Lecture IV

Physical spectrum
of the BOSONIC
STRING
and
BOSONIC D-BRANES

RECAP

BOSONIC STRING $\nearrow$ GHOST CFT $c=-26$
$\searrow$ MATTER CFT $c=c^{(m)}$

$$\boxed{Q_B^2=0} \Longleftrightarrow \boxed{c^{(m)}-26=0}$$

Gauge invariant operators

$\nearrow$ NON INTEGRATED $c\,V_1^{(m)}(z)$

$\searrow$ INTEGRATED $\mathcal{V}^m=\int dz\, V_1^{(m)}(z)$

They are BOTH BRST invariant

$$\left(Q_B V_1^{(m)}(z)=\partial\left(cV_1^{(m)}(z)\right)\right)$$

and manifestly conformal invariant fields.

Therefore:

PHYSICS → MATTER CFT

↳ weight 1 PRIMARIES

BOSONIC STRING BACKGROUND

$$\boxed{\text{"ANY } c=26 \text{ CFT"}}$$

∋

For example 26 Free bosons

[Flat Minkowski space]

Matter CFT (26 Free Bosons $X^\mu(z,\bar{z})$)

① $U(1)$ - currents

$$j^\mu(z) = i\sqrt{\frac{2}{\alpha'}}\,\partial X^\mu \qquad h = (1,0)$$

$$= \sum_{n\in\mathbb{Z}} \alpha_n^\mu z^{-n-1} \qquad [\alpha_n^\mu, \alpha_m^\nu] = n\eta^{\mu\nu}\delta_{n+m}$$

$$\boxed{j^\mu(z)\, j^\nu(w) = \frac{\eta^{\mu\nu}}{(z-w)^2}}$$

$$\boxed{\begin{aligned} X(z)X(w) &= -\frac{\alpha'}{2}\ln(z-w) \\ \partial X(z)X(w) &= -\frac{\alpha'}{2}\frac{1}{z-w} \\ \partial X(z)\partial X(w) &= -\frac{\alpha'}{2}\frac{1}{(z-w)^2} \end{aligned}}$$

$$T^{(X)}(z) \to (2,0)$$

$$\boxed{T(z)^{(X)} = \frac{1}{2} :j_\mu j^\mu:(z)}$$

EXERCISE Use $j^\mu j^\nu$ OPE

$$\underbrace{T^{X}(z)\,T^{w}}(w) = \frac{D/2}{(z-w)^4} + \frac{2T^{X}(w)}{(z-w)^2} + \frac{\partial T^{X}(w)}{z-w}$$

→ Wick theorem

$$\underbrace{:jj:\qquad :jj:} = (2\text{ contract.}) + (1\text{-contr.})\, :jj:$$

$$D = \eta_\mu{}^\mu \qquad\qquad c = D = 26$$

$$j^\mu(0)\,|0\rangle_{SL(2,\mathbb{C})} = \alpha_{-1}^\mu\,|0\rangle$$

$$\partial j^\mu(0)\,|0\rangle_{SL(2,\mathbb{C})} = \alpha_{-2}^\mu\,|0\rangle$$

$$\vdots \qquad\qquad \vdots$$

MOMENTUM MODES

$|0, p\rangle$

$$|0,p\rangle = \lim_{z,\bar z \to 0} :e^{ip\cdot X}:(z,\bar z)\,|0\rangle_{SL(2,\mathbb{C})}$$

$$:e^{ip\cdot X}: = \sum_{n=0}^{\infty} \frac{1}{n!} :ip\cdot X:^n$$

X_L, X_R REGULAR

$$:e^{ip\cdot X}:(z,\bar z) = :e^{ipX_L}:(z)\,:e^{ipX_R}:(\bar z)$$

$$\boxed{j^\mu(z)\,:e^{ip\cdot X_L}:(w) = \sqrt{\frac{\alpha'}{2}}\,\frac{p^\mu}{z-w}:e^{ipX}:(w)}$$

$$\underbrace{T(z)\,:e^{ip\cdot X}:(w)} = \frac{\alpha' p^2}{4}\frac{:e^{ip\cdot X}:(w)}{(z-w)^2} + \frac{\partial :e^{ip\cdot X}:(w)}{z-w}$$

$$\boxed{e^{ip\cdot X}(z,\bar z) \longrightarrow \left(\frac{\alpha' p^2}{4}, \frac{\alpha' p^2}{4}\right)}$$

PHYSICAL STATES

$\Rightarrow$ MATTER PRIMARIES OF $h=(1,1)$

$\rightarrow$ INTRODUCE LEVEL

$$\overset{(-)}{L}_0 = \oint \frac{d\overset{(-)}{z}}{2\pi i}\, \overset{(-)}{z}\, \overset{(-)}{T}(\overset{(-)}{z}) = \frac{\alpha' p^2}{4} + \underbrace{\sum_{n\geq 1} \overset{(-)}{\alpha}_{-n} \cdot \overset{(-)}{\alpha}_n}_{\text{LEVEL} = N}$$

∂X has $L=1$

$:\partial X \partial X:$ has $L=2$ $\qquad :\partial^2 X:$

$\hookrightarrow \alpha_{-1}^2 |0\rangle \qquad\qquad \downarrow$

$\qquad\qquad\qquad\qquad \alpha_{-2}|0\rangle$

$L_0 = 1$

$\bar{L}_0 = 1$

$\Rightarrow$

① $L_0 - \bar{L}_0 = 0 = N - \bar{N} = 0$ | LEVEL MATCHING

② $L_0 + \bar{L}_0 = 2 \Rightarrow \boxed{\frac{\alpha' p^2}{4} = 1 - N}$

$p^2 = -m^2$

$$\boxed{\frac{\alpha' m^2}{4} = N - 1}$$

- LEVEL ZERO $\quad N = \bar{N} = 0$

$$\boxed{\frac{\alpha' m^2}{4} = -1} \qquad \boxed{e^{ip\cdot X}(z,\bar{z})\, t(p)}$$

INFAMOUS CLOSED STRING TACHYON

- LEVEL ONE $\quad N = \bar{N} = 1$

$$G_{\mu\nu}(p) :\partial X^\mu \bar{\partial} X^\nu e^{ip\cdot X}:(z,\bar{z}) = V_G(z,\bar{z})$$

$$L_0 = 1 = \bar{L}_0 \quad \Rightarrow \quad \boxed{\frac{\alpha' m^2}{4} = 1 - 1 = 0}$$

$$T(z)\, V_G(w,\bar{w}) = \frac{p^\mu G_{\mu\nu}}{(z-w)^3} + \frac{V_G(w,\bar{w})}{(z-w)^2} + \frac{\partial V_G(w,\bar{w})}{z-w}$$

$$\bar{T}(\bar{z})\, V_G(w,\bar{w}) = \frac{G_{\mu\nu} p^\nu}{(\bar{z}-\bar{w})^3} + \frac{V_G(w,\bar{w})}{(\bar{z}-\bar{w})^2} + \frac{\bar{\partial} V_G(w,\bar{w})}{\bar{z}-\bar{w}}$$

$$\begin{cases} p^2 G_{\mu\nu}(p) = 0 \\ p_\mu G^{\mu\nu} = G^{\mu\nu} p_\nu = 0 \end{cases}$$

$$G_{\mu\nu} = \underbrace{h_{\mu\nu}}_{\text{SYMM TRACELESS}} + \underbrace{B_{\mu\nu}}_{\text{ANTISYMM.}} + \underbrace{\eta_{\mu\nu}\Phi}_{\text{TRACE}}$$

$L = 1 \Rightarrow$ 3 LORENTZ IRREPS MASSLESS

$h_{\mu\nu} \rightarrow$ spin 2 particle $\Rightarrow$ GRAVITON

$B_{\mu\nu} \rightarrow$ 2-FORM FIELD $\rightarrow$ KALB-RAMOND FIELD

$\Phi \rightarrow$ SCALAR $\rightarrow$ DILATON

$\Rightarrow$ BOSONIC STRING THEORY CAN DESCRIBE GRAVITY

• LEVEL ≥ 1

$$\frac{\alpha' m^2}{4} = N - 1 \quad \Rightarrow \infty \text{ MASSIVE PARTICLES WITH NON TRIVIAL LORENTZ STRUCTURE}$$

$\Rightarrow \infty$ - TOWER OF MASSIVE HIGHER SPIN PARTICLES

→ <u>REGGE TRAJECTORIES</u>

$\Rightarrow$
- <u>TACHYON</u> ☹
- <u>GRAVITON + other massless excit.</u> ☺
- ∞ - MASSIVE HIGHER SPINS !

NON-LINEAR σ-MODEL

Massless spectrum directly couple to WS

$$S = \frac{1}{2\pi\alpha'}\int_\Sigma d^2 z \Big(g_{\mu\nu}(X)\partial X^\mu \bar\partial X^\nu + i B_{\mu\nu}(X)\partial X^\mu \bar\partial X^\nu + \alpha' \Phi(X) R^{(2)} \Big)$$

$\Rightarrow g, B, \Phi$ GENERALIZED COUPLINGS

Diff -invariant

WEYL : CLASSICALLY OK $\left[\sqrt{h} h^{\alpha\beta} \text{ is WEYL INVARIANT} \right]$

QUANTUM : CONTACT UV DIVERGENCES

$\Rightarrow$ DIM REGULARIZATION $D = 2 + \varepsilon$

$\varepsilon \times \frac{1}{\varepsilon}$ effect $\Rightarrow$ β-FUNCTIONS

$$T_{z\bar z} = -\frac{1}{2\alpha'}\beta^{(g)}_{\mu\nu}\partial X^\mu \bar\partial X^\nu - \frac{i}{2\alpha'}\beta^{(B)}_{\mu\nu}\partial X^\mu \bar\partial X^\nu - \frac{1}{2}\beta^{(\Phi)} R^{(2)}$$

$$\boxed{\begin{aligned} \beta^{(g)}_{\mu\nu} &= \alpha'\Big(R_{\mu\nu} + 2D_\mu D_\nu \Phi - \frac{1}{4}H_{\mu\lambda\sigma}H_\nu{}^{\lambda\sigma}\Big) + O(\alpha'^2) \\ \beta^{(B)}_{\mu\nu} &= \alpha'\Big(-\frac{1}{2}D^\lambda H_{\lambda\mu\nu} + D^\lambda\Phi\, H_{\lambda\mu\nu}\Big) + O(\alpha'^2) \\ \beta^{(\Phi)} &= \frac{D-26}{6} + \alpha'\Big(-\frac{1}{2}D^2\Phi + D_\mu\Phi D^\mu\Phi - \frac{1}{24}H^2\Big) + O(\alpha'^2) \end{aligned}}$$

↳ SPACETIME EOMS !!! GRAVITY + ...

EXACT STRING BACKGROUND

$G_{\mu\nu}(X)$, $B_{\mu\nu}(X)$, $\Phi(X)$, ...

$\beta \equiv 0$ of ALL ORDERS in α'

SIMPLEST EXAMPLE ($D=26$)

$G_{\mu\nu} = \eta_{\mu\nu}$, $B_{\mu\nu} = 0$, $\Phi = \Phi_0$ CONSTANT

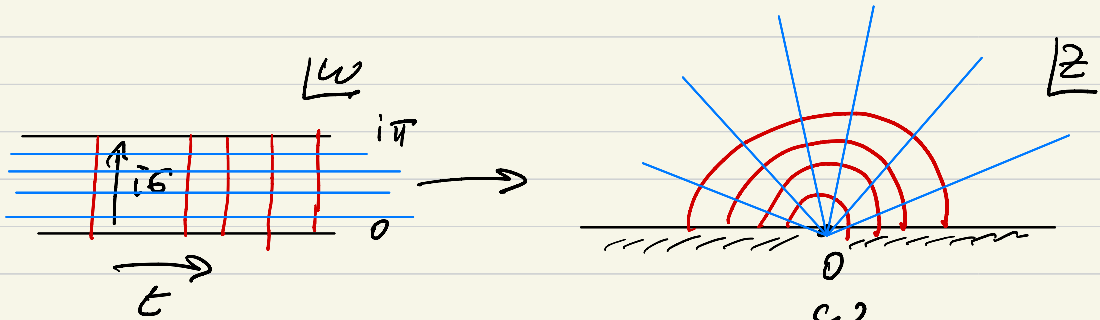
$$S^{\sigma\text{-model}} \longrightarrow \Phi_0 \int_\Sigma d^2 z\, R^{(2)} = \Phi_0 \chi$$

$$\rightarrow \lambda \int_\Sigma d^2 z\, R^{(2)} \qquad e^\lambda = g_s$$

$$\boxed{g_s = e^{\langle \Phi \rangle}}$$

OPEN STRINGS

• D-branes $\Rightarrow$ b.c. for open strings



$w = t + i\sigma$ $\qquad$ $z = e^{w}$

$\Rightarrow$ BOUNDARY MUST PRESERVE CONFORMAL INVARIANCE

$$T(z) = \bar{T}(\bar{z})\Big|_{z=\bar{z}}$$ GLUING CONDITION

$\Rightarrow L_n = \bar{L}_n \rightarrow$ 1 set of VIRASORO OPERATORS

$$\delta_\varepsilon \phi(z,\bar z) = -[T_\varepsilon, \phi(z,\bar z)] - [\bar T_\varepsilon, \phi(z,\bar z)] =$$

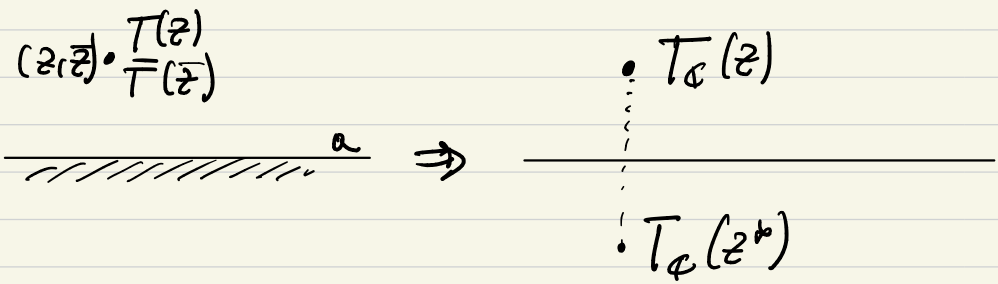
$$= -\oint_z \frac{dw}{2\pi i}\,\varepsilon T(w)\,\phi(z,\bar z) - \oint_{\bar z} \frac{d\bar w}{2\pi i}\,\bar\varepsilon \bar T(\bar w)\,\phi(z,\bar z)$$

DOUBLING TRICK

$(T(z), \bar T(\bar z))$ / GLUING-CONDITION

$$T_{\mathbb{C}}(z) = \begin{cases} T_{UHP}(z) & \mathrm{Im}\, z > 0 \\ \bar T_{UHP}(z^*) & \mathrm{Im}\, z < 0 \end{cases}$$

$z^* = \bar z$

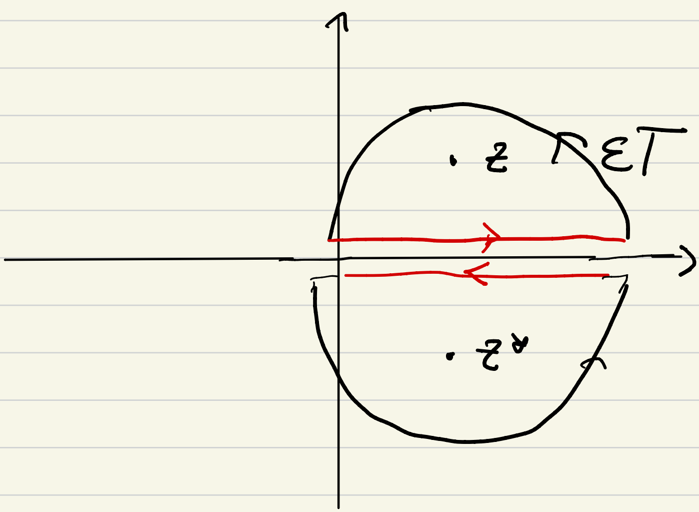


$$\phi(z,\bar z) = \sum_{i,j} \Phi_{ij}\, V_i(z)\, \underbrace{\bar V_j(\bar z)}_{\downarrow\; V_j(z^*)}$$

$\Downarrow$ DOUBLING

$$\phi(z,\bar z) \to \phi^{(a)}(z,z^*) = \sum_{i,j} \Phi^{e}_{ij}\, V_i(z)\, V_j(z^*)$$

$$\delta_\varepsilon \phi^{(a)}(z,z^*) = \oint_{z,z^*} \frac{dw}{2\pi i}\, \varepsilon(w)\, T(w)\, \phi^{(a)}(z,z^*)$$



X^μ - BCFT $\begin{cases} N \\ D \end{cases}$

<u>Dp-brane</u>

$N \Rightarrow \mu = 0, \dots, p$
$D \Rightarrow i = p+1, \dots, 25$

$$j^\mu(z) = \Omega^\mu{}_\nu \bar{j}^\nu(\bar{z})\Big|_{z=\bar{z}} \qquad \underbrace{\Omega^\mu{}_\nu}_{\text{GLUING MAP}} = \left(\begin{array}{c|c} \delta^\mu{}_\nu & 0 \\ \hline 0 & -\delta^i{}_j \end{array}\right)$$

$$\Omega^2 = 1$$

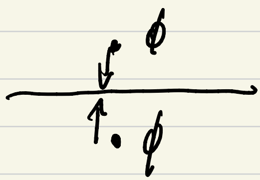
$$T = \frac{1}{2} :jj: \qquad T(z) = \bar{T}(\bar{z}) \quad z = \bar{z}$$

$$j^\mu_{\mathbb{C}}(z) = \begin{cases} j^\mu_{UHP}(z) & \mathrm{Im}\, z > 0 \\ \Omega^\mu{}_\nu \bar{j}^\nu(z^*) & \mathrm{Im}\, z < 0 \end{cases}$$

$\hookrightarrow$ SINGLE HOLOMORPHIC CURRENT

OPEN STRING STATES

$\Phi(z,\bar{z})$ → WE CANNOT PUT THEM ON THE BOUNDARY

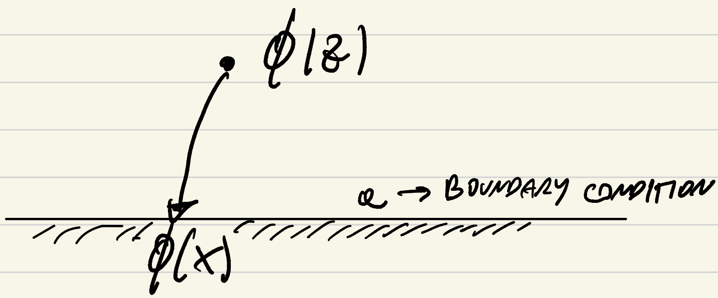


COLLISION!

- On the boundary → BOUNDARY FIELDS

→ (CHIRAL / ANTICHIRAL REPRESENTATIONS OF THE VIRASORO ALGEBRA)

- Concretely: A BOUNDARY FIELD $\Updownarrow$ CHIRAL FIELD ON THE BOUNDARY



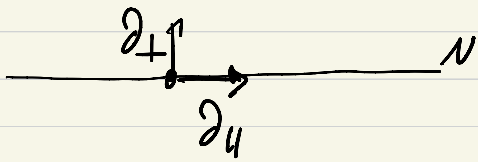
$$\phi(x) = \lim_{z \to x} \phi(z)$$

Example

$j(z)$ on the boundary

① Neumann b.c. $\quad (\partial - \bar\partial) X \big|_{z=\bar z} = 0$

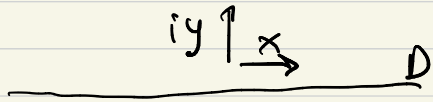
$$\partial X(z=x) = \partial_\parallel X(x) = \partial_x X(x)$$



② Dirichlet b.c. $\quad (\partial + \bar\partial) X \big|_{z=\bar z} = 0$

$\partial X(z=x)$

$z = x + iy$



$$\partial X(z=x) = -i\,\partial_y X(z)\big|_{y=0} = \partial_\perp X(x)$$

NEUMANN B.C. $\Leftrightarrow$ SPACETIME MOMENTUM

$$:e^{ip\cdot X}:(x) = \lim_{z\to x} :e^{2ip\cdot X_L}:(z)$$

$$\downarrow$$

$$e^{ip\cdot x_0 + \cdots} \qquad X_L(z) = \frac{x_0}{2} + \cdots$$

$$:e^{ip\cdot X}:(x) \Rightarrow \text{BOUNDARY PRIMARY}$$

$$L_0 \to T(z) \qquad e^{ik\cdot X_L} \to \frac{\alpha' k^2}{4}$$

$$:e^{ip\cdot X}:(x) = :e^{2ip\cdot X_L}:(z=x) \Rightarrow \alpha' \frac{(2p)^2}{4} = \alpha' p^2$$

$$e^{ip\cdot X}(z,\bar{z}) \to \left(\frac{\alpha' p^2}{4}, \frac{\alpha' p^2}{4}\right)$$

$$e^{ip\cdot X}(x) \to \alpha' p^2$$

OPEN STRINGS ON A Dp BRANE

$$Q_B = \oint \frac{dz}{2\pi i}\, j_B(z) \qquad \to \text{A SINGLE } Q_B$$

$$Q_B = \overline{Q_B}$$

(OPEN STRINGS)

· **NON INTEGRATED VERTEX OPERATORS**

$$c\, \underbrace{V_1^{(m)}(x)}_{\text{BOUNDARY PRIMARY OF } h=1} \qquad (1)$$

· **INTEGRATED VERTEX OPERATOR**

$$\int_{-\infty}^{+\infty} dx\, V_1^{(m)}(x) \qquad (2)$$

(1) and (2) ARE BOTH BRST INVARIANT

((2) UP TO POSSIBLE BOUNDARY TERMS)

DP- brane

$$\boxed{L_0 = 1}$$

$$L_0 = \alpha' p^2 + N = \oint \frac{dz}{2\pi i} z T(z)$$

$$\alpha_0^{(o.s)} = \sqrt{2\alpha'}\, p$$

$$\alpha_0^{(c.s)} = \sqrt{\frac{\alpha'}{2}}\, p$$

$$N=0 \quad t^{(0)}(p)\, e^{ip\cdot X}(x) \rightarrow \boxed{\alpha' m^2 = -1}$$

OPEN STRING TACHYON

↳ DP brane is UNSTABLE

$$N=1 \quad \underline{A_\mu}(p) :j^\mu e^{ip\cdot X}:(x) + \underline{\Phi_i}(p)\, j^i e^{ip\cdot X}(x)$$

$\downarrow$ GAUGE BOSON

$\downarrow$ TRANSVERSE SCALARS

$$L_0 = 1 \rightarrow \boxed{\alpha' m^2 = 0}$$

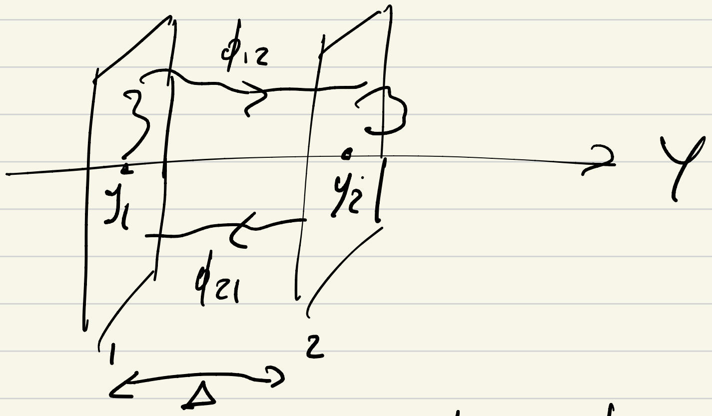
$$L_{n>0} = 0 \rightarrow p_\mu A^\mu(p) = 0$$

$$A_\mu(p)\, c j^\mu e^{ip\cdot X} + Q\left(\Lambda(p)\, e^{ip\cdot X}\right)$$

$$\Rightarrow A_\mu(p) \rightarrow A_\mu(p) + p_\mu \Lambda(p)$$

$$\boxed{\text{U(1) GAUGE TRANSF.}}$$

Two parallel Dp-branes



DIRICHLET CONDITIONS

OPEN STRINGS
→ CHAN-PATON MATRIX

$$\begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix}$$

$$\Delta_{ij} = y_i - y_j$$

OFF-DIAGONAL STRINGS → (STRETCHED STRINGS)

$i, j = 1, 2$

N=0

$$\underbrace{t_{ij}(p)\, e^{ip\cdot X}_{(z=0)}\, e^{i \frac{\Delta_{ij}}{2\pi\alpha'} \tilde{Y}}_{(z=0)}}_{h = \alpha' p^2 + \left(\frac{\Delta_{ij}}{2\pi\sqrt{\alpha'}}\right)^2 = 1} |0\rangle \qquad \tilde{Y}(x) = \lim_{z\to x} 2 Y_L(z)\Big|_D$$

$$\alpha' m_{ij}^2 = -1 + \left(\frac{\Delta_{ij}}{2\pi\sqrt{\alpha'}}\right)^2 \rightarrow \text{CRITICAL DISTANCE}$$

Level 1

① Gauge bosons

$$\tilde{\Delta} = \frac{\Delta}{2\pi\sqrt{\alpha'}}$$

$$\begin{pmatrix} A^{11}_\mu j^\mu & W^+_\mu j^\mu e^{i\tilde{\Delta}\tilde{Y}} \\ W^-_\mu j^\mu e^{-i\tilde{\Delta}\tilde{Y}} & A^{22}_\mu j^\mu \end{pmatrix} e^{iP\cdot X}(x)$$

$L_0 = 1 \Rightarrow \alpha' M_{ii}^2 = 0 \rightarrow A^i_\mu$ MASSLESS VECTORS

$\alpha' M_{ij}^2 = \tilde{\Delta}_{ij}^2 \rightarrow$ MASSIVE VECTORS!

$L_{n>0} = 0 \quad P_\mu A^\mu = 0 \ ; \ P_\mu W^\pm_\mu = 0$

$\Rightarrow$ HIGGS MECHANISM

$\hookrightarrow M^2 = \Delta$

② TRANSVERSE SCALARS

$$\Phi^i(x) = \begin{pmatrix} \Phi_{11}^{i}\, j^i & \Phi_{12}^{i}{:}j^i e^{i\tilde{\Delta}\tilde{Y}}{:} \\ \Phi_{21}^{i}{:}j^i e^{-i\tilde{\Delta}\tilde{Y}}{:} & \Phi_{22}^{i}\, j^i \end{pmatrix} e^{ip\cdot X(x)}$$

- $L_0 = 1 \qquad \alpha' M_{ij}^2 = \tilde{\Delta}_{ij}^2 \qquad i = j \quad \tilde{\Delta} = 0$

- $L_{n>0} = 0 \qquad i \neq y \rightarrow$ <u>OK</u> <u>ALL PRIMARIES</u>

For $i = y$

$$\Phi^{i=y} = \begin{pmatrix} \Phi_{11}^{y}\, j^y & \Phi_{12}^{y}{:}j^y e^{i\tilde{\Delta}\tilde{Y}}{:} \\ \Phi_{21}^{y}{:}j^y e^{-i\tilde{\Delta}\tilde{Y}}{:} & \Phi_{22}^{y}\, j^y \end{pmatrix} e^{ip\cdot X}$$

● → <u>NOT PHYSICAL</u>

SYMMETRY ENHANCEMENT

$\Delta y \Rightarrow 0 \qquad W_\mu^\pm \to$ MASSLESS
(THEY LOOSE 1 D.O.F.)

$\Phi^y_\pm \to$ PHYSICALS

$\Rightarrow$ 4 GAUGE BOSONS $\to \boxed{U(2)}$ GAUGE GROUP

CLOSED STRINGS $\to$ GRAVITY + extra dynamical matter

OPEN STRINGS (D-BRANES) $\to$ YM + extra dynamical

$\Rightarrow$ TACHYONS ?? $\to$ INSTABILITY

NO SPACE-TIME FERMIONS

$\Rightarrow \boxed{\text{SUPERSTRING}}$