STRINGS & SUPERSTRINGS

LACES 2020

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• Ref's

- POLCHINSKI I & II
- BLUMENHAGEN-LUEST-THEISEN
- GSW I & II
- DAVID TONG lecture notes (bosonic)

PLAN OF THE LECTURES

1 - POLYAKOV PATH INTEGRAL

2 - 2D CONFORMAL FIELD THEORY

3 - BOSONIC STRINGS AND D-BRANES

4 - TYPE II SUPERSTRINGS

5 - D-BRANES and RR-charges

6 - Tree-level amplitudes

7 - 1 LOOP (vacuum) amplitudes

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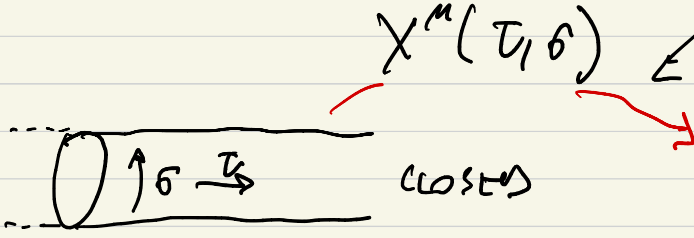
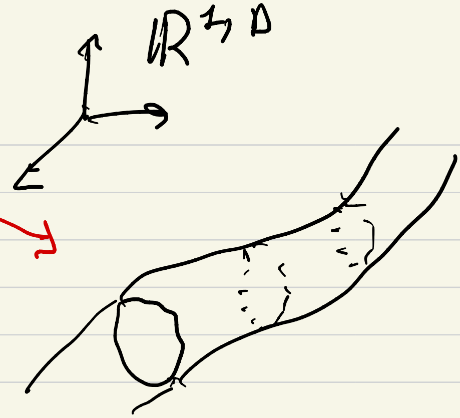
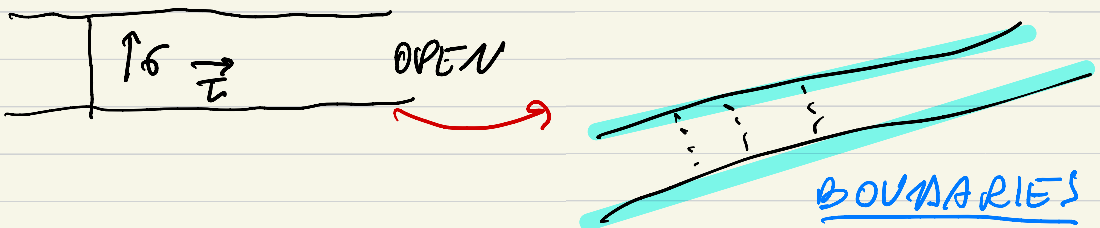
Lecture I

POLYAKOV PATH INTEGRAL

Strings







WORLD SHEET

→ NAMBU-GOTO ACTION

$$S = -\frac{1}{2\pi\alpha'}\int d^2\sigma \sqrt{-\det G} \qquad G_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

→ CLASSICAL SOLUTION : MINIMAL AREA SURFACES

* DIFFICULT TO QUANTIZE !

$$S_P = -\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma \sqrt{h}\, h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

→ TWO FIELDS $\begin{cases} X^\mu \\ h_{\alpha\beta} \end{cases}$

EOM: $\delta_h S$: $\boxed{T_{\alpha\beta} = G_{\alpha\beta} - \frac{1}{2} h_{\alpha\beta} (h \cdot G) = 0}$

$\delta_X S$: $\partial_\alpha \left(\sqrt{-h}\, h^{\alpha\beta} \partial_\beta X^\mu \right) = 0$ (BULK)

$$h^{\sigma\beta} \partial_\beta X^\mu \delta X_\mu \Big|_{\partial\Sigma} = 0 \quad \text{(BOUNDARY)}$$

Exercise: Use $T_{\alpha\beta} = 0$, to show that

$S_P = S_{N.G.}$ $\qquad \delta h = -h\, h_{\alpha\beta} \delta h^{\alpha\beta}$

$$S_P = -\frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \sqrt{h}\, h^{\alpha\beta} \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu}$$

GAUGE SYMMETRIES

① Diff

$$\delta_D \sigma^\alpha = \xi^\alpha(\sigma)$$

$$\delta_D X^\mu = \xi^\alpha \partial_\alpha X^\mu$$

$$\delta_D h_{\alpha\beta} = D_\alpha \xi_\beta + D_\beta \xi_\alpha$$

② Weyl

$$\delta_W \sigma = 0$$

$$\delta_W X^\mu = 0$$

$$\delta_W h_{\alpha\beta} = \omega h_{\alpha\beta} \quad (h \to e^\omega h)$$

in <u>D=2</u> $\sqrt{h}\, h^{\alpha\beta}$ <u>is WEYL INVARIANT</u>

GLOBAL SYMMETRIES

Poincaré

$$\delta X^\mu = \Lambda^\mu + M^\mu{}_\nu X^\nu$$

$$\delta h_{\alpha\beta} = 0$$

GAUGE FIXING

- CORRELATION FUNCTIONS OF GAUGE INVARIANT OPERATORS!

$\mapsto$ (SCATTERING AMPLITUDES IN SPACE-TIME)

$$\langle\langle V_1 \cdots V_n \rangle\rangle = \int \frac{\mathcal{D}h \mathcal{D}X}{V_{gauge}} V_1 \cdots V_n \, e^{-S[h,X]}$$

$$V_{gauge} = \int \underbrace{\mathcal{D}\xi}_{\text{DIFF}} \underbrace{\mathcal{D}\omega}_{\text{WEYL}}$$

in $D=2$ $h_{\alpha\beta}$ has no PROPAGATING D.O.F.

$D-2=0$

$$\delta h_{\alpha\beta} = \underbrace{\delta_{\text{GAUGE}} h_{\alpha\beta}}_{\substack{\text{GAUGE} \\ \text{VARIATION}}} + \underbrace{\delta_{m} h_{\alpha\beta}}_{\substack{\text{MODULI} \\ \text{VARIATION}}}$$

$$\delta_{GAUGE} h_{\alpha\beta} = D_\alpha \xi_\beta + D_\beta \xi_\alpha - \omega h_{\alpha\beta} =$$

$$= 2(P_1 \xi)_{\alpha\beta} - \Omega h_{\alpha\beta}$$

$$(P_1 \xi)_{\alpha\beta} = \frac{1}{2}\left(D_\alpha \xi_\beta + D_\beta \xi_\alpha - h_{\alpha\beta} D\cdot\xi\right)$$

$$\Omega = \omega - \frac{1}{2} D\cdot\xi$$

P_1 : VECTOR FIELDS $\longrightarrow$ SYMM. TRACELESS RANK 2 TENSORS

- δ_m ? VARIATION WHICH IS ORTHOGONAL TO δ_{GAUGE}

$\forall$ chosen $\hat{h}_{\alpha\beta}$ $\qquad h_{\alpha\beta} = \hat{h}_{\alpha\beta} + \delta h_{\alpha\beta}$

$$\langle \delta h_1, \delta h_2 \rangle \equiv \int d^2\sigma \sqrt{h}\, (\delta h_1)_{\alpha\beta} (\delta h_2)^{\alpha\beta}$$

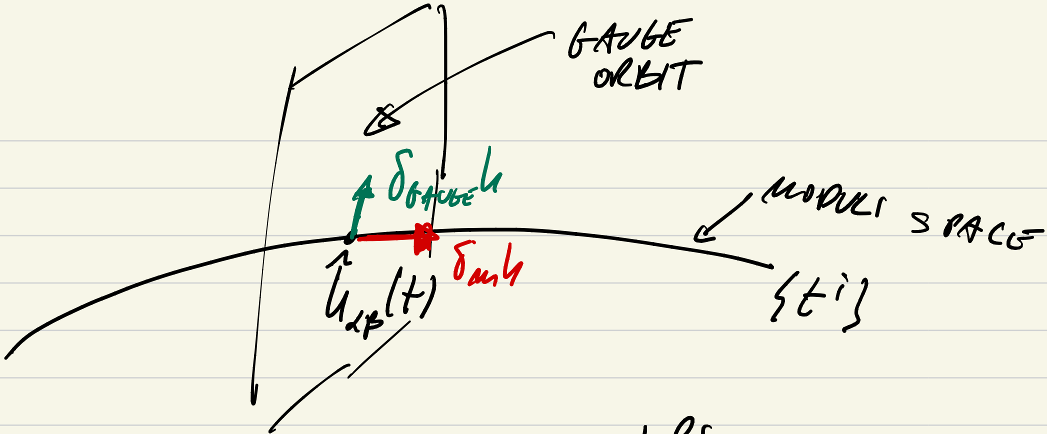
$$\delta_m h \,/\, \langle \delta_{GAUGE} h, \delta_m h \rangle = 0 = \langle 2P_1\xi - \Omega h, \delta_m h \rangle$$

P_1^T

$$\hat{h}^{\alpha\beta} (\delta_m h)_{\alpha\beta} = 0$$

$$(P_1^T \delta_m h)^\alpha = 0 \equiv D^\beta \delta_m h_{\beta\alpha} = 0$$

$\to$ FINITE DIMENSIONAL SPACE



$$\mathcal{D}h_{\alpha\beta} = \mathcal{D}h^{GAUGE}\,\mathcal{D}h^{moduli}$$

$$\mathcal{D}h^{GAUGE} = \left|\frac{\mathcal{D}\delta_{GAUGE} h}{\mathcal{D}(\xi,\omega)}\right| \mathcal{D}\xi^\alpha \mathcal{D}\omega \rightarrow \infty \text{ DIM. INTEGRAL}$$

$$\mathcal{D}h^{moduli} = \left|\frac{\partial \delta_m h}{\partial t^i}\right| dt^i \rightarrow \text{FINITE DIM INTEGRAL}$$

MODULI: $P_1^T \delta_m h_{\alpha\beta} = 0 \Leftrightarrow \nabla^\beta (\delta_m h)_{\alpha\beta} = 0$

CONFORMAL KILLING VECTORS: $P_1 \xi^\alpha = 0 \rightarrow \nabla_\alpha \xi_\beta + \nabla_\beta \xi_\alpha - h_{\alpha\beta} \nabla\cdot\xi = 0$

RIEMANN-ROCH THEOREM

$$\dim \mathrm{Ker}\, P_1^T - \dim \mathrm{Ker}\, P_1 = -3\chi$$

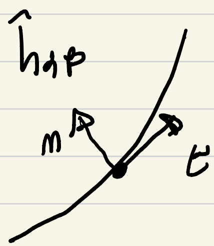
$\chi \equiv$ EULER CHARACTERISTIC $\boxed{\text{in } D=2}$

$$\chi = \frac{1}{4\pi}\int_\Sigma d^2\sigma \sqrt{h}\, \underline{R^{(2)}} + \frac{1}{2\pi}\int_{\partial\Sigma} ds\, \underline{\kappa}$$

RICCI SCALAR in D=2

GEODESIC CURVATURE OF THE BOUNDARY

$$\kappa = t^\alpha t^\beta \nabla_\alpha n_\beta$$



EXERCISE

Disk of radius R with $h_{\alpha\beta} = \delta_{\alpha\beta}$

$$\Rightarrow \kappa = \frac{1}{R}$$

χ CONTROLS THE PERTURBATION EXPANSION OF STRING SCATTERING AMPLITUDES

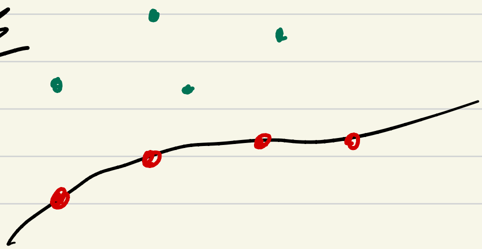
↳ CORR. FUNCTIONS OF GAUGE INVARIANT OPERATORS IN $D=2$

GAUGE INVARIANT OPERATORS

→ CLOSED STRINGS G.I. OPERATORS → BULK PUNCTURES

→ OPEN STRINGS G.I. OPERATORS → BOUNDARY PUNCTURES

Σ

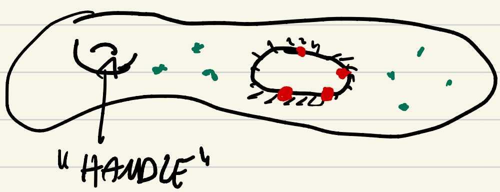


⇒ LOCAL GAUGE INVARIANT OPERATORS

FOR ORIENTED 2D SURFACES

$$\chi = 2(1-g) - b - M_c - \frac{1}{2} M_o$$

$g \equiv$ GENUS → # OF "HANDLES"



$\chi \to$ TOPOLOGICAL

$$S_p[h,X] \longrightarrow \underbrace{S_p[h,X] + \lambda\chi}_{}$$ | SAME EOMS!

$$\int \mathcal{D}h = \sum_{\text{TOPOLOGIES}} \mathcal{D}h^{\lambda}$$

$h^{\lambda} \to$ FIXED TOPOLOGY

($V \to$ CLOSED STRINGS)

$$\langle\langle V_1 \cdots V_{M_c} \rangle\rangle = \sum_{g=0}^{\infty} e^{-\lambda(2(1-g)-M_c)} \langle\langle V_1 \cdots V_M \rangle\rangle^{(g)}$$

$$\langle\langle V_1 \cdots V_{M_c} \rangle\rangle^{(g)} = \int \frac{\mathcal{D}h^{(g)}\mathcal{D}X}{V_{gauge}} V_1 \cdots V_{M_c} e^{-S[h,X]}$$

$$\boxed{g_s = e^{\lambda}}$$

WE WILL SEE $\Longrightarrow \lambda = \langle \Phi \rangle$

$\Phi \to$ MASSLESS EXCITATION OF A CLOSED STRING $\Longrightarrow$ <u>DILATON</u>

ξ IS FIXED

$$\int \frac{\mathcal{D}h\,\mathcal{D}X}{V_{gauge}} \left(\begin{matrix}\text{IGNORE}\\ V\text{'s}\end{matrix}\right) e^{-S[h,X]} =$$

$$= \int \frac{\mathcal{D}h^{(m)}\mathcal{D}h^{(gauge)}\mathcal{D}X}{V_{gauge}}\, e^{-S[h,X]} = (*)$$

GAUGE TRANSFORM.

$$h_{\alpha\beta} = \left(\hat{h}_{\alpha\beta}(t)\right)^{(\xi,\omega)} = \hat{h}^{(1)}_{\alpha\beta}(t)$$

$$(*) = \int_M dt^i \left|\underbrace{\frac{\partial \delta_m h}{\partial t_i}}_{\text{G.F.}}\right| \frac{\cancel{\mathcal{D}\xi^a\mathcal{D}\omega}}{\cancel{V_{gauge}}} \left|\underbrace{\frac{\mathcal{D}(\delta_\sigma h)}{\mathcal{D}\xi\,\mathcal{D}\omega}}_{\text{G.F.}}\right| \underbrace{\mathcal{D}X'}_{\text{G.I.}}\, \underbrace{e^{-S[\hat{h}'(t),X']}}_{\text{G.I.}}$$

G.I. [only in D=26]

$$= \int_M dt^i \int \mathcal{D}X \left|\frac{\partial \delta_m h}{\partial t_i}\right| \left|\frac{\mathcal{D}(\delta_\sigma h)}{\mathcal{D}\xi\,\mathcal{D}\omega}\right| e^{-S[\hat{h}(t),X]} = ☆ \rightarrow$$

GAUGE-FIXED POLYAKOV ACTION

⇒ "EXPONENTIATE" THE DETERMINANTS

⇒ FADDEEV-POPOV TRICK

$$\det M^I{}_j = \int [db_I][dc^j] \exp[-b_I M^I{}_j c^j]$$

GRASSMANN VARIABLES

→ TRY FOR A 2×2 MATRIX!

→ BEREZIN INTEGRAL θ $\theta^2=0$ $\int d\theta\, \theta = 1$; $\int d\theta \cdot 1 = 0$

$b_I \to$ ANTI-GHOST , $c^j \to$ GHOST

$$\left|\frac{\partial \delta_m h}{\partial t^i}\right| \left|\frac{\mathcal{D}\delta_\sigma h}{\mathcal{D}(\xi, \omega)}\right| \qquad \begin{array}{l} t^i \to \psi^i \\ \xi^\alpha(\sigma) \to c^\alpha(\sigma) \oplus b_{\alpha\beta}(\sigma) \\ \omega(\sigma) \to \lambda(\sigma) \end{array}$$

$$\star = \int_M dt^i \int \mathcal{D}X\, d\psi^i \mathcal{D}c^\alpha \mathcal{D}\lambda \mathcal{D}b_{\alpha\beta} \,.$$

$$\cdot\, e^{-\left(\int d^2\sigma \sqrt{\hat h}\, b_{\alpha\beta}\, \partial_i \hat h^{\alpha\beta}(t)\right)\psi^i}$$

$$\cdot\, e^{-\frac{1}{2\pi}\int d^2\sigma \sqrt{\hat h}\, b_{\alpha\beta}\left[(P_1 c)^{\alpha\beta} - \frac{1}{2}\left(\lambda - \frac{1}{2}\nabla\cdot c\right)\hat h^{\alpha\beta}\right]}$$

$$\cdot\, e^{-S[\hat h(t), X]}$$

$$\int d\varphi^i \to \left(\int d^2\sigma \sqrt{\hat{h}}\, b_{\alpha\beta}\, \partial_i h^{\alpha\beta} \right) = \langle b_{\alpha\beta}, \partial_i h^{\alpha\beta} \rangle$$

$$\int \mathcal{D}\lambda \to \hat{h}^{\alpha\beta} b_{\alpha\beta} = 0 \to$$ $b_{\alpha\beta}$ IS TRACELESS

$$\langle\langle V_1 \cdots V_{n_c} \rangle\rangle^{(g)} =$$

$$= \int_{\mathcal{M}_g} dt^i \int \mathcal{D}X\, \mathcal{D}b\, \mathcal{D}c\, [V_1 \cdots V_n] \prod_{i=1}^{\dim \mathcal{M}_g} \langle b, \partial_i \hat{h} \rangle \, e^{-S[\hat{h}, X] - S_{F.P.}[b,c]}$$

$$\boxed{S_{FP}[b,c] = \frac{1}{2\pi} \int d^2\sigma \sqrt{\hat{h}}\, b_{\alpha\beta} (P_1 c)^{\alpha\beta}}$$

valid for "any" choice of $\hat{h}_{\alpha\beta}$

GAUGE FIXING : CONFORMAL GAUGE

$$\hat{h}_{\alpha\beta} = \delta_{\alpha\beta} \qquad \alpha = 0,1$$ [EUCLIDEAN SIGNATURE ON THE WS]

$$(\tau, \sigma) \to (-it, \sigma) \quad \begin{cases} w = t + i\sigma \\ \bar{w} = t - i\sigma \end{cases}$$

$$\hat{h} = dt^2 + d\sigma^2 = -d\tau^2 + d\sigma^2 = dw\, d\bar{w}$$

$$b_{\alpha\beta} \to \begin{cases} b_{ww} = b \\ b_{\bar{w}\bar{w}} = \bar{b} \\ b_{w\bar{w}} = 0 \end{cases} \quad c^{\alpha} \to \begin{cases} c^{w} \equiv c \\ c^{\bar{w}} \equiv \bar{c} \end{cases} \quad \begin{matrix} \nabla_w = \partial_w = \partial \\ \nabla_{\bar{w}} = \partial_{\bar{w}} = \bar{\partial} \end{matrix}$$

$$S_{MATTER} = \frac{1}{2\pi\alpha'} \int d^2w\, \partial X^{\mu} \bar{\partial} X_{\mu}$$

$$S_{GHOST} = \frac{1}{2\pi} \int d^2w \left(b \bar{\partial} c + \bar{b} \partial \bar{c} \right)$$

$\Rightarrow$ GAUGE SYMMETRY HAS BEEN "FIXED"

$\Rightarrow$ NEW GLOBAL SYMMETRY $\Rightarrow$ $\boxed{\text{BRST symmetry}}$

BRST symmetry is "FERMIONIC"

$\epsilon\delta_B \to$ BOSONIC

$\delta_B \to$ FERMIONIC

$$\epsilon\delta_B X^\mu = \epsilon c\,\partial X^\mu + \epsilon\bar{c}\,\bar{\partial} X^\mu$$

$$\epsilon\delta_B c = \epsilon c\,\partial c \qquad \delta_B \bar{c} = \bar{c}\,\bar{\partial}\bar{c}$$

$$\epsilon\delta_B b = -\frac{1}{\alpha'}\partial X\cdot\partial X + (2b\,\partial c + (\partial b)c)$$

$$\epsilon\delta_B \bar{b} = -\frac{1}{\alpha'}\bar{\partial} X\cdot\bar{\partial} X + (2\bar{b}\,\bar{\partial}\bar{c} + (\bar{\partial}\bar{b})\bar{c})$$

EXERCISE

① $\delta_B S^{TOT} = 0 \qquad (S^{TOT} = S_{MATTER} + S_{gh})$

② $\delta_B^2 = 0$

$\Rightarrow$ EOMS $\boxed{\text{USE THEM!}}$

$$\begin{cases} \partial\bar{\partial} X^\mu = 0 & (1) \\ \partial\bar{c} = \bar{\partial} c = 0 \\ \bar{\partial} b = \partial\bar{b} = 0 \end{cases}$$

$$X^\mu(w,\bar{w}) = X_L^\mu(w) + X_R^\mu(\bar{w})$$

$$\left.\begin{matrix} c = c(w) & \bar{c} = \bar{c}(\bar{w}) \\ b = b(w) & \bar{b} = \bar{b}(\bar{w}) \end{matrix}\right\} \to \text{"HOLOMORPHIC" FIELDS}$$