LECTURE VI

Type II A/B

D- bromes

RR- chorpes

RECAP

- NS & R states
- BOSONIZATION

$$\psi_\pm^I = e^{\pm i H_I} \qquad \begin{cases} \beta = e^{-\phi}\partial\xi \\ \gamma = \eta e^{\phi} \end{cases}$$

NS STATES $\rightarrow e^{i\nu_I H_I + \nu_6 \phi}$ $\quad \nu_I, \nu_6 \in \mathbb{Z}$

R STATES $\rightarrow e^{i s_I H_I + s_6 \phi}$ $\quad s_I, s_6 \in \mathbb{Z}+\frac{1}{2}$

- PICTURE $\quad J_P = :\xi\eta: - \partial\phi$
- PICTURE CHANGING OPERATORS

$$X(z) = [Q, \xi(z)]$$

$$Y(z) = \left[\eta, -c\xi\partial\xi e^{-2\phi}\right] = c\partial\xi e^{-2\phi}$$

$$\phi^{(p+1)}(z) = [Q_B, \xi\phi^{(p)}(z)] \rightarrow$$ INFINITE DEGENERACY OF PHYSICAL STATES

- CANONICAL VACUA

$$|0\rangle_{NS} = e^{-\phi}|0\rangle_{SL(2,\mathbb{C})} \quad (\text{pict} = -1)$$

$$|d\rangle_R = S_d e^{-\phi/2}|0\rangle_{SL(2,\mathbb{C})} \quad (\text{pict} = -\tfrac{1}{2})$$

GSO PROJECTION

NS $\quad e^{i\nu_e H_e + \nu_6 \phi} \qquad (\nu_e, \nu_6) \in \mathbb{Z}$

R $\quad e^{i s_e H_e + s_6 \phi} \qquad (s_e, s_6) \in \mathbb{Z} + \frac{1}{2}$

$NS \times NS \to NS$

$NS \times R \to R$

$R \times R \to NS$

NS $\to$ $GSO_{NS}^{(+)} \Rightarrow \sum_e \nu_e + \nu_6 = \text{EVEN}$ — CLOSED UNDER OPE

NS $\to$ $GSO_{NS}^{(-)} \Rightarrow \sum_e \nu_e + \nu_6 = \text{ODD}$

R $\to$ $GSO_R^{(+)} \to \sum_e s_e + s_6 = \text{EVEN}$

R $\to$ $GSO_R^{(-)} \to \sum_e s_e + s_6 = \text{ODD}$

FOR FIXED s_6 → CHIRALITY OF THE SPACETIME FERMION $|\{s_e\}\rangle$

OPE's

$$e^{i P_e H_e + P_\sigma \phi}(z)\, e^{i q_e H_e + q_\sigma \phi}(w) =$$

$$= (z-w)^{-P_e q_e + P_\sigma q_\sigma}\, e^{i(P_e+q_e)H_e + (P_\sigma+q_\sigma)\phi}(w) + \cdots$$

$\Rightarrow$ LOCALITY (W THE CHIRAL VECTOR)

$$-P_e q_e + P_\sigma q_\sigma \in \mathbb{Z}$$

PROBLEMS

$$GSO^{(-)}_{NS}(z)\; GSO^{(\pm)}_{R}(w) \sim \frac{1}{\sqrt{z-w}}(\cdots) \quad \text{NO}$$

$$GSO^{(-)}_{R}(z)\; GSO^{(+)}_{R}(w) \sim \frac{1}{\sqrt{z-w}}(\cdots) \quad \text{NO}$$

A MINIMAL CONSISTENT SECTOR IS

$$GSO_{NS}^{(+)} \oplus \begin{cases} \to GSO_R^{(+)} \\ \to GSO_R^{(-)} \end{cases} \to \text{2 POSSIBLE CHOICES}$$

- $\to$ GIVES RISE TO SPACETIME SUPERSYMMETRY !

- LOWEST STATES in the natural picture $(-1, -\frac{1}{2})$

NS $\quad c\psi^\mu e^{-\phi} e^{ip\cdot X}$

$GSO_{NS}^{(+)} \to 10_V$

$SO(1,9)$

$$R \to \begin{cases} \to c S_\alpha e^{-\phi/2} e^{ip\cdot X} \to 16_C \quad GSO_R^{+} \\ \to c S_{\dot\alpha} e^{-\phi/2} e^{ip\cdot X} \to 16_S \quad GSO_R^{-} \end{cases}$$

$\Rightarrow$ NO TACHYON

$\sim c e^{-\phi} e^{ip\cdot X} \in GSO_{NS}^{(-)} \to$ PROJECTED AWAY

PHYSICAL CONDITIONS - BRST

→ STATES / OPERATOR - OSCILLATOR FORMALISM

NS: $c_1 \psi^\mu_{-1/2} |0,p\rangle_{NS}$ $\quad |q,p\rangle_{NS} \equiv e^{ip\cdot X} e^{-\phi}(0)|0\rangle_{SL(2,\mathbb{R})}$

$\gamma_r |0,p\rangle_{NS} = 0 \quad r \geq \frac{1}{2}$ $\quad \psi_r |q,p\rangle_{NS} = 0 \quad r \geq \frac{1}{2}$

$\beta_r |q,p\rangle_{NS} = 0 \quad r \geq -\frac{1}{2}$

R: $\begin{cases} c_1 |d,p\rangle_R \\ c_1 |\bar{d},p\rangle_R \end{cases}$ $\quad |d^{(i)},p\rangle_R = e^{ip\cdot X} S_{(i)} e^{-\phi/2}|0\rangle$

$\gamma_m |d^{(i)},p\rangle_R = 0 \quad m \geq 1$ $\quad \psi_m |d^{(i)},p\rangle_R = 0 \quad m \geq 1$

$\beta_m |d^{(i)},p\rangle_R = 0 \quad m \geq 0$

NS: $G(z) = \sum_{r \in \mathbb{Z}+\frac{1}{2}} G_r z^{-r-\frac{1}{2}}$

R: $G(z) = \sum_{m \in \mathbb{Z}} G_m z^{-m-\frac{1}{2}}$

$Q \; \xi_\mu \psi^\mu_{-1/2} c_1 |q p\rangle^{(-1)}_{NS} = 0$

$L_0^{TOT} = \frac{d}{4} + (-1) + (\frac{1}{2}) + (\frac{1}{2})$

$c_1 \quad e^{-\phi} \quad \psi^\mu_{-1/2}$

$\Rightarrow \begin{cases} L_0^{TOT} = 0 \to p^2 = 0 = m^2 \\ L_m^{(m)} = 0 \quad m \geq 1 \to V \\ G_r^{(m)} = 0 \quad r \geq \frac{1}{2} \to \boxed{p \cdot \xi = 0} \end{cases}$

$G_r^{(m)} = \sum_{m \in \mathbb{Z}} \alpha_m \cdot \psi_{r-m} \to G^{(m)}_{1/2} \to \alpha_0 \cdot \psi_{1/2}$

$1 Q_V \to \boxed{\delta_V \text{ of } SO(8)}$

$Q \; u^a c_1 |q p\rangle^{(-1)}_R = 0$

$L_0^{TOT} = \frac{d}{4} + (-1) + \frac{5}{8} + \frac{3}{8}$

$c \quad s \quad e^{-\phi/2}$

$\Rightarrow \begin{cases} L_0^{TOT} = 0 \to p^2 = 0 \\ L_m^{(m)} = 0 \quad m \geq 1 \to V \\ G_m^{(m)} = 0 \quad m \geq 0 \to (\infty) \end{cases}$

$G_m^{(m)} = \sum_{m \in \mathbb{Z}} \alpha_m \cdot \psi_{m-m} \to G_0^{(m)} \to \alpha_0 \cdot \psi_0 = \not{p}$ (p ↑, Γ →)

DIRAC OPERATOR

$(\infty) \quad \boxed{p_\mu \Gamma^\mu_{\alpha\beta} u^\beta = 0} \qquad G_0^2 \propto L_0$

$1 \delta_C \to \boxed{\delta_C \text{ of } SO(8)}$ ↔ $\delta_V + \delta_C$ ⇒ SPACETIME SUSY!

$1 \delta_S \to \boxed{\delta_S \text{ of } SO(8)}$ ↔ $\delta_V + \delta_S$

Type II SUPERSTRINGS

Left × Right

II A

$$\left(GSO_{NS}^{(+)} \oplus GSO_{R}^{(+)}\right) \otimes \overline{\left(GSO_{NS}^{(+)} \oplus GSO_{R}^{(-)}\right)}$$

II B

$$\left(GSO_{NS}^{(+)} \oplus GSO_{R}^{(+)}\right) \otimes \left(GSO_{NS}^{(+)} \oplus GSO_{R}^{(+)}\right)$$

II A

Massless: $\underset{NS}{(g_\mu + g_c)} \otimes \underset{\overline{NS}}{(g_\mu + g_6)}$ $\quad \underset{R}{\quad} \quad \underset{\overline{R}}{\quad}$

$NS\text{-}\overline{NS} \to 8_V \times 8_V = 1 + 28_A + 35_S$

$1 \to \Phi$ $\quad 8_A \to B_{\mu\nu}$ $\quad 35_S \to G_{\mu\nu}$

$NS\text{-}R \to 8_V \times 8_S = 8_C + 56_C$

$8_C \to \chi_\alpha + \Psi_\alpha^\mu$

↳ DILATINO

$R\text{-}\overline{NS} \to 8_C \times 8_V = 8_S + 56_S$

$8_S \to \chi_\alpha + \Psi_\alpha^\mu$

↳ DILATINO

$R\text{-}\overline{R} \to 8_C \times 8_S = 8_V + 56_A$

$8_V \to C_\mu$ $\quad 56_A \to C_{\mu\nu\rho}$ → RR FORMS

TYPE IIA SUGRAVITY

$G_{\mu\nu}, B_{\mu\nu}, \Phi$; C_μ $C_{\mu\nu\rho}$; FERMIONS

Type IIB case

$$(8_v + 8_c) \otimes (8_v + 8_c)$$

$\Rightarrow$ Φ $B_{\mu\nu}$ $G_{\mu\nu}$ NS NS

χ_α $\psi^\mu_{\dot\alpha}$ } R-NS

$\chi'_{\dot\alpha}$ ψ'^μ_α } NS-R

$C^{(0)}$ $C_{\mu\nu}$ $C^{(4)}_{\mu\nu\rho\sigma} = C_4$

$(4) \rightarrow$ SELF-DUAL

LOP: $F_5 = d C_4$ $\boxed{*F_5 = F_5}$

$\Rightarrow$ Type IIB SUPERGRAVITY

Let's look closer at the RR spectrum of, say, Type IIA.

$(-\frac{1}{2},-\frac{1}{2})$ picture

$$F_{\mu_1\dots\mu_n}\,\tilde{S}^T C\Gamma^{\mu_1\dots\mu_n} S \qquad \tilde{S}\in 16_c \quad S\in 16_s$$

$S_2 e^{-\phi/2}$

Using $\Gamma_\mu^T C = C\Gamma_\mu$ and $\Gamma_{11}^T C = -C\Gamma_{11}$ we find that $n=2p$ $\Rightarrow$ $F^{(2p)}$

$$G_0 \to p_\mu F_{\mu_1\dots\mu_n}\tilde{S}^T C\Gamma^{\mu_1\dots\mu_n}\Gamma^\mu S = 0$$

$$\bar{G}_0 \to p_\mu F_{\mu_1\dots\mu_n}\tilde{S}^T\Gamma^{\mu T} C\Gamma^{\mu_1\dots\mu_n} S$$

$$\Gamma^\mu\Gamma^{\mu_1\dots\mu_p} = \Gamma^{\mu_1\dots\mu_p} + p\,\eta^{\mu[\mu_1}\Gamma^{\mu_2\dots\mu_p]}$$

$$\begin{cases} p^\mu F_{\mu\mu_1\dots\mu_p} = 0 \\ p_{[\mu} F_{\mu_1\dots\mu_p]} = 0 \end{cases} \Longrightarrow \begin{cases} d{*}F = 0 \quad \text{(MAXWELL)} \\ dF = 0 \quad \text{(BIANCHI)} \end{cases}$$

$$\Rightarrow F^{(2p)} \underset{\text{loc}}{=} dC^{(2p-1)}$$

ELECTRIC AND MAGNETIC FIELD STRENGTHS

- Γ matrices are "self-dual"

$$\Gamma_{11}\,\Gamma^{\mu_1\cdots\mu_p} = \varepsilon^{\mu_1\cdots\mu_p\nu_1\cdots\nu_{10-p}}\,\Gamma_{\nu_1\cdots\nu_{10-p}}$$

$$\boxed{F_{(10-p)} = *_{10} F_p} \Rightarrow \left(\begin{array}{c}\text{type IIB} \\ F_5 = *_{10} F_5\end{array}\right)$$

How to couple these new d.o.f.??

$$A_\mu^{(1)} \rightarrow \text{PARTICLE}$$

$$q\int d\tau\, \dot{X}^\mu A_\mu = q\int_{M_1} A \;\rightarrow\; q = \int_{S_{D-p}} *F_2$$

GAUSS LAW

$C^{(p)} \to (p-1)$ DIMENSIONAL OBJECT

$$Q_{p-1} \int_{M_p} C_p \quad \longrightarrow \quad Q_{p-1} = \int_{S^{9-p}} \star F^{(p+1)}$$

$\Rightarrow$ p-DIMENSIONAL CHARGED OBJECTS ARE THE Dp-branes! (POLCHINSKI)

- In type II theories Dp-branes are charged under RR-potentials

  → TypeIIA → CHARGED Dep-branes

  → TypeIIB → CHARGED D(2p+1)-branes

$$Q_p = \frac{1}{g_s (2\pi)^p \alpha'^{\frac{p+1}{2}}} = \frac{\sqrt{2\pi}\,(2\pi\sqrt{\alpha'})^{3-p}}{\sqrt{2}\,\kappa_{10}}$$

$$\kappa_{10} = 8\pi^{7/2} \alpha'^{2} g_s$$

- They also have a TENSION

$T_p = Q_p$!!!

- Dp is ELECTRICALLY CHARGED UNDER F_{p+2}

$Q_p \int_{M_{p+1}} C_{p+1}$ $\qquad Q_p = \int_{S_{8-p}} * F_{p+2}$

- D(6-p) is MAGNETICALLY CHARGED UNDER F_{p+2}

$Q_{6-p} \int_{M_{7-p}} C_{7-p}$ $\qquad Q_{6-p} = \int_{S_{p+2}} * F_{8-p} = \int_{S_{p+2}} F_{p+2}$

- SIMULTANEOUS PRESENCE OF MAGNETIC AND ELECTRIC CHARGE

$\Rightarrow$ DIRAC QUANTIZATION CONDITION

$$q_e q_m = \frac{2\pi M}{\#} \qquad M \in \mathbb{Z}$$

$$\rightarrow \frac{1}{\#(2p)!} \int F_{2p} \wedge \dots F_{2p}$$

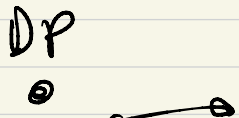
$$\# = 2\kappa_{10}^2 = (2\pi)^7 \alpha'^4 g_s^2 \sim G_N$$

$$\boxed{Q_p Q_{6-p} = \frac{2\pi}{(2\kappa_{10}^2)}} \quad !!! \quad M = 1 \; !!$$

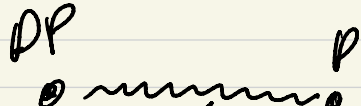
$\Rightarrow$ Dp branes are the MINIMAL QUANTA OF RR-charge

- We can have Dp-branes, we can have ANTI-Dp branes (same tension, opposite RR charge)

- 2 Dp branes DON'T FEEL ANY FORCE



NO FORCE $\rightarrow$ $T_p = Q_p$



exchange of CLOSED STRINGS
$\rightarrow$ (NS NS) OR (R R)

$$RR \sim Q_p^2 \frac{1}{r^{d-2}}$$ (COULOMB POTENTIAL) (REPULSIVE)

$$NS\,NS \sim -T_p^2 \frac{1}{r^{d-2}}$$ (NEWTON POTENTIAL) (ATTRACTIVE)

- $T_p \sim \frac{1}{g_s} \Rightarrow$ NON-PERTURBATIVE

$\Rightarrow e^{-\frac{1}{g_s}}$ EFFECTS IN STRING'S AMPLITUDES

D-BRANES AND SUPERSYMMETRY

→ BREAKS HALF OF SPACE-TIME SUSY

(IIA)

$$Q_\alpha(z), \quad \bar{Q}_{\dot\alpha}(\bar z)$$

$$\| \qquad\qquad \|$$

$$S_\alpha e^{-\phi/2}(z) \qquad \bar S_{\dot\alpha} e^{-\bar\phi/2}(\bar z)$$

→ SPACE-TIME FERMIONIC SUPERCHARGES

$$Q_\alpha(z)\, Q_\beta(w) = \frac{1}{\sqrt{2}} \frac{(\Gamma_\mu)_{\alpha\beta}}{z-w} \underbrace{\psi^\mu e^{-\phi}}$$

$$X_0^{PICT}\left(\psi^\mu e^{-\phi}\right) = \int j^\mu$$

MOMENTUM CHARGE

$$Q_\alpha\, Q_\beta \sim \Gamma_\mu\, p^\mu$$

→ PICTURE CHANGING IS NEEDED TO FULLY IMPLEMENT SPACE-TIME SUSY!

FORBIDDEN BY GSO

$$Q_\alpha(z)\, Q_{\bar\beta}(w) = \frac{C_{\alpha\dot\beta}}{(z-w)^{3/2}} e^{-\phi} + \frac{1}{2} \frac{(\Gamma_\mu \Gamma_\nu)}{(z-w)^{1/2}} :\psi^\mu \psi^\nu:$$

→ NON LOCAL!

To understand this we need the following

OPE's

$$S_\alpha(z)\, S_\beta(w) = \frac{1}{\sqrt{2}} \frac{(\Gamma^m)_{\alpha\beta}}{(z-w)^{3/4}} \psi^m(w)$$

$$S_\alpha(z)\, S_\beta(w) = \frac{C_{\alpha\beta}}{(z-w)^{5/4}} + \frac{1}{2} \frac{(\Gamma^m \Gamma^n)_{\alpha\beta}}{(z-w)^{1/4}} :\psi^m \psi^n:(w)$$

$$e^{-\phi/2}(z)\, e^{-\phi/2}(w) = \frac{1}{(z-w)^{1/4}}\, e^{-\phi}(w)$$

BOUNDARY CONDITIONS Dp-branes

Type IIB $\quad Q_\alpha, \overline{Q}_\beta$

Type IIA $\quad Q_\alpha, \overline{Q}_{\dot\beta}$

→ GLUING CONDITION FOR THE Q's

Type IIB

$$Q_\alpha(z) = P_\alpha{}^\beta \overline{Q}_\beta(\bar z) \qquad z=\bar z$$

$$Q_\alpha(z)\, Q_\beta(w) = \frac{1}{\sqrt{2}} \frac{(\Gamma_M)_{\alpha\beta}}{z-w} \psi^M e^{-\phi}(w)$$

$$\downarrow$$

$$P_\alpha{}^\gamma \overline{Q}_\gamma(\bar z)\, P_\beta{}^\delta \overline{Q}_\delta(\bar w) = \frac{1}{\sqrt{2}} \frac{(\Gamma_M)_{\alpha\beta}}{\bar z-\bar w} \Omega^M{}_N \bar\psi^N e^{-\bar\phi}$$

$$\begin{cases} j^M(z) = \Omega^M{}_N \bar j^N(\bar z) \\ \psi^M(z) = \Omega^M{}_N \bar\psi^N(\bar z) \end{cases}{}_{z=\bar z} \qquad \Omega^M{}_N = \left(\begin{array}{c|c} \delta^\mu_\nu & \\ \hline & -\delta^i_j \end{array}\right)$$

$$\Rightarrow \boxed{P_\alpha{}^\gamma P_\beta{}^\delta (\Gamma_N)_{\gamma\delta} = (\Gamma_M)_{\alpha\beta} \Omega^M{}_N}$$

$$P \Gamma_N C^{-1} P^T = \Lambda^M{}_N \Gamma_M C^{-1}$$

check $\qquad P_p^{\pm} = \pm \prod_{i=p+2}^{9} (\Gamma_i \Gamma)$

IIB) $\quad P = P_+{}^{\beta}$ $\longrightarrow$ P is ODD

(P DON'T CHANGE THE CHIRALITY)

$\hookrightarrow$ CONSISTENT GLUING CONDITION FOR THE SUPERCHARGE FOR D(2+1) BRANES!

IIA) $\quad \tilde{P} = P_{\alpha}{}^{\dot{\beta}}$ (P CHANGES THE CHIRALITY)

$$Q_{\alpha}(z) = \tilde{P}_{\alpha}{}^{\dot{\beta}} \bar{Q}_{\dot{\beta}}(\bar{z}) \qquad z = \bar{z}$$

$$\tilde{P}_p^{\pm} = \pm \prod_{i=p+2}^{9} (\Gamma_i \Gamma) \rightarrow$$ P IS EVEN

IIA / IIB There are CHARGED Dp-branes which PRESERVE HALF SPACE-TIME SUPERSYMMETRY

$$Q(\xi) = P\,\bar{Q}(\bar{\xi}) \qquad \xi = \bar{\xi}$$

$$P = \pm \prod_{i=p+2} \Gamma^i \Gamma_{11}$$

$\pm$ → BRANES
→ ANTI-BRANES

$\int_{M_{p+1}} C^{(RR)}_{p+1} \rightarrow C_{p+1}$ IS A VOLUME FORM ON THE WORLD-VOLUME

→ ≥ 0 (POSITIVE ORIENTATION) → BRANE

→ < 0 (NEGATIVE ORIENTATION) → ANTI BRANE

PRESERVED SUSY

$D_p \rightarrow Q + P_p \bar{Q}$ $\rightarrow \frac{1}{2}$ BPS

$\bar{D}_p \rightarrow Q - P_p \bar{Q}$ $\rightarrow \frac{1}{2}$ BPS

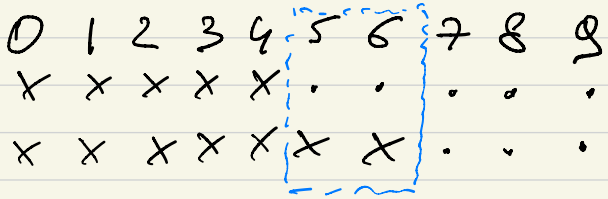
OPPOSITE

$D_p - \bar{D}_p$ BROKEN SUSY COMPLETELY !!!

↳ TACHYON CONDENSATION

$\rightarrow D\bar{D}$ DECAY

DP - DP' SYSTEM



$$Dp \to Q + \Pi_p \bar{Q}$$

$$Dp' \to Q + \Pi_{p'} \bar{Q} = Q + \Pi_p \left(\Pi_p^{-1} \Pi_{p'}\right) \bar{Q}$$

EXERCISE $\quad \Pi_p^{-1} \Pi_{p'} = \prod_{i \in ND} \Gamma_i$

$$\Rightarrow \Pi_p^{-1} \Pi_{p'} \bar{Q} = \bar{Q}$$

NEED SOME EIGENVALUE = 1

$$\left(\Gamma_i \Gamma_{i+1}\right)^2 = -1 \to \Gamma_i \Gamma_{i+1} \text{ HAS EIGENVALUES } \pm i$$

$$\Rightarrow \boxed{p - p' = 4M}$$

$DP/D(P+4)$ $(\frac{1}{4}$ BPS$)$

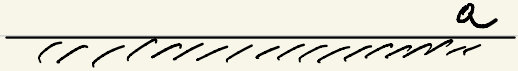
$DP/D(P+2) \Rightarrow$ BROKEN SUSY !

BOUNDARY STATES

- D-brane can be represented as a STATE in the CLOSED STRING HILBERT SPACE

- D-brane → BOUNDARY CONDITION ON THE WORLDSHEET

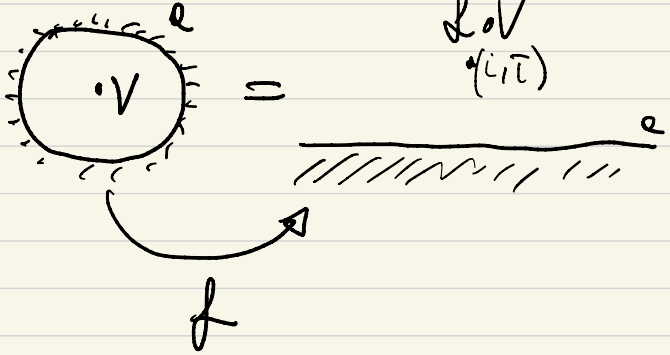
$|B^{(a)}\rangle$

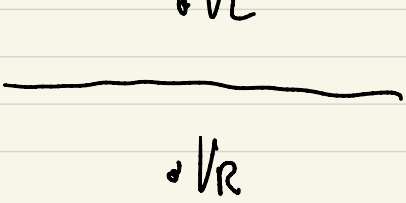


$\langle\, 1 \,\rangle$ → NON DEGENERATE

$\langle B^{(a)}|V\rangle$ V is ANY CLOSED STRING STATE (OFF-SHELL)

$\langle V(0,0)\rangle_{\text{DISK}}^{(a)}$





For Dp branes we have:

- CHARGED Dp-brane

$$|Dp\rangle = T_p |NSNS\rangle^{(p)} \pm Q_p |RR\rangle^{(p)}$$

- NON BPS D-branes ("WRONG" DIMENSIONALITY)

→ NON CHARGED

$$|Dp\rangle^{(non\text{-}BPS)} = \sqrt{2}\, T_p |NSNS\rangle^{(p)} \quad (\text{NO RR SECTOR!!})$$

- $D\bar{D}$ pair (COINCIDENT)

$$|D\bar{D}p\rangle = 2 T_p |NSNS\rangle^{(p)} \quad (\text{NO RR SECTOR!!})$$