Latitude Wilson loop in ABJM & dual line operators

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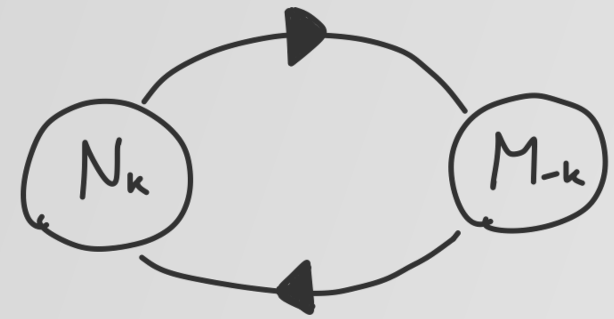
General context

ABJ(M): $\mathcal{N}=6$ supersymmetric
Chern-Simons matter theory



Supersymmetric Wilson loops: *[Gaiotto, Yin]*

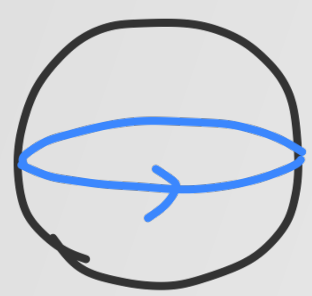
$$W = Tr\ Pexp\left(-i\oint_\gamma d\tau\left(\dot{x}^\mu A_\mu - \frac{2\pi i}{k}M_J{}^I C_I \bar{C}^J\right)\right)$$

Standard example

The great circle, 1/6-BPS:

[Drukker, Plefka, Young]

$$M_J{}^I = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



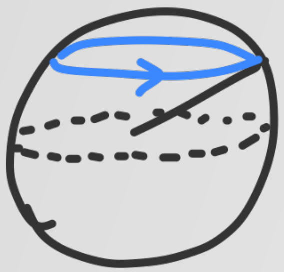
It preserves the $\mathfrak{su}(1,1\,|\,1)$ superconformal algebra on the circle

Latitude Wilson loop

Latitude, 1/12-BPS

[Bianchi, Griguolo, Mauri, Penati, Seminara]

$$M_J^{\ I} = \begin{pmatrix} -\nu & e^{-i\tau}\sqrt{1-\nu^2} & 0 & 0 \\ e^{i\tau}\sqrt{1-\nu^2} & \nu & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



- The loop preserves an $\mathfrak{su}(1\,|\,1)$ Poincaré superalgebra
- ν breaks the conformal invariance

Latitude & Brehmmstrahlung

- The Brehmmstrahlung function B is the two point function of the displacement operator [Correa, Maldacena, Sever]
- It can be computed from the latitude Wilson loop:

$$B_{\frac{1}{6}} = \frac{1}{4\pi^2}\frac{\partial}{\partial\nu}\log|\langle W_L(\nu)\rangle|\Big|_{\nu=1}$$

- Crucial parameter of the dCFT, bridge with integrability (open question)

ABJ(M) matrix models

Great circle: $\nu = 1$
(localization)

[Kapustin, Willett, Yaakov]

Latitude: $\nu \in (0,1)$
(conjecture)

[Bianchi, Griguolo, Mauri, Penati, Seminara]

$$\langle W_c \rangle = \langle \frac{1}{N} \sum_i e^{2\pi\lambda_i} \rangle_{MM} \qquad \langle W_L \rangle = \langle \frac{1}{N} \sum_i e^{2\sqrt{\nu}\pi\lambda_i} \rangle_{MM}$$

$$\langle \quad \rangle_{MM} = \frac{1}{N!M!} \int d\lambda_i \, d\mu_j \, e^{i\pi k(\lambda_i^2 - \mu_i^2)} Z_{vec}(\lambda_i) \, Z_{vec}(\mu_j) \, Z_{mat}(\lambda_i, \mu_j)$$

$$Z_{vec}^{ABJM}(\lambda_i) = \prod_{i<j} \sinh^2 \pi(\lambda_i - \lambda_j) \longrightarrow Z_{vec}^{lat}(\lambda_i) = \prod_{i<j} \sinh \pi\sqrt{\nu}(\lambda_i - \lambda_j) \sinh \pi \frac{(\lambda_i - \lambda_j)}{\sqrt{\nu}}$$

$$Z_{mat}^{ABJM}(\lambda_i) = \frac{1}{\prod_{i,j} \cosh^2 \pi(\lambda_i - \mu_j)} \longrightarrow Z_{mat}^{lat}(\lambda_i) = \frac{1}{\prod_{i,j} \cosh \pi\sqrt{\nu}(\lambda_i - \mu_j) \cosh \pi \frac{(\lambda_i - \mu_j)}{\sqrt{\nu}}}$$

$\langle 1 \rangle_{MM}^{ABJM} = \langle 1 \rangle_{MM}^{lat}$ but $\langle W_c \rangle_{MM}^{ABJM} \neq \langle W_L \rangle_{MM}^{lat}$

Localization result

- Z^{lat}_{vec} and Z^{lat}_{mat} seems to come from an ellipsoid geometry *[Hama, Hosomichi, Lee]*
- The bosonic symmetry Q^2 guides the localization scheme

$$Q^2 \propto -\sqrt{\nu}\partial_\varphi + \frac{1}{\sqrt{\nu}}\partial_\tau + G + R$$

- We close the SUSY off-shell using cohomological multiplets *[Kallen]*
- On the loop G coincides with the connection of the Wilson loop
- Squashing isometry -> It explains Z^{lat}_{vec}
- R is a mass deformation -> It reproduces Z^{lat}_{mat} *[Gaiotto, Okazaki]*
- The precise mapping to the cohomological multiplet is missing

Interpolating supercharge

The latitude supercharge can be embedded into a $\mathcal{N}=4$ subalgebra

$$(\xi_\nu)_{IJ} \to (\xi_\nu)_{a\dot{a}}$$

There is an interesting ν-interpolating structure:

- $\nu = 1$ is the usual $\mathcal{N}=2$ supercharge (KWY)
- $\nu = 0$ is related to new topological QM inside 3d $\mathcal{N}=4$ theories, like chiral algebra in 4d theories *[Pufu et al.]*

A dual relation with ABJM

ABJM at k=1 ⟷ (IR dual) $\mathcal{N}=4$ SYM theory with an adjoint hyper and a fundamental hyper

More line operators

Disorder line operators -> Vortex loops

- They are singular classical configuration defined by the holonomy around the curve
- We can impose a BPS condition $D_{sing.} = iF_{sing.}$ *[Kapustin, Willett, Yaakov]*
- They are the mirror symmetric duals of the Wilson loops

BPS-Wilson loops $\longleftrightarrow$ BPS-Vortex loops

Mirror symmetry

[Assel, Gomis]

A vortex loop has a defect description in terms of:

- $U(1)$ gauge field
- N charge 1 chiral multiplets
- N charge -1 chiral multiplets

A bound state of line operators

- The IR duality of ABJM is not only S-duality, it involves T-transformation
- Conjecture: the dual operator is a mixed Wilson-vortex loop

How can we realize it?

- Add a 1d level 1 Chern Simons term in the vortex setup for the $U(1)$ gauge field
- Turn on a flavor gauge field for the $U(1)_C$ R-symmetry

Consistency check:

- The limit $\nu \to 1$ exhibit the correct SUSY enhancement
- The ν-refined Witten index $I(\nu)$ reproduces the latitude matrix model

$$Z_{3d/1d} = \int d\sigma\, Z_{3d}(\sigma)\, I(\nu) \equiv \langle W_L \rangle$$

Conclusion

- A new localization for the latitude supercharge
- Dual operators for (bosonic) Wilson loops in ABJ(M)
- Novel type of mixed Wilson-vortex loop

Thank you!