla_i D$$

Linearized scalar equations: orientifold case

$$L_3^2 \,\Box\, A - \left[4 + 3\,\sigma_3 + \frac{\ell_3}{3}\,(\sigma_3 - 1)\right] A + \frac{7}{2}\,\alpha\,\sigma_3\,\delta\phi - \frac{\ell_3}{2}\,(\sigma_3 - 1)\,B = 0$$

$$L_3^2 \,\Box\, \delta\phi + 2\,\alpha\,\sigma_3\,A - \left[2\,\alpha^2\,\sigma_3 + \tau_3 + \frac{\ell_3}{3}\,(\sigma_3 - 1)\right] \delta\phi + \alpha\,\frac{\ell_3}{3}\,(\sigma_3 - 1)\,B = 0$$

$$L_3^2 \,\Box\, B - 8\,\sigma_3\,A + 4\,\alpha\,\sigma_3\,\delta\phi - \frac{\ell_3}{3}\,(\sigma_3 - 1)\,B = 0$$

where:

$$\ell_3 = \ell\,(\ell + 6)\,, \qquad \sigma_3 = 1 + 3\,\frac{L_3^2}{R_7^2}\,, \qquad \tau_3 = L_3^2\,V_0''$$

Linearized scalar equations: heterotic case

$$L_7^2 \,\Box\, A - [\ell_7\,(\sigma_7 - 3) + 5\,\sigma_7 + 12]\, A + \frac{5}{2}\,\alpha\,\sigma_7\,\delta\phi - \frac{3\,\ell_7}{2}\,(\sigma_7 - 3)\, B = 0$$

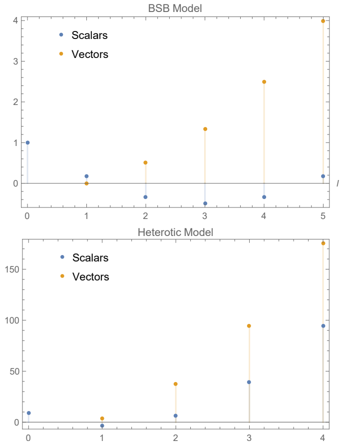
$$L_7^2 \,\Box\, \delta\phi + 6\,\alpha\,\sigma_7\, A - \left[2\,\alpha^2\,\sigma_7 + \tau_7 + \ell_7\,(\sigma_7 - 3)\right] \delta\phi + \alpha\,\ell_7\,(\sigma_7 - 3)\, B = 0$$

$$L_7^2 \,\Box\, B - 8\,\sigma_7\, A + 4\,\alpha\,\sigma_7\,\delta\phi - \ell_7\,(\sigma_7 - 3)\, B = 0$$

where:

$$\ell_7 = \ell\,(\ell + 2)\,, \qquad \sigma_7 = 3 + \frac{L_7^2}{R_3^2}\,, \qquad \tau_7 = L_7^2\, V_0''$$

Results: violations of BF bounds



BSB Model $(AdS_3 \times S^7)$

- Scalars: $\ell = 2, 3, 4$
- Vectors: $\ell = 1$ *massless (KK)*

Heterotic Model $(AdS_7 \times S^3)$

- Scalars: $\ell = 1$
- Vectors: $\ell = 1$ *massless (KK)*

Orbifolds: *can get rid of unstable modes...*

$\rightarrow$ *vacuum bubbles?* (Horowitz, Orgera, Polchinski, 2008)

[also **cosmological** vacuum: stable, but **isotropy** breaking?] (IB, Mourad, Sagnotti, 2018)

Non-perturbative instabilities: flux tunneling

- Gravity in $D = p + 2 + q$ dims + fluxes: S^q reduction

$$ds^2 = R^{-\frac{2q}{p}}\, ds^2_{p+2} + R^2\, d\Omega_q^2$$

- Reduced action: $(p+2)$-Einstein frame

$$S_{p+2} = \frac{1}{2\kappa^2_{p+2}} \int d^{p+2}x \sqrt{-g_{p+2}} \left(\mathcal{R}_{p+2} - 2\Lambda\, R^{-\frac{2q}{p}} \right)$$

vacuum energy $\longrightarrow \mathcal{E}_{\text{vac}} \propto -R^{-\frac{2q}{p}-2}$

$\mathcal{E}_{vac}$ *depends on flux...*

...higher-dim. instantons, flux transitions (Blanco-Pillado, Schwartz-Perlov, Vilenkin, 2009)

Many branes: background geometry

$SO(1,p) \times SO(q)$ symmetry: $\phi(r)\,,\ v(r)\,,\ b(r)$

$$
\begin{aligned}
ds^2 &= e^{\frac{2}{p+1}v - \frac{2q}{p}b}\, dx_{1,p}^2 + e^{2v - \frac{2q}{p}b}\, dr^2 + e^{2b}\, R_0^2\, d\Omega_q^2\,,\\
\phi &= \phi(r)\,,\\
H_{p+2} &= c\, e^{2v - \frac{q}{p}(p+2)b}\, d^{p+1}x \wedge dr\,, \qquad c \equiv \frac{N}{e^{\alpha\phi}(R_0\, e^b)^q}
\end{aligned}
$$

(Constrained) Toda-like system: $(+,-,+)$ kinetic term w/ potential

$$
\boxed{U = -\,T\, e^{\gamma\phi + 2v - \frac{2q}{p}b} - \frac{n^2}{2R_0^{2q}}\, e^{-\alpha\phi + 2v - \frac{2q(p+1)}{p}b} + \frac{q(q-1)}{R_0^2}\, e^{2v - \frac{2(D-2)}{p}b}}
$$

Geometry: near-horizon

Recover original $\mathrm{AdS}_{p+2} \times S^q$ with $[r < 0]$

$$\phi = \phi_0 \,, \qquad e^v = \frac{L}{p+1} \left(\frac{R}{R_0}\right)^{-\frac{q}{p}} \frac{1}{-r} \,, \qquad e^b = \frac{R}{R_0}$$

Radial perturbations: $\delta\phi \,, \, \delta v \,, \, \delta b \propto (-r)^\lambda$

$$\{\lambda\}_{\mathrm{BSB}} = \left\{-1 \,, \, \frac{1 \pm \sqrt{13}}{2} \,, \, \frac{1 \pm \sqrt{5}}{2}\right\} \,, \; \{\lambda\}_{\mathrm{het}} = \left\{-1 \,, \, \pm 2\sqrt{\frac{2}{3}} \,, \, 1 \pm 2\sqrt{\frac{2}{3}}\right\}$$

$\longrightarrow$ two *extremality-breaking* deformations

[two asymptotic fine-tunings?]

Geometry: "far-horizon"

Away from branes $[r > 0]$**:** assume $U \sim U_T = -\, T\, e^{\gamma\phi + 2v - \frac{2q}{p}\, b}$

Solutions as $r \to \infty$: $\phi\,, v\,, b \propto y(r) + \text{subleading}$

$$y'' \sim \hat{T}\, e^{\Omega\, y + L\, r}$$

$$\frac{1}{2}\,\Omega\, {y'}^2 + L\, y' \sim \hat{T}\, e^{\Omega\, y + L\, r} - M$$

where $\Omega = \frac{D-2}{8}\,\gamma^2 - \frac{2(D-1)}{D-2} = \frac{D-2}{8}\left(\gamma^2 - \gamma^2_{\text{crit}}\right)$ (IB, Mourad, Sagnotti, 2018)

- Orientifolds: $\phi\,,\ v\,,\ b \propto r^2$ (due to $\Omega = 0$)
- Heterotic: $\phi\,,\ v\,,\ b \propto \log(r_0 - r)$

Holography of vacuum bubbles ("Bubbleography")

Non-SUSY brane instantons at low energy: **vacuum bubbles**

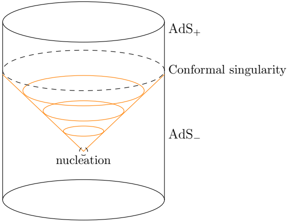
Spontaneous, irreversible process

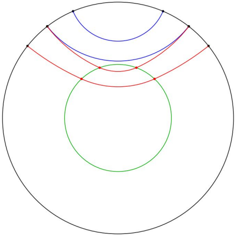
$$\mathrm{AdS}_3(L_-) \quad \longrightarrow \quad \mathrm{AdS}_3(L_+)$$

dual "central charge" $c_- < c_+$

...holographic description? (Antonelli, IB, Bombini, 2018)

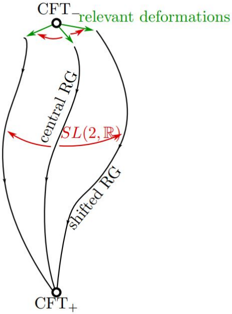
- (AdS) vacuum bubbles $\longleftrightarrow$ boundary RG flow
- Bubble nucleation $\longleftrightarrow$ (**non-local**) RG trigger
- Displaced bubble $\longleftrightarrow$ entanglement pattern of boundary





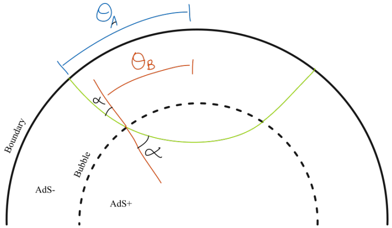
Bulk $\to$ expanding bubble and geodesics

Boundary $\to$ relevant deformations and RG



Our check: bubble entanglement entropy (in 3d)

$$ds_\pm^2 = L_\pm^2 \left(-\cosh^2\rho_\pm \, d\tau_\pm^2 + d\rho_\pm^2 + \sinh^2\rho_\pm \, d\phi_\pm^2\right)$$

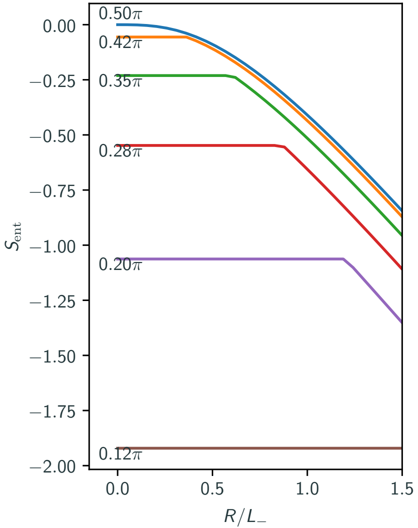


- **Thin-wall:** geodesic is hyperbolic polygonal

$$\text{length} = 2L_-\Lambda + 2L_- \log(\cosh\rho_- - \sinh\rho_- \cos(\theta_B - \theta_A)) + L_+ \cosh^{-1}(\cosh^2\rho_+ - \sinh^2\rho_+ \cos(2\theta_B)) + \mathcal{O}(\Lambda^{-1})$$

- Angle at bubble θ_B: **no-kink condition**

gluing condition: $L_- \sinh\rho_- = L_+ \sinh\rho_+$



Finite part of $S_{\text{ent}}(\theta_A, R)$ decreases with bubble radius.

candidate c-**function**!

Check: trace anomaly

(Henningson, Skenderis, 1998)

$$\langle T^{\mu}{}_{\mu} \rangle = \frac{c_{\rm ent}(R)}{12}\mathcal{R}$$

Moving the bubble:
can use integral geometry

(Antonelli, IB, Bombini, 2018)

Moving the bubble: pills of integral geometry [backup]

Crofton theorem: *space of all lines* $\mathcal{K}$ (Crofton, 1968)

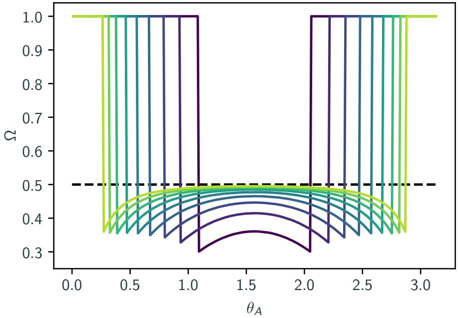
$$\text{length}(\gamma) = \frac{1}{4}\int_{\mathcal{K}} n_{\gamma,\kappa}\,\omega(\kappa) \leftarrow \text{Crofton form}$$

Asymptotically $\mathbb{H}^2$ slices

$$\omega_{\text{bubble}} = \Omega\,\omega_{\mathbb{H}^2}$$

moving bubble via isometry

$\longrightarrow$ Ω **scalar**!



Conclusions

Summary

- **Non-SUSY** AdS flux vacua: *controlled for large fluxes*
- Perturbative instabilities: violation of BF bounds
- Non-perturbative instabilities: flux tunneling
- Brane description $\longrightarrow$ dual framework *("bubble RG")*

Outlook

- Brane dynamics: small N potential, gauge theory, ...
- Explore bubble RG: structure of RG trigger?
  (closer to SUSY: $\mathbb{Z}_k$ orbifolds of $\mathcal{N} = 4$) (Horowitz, Orgera, Polchinski, 2008)
- Time dependence: analog of 10d cosmological vacuum?