BRST-Lagrangian Double Copy of Yang-Mills Theory

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21 December 2020

Based on joint work with L Borsten, B Jurčo, H Kim, C Saemann, M Wolf
2007.13803, 2101.?????
Naive idea: \[ A_\mu \otimes A_\nu = g_{\mu\nu} \oplus B_{\mu\nu} \oplus \phi \]

First concrete incarnation: KLT relations

A purely field theoretic approach: BCJ color–kinematic duality and double copy prescription
Flash review of BCJ duality and double-copy

- Every Lagrangian field theory is equivalent to a theory with only cubic interactions

- n-points L-loops YM amplitude as sums of trivalent graphs

\[ \mathcal{A}_{n,L}^{YM} = \sum_i \int \prod_{l=1}^{L} d^d p_l \frac{1}{S_i} \frac{C_i N_i}{D_i} \]

- \( i \) ranges over all trivalent \( L \)-loops graphs

- \( C_i \): color factor, composed of gauge group structure constants

- \( N_i \): kinematic factor, composed of Lorentz-invariant contractions of polarisations and momenta
Flash review of BCJ duality and double-copy

**BCJ color–kinematic duality**

There is a choice of kinematic factors such that $N_i$s obey the same algebraic relations (e.g., Jacobi identity) of the correspondent $C_i$

- True at tree-level, conjectured for loop-level
- Gravity amplitudes can be represented as sum over trivalent graphs, too

**Yang–Mills double copy**

If BCJ duality holds true, replacing the color factor with a copy of the kinematic factor in $A_{n,L}^{YM}$ produces a $\mathcal{N} = 0$ supergravity amplitude

- All-loop statement, the problem is then to validate BCJ duality at loop level
 Until now, on-shell scattering amplitude approach

 An off-shell Lagrangian realization of color-kinematic duality and double-copy could solve the all-loop conundrum

 **Our approach**

 Double-copy YM Lagrangian and BRST operator to obtain a theory equivalent to $\mathcal{N} = 0$ supergravity
There exist a non-local YM Lagrangian with manifest tree-level BCJ duality for on-shell physical gluons (Tolotti, Weinzierl, ’13).

We can insert auxiliary fields to make it local, and strictify to a Lagrangian with only cubic interaction

\[ \mathcal{L} = \frac{1}{2} \Phi^\alpha_i g_{\alpha\beta} G_{ij} \Box \Phi^\beta_j + \frac{1}{3!} \Phi^\alpha_i f_{\alpha\beta\gamma} F_{ijk} \Phi^\beta_j \Phi^\gamma_k + \text{ghosts + gauge fixing} \]

We want to double-copy the BRST-extended field space.

BCJ duality is satisfied only on-shell for physical gluons: eventual BCJ violations due to unphysical gluons and ghosts!
We compensate for these eventual BCJ violations with suitable field redefinitions.

We start with 4-gluon tree amplitude.

Unphysical gluons ruin BCJ duality.

We compensate the violation of BCJ duality adding terms to the gauge-fixing fermion and shifting the Nakanishi–Lautrup fields.
Next, gluon-gluon-ghost-antighost tree amplitude

Thanks to the Ward identity, we can write this amplitude in terms of 4-gluon tree amplitudes: we redefine the ghost sector to compensate for BCJ violations.

Recursive construction
For every tree amplitude with $n$ legs and $k$ pairs ghost-antighost, recursively redefine gauge-fixing sector and ghost sector.
We obtain a cubic action that manifest on-shell tree-level BCJ duality for the BRST-extended field space:

\[ \mathcal{L} = \frac{1}{2} \Phi^\alpha i g_{\alpha\beta} G_{ij} \Box \Phi^\beta j + \frac{1}{3!} \Phi^\alpha i f_{\alpha\beta\gamma} F_{ijk} \Phi^\beta j \Phi^\gamma k \]

BRST operator

\[ (Q \Phi)^\alpha i = \delta^\alpha_\beta q^i_\beta \Phi^\beta j + \frac{1}{2} f^\alpha_{\beta\gamma} Q^i_{jk} \Phi^\beta j \Phi^\gamma k + \frac{1}{3!} f^\alpha_{\beta\gamma\delta} Q^i_{jkl} \Phi^\beta j \Phi^\gamma k \Phi^\delta l \]

Double-copy (both Lagrangian and \(Q\))

\[ \mathcal{L}_{dc} = \frac{1}{2} \Phi^{i'i'} G_{i'j'} G_{ij} \Box \Phi^{j'j} + \frac{1}{3!} \Phi^{i'i'} F_{i'j'k'} F_{ijk} \Phi^{j'j} \Phi^{k'k} \]

\[ (Q_{dc} \Phi)^{i'i} = \ldots \]
If $F_{ijk}$ satisfies the same algebraic properties of $f_{\alpha \beta \gamma}$, then $Q^{2}_{dc} = 0$ and $Q_{dc} S_{dc} = 0$.

We have $Q^{2}_{dc} = 0$ and $Q_{dc} S_{dc} = 0$ on-shell, and that’s enough: it ensures the correct Ward identities and a consistent quantization.

$L_{dc}$ is related to $\mathcal{N} = 0$ supergravity by local field redefinitions.

We obtain a double-copy action $L_{dc}$ that is (perturbatively) quantum equivalent to $\mathcal{N} = 0$ supergravity.
Future

- Further clarification of the algebraic structure of Lagrangian double-copy in terms of homotopy algebras (*paper to appear, hopefully January*)
- Extend our approach to supersymmetric theories
- Homotopic description of open–closed string duality?
Thank you for listening!