BRST-Lagrangian Double Copy of Yang-Mills Theory

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Introduction: gravity = gauge × gauge

- Naive idea: "$A_\mu \otimes \bar{A}_\nu \; = \; g_{\mu\nu} \oplus B_{\mu\nu} \oplus \phi$"
- First concrete incarnation: KLT relations
- A purely field theoretic approach: BCJ color–kinematic duality and double copy prescription

Flash review of BCJ duality and double-copy

- Every Lagrangian field theory is equivalent to a theory with only cubic interactions
- n-points L-loops YM amplitude as sums of trivalent graphs

$$\mathcal{A}_{n,L}^{\text{YM}} = \sum_i \int \prod_{l=1}^{L} \mathrm{d}^d p_l \frac{1}{S_i} \frac{C_i N_i}{D_i}$$

- i ranges over all trivalent L-loops graphs
- C_i: color factor, composed of gauge group structure constants
- N_i: kinematic factor, composed of Lorentz-invariant contractions of polarisations and momenta

Flash review of BCJ duality and double-copy

BCJ color–kinematic duality

There is a choice of kinematic factors such that N_is obey the same algebraic relations (e.g., Jacobi identity) of the correspondent C_i

- True at tree-level, conjectured for loop-level
- Gravity amplitudes can be represented as sum over trivalent graphs, too

Yang–Mills double copy

If BCJ duality holds true, replacing the color factor with a copy of the kinematic factor in $\mathcal{A}_{n,L}^{\text{YM}}$ produces a $\mathcal{N} = 0$ supergravity amplitude

- All-loop statement, the problem is then to validate BCJ duality at loop level

BRST-Lagrangian perspective on double copy

- Until now, on-shell scattering amplitude approach
- An off-shell Lagrangian realization of color-kinematic duality and double-copy could solve the all-loop conundrum

Our approach

Double-copy YM Lagrangian and BRST operator to obtain a theory equivalent to $\mathcal{N} = 0$ supergravity

BRST-Lagrangian perspective on double copy

- There exist a non-local YM Lagrangian with manifest tree-level BCJ duality for on-shell physical gluons (Tolotti, Weinzierl, '13)
- We can insert auxiliary fields to make it local, and strictify to a Lagrangian with only cubic interaction

$$\mathcal{L} \;=\; \tfrac{1}{2}\Phi^{\alpha i}\mathsf{g}_{\alpha\beta}\mathsf{G}_{ij}\Box\Phi^{\beta j}+\tfrac{1}{3!}\Phi^{\alpha i}\mathsf{f}_{\alpha\beta\gamma}\mathsf{F}_{ijk}\Phi^{\beta j}\Phi^{\gamma k}+\text{ghosts}+\text{gauge fixing}$$

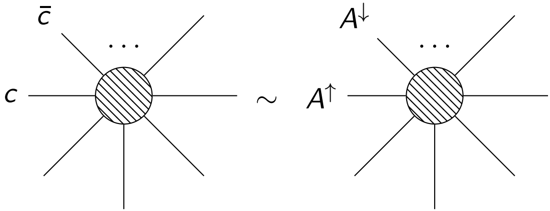
- We want to double-copy the BRST-extended field space
- BCJ duality is satisfied only on-shell for physical gluons: eventual BCJ violations due to unphysical gluons and ghosts!

BRST-Lagrangian perspective on double copy

- We compensate for these eventual BCJ violations with suitable field redefinitions
- We start with 4-gluon tree amplitude
- Unphysical gluons ruin BCJ duality
- We compensate the violation of BCJ duality adding terms to the gauge-fixing fermion and shifting the Nakanishi–Lautrup fields

BRST-Lagrangian perspective on double copy

- Next, gluon-gluon-ghost-antighost tree amplitude
- Thanks to the Ward identity, we can write this amplitude in terms of 4-gluon tree amplitudes: we redefine the ghost sector to compensate for BCJ violations



Recursive construction

For every tree amplitude with n legs and k pairs ghost-antighost, recursively redefine gauge-fixing sector and ghost sector

BRST-Lagrangian perspective on double copy

- We obtain a cubic action that manifest on-shell tree-level BCJ duality for the BRST-extended field space:

$$\mathcal{L} = \frac{1}{2}\Phi^{\alpha i} g_{\alpha\beta} G_{ij} \Box \Phi^{\beta j} + \frac{1}{3!}\Phi^{\alpha i} f_{\alpha\beta\gamma} F_{ijk} \Phi^{\beta j} \Phi^{\gamma k}$$

- BRST operator

$$(Q\Phi)^{\alpha i} = \delta^{\alpha}_{\beta} q^{i}_{j} \Phi^{\beta j} + \frac{1}{2} f^{\alpha}_{\beta\gamma} Q^{i}_{jk} \Phi^{\beta j} \Phi^{\gamma k} + \frac{1}{3!} f^{\alpha}_{\beta\gamma\delta} Q^{i}_{jkl} \Phi^{\beta j} \Phi^{\gamma k} \Phi^{\delta l}$$

- Double-copy (both Lagrangian and Q)

$$\begin{aligned} \mathcal{L}_{\mathrm{dc}} &= \frac{1}{2}\Phi^{i' i} G_{i'j'} G_{ij} \Box \Phi^{j' j} + \frac{1}{3!}\Phi^{i' i} F_{i'j'k'} F_{ijk} \Phi^{j' j} \Phi^{k' k} \\ (Q_{\mathrm{dc}}\Phi)^{i' i} &= \dots \end{aligned}$$

BRST-Lagrangian perspective on double copy

- If F_{ijk} satisfies the same algebraic properties of $\mathsf{f}_{\alpha\beta\gamma}$, then $Q_{\text{dc}}^2 = 0$ and $Q_{\text{dc}} S_{\text{dc}} = 0$
- We have $Q_{\text{dc}}^2 = 0$ and $Q_{\text{dc}} S_{\text{dc}} = 0$ on-shell, and that's enought: it ensures the correct Ward identities and a consistent quantization
- $\mathcal{L}_{\text{dc}}$ is related to $\mathcal{N} = 0$ supergravity by local field redefinitions
- We obtain a double-copy action $\mathcal{L}_{\text{dc}}$ that is (perturbatively) quantum equivalent to $\mathcal{N} = 0$ supergravity

Future

- Further clarification of the algebraic structure of Lagrangian double-copy in terms of homotopy algebras (*paper to appear, hopefully January*)
- Extend our approach to supersymmetric theories
- Homotopic description of open–closed string duality?

Thank you for listening!