Thermalization in large-N CFTs

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Motivation

Finite-temperature CFTs are important for several reasons:

- Quantum critical points in the laboratory have finite temperature.
- In holography, they are dual to black holes and black branes.

- We consider finite-temperature CFT on Euclidean manifold $S^1_\beta \times S^{d-1}_R$. β and R are radii of thermal circle and spatial sphere.
- We take the large volume limit $\beta/R \to 0$.
- We consider theories with large central charge $C_T \propto N^2 \to \infty$ in spacetime with dimension greater then two $d > 2$.

Thermalization and universality

- We focus on the stress tensor sector of large-N CFT:

$$T^k_{\tau,s} =: T_{\mu_1\nu_1} T_{\mu_2\nu_2} (\partial_\alpha)^l \partial^{2n} \ldots T_{\mu_k\nu_k} :,$$

where s denotes spin and twist is defined as $\tau = \Delta - s$.

- By thermalization of stress tensor sector we mean

$$\langle T^k_{\tau,s} \rangle_\beta \approx \langle \mathcal{O}_H T^k_{\tau,s} \mathcal{O}_H \rangle_{\mathbb{R}^1 \times S^{d-1}_R},$$

where $\mathcal{O}_H$ is a scalar primary operator with conformal dimension $\Delta_H \propto C_T$, that is related to temperature $1/\beta$ by considering the case $k = 1$ (stress tensor, itself).

- Equivalently (with suppressed index structure)

$$\langle T^k_{\tau,s} \rangle_\beta = \frac{1}{R^{\tau+s}} \lambda_{\mathcal{O}_H \mathcal{O}_H T^k_{\tau,s}} \Big|_{\Delta^k_H},$$

where $|_{\Delta^k_H}$ denotes that we keep only the leading term in large-Δ limit of the OPE coefficient.

Thermalization and universality

- Large-N (or large-C_T) factorization implies

$$\langle T^k_{\tau,s}\rangle_\beta = c^k_{\tau,s}\left(\langle T^1_{d-2,s}\rangle_\beta\right)^k ,$$

where $c^k_{\tau,s}$ are theory-independent coefficients.

- Thermalization implies the universality of the leading term in large-Δ limit of multi stress tensor OPE coefficient

$$\lambda_{\mathcal{O}_H\mathcal{O}_H T^k_{\tau,s}}\Big|_{\Delta^k_H} = \langle T^k_{\tau,s}\rangle_\beta = c^k_{\tau,s}\left(\langle T^1_{d-2,s}\rangle_\beta\right)^k = c^k_{\tau,s}\left(\lambda_{\mathcal{O}_H\mathcal{O}_H T^1_{d-2,2}}\right)^k .$$

- We show that in the free theory the stress tensor sector thermalizes, therefore, the universality is equivalent to the thermalization.

OPE coefficients

- We consider free scalar adjoint model of group $SU(N)$ on $\mathbb{R} \times S^3_R$, with central charge $C_T = \frac{4}{3}(N^2 - 1)$.
- The correlation functions (and OPE coefficients) can be computed via Wick contractions.
- We consider several types of multi-stress tensors and their three-point function with operators $\mathcal{O}_\Delta = \frac{1}{\sqrt{\Delta} N^{\frac{\Delta}{2}}} : Tr(\phi^\Delta) :$.
- Some of the OPE coefficients we computed in large-C_T limit are given by

$$\lambda_{\mathcal{O}_\Delta \mathcal{O}_\Delta T^k_{2k,2k}} = \left(-\frac{4}{3}\right)^k \frac{1}{\sqrt{k!} C_T^{k/2}} \frac{\Gamma(\Delta+1)}{\Gamma(\Delta-k+1)},$$

$$\lambda_{\mathcal{O}_\Delta \mathcal{O}_\Delta T^2_{6,2}} = \frac{4\sqrt{2}\Delta(\Delta-1)}{9C_T}, \qquad \lambda_{\mathcal{O}_\Delta \mathcal{O}_\Delta T^2_{8,0}} = \frac{2\sqrt{2}\Delta(\Delta-1)}{9C_T}.$$

- By comparing these OPE coefficients with the ones computed in holography (at strong coupling) [A. L. Fitzpatrick, K. W. Huang, '19] one can see that they are the same in the large-Δ limit, which confirms the universality!

Thermal one-point functions

- We consider one-point functions of multi stress tensors on $S^1_\beta \times S^3_R$, when $\beta/R \to 0$.

$$\langle \mathcal{O}_{\mu_1 \cdots \mu_{J_\mathcal{O}}} \rangle_\beta = \frac{b_\mathcal{O}}{\beta^{\Delta_\mathcal{O}}} \left(e_{\mu_1} \cdots e_{\mu_{J_\mathcal{O}}} - (\text{traces}) \right),$$

  where e_μ is a unit vector along the thermal circle.

- We compute $b_\mathcal{O}$ explicitly for several multi stress tensors in the large-N and large volume limit

$$b_{T^k_{2k,2k}} = \frac{(-\frac{2}{5})^k N^k \pi^{4k}}{3^{\frac{3k}{2}} \sqrt{k!}}, \quad b_{T^2_{6,2}} = \frac{\sqrt{2} N^2 \pi^8}{675}, \quad b_{T^2_{8,0}} = \frac{\pi^8 N^2}{675\sqrt{2}}.$$

- Assuming the thermalization of stress tensor $T^1_{2,2}$, we relate the conformal dimension of the heavy operator Δ_H with temperature $1/\beta$

$$\frac{1}{\beta^4} b_{T^1_{2,2}} = \frac{1}{R^4} \lambda_{\mathcal{O}_H \mathcal{O}_H T^1_{2,2}} \implies \frac{\Delta_H}{C_T} = \frac{1}{20} \left(\frac{\pi R}{\beta} \right)^4.$$

Thermalization in the free theory

- Now, it is easy to check

$$\beta^{-4k} b_{T^k_{2k,2k}} = R^{-4k} \lim_{\Delta_H=\Delta\to\infty} \lambda_{\mathcal{O}_\Delta \mathcal{O}_\Delta T^k_{2k,2k}},$$
$$\beta^{-8} b_{T^2_{6,2}} = R^{-8} \lim_{\Delta_H=\Delta\to\infty} \lambda_{\mathcal{O}_\Delta \mathcal{O}_\Delta T^2_{6,2}},$$
$$\beta^{-8} b_{T^2_{8,0}} = R^{-8} \lim_{\Delta_H=\Delta\to\infty} \lambda_{\mathcal{O}_\Delta \mathcal{O}_\Delta T^2_{8,0}}.$$

- Therefore, all these operators thermalize!
- The thermalization is equivalent to the eigenstate thermalization hypothesis condition for multi stress tensors.
- **Note**: Not all operators in the spectrum of free theory thermalize in the following sense. For example: $\mathcal{O}_2 = \frac{1}{\sqrt{2}N} Tr(\phi^2)$

$$\beta^{-2} b_{\mathcal{O}_2} \neq R^{-2} \lim_{\Delta_H=\Delta\to\infty} \lambda_{\mathcal{O}_\Delta \mathcal{O}_\Delta \mathcal{O}_2}.$$

Conclusions and future developments

- Stress tensor sector of large-N CFTs thermalize for all values of coupling (or Δ_{gap}) including the strong coupling in holographic theory.
- The leading term of OPE coefficients of multi stress tensor operators with scalars in large-Δ limit do not depend on coupling in the theory.
- Stress tensor sector of large-N CFTs satisfy the (diagonal part of) ETH condition.

In future:

- What happens with operators that do not thermalize when the weak coupling is introduced?
- Can the universality of the leading term in OPE coefficients in large-Δ limit be proven by the conformal bootstrap technique?

THANK YOU.