The Holographic Swampland

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Based on 2006.01021 with Joseph Conlon

Introduction

Swampland program: criteria to distinguish low energy Lagrangians admitting a UV completion in ST (QG)

Topic of this talk: AdS/CFT

Swampland constraints on 4d EFTs ⟷ ? ⟷ CFT inconsistencies or reformulation

Why?

- Develop independent framework to test debated constructions
- AdS/CFT universal tool for QG, not just ST
- Bootstrap very successful in mapping space of allowed theories

Plan of the talk

Intro: Holographic CFTs

CFT positivity bounds and how to apply them

LVS dual as a motivating scenario

A conjecture on mixed anomalous dimensions

Other examples (E.g. KKLT)

Relation to distance conjecture

WORK IN PROGRESS

A new inequality on <u>mixed & identical</u> anomalous dimensions

Applications: DBI

<u>Conclusions and Outlook</u>

Holographic CFTs

Gravity dual is weakly coupled and amenable to perturbative analysis

$$\lambda = g^2 N \longrightarrow (R/\ell_S)^d >> 1$$

Expansion in large N parameter or equivalent

Large gap in the spectrum $\Delta_{gap} >> 1$

Single trace primaries $\mathcal{O}_1, \mathcal{O}_2$ Dimension Δ_1, Δ_2

OPE: $\mathcal{O}_1 \times \mathcal{O}_2 \supset \mathbb{1}, \mathcal{O}_1, \ \mathcal{O}_2, \ [\mathcal{O}_1 \mathcal{O}_2]_{n,l}$

Double trace operators: $[\mathcal{O}_1 \mathcal{O}_2]_{n,l} \sim \mathcal{O}_1 \Box^n \partial_{\mu_1} \partial_{\mu_2 \ldots} \partial_{\mu_l} \mathcal{O}_2$

$\Delta = \Delta_1 + \Delta_2 + 2n + \ell + \gamma(n,l)$ ⟶ anomalous dimension

The bootstrap

conformal blocks

$$\langle \mathcal{O}_1(x_1)\,\mathcal{O}_1(x_2)\,\mathcal{O}_2(x_3)\,\mathcal{O}_2(x_4)\rangle = \sum_{\mathcal{O}} C_{11\mathcal{O}} C_{22\mathcal{O}} \frac{G_{\Delta,\ell}(u,v)}{|x_{12}|^{2\Delta_1}\,|x_{34}|^{2\Delta_2}}$$

OPE coefficients

Bootstrap Equation:

$$u^{\Delta}\left(1+\sum_{\Delta,\ell} c^2_{\Delta,\ell} G_{\Delta,\ell}(v,u)\right) = v^{\Delta}\left(1+\sum_{\Delta,\ell} c^2_{\Delta,\ell} G_{\Delta,\ell}(u,v)\right)$$

No constraints from crossing symmetry alone

[Heemskerk,Penedones, Polchinski,Sully '05]

Solution to bootstrap equations

Quartic (derivative) vertices

CFT positivity bounds

Minimal twist operators dominate Lorentzian OPE on the light cone

$\tau = \Delta - \ell$

$\gamma(0,\ell)$ convex & negative for double traces made out of identical operators if $\ell_c \geq 2$

Analytical bootstrap [Komargodski, Zhiboedov '12]

Inversion formula [Caron-Huot '17] + [Costa, Hansen, Penedones '17]

Causality arguments in the CFT:

$$\gamma(0,2) \leq 0$$

[Hartman, Jain, Kundu '16]

$$\mathcal{L} = \frac{g}{\Lambda^4}(\nabla\varphi)^4 \quad g > 0$$

on AdS

Generalization of flat space S-matrix bounds [Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06]

The Large Volume Scenario

Motivating scenario: Low energy dynamics of moduli in ST

LVS: Type IIB flux compactification, with all moduli stabilised at an exponentially large volume

$$V = V_0 e^{-\lambda\Phi/M_P}\left(-\left(\frac{\Phi}{M_P}\right)^{3/2} + A\right) \qquad \Phi = \sqrt{\frac{2}{3}}\ln\mathcal{V}$$

canonically normalised volume modulus

AdS vacuum:

$$V_{\rm min} = -3M_P^2 R_{AdS}^{-2} \qquad m_\Phi = \sqrt{\frac{3}{2}}\frac{\lambda}{R_{AdS}}$$

[Balasubramanian,Berglund,Conlon,Quevedo ’05]

[Conlon,Quevedo,Suruliz ’05]

Holographic reformulation of LVS

AdS/CFT

4d moduli EFT in AdS

Crucial property of LVS:

$m_\phi << m_{3/2}$ as $\mathcal{V} \to \infty$

+

Putative 3d CFT dual

$$m^2 R_{AdS}^2 = \Delta(\Delta - 3)$$

Small number of low lying operators:

Mode	Spin	Parity	Conformal dimension
$T_{\mu\nu}$	2	+	3
a	0	-	3
Φ	0	+	$8.038 = \frac{3}{2}\left(1 + \sqrt{19}\right)$

[Conlon,Quevedo '18]

LVS Effective Lagrangian

All interactions fixed in term of R_{AdS} only-
uniquely determined theory in large $\mathcal{V}$ limit

$$\mathcal{L}_{(\delta\Phi)^n} = (-\lambda)^n \frac{3M_P^2}{R_{AdS}^2}\frac{n-1}{n!}\left(\frac{\delta\Phi}{M_P}\right)^n\left(1+\mathcal{O}\left(\frac{1}{\lambda\langle\Phi\rangle}\right)\right)$$

$$\mathcal{L}_{(\delta\Phi)^{n-2}aa} = \left(-\sqrt{\frac{8}{3}}\right)^{(n-2)}\frac{1}{2(n-2)!}\left(\frac{\delta\Phi}{M_P}\right)^{n-2}\partial_\mu a\partial^\mu a$$

Some modifications certainly in the Swampland

Sign flip in axion kinetic term → Divergent f_a as $\mathcal{V}\to\infty$

$$\mathcal{L}\supset\frac{3}{4}e^{-\sqrt{\frac{8}{3}}\frac{\Phi}{M_P}}\partial_\mu a\partial^\mu a$$

What happens to the CFT?

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What happens to the CFT?

Can we apply the positivity bounds?

<u>Contact diagrams:</u>

Finite support in the spin

$$\ell \leq \frac{\#\partial}{2}$$

[Heemskerk,Penedones, Polchinski,Sully '05]

[Fitzpatrick,Katz,Poland, Simmons-Duffin '12]

VS

<u>Exchange diagrams:</u>

Arbitrary large spin

BUT

Manifestly negative

$$\gamma(0,\ell) \propto -\lambda^2$$

For identical operators, not quite...

Claim: $\gamma^{\varphi a}(0,\ell)$ correlates with swampland conditions

Consequences for LVS

$$\gamma^{\varphi a}(0,\ell) \propto -\frac{g\mu(\Delta_\varphi - 6)}{\Gamma\left(\frac{6-\Delta_\varphi}{2}\right)} \frac{1}{M_P^2 R_{AdS}^2} \frac{1}{\ell^{\Delta_\varphi}}$$

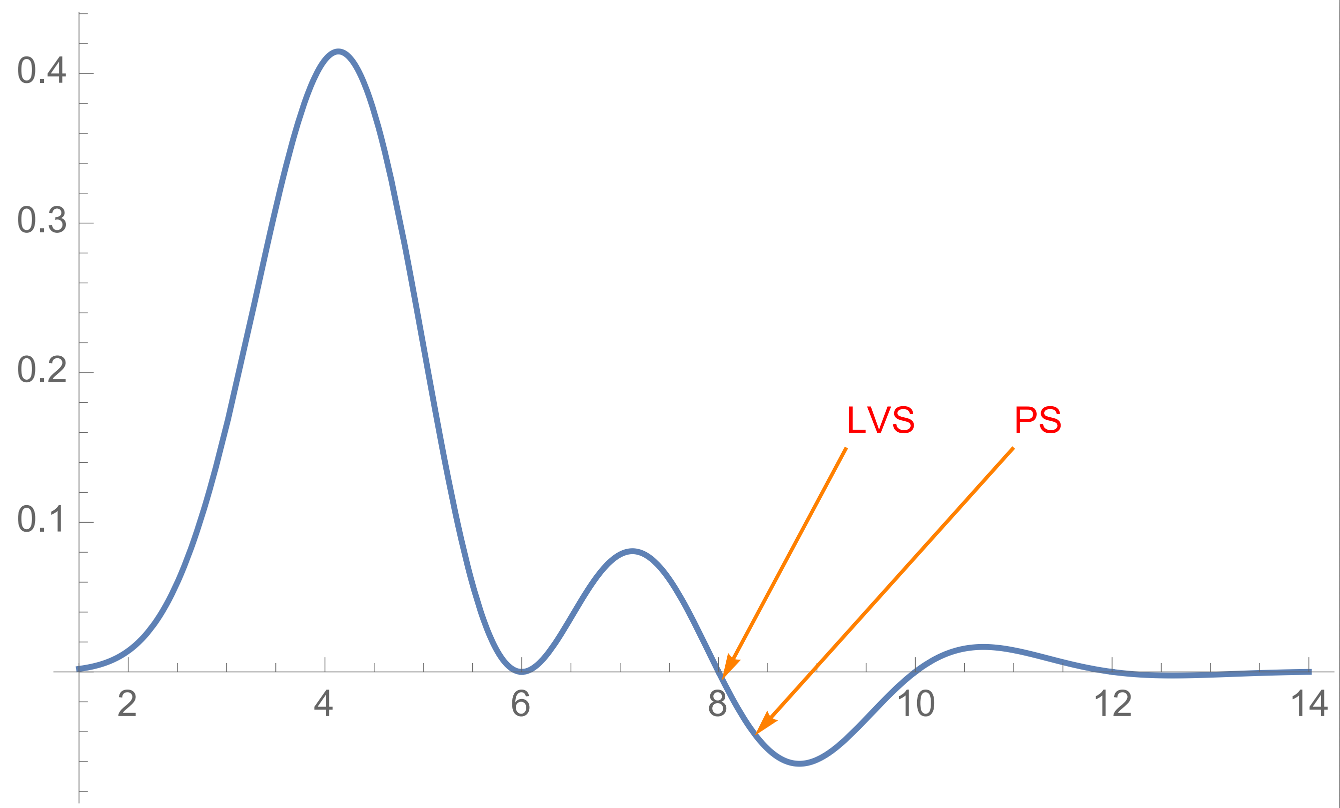
decouples higher order contributions at large volume

With LVS couplings, $\gamma^{\varphi a}(0,\ell) < 0$

Swampland

Sign flip in $g\mu$ $\longleftrightarrow$ $f_a \sim M_P \mathcal{V}^{\frac{2}{3}}$

$\Delta_\varphi \leq 6$ $\longleftrightarrow$ $V \gtrsim M_S^4$



LVS very close to boundary of the allowed region!

Other examples

Perturbative Stabilisation:
[Berg, Haack,Kors '06,
von Gersdorff, Hebecker '05]

$$V_{eff} = Ae^{-\lambda_1\varphi} - Be^{-\lambda_2\varphi}$$

LVS-like: Sign flip has same effect

$\gamma(0,\ell)_{\varphi a} < 0$ for $\lambda_1 = \frac{10}{3}\sqrt{\frac{3}{2}}, \lambda_2 = 3\sqrt{\frac{3}{2}}$

KKLT (before uplifting): [Kachru,Kallosh,Linde,Trivedi '03]

$$\mathcal{K} = -3\log\left[-i(T-\bar{T})\right] \quad \mathcal{W} = W_0 + Ae^{-\alpha T}$$

Racetrack Stabilisation: [Krasnikov '87, Taylor '90,
De Carlos, Casas, Munoz '93]

$$\mathcal{K} = -3\log[-i(T-\bar{T})] \quad \mathcal{W} = Ae^{-\alpha T} - Be^{-\beta T}$$

For both $\gamma(0,\ell)_{\varphi a} < 0$ in their regime of validity

$$\sigma_c >> 1, \sigma_c\alpha >> 1, (\sigma_c\beta >> 1)$$

Qualitatively different from LVS

Potential for a → $\Delta_a > 3$

$$\mathcal{L} \supset \sigma^3, \sigma a^2, \sigma\partial_\mu a\partial^\mu a$$

Connection to the distance conjecture

Interactions with heavy modes:

$$\mathcal{L}_{\psi\bar{\psi}} = K_{\psi\bar{\psi}}\partial_\mu\psi\partial^\mu\bar{\psi} + \frac{1}{\mathcal{V}^2}K^{\psi\bar{\psi}}\psi\bar{\psi}$$

Since superpotential W does depend on T

E.g. KK mode: $K_{\psi\bar{\psi}} \sim \mathcal{V}^{-1/3}$

$$f_{\varphi\psi\psi} = \frac{\Gamma\left(\frac{2\Delta_\psi+\Delta_\varphi-3}{2}\right)}{2\Gamma\left(\Delta_\varphi\right)\Gamma\left(\Delta_\psi\right)^2}\left(\sqrt{\frac{1}{6}}\left(\Delta_\varphi+2\Delta_\psi-\Delta_\psi^2-3\right)+\frac{5}{\sqrt{6}}\Delta_\psi\left(\Delta_\psi-3\right)\right)$$

$$f_{\varphi\psi\psi} > 0 \equiv \frac{\partial m^2(\psi)}{\partial\varphi} < 0$$

$$f_{\varphi\psi\psi} < 0 \equiv \frac{\partial m^2(\psi)}{\partial\varphi} > 0$$

Negative $\gamma(0,\ell)$ requires heavy states to become light in LV limit

↓

Distance Conjecture

Conclusions and outlook

CFT techniques promising tool to address AdS stringy EFTs

Not proven, but very similar to known positivity bounds

- Sign of $\gamma^{\mathrm{mix}}(0,\ell)$ correlates with Swampland conditions
- Applications: LVS, KKLT, Racetrack, Perturbative stabilisation
- Connection to the distance conjecture
- Just a feature of volume modulus?

Work in progress on more examples: Fibred models, Type IIA…

Other work in progress: explore consequences of established bounds

Long term goal: invert the relationship to "navigate" the Swampland

Thank you for your attention!