LECTURE V

Type II Superstrings

- Bosonic string is not satisfactory
  1) NO SPACE-TIME FERMIONS
  2) Closed and open string tachyons

  $\Longrightarrow$ INSTABILITY OF THE VACUUM

  IS THERE A STABLE VACUUM?

BIG MISTERY ($\Longrightarrow$ STRING FIELD THEORY)

$\Longrightarrow$ SUPERSTRING SOLVES 1) & 2)

- Add supersymmetry to the world-sheet

$X^\mu \longrightarrow \psi^\mu$ 2-D Majorana spinor $\begin{pmatrix} \psi^\mu_+ \\ \psi^\mu_- \end{pmatrix}$

- Since X^μ was coupled to 2D-metric $h_{\alpha\beta}$ this requires to add the susy partner of $h_{\alpha\beta} = e^a_\alpha \delta_{ab} e^b_\beta$ (ZWEIBEIN) ($\sqrt{h} = e = \det e$)

GRAVITINO χ_α (SPINOR-VECTOR IN 2D)

$[\alpha'=1]$

$$S = -\frac{1}{4\pi}\int d^2\sigma\, e\left(h^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X_\mu - i\bar\psi^\mu \rho^\alpha \partial_\alpha \psi_\mu + \frac{i}{\sqrt{2}}\bar\chi_\alpha \rho^\beta\rho^\alpha \psi^\mu \partial_\beta X_\mu - \frac{1}{8}\bar\psi^\mu\psi_\mu \bar\chi_\alpha \rho^\beta\rho^\alpha \chi_\beta \right)$$

$h_{\alpha\beta} = e_\alpha^a \delta_{ab} e_\beta^b$

$\rho^\alpha = e^\alpha_a \rho^a$; $\rho^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ $\rho^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$; $\rho^0\rho^1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

[2D-GAMMA MATRICES]

Diffs:

$\delta_\xi X^\mu = \xi^\beta \partial_\beta X^\mu$; $\delta_\xi \psi^\mu = \xi^\beta \partial_\beta \psi^\mu$

$\delta_\xi e_\alpha^a = \xi^\beta \partial_\beta e_\alpha^a + e_\beta^a \partial_\alpha \xi^\beta$

$\delta_\xi \chi_\alpha = \xi^\beta \partial_\beta \chi_\alpha - \chi_\beta \partial_\alpha \xi^\beta$

Weyl:

$\delta_\Lambda X^\mu = 0$; $\delta_\Lambda \psi^\mu = -\frac{1}{2}\Lambda \psi^\mu$

$\delta_\Lambda e_\alpha^a = \Lambda e_\alpha^a$

$\delta_\Lambda \chi_\alpha = \frac{1}{2}\Lambda \chi_\alpha$

Local susy:

$\delta_\varepsilon X^\mu = i\sqrt{\frac{\alpha'}{2}}\,\bar\varepsilon \psi^\mu$ $\quad \delta_\varepsilon \psi^\mu = \frac{1}{2}\rho^\alpha\left(\sqrt{\frac{2}{\alpha'}}\partial_\alpha X^\mu - \frac{i}{2}\bar\chi_\alpha \psi^\mu\right)\varepsilon$

$\delta_\varepsilon e_\alpha^a = \frac{i}{2}\bar\varepsilon \rho^a \chi_\alpha$; $\delta_\varepsilon \chi_\alpha = 2 D_\alpha \varepsilon$

Super Weyl

$\delta_\eta \chi_\alpha = \rho_\alpha \eta$

$\delta_\eta(\text{others}) = 0$

SUPER CONFORMAL GAUGE

$$e^a_\alpha = \delta^a_\alpha$$

$$\chi_\alpha = 0$$

SUPER CONFORMAL GAUGE

$\Longrightarrow$ MISSING EOMS FOR $e_\alpha^{\ a}$ and χ_α

- ENERGY-MOMENTUM TENSOR

$$\frac{1}{e}\frac{\delta S}{\delta h^{\alpha\beta}} = T_{\alpha\beta} = \partial_\alpha X^\mu \partial_\beta X_\mu + \frac{i}{2}\bar{\psi}^\mu \rho_{(\alpha}\partial_{\beta)}\psi_\mu - \frac{1}{2}h_{\alpha\beta}(\text{trace}) = 0$$

- SUPER CURRENT

$$\frac{\delta S}{\delta \bar{\chi}_\alpha} = G^\alpha = \frac{1}{2}\rho^\beta\rho^\alpha\psi^\mu\partial_\beta X_\mu = 0$$

$\Rightarrow$ These MISSING EOMS WILL BE THE CONSTRAINTS TO DEAL WITH IN THE BRST QUANTIZATION

SUPERCONFORMAL FIELD THEORY

In conformal gauge

$$S = \frac{1}{4\pi}\int d^2w \left(\frac{2}{\alpha'}\partial X^\mu \bar\partial X_\mu + \psi^\mu \bar\partial \psi_\mu + \bar\psi^\mu \partial \bar\psi_\mu\right)$$

$\Downarrow$ CYL $\to \mathbb{C}$

$$= \frac{1}{4\pi}\int d^2z \left(\frac{2}{\alpha'}\partial X^\mu \bar\partial X_\mu + \psi^\mu \bar\partial \psi_\mu + \bar\psi^\mu \partial \bar\psi_\mu\right)$$

$\psi^\mu \to \left(\frac{1}{2}, 0\right)$ PRIMARY $\qquad \bar\psi^\mu \to \left(0, \frac{1}{2}\right)$ PRIMARY

OPE

$$X^\mu(z,\bar z)\, X^\nu(0,0) = -\frac{\alpha'}{2}\eta^{\mu\nu}\ln|z|^2$$

$$\underbrace{\psi^\mu(z)\,\psi^\nu(0)} = \frac{\eta^{\mu\nu}}{z} \qquad \bar\psi^\mu(\bar z)\,\bar\psi^\nu(0) = \frac{\eta^{\mu\nu}}{\bar z}$$

$$T(z) = -\frac{1}{\alpha'} :\partial X_\mu \partial X^\mu: - \frac{1}{2}\psi^\mu \partial \psi_\mu$$

$$\underbrace{T(z)\,T(w)} = \frac{\frac{1}{2}\left(\frac{3D}{2}\right)}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

$$c = \frac{3D}{2} = \underset{\substack{\uparrow\\ X}}{D} + \underset{\substack{\uparrow\\ \psi}}{\frac{D}{2}}$$

SUPER CURRENT

$j = i\sqrt{\frac{2}{\alpha'}}\,\partial X$

$$G(z) = \psi^\mu j_\mu$$

$$\rightarrow \quad h = \left(\tfrac{3}{2}, 0\right)$$

$$\boxed{\mathcal{N}=1 \text{ SUPER CONFORMAL ALGEBRA}}$$

$$T(z)\,T(w) = \frac{\tfrac{1}{2}\left(\tfrac{3D}{2}\right)}{(z-w)^4} + \frac{2T(w)}{(z-w)^2} + \frac{\partial T(w)}{z-w}$$

$$T(z)\,G(w) = \frac{\tfrac{3}{2}G(w)}{(z-w)^2} + \frac{\partial G(w)}{z-w}$$

$$G(z)\,G(w) = \frac{\tfrac{2}{3}\left(\tfrac{3D}{2}\right)}{(z-w)^3} + \frac{2T(w)}{z-w}$$

ψ^μ, X^μ is a $\mathcal{N}=1$ SCFT $\quad c = \frac{3D}{2}$

$(T, G) \Rightarrow$ CONSTRAINTS FOR PHYSICAL STATES

SUPERCONFORMAL PRIMARIES

$\phi^{(h)}$ is superconf. primary

① It is a primary of weight h

② → SIMPLE POLE WITH G

$$\underbrace{G(z)\,\phi^{(h)}}(w) = \frac{\psi^{(h+\frac{1}{2})}(w)}{z-w}$$

<u>Ex</u>

$$\underbrace{G(z)\,\psi^{\mu}}(w) = j_\nu\,\underbrace{\psi^{\nu}(z)\,\psi^{\mu}}(w) = \eta^{\nu\mu}\frac{j_\nu(z)}{z-w} \simeq \frac{j^{\mu}(w)}{z-w} + \text{reg.}$$

$$\underbrace{G(z)\,j}^{\mu}(w) = \frac{\eta^{\mu\nu}}{(z-w)^2}\,\psi_\nu(w)$$

$\Rightarrow j^{\mu}$ IS A SUPER-DESCENDENT OF ψ^{μ}

(IT IS STILL PRIMARY)

· Momentum modes

$$G(z) :e^{ip\cdot X_L}:(w) = \psi_\mu j^\mu(z) :e^{ip\cdot X_L}:(w) =$$

$$= \sqrt{\frac{\alpha'}{2}}\, \frac{p_\mu \psi^\mu e^{ip\cdot X_L}(w)}{z-w} \rightarrow \left[\begin{array}{l}\text{PICTURE}\\ \text{CHANGING}\end{array}\right]$$

$e^{ip\cdot X_L}(w)$ is a $\left(\frac{\alpha' p^2}{4}, 0\right)$ SUPER-CONFORMAL PRIMARY

EXERCISE

WS TRANSLATIONS : $\partial_z \longrightarrow \oint \frac{dz}{2\pi i} T(z) = L_{-1}$

WS SUPERSYMMETRY: $\delta_s \longrightarrow \oint \frac{dz}{2\pi i} G(z)$

$$\delta_s \left(i\sqrt{\tfrac{2}{\alpha'}}X^\mu\right) = \psi^\mu$$

$$\delta_s \psi^\mu = j^\mu$$

$$\delta_s j^\mu = \partial \psi^\mu$$

$$[\delta_s, \delta_s] = \partial_z$$

SUPERGHOST'S AND BRST

Diff × Weyl $\rightarrow$ $b_{\alpha\beta}$, c^{α} $\rightarrow$ $(b(z), c(z))$ + h.c.

Susy × Superweyl $\rightarrow$ β_{α} , γ $\rightarrow$ $(\beta(z), \gamma(z))$ + h.c.

SPINOR VECTOR SPINOR

$$S^{ghost} = \frac{1}{2\pi} \int d^2 z \left(b \bar{\partial} c + \beta \bar{\partial} \gamma + h.c. \right)$$

$b \rightarrow (2, 0)$

$c \rightarrow (-1, 0)$

$\beta \rightarrow \left(\frac{3}{2}, 0\right)$

$\gamma \rightarrow \left(-\frac{1}{2}, 0\right)$

$$\underbrace{b(z) \, c}(w) = \frac{1}{z-w} = c(z) \, b(w)$$

$$\gamma(z) \, \beta(w) = \frac{1}{z-w} = -\beta(z) \gamma(w)$$

$$T_{gh} = T^{bc}(z) + T^{\beta\gamma}(z) =$$

$$= :\partial b c:(z) - 2 :\partial(bc): + :\partial \beta \gamma: - \frac{3}{2} :\partial(\beta\gamma):$$

$$T^{bc}(z)\,T^{bc}(w) \to \frac{\frac{1}{2}(-26)}{(z-w)^4} + \cdots$$

$$T^{\beta\gamma}(z)\,T^{\beta\gamma}(w) \to \frac{\frac{1}{2}(+11)}{(z-w)^4} + \cdots$$

$$c^{ghost} = -26 + 11 = -15$$

$$G^{gh} = -\frac{1}{2}\partial\beta c + \frac{3}{2}\partial(\beta c) - 2b\gamma$$

EXERCISE

$$T^{gh}(z)\,G^{gh}(w) = \frac{3/2\, G^{gh}(w)}{(z-w)^2} + \frac{\partial G^{gh}(w)}{z-w}$$

$$G^{gh}(z)\,G^{gh}(w) = \frac{\frac{2}{3}(-15)}{(z-w)^3} + \frac{2T^{gh}(w)}{z-w}$$

Matter + ghost (GAUGE FIXING)

$$C^{TOT} = \frac{3D}{2} - 15$$

$$C^{TOT} = 0 \iff \boxed{D = 10}$$

$$j_B(z) = c\,T^{m}(z) + \gamma\,G^{m}(z) + $$
$$+ \frac{1}{2}\left(:c\,T^{gh}:(z) + :\gamma\,G^{gh}:(z) \right)$$

$$Q_B = \oint \frac{dz}{2\pi i}\, j_B(z)$$

$$[Q_B, Q_B] = 0 \iff C_{TOT} = 0$$

EXERCISE

$$[Q_B, b(z)] = T^{tot}(z)$$

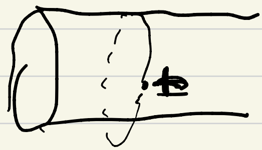
$$[Q_B, \beta(z)] = G^{TOT}(z) \leftarrow \text{NEW!}$$

RAMOND AND NEVEU-SCHWARZ SECTORS

WS SPINORS ON THE CYLINDERS

PERIODIC $\rightarrow$ (R)

ANTI-PERIODIC $\rightarrow$ (NS)



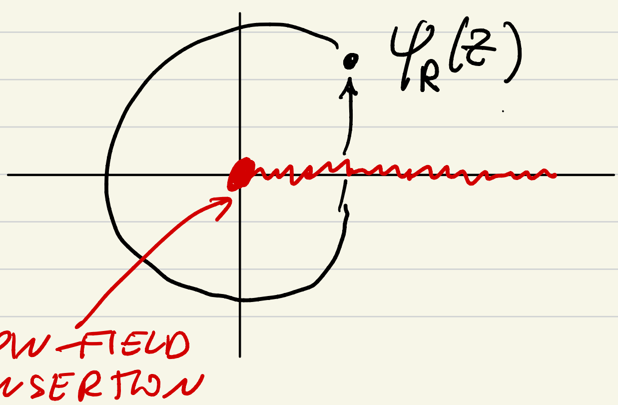
$$(\psi^\mu, \beta, \gamma; G^{(m)}; G^{(gh)})$$

$$\Downarrow \text{cyl} \rightarrow \mathbb{C}$$

$$\psi(z)\, dz^{1/2} = \psi(w)\, dw^{1/2} \qquad \psi_{\mathbb{C}}(z) = \left(\frac{dw}{dz}\right)^{1/2} \psi_{cyl}(w)$$

$$\psi_{NS}(z) = \sum_{r \in \mathbb{Z}+\frac{1}{2}} \psi^\mu_r z^{-r-\frac{1}{2}}$$ SINGLE-VALUED FUNCTION

$$\psi_R(z) = \sum_{n \in \mathbb{Z}} \psi^\mu_n z^{-n-\frac{1}{2}}$$ $\rightarrow$ DOUBLE-VALUED FUNCTION



$$[\psi^\mu_r, \psi^\nu_s] = \eta^{\mu\nu}\delta_{r+s} \quad (NS) \quad r \in \mathbb{Z}+\frac{1}{2}$$

$$[\psi^\mu_n, \psi^\nu_m] = \eta^{\mu\nu}\delta_{n+m} \quad (R) \quad n \in \mathbb{Z}$$

<u>NS- VACUUM</u>

$$\psi^\mu_r |0\rangle_{NS} = 0 \quad r \geq \frac{1}{2} \quad {}_{NS}\langle 0| \psi^\mu_{-r} = 0$$

<u>R- VACUUM</u>

$$\psi^\mu_n |\alpha\rangle_R = 0 \quad , \langle\alpha| \psi^\mu_{-n} \quad n \geq 1$$

$\Rightarrow$ ZERO MODES $\quad [\psi^\mu_0, \psi^\nu_0] = \eta^{\mu\nu} \rightarrow$ CLIFFORD ALGEBRA

$$\Gamma^\mu \equiv \sqrt{2}\,\psi^\mu_0 \qquad \{\Gamma^\mu, \Gamma^\nu\} = 2\eta^{\mu\nu}$$

10-Dim. Γ-algebra!!

$$\boxed{|\alpha\rangle_R \rightarrow \text{IS A SPACE-TIME SPINOR in } D=10}$$

RAMOND-VACUUM DEGENERACY

$$\psi_{\underline{0}}^{\pm} = \frac{i}{\sqrt{2}} \underbrace{\left(\psi_0^{\underline{0}} \pm \psi_0^{\underline{D-1}} \right)}_{\text{LIGHTCONE}}$$

$-$ → LORENTZ

$$\psi_{\underline{j}}^{\pm} = \frac{1}{\sqrt{2}} \underbrace{\left(\psi_0^{2j} \pm i \psi_0^{2j+1} \right)}_{\text{SPACE-LIKE}} \qquad j = 1,2,3,4$$

$$\begin{cases} [\psi_I^{+}, \psi_J^{-}] = \delta_{IJ} \\ [\psi_I^{\pm}, \psi_J^{\pm}] = 0 \end{cases} \qquad I = (0, j)$$

→ 5 independent FERMIONIC OSCILLATORS

$\forall I, \left(\psi_I^{+}, \psi_I^{-}\right)$ span a spin $\frac{1}{2}$ IRREP OF SU(2)

$$|\alpha\rangle_R = \left|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \pm\tfrac{1}{2}\right\rangle$$

→ $\boxed{2^5 = 32 \text{ DIMENSIONS}}$

→ 10 Dimensional spinor

CHIRALITY MATRIX

$$\chi_J \equiv \psi_J^+ \psi_J^- - \frac{1}{2}$$

$$\chi\,|+\tfrac{1}{2}\rangle = +\tfrac{1}{2}\,|+\tfrac{1}{2}\rangle$$
$$\chi\,|-\tfrac{1}{2}\rangle = -\tfrac{1}{2}\,|-\tfrac{1}{2}\rangle$$

$$\Gamma^{11} = \Gamma = 2^5 \prod_{J=0}^{4} \chi_J = -\Gamma^0 \Gamma^1 \cdots \Gamma^9$$

$$\Gamma\,|\pm\tfrac{1}{2}, \pm\tfrac{1}{2}, \dots, \pm\tfrac{1}{2}\rangle = (-)^{\#}\,|\%\rangle$$

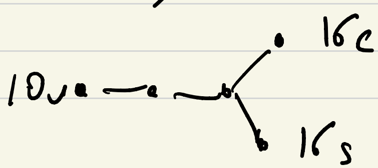
$\# \to$ NUMBER OF "$-\frac{1}{2}$"

$\alpha \to 16_c \to$ EVEN # OF "$-\frac{1}{2}$" (α)

$\alpha \to 16_s \to$ ODD # OF "$-\frac{1}{2}$" ($\dot{\alpha}$)

$$|\ \rangle_R = \underset{\substack{\downarrow \\ \Gamma = 1 \\ 16_c}}{|\alpha\rangle} \oplus \underset{\substack{\downarrow \\ \Gamma = -1 \\ 16_s}}{|\dot{\alpha}\rangle}$$

SO(1,9)



SPIN FIELDS

$$|\alpha\rangle_R = S_\alpha(0)\,|0\rangle_{SL(2,\mathbb{C})}$$

$$|\dot\alpha\rangle_R = S_{\dot\alpha}(0)\,|0\rangle_{SL(2,\mathbb{C})}$$

$$T_H = \frac{1}{2}:\partial H \partial H:$$

$$S = ?$$

→ BOSONIZATION

$$H_I(z)\,H_J(w) = \delta_{IJ}\ln(z-w)$$

$$\psi_I^{\pm}(z) \qquad e^{\pm i H_I}(z)$$

$$\psi_I^{+}(z)\,\psi_J^{-}(w) = \frac{\delta_{IJ}}{z-w} \qquad e^{+iH_I}(z)\,e^{-iH_J}(w) = \frac{\delta_{IJ}}{z-w}$$

$$\psi_I^{+}(z)\,\psi_J^{+}(w) = 0 \qquad e^{iH}(z)\,e^{iH}(w) = 0$$

$$\boxed{\psi_I^{\pm}(z) = e^{\pm i H_I}(z)}$$

$$S_\alpha(z) = e^{i\sum_I s_I H_I}(z) \qquad S_{\dot\alpha} = e^{i\sum \dot s_I H_I}(z)$$

SPIN FIELDS (MATTER) $s_I = \left\{\pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}, \pm\frac{1}{2}\right\}\Big|_{\text{even }(-)}$

$\dot s_I = \{\ - - - - \ \}\Big|_{\text{odd }(-)}$

$$h\left(e^{\frac{i}{2}H}\right) = \frac{1}{8} \quad \rightarrow \quad \boxed{h(S_\alpha) = \frac{5}{8}}$$

→ BRANCH CUTS!! BAD!

→ **$\beta\gamma$ - system** $\qquad \gamma(z)\beta(w) = \frac{1}{z-w}$

→ "BOSONIZE" the $\beta\gamma$- system !

$$(\beta,\gamma) \longleftrightarrow (\eta,\xi) \oplus \phi$$

2 - DECOUPLED CFT's

$$\beta(z) = e^{-\phi}\partial\xi(z)$$
$$\gamma(z) = \eta\, e^{\phi}(z)$$

(η,ξ) → FERMIONIC "bc" system

$$h(\eta)=1 \qquad h(\xi)=0 \qquad \xi(z)\eta(w) = \frac{1}{z-w}$$

$c=-2$ $\qquad T_{\eta\xi} = :\partial\eta\,\xi: - \partial:\eta\xi:$

$$\langle 0|\xi_0|0\rangle = 1$$

ϕ → is CHIRAL "TWISTED" FREE BOSON WITH NEGATIVE SIGNATURE

$$\phi(z)\phi(w) = -\ln(z-w)$$

$c=+13$ $\qquad T_\phi = -\frac{1}{2}\partial\phi\partial\phi - \partial^2\phi$

EXERCISE

$$h(e^{q\phi}) = -\frac{q}{2}(q+2) \qquad \begin{cases} 1 \to h=0 & (1) \\ e^{-2\phi} \to h=0 & (c\partial c\partial^2 c) \end{cases}$$

$$\langle c\partial c\partial^2 c\rangle = 1$$

$$\boxed{\langle e^{-2\phi} \rangle \neq 0} \qquad \langle 1 \rangle = 0$$

→ ANOMOLOUS "MOMENTUM" CONSERVATION

$$Q_\phi = \oint \frac{dz}{2\pi i} (-\partial\phi) \qquad J_\phi(z) \equiv -\partial\phi(z)$$

$$[Q_\phi, e^{q\phi}] = q\, e^{q\phi} \qquad T_\phi(z) J_\phi(0) = \frac{-2}{z^3} + \frac{J_\phi}{z^2} + \frac{\partial J_\phi}{z}$$

$$Q_\phi |0\rangle_{SL(2,\mathbb{C})} = 0 \xrightarrow[z' = -\frac{1}{z}]{} \langle 0| (Q_\phi + 2) = 0$$

$$\langle \prod_i e^{q_i \phi}(z_i) \rangle \neq 0 \qquad \boxed{\sum_i q_i = -2}$$

$$|0\rangle_{NS} = e^{-\phi}(0) |0\rangle_{SL(2,\mathbb{C})} \rightarrow \boxed{h = \frac{1}{2}}$$

$$|\alpha\rangle_R = \underset{\substack{\downarrow \\ h = \frac{5}{8}}}{S_\alpha}\, \underset{\substack{\downarrow \\ h = \frac{3}{8}}}{e^{-\phi/2}} |0\rangle_{SL(2,\mathbb{C})} \rightarrow \boxed{h = 1}$$

PICTURE CHARGE:

$$J_P = :\xi\eta: - \partial\phi$$
$$= :\xi\eta: + J_\phi$$

	PICT		
β	0	$\lvert 0\rangle_{NS}$	(-1)
γ	0		
$e^{m\phi}$	$-m$	$\lvert 0\rangle_R$	$(-\frac{1}{2})$
ξ	1		
η	-1		

The total anomalous conservations in the ghost sector gives rise in the $(\eta|\xi), \phi, (bc)$ theory

$$\left\langle \xi e^{-2\phi} \frac{1}{2}\partial^2 c\, \partial c\, c(z) \right\rangle_L^{\text{CHIRAL}} = 1$$

$\langle \quad \rangle_L \Rightarrow$ "LARGE" HILBERT SPACE CORRELATOR

BUT $\beta = \partial\xi e^{-\phi}$ $\gamma = \eta e^{\phi}$

$$\xi = \sum_n \xi_n z^{-n} = \xi_0 + \sum_{n\neq 0} \xi_n z^{-n}$$

$\Rightarrow$ SMALL HILBERT SPACE

$$\langle (\cdots) \rangle_S \equiv \langle \xi (\cdots) \rangle_L$$

$$\left\langle e^{-2\phi} \frac{1}{2}\partial^2 c\, \partial c\, c(z) \right\rangle_S = 1 \Rightarrow \begin{cases} \text{PICT} = -2 \\ \text{GHOST} = 3 \end{cases}$$

LARGE HILBERT SPACE AND PICTURE

$$\xi_0 = \oint \frac{dz}{2\pi i} \frac{1}{z} \xi(z) \qquad [\eta_0, \xi_0] = 1$$

$$\eta_0 = \oint \frac{dz}{2\pi i} \eta(z) \qquad Q_B = \oint \frac{dz}{2\pi i} j_B(z)$$

SMALL HILBERT SPACE (NO ξ_0-DEPENDENCE)

$$\frac{\partial}{\partial \xi_0} = 0 \qquad \boxed{H_S \equiv \mathrm{Ker}\,\eta_0}$$

$$\eta_0^2 = 0 \qquad Q_B^2 = 0 \qquad [\eta_0, Q_B] = 0$$

$$[\eta_0, \xi(z)] = 1 \qquad [Q_B, -c\xi\partial\xi e^{-2\phi}(z)] = 1$$

↳ CONTRACTING HOMOTOPY OPERATOR FOR η

CONTRACTING HOMOTOPY OPERATOR FOR Q $\not\in H_S$!!

→ THE COHOMOLOGY OF Q_B IS SAVED IN THE SHS!

$$\begin{array}{ccc} \eta & & \xi \\ Q_B & & -c\xi\partial\xi e^{-2\phi} \end{array}$$

$$X(z) = [Q_B, \xi(z)]$$

$$Y(z) = [\eta, -c\xi\partial\xi e^{-2\phi}(z)] = c\partial\xi e^{-2\phi}(z)$$

$$\left.\begin{array}{l} [Q, X] = 0 \quad \checkmark \\ [\eta, X] = 0 \quad \checkmark \end{array}\right\} \longrightarrow \boxed{X \in H_s} \quad \boxed{\text{picture} = +1}$$

$$\left.\begin{array}{l} [Q, Y] = 0 \\ [\eta, Y] = 0 \end{array}\right\} \longrightarrow \boxed{Y \in H_s} \quad \boxed{\text{picture} = -1}$$

$X \to$ PICTURE RAISING OPERATOR

$Y \to$ PICTURE LOWERING OPERATOR

(3) $\lim_{z \to 0} X(z)\, Y(0) = 1$

$$X_0 = \oint \frac{dz}{2\pi i} \frac{1}{z} X(z)$$

ϕ_p p = PICTURE

$$Q\phi_p = 0 \quad \to \quad X_0 \phi_p = \phi_{p+1} \quad Q\phi_{p+1} = 0$$

$$\overset{X}{\longrightarrow}$$

$$\cdots \to \phi_{p-1} \to \phi_p \to \phi_{p+1} \to$$

$$\underset{Y}{\longleftarrow}$$

SUPERSTRING PHYSICAL STATES ARE INFINITELY DEGENERATED

$\updownarrow$

$\mathbb{Z}_2$-degeneration $\begin{cases} \text{INTEGRATED} \\ \text{UN-INTEGRATED VERTEX OPERATORS} \end{cases}$

$$\langle (- - - -) \rangle_S \equiv \text{AMPLITUDE}$$

TREE-LEVEL

$$(- - - -) \to \text{pict} = -2$$

$$(- - - -) \to \text{ghost} = +3$$

CONTRACTING HOMOTOPY OPERATOR α

$$Q^2 = 0 \qquad \alpha \,/\, [Q, \alpha] = 1$$

$$Q\Psi = 0 \qquad Q\,\alpha\Psi = \Psi - \alpha \cancel{Q\Psi} = 0$$

$$\Psi = Q(\alpha\Psi)$$