Light-ray operators and detector algebra

Riccardo Gonzo based on hep-th/2012.01406
with A.Pokraka

SAGEX
Scattering Amplitudes:
from Geometry to Experiment

Trinity College Dublin
Coláiste na Tríonóide, Baile Átha Cliath
The University of Dublin

Hamilton Mathematics Institute – HMI
Trinity College Dublin – TCD

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 764850

21 December 2020

Content

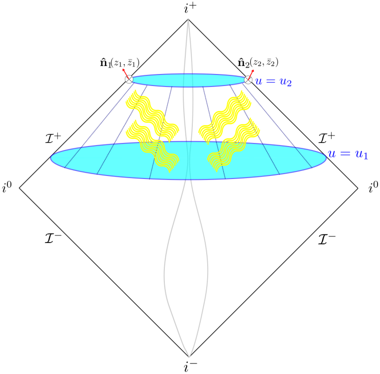
Why light-ray operators?

- Interesting non-local operators which arise naturally by integrating Einstein equations at null infinity along the light-cone time ("charge densities")

Why light-ray operators?

- Interesting non-local operators which arise naturally by integrating Einstein equations at null infinity along the light-cone time ("charge densities")

  - Phenomenologically relevant for gravitational physics: similarly to jets in QCD physics, one can define a notion of localized detector, which is collecting quanta of radiation in a direction n̂



Definition of light-ray operators in QFT

We interpret physically the insertion of our light-ray operators as an insertion of a physical detector on the boundary of Minkowski space and we consider only hard modes contributions.

In flat-null coordinates, our family of light-ray operators is defined by

$$\mathcal{E}(\hat{n}) = \int_{-\infty}^{+\infty} \mathrm{d}u \lim_{r\to\infty} r^2 T_{uu}(u, r, z_{\hat{n}}, \bar{z}_{\hat{n}}),$$

$$\mathcal{K}(\hat{n}) = \int_{-\infty}^{+\infty} \mathrm{d}u\, u \lim_{r\to\infty} r^2 T_{uu}(u, r, z_{\hat{n}}, \bar{z}_{\hat{n}}),$$

$$\mathcal{N}_z(\hat{n}) = \int_{-\infty}^{+\infty} \mathrm{d}u \lim_{r\to\infty} r^2 T_{uz}(u, r, z_{\hat{n}}, \bar{z}_{\hat{n}}),$$

$$\mathcal{N}_{\bar{z}}(\hat{n}) = \int_{-\infty}^{+\infty} \mathrm{d}u \lim_{r\to\infty} r^2 T_{u\bar{z}}(u, r, z_{\hat{n}}, \bar{z}_{\hat{n}}).$$

We will collectively denote these operators by $L(\hat{n}) = \{\mathcal{E}(\hat{n}), \mathcal{K}(\hat{n}), \mathcal{N}_z(\hat{n}), \mathcal{N}_{\bar{z}}(\hat{n})\}$.

Saddle-point evaluation of detector light-ray operators

- Use the Hilbert stress tensor

$$T_{\mu\nu}^{\text{matter}} = -\frac{2}{\sqrt{|g|}}\frac{\delta S^{\text{matter}}}{\delta g^{\mu\nu}}$$

Saddle-point evaluation of detector light-ray operators

- Use the Hilbert stress tensor

$$T_{\mu\nu}^{\text{matter}} = -\frac{2}{\sqrt{|g|}}\frac{\delta S^{\text{matter}}}{\delta g^{\mu\nu}}$$

- Evaluate the light-ray operators by taking the saddle point estimate of the mode expansion of on-shell quantum fields, which is justified at large r. Using

$$x^\mu = \frac{r}{2}\left(1 + z\bar{z} + \frac{u}{r}, z + \bar{z}, -i(z - \bar{z}), 1 - z\bar{z} - \frac{u}{r}\right),$$
$$p^\mu = \frac{\omega_p}{2}(1 + w\bar{w}, w + \bar{w}, -i(w - \bar{w}), 1 - w\bar{w}).$$

we have

$$ip \cdot x = -i\frac{\omega_p u}{2} - i\frac{\omega_p r}{2}|z - w|^2 \to \text{localize at } (z, \bar{z}) = (w, \bar{w})$$

Scalar light-ray detector operators (1)

In the simplest case of a self-interacting massless real scalar without derivative interactions

$$T_{uu}^{\text{scalar}}(x)=(\partial_u\phi)(\partial_u\phi), \qquad T_{uz}^{\text{scalar}}(x)=(\partial_u\phi)(\partial_z\phi), \qquad T_{u\bar{z}}^{\text{scalar}}(x)=(\partial_u\phi)(\partial_{\bar{z}}\phi).$$

Scalar light-ray detector operators (1)

In the simplest case of a self-interacting massless real scalar without derivative interactions

$$T_{uu}^{\text{scalar}}(x)=(\partial_u\phi)(\partial_u\phi), \qquad T_{uz}^{\text{scalar}}(x)=(\partial_u\phi)(\partial_z\phi), \qquad T_{u\bar{z}}^{\text{scalar}}(x)=(\partial_u\phi)(\partial_{\bar{z}}\phi).$$

where

$$\phi(u,\hat{\mathrm{n}}) = \frac{i}{(8\pi^2)r}\int_0^{+\infty} \mathrm{d}\omega_p \left[b(\omega_p, z_{\hat{\mathrm{n}}}, \bar{z}_{\hat{\mathrm{n}}})e^{i\frac{\omega_p u}{2}} - b^\dagger(\omega_p, z_{\hat{\mathrm{n}}}, \bar{z}_{\hat{\mathrm{n}}})e^{-i\frac{\omega_p u}{2}}\right],$$

$$\partial_z\phi(u,\hat{\mathrm{n}}) = \frac{i}{(8\pi^2)r}\int_0^{+\infty} \mathrm{d}\omega_p \left[\partial_{z_{\hat{\mathrm{n}}}} b(\omega_p, z_{\hat{\mathrm{n}}}, \bar{z}_{\hat{\mathrm{n}}})e^{i\frac{\omega_p u}{2}} - \partial_{z_{\hat{\mathrm{n}}}} b^\dagger(\omega_p, z_{\hat{\mathrm{n}}}, \bar{z}_{\hat{\mathrm{n}}})e^{-i\frac{\omega_p u}{2}}\right].$$

Scalar light-ray detector operators (2)

- The explicit structure of the scalar light-ray operators is

$$\mathcal{E}_{\text{scalar}}(\hat{n}) = \int \widetilde{\mathrm{d}^3 p}\, \omega_p \;: b^\dagger(\omega_p \hat{n}) b(\omega_p \hat{n}) :\, \delta^2(z_{\hat{n}} - z_{\hat{p}}),$$

$$\mathcal{K}_{\text{scalar}}(\hat{n}) = (-i) \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega_-}{2(2\pi)^3} \delta^{(1)}(\omega_-) \int_{\max\{-\omega_-, \omega_-\}}^{+\infty} \mathrm{d}\omega_+ \left((\omega_+)^2 - (\omega_-)^2\right)$$
$$\times : \left[b^\dagger\left((\omega_+ + \omega_-)\hat{n}\right) b\left((\omega_+ - \omega_-)\hat{n}\right) + h.c.\right] :,$$

$$\mathcal{N}_{z,\text{scalar}}(\hat{n}) = i \int \widetilde{\mathrm{d}^3 p}\, \delta^2\left(z_{\hat{n}} - z_{\hat{p}}\right) : \left[b^\dagger(\omega_p \hat{n}) \overleftrightarrow{\partial}_{z_{\hat{n}}} b(\omega_p \hat{n}) \right] : .$$

Scalar light-ray detector operators (2)

- The explicit structure of the scalar light-ray operators is

$$\mathcal{E}_{\text{scalar}}(\hat{\mathrm{n}}) = \int \widetilde{\mathrm{d}^3 p}\, \omega_p \ : b^\dagger(\omega_p \hat{\mathrm{n}}) b(\omega_p \hat{\mathrm{n}}) : \delta^2(z_{\hat{\mathrm{n}}} - z_{\hat{\mathrm{p}}}),$$

$$\mathcal{K}_{\text{scalar}}(\hat{\mathrm{n}}) = (-i) \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega_-}{2(2\pi)^3} \delta^{(1)}(\omega_-) \int_{\max\{-\omega_-,\omega_-\}}^{+\infty} \mathrm{d}\omega_+ \left((\omega_+)^2 - (\omega_-)^2\right) \times : \left[b^\dagger\left((\omega_+ + \omega_-)\hat{\mathrm{n}}\right) b\left((\omega_+ - \omega_-)\hat{\mathrm{n}}\right) + h.c. \right] :,$$

$$\mathcal{N}_{z,\text{scalar}}(\hat{\mathrm{n}}) = i \int \widetilde{\mathrm{d}^3 p}\, \delta^2\left(z_{\hat{\mathrm{n}}} - z_{\hat{\mathrm{p}}}\right) : \left[b^\dagger(\omega_p \hat{\mathrm{n}}) \overleftrightarrow{\partial}_{z_{\hat{\mathrm{n}}}} b(\omega_p \hat{\mathrm{n}}) \right] : .$$

- Note: the action of $\mathcal{E}_{\text{scalar}}(\hat{\mathrm{n}})$ on an on-shell state is

$$\mathcal{E}_{\text{scalar}}(\hat{\mathrm{n}}) \left| p_1 \dots p_n \right\rangle = \sum_{i=1}^{n} (\omega_i)\, \delta^2(z_{\hat{\mathrm{n}}} - z_{\hat{\mathrm{p}}_i}) \left| p_1 \dots p_n \right\rangle .$$

Connection with energy event shapes!

Scalar light-ray detector algebra

The scalar light-ray algebra is

$$
\begin{aligned}
&[\mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = 0, \\
&[\mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = 0, \\
&[\mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = -2i\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2), \\
&[\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = -2i\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\partial_z\mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2) \\
&\qquad\qquad\qquad\qquad + 2i\partial_z\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2), \\
&[\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = -2i\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\partial_z\mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_2) \\
&\qquad\qquad\qquad\qquad + 2i\partial_z\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_2), \\
&[\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{N}_{\bar{z},\text{scalar}}(\hat{\mathsf{n}}_2)] \\
&\quad = +2i\partial_{z_{\hat{\mathsf{n}}_1}}\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{N}_{\bar{z},\text{scalar}}(\hat{\mathsf{n}}_2) - 2i\partial_{\bar{z}_{\hat{\mathsf{n}}_2}}\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_2) \\
&\qquad - 2i\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})(\partial_{z_{\hat{\mathsf{n}}_2}}\mathcal{N}_{\bar{z},\text{scalar}}(\hat{\mathsf{n}}_2) - \partial_{\bar{z}_{\hat{\mathsf{n}}_2}}\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_2)).
\end{aligned}
$$

Scalar light-ray detector algebra

The scalar light-ray algebra is

$$
\begin{aligned}
&[\mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = 0, \\
&[\mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = 0, \\
&[\mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = -2i\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2), \\
&[\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = -2i\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\partial_z\mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2) \\
&\qquad\qquad + 2i\partial_z\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{E}_{\text{scalar}}(\hat{\mathsf{n}}_2), \\
&[\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_2)] = -2i\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\partial_z\mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_2) \\
&\qquad\qquad + 2i\partial_z\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{K}_{\text{scalar}}(\hat{\mathsf{n}}_2), \\
&[\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_1), \mathcal{N}_{\bar{z},\text{scalar}}(\hat{\mathsf{n}}_2)] \\
&\quad = +2i\partial_{z_{\hat{\mathsf{n}}_1}}\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{N}_{\bar{z},\text{scalar}}(\hat{\mathsf{n}}_2) - 2i\partial_{\bar{z}_{\hat{\mathsf{n}}_2}}\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_2) \\
&\qquad - 2i\delta^2(z_{\hat{\mathsf{n}}_1} - z_{\hat{\mathsf{n}}_2})(\partial_{z_{\hat{\mathsf{n}}_2}}\mathcal{N}_{\bar{z},\text{scalar}}(\hat{\mathsf{n}}_2) - \partial_{\bar{z}_{\hat{\mathsf{n}}_2}}\mathcal{N}_{z,\text{scalar}}(\hat{\mathsf{n}}_2)).
\end{aligned}
$$

Consistent with complexified Cordova-Shao algebra!

Yang mills (spin 1) light-ray operators

The light-ray spin 1 family is given by

$$\mathcal{E}_{\text{gluon}}(\hat{n}) = \int \widetilde{\mathrm{d}^3 p}\, \omega_p\, \delta^2(z_{\hat{n}} - z_{\hat{p}}) \sum_{\sigma=\pm} : \left[a_\sigma^{\dagger a}(\omega_p \hat{n}) a_\sigma^a(\omega_p \hat{n}) \right] :,$$

$$\mathcal{K}_{\text{gluon}}(\hat{n}) = (-i) \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega_-}{2(2\pi)^3} \delta^{(1)}(\omega_-) \int_{\max\{-\omega_-,\omega_-\}}^{+\infty} \mathrm{d}\omega_+ \left((\omega_+)^2 - (\omega_-)^2\right)$$
$$\times \sum_{\sigma=\pm} : \left[a_\sigma^{\dagger a}\left((\omega_+ + \omega_-)\hat{n}\right) a_\sigma^a\left((\omega_+ - \omega_-)\hat{n}\right) \right] :,$$

$$\mathcal{N}^{\text{orb}}_{z,\text{gluon}}(\hat{n}) = i \int \widetilde{\mathrm{d}^3 p}\, \delta^2(z_{\hat{n}} - z_{\hat{p}}) \sum_{\sigma=\pm} : \left[a_\sigma^{\dagger a}(\omega_p \hat{n}) \overleftrightarrow{\partial}_{z_{\hat{n}}} a_\sigma^a(\omega_p \hat{n}) \right] :,$$

$$\mathcal{N}^{\text{spin}}_{z,\text{gluon}}(\hat{n}) = i \int \widetilde{\mathrm{d}^3 p}\, \partial_{z_{\hat{n}}} \delta^2(z_{\hat{n}} - z_{\hat{p}}) \sum_{\sigma=\pm} \sigma : \left[a_\sigma^{\dagger a}(\omega_p \hat{n}) a_\sigma^a(\omega_p \hat{n}) \right] : .$$

Yang mills (spin 1) light-ray operators

The light-ray spin 1 family is given by

$$\mathcal{E}_{\text{gluon}}(\hat{n}) = \int \widetilde{\mathrm{d}^3 p}\, \omega_p\, \delta^2(z_{\hat{n}} - z_{\hat{p}}) \sum_{\sigma=\pm} :\left[a^{\dagger a}_{\sigma}(\omega_p \hat{n}) a^{a}_{\sigma}(\omega_p \hat{n})\right]:,$$

$$\mathcal{K}_{\text{gluon}}(\hat{n}) = (-i) \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega_-}{2(2\pi)^3} \delta^{(1)}(\omega_-) \int_{\max\{-\omega_-,\omega_-\}}^{+\infty} \mathrm{d}\omega_+ \left((\omega_+)^2 - (\omega_-)^2\right) \times \sum_{\sigma=\pm} :\left[a^{\dagger a}_{\sigma}\left((\omega_+ + \omega_-)\hat{n}\right) a^{a}_{\sigma}\left((\omega_+ - \omega_-)\hat{n}\right)\right]:,$$

$$\mathcal{N}^{\text{orb}}_{z,\text{gluon}}(\hat{n}) = i \int \widetilde{\mathrm{d}^3 p}\, \delta^2(z_{\hat{n}} - z_{\hat{p}}) \sum_{\sigma=\pm} :\left[a^{\dagger a}_{\sigma}(\omega_p \hat{n}) \overleftrightarrow{\partial}_{z_{\hat{n}}} a^{a}_{\sigma}(\omega_p \hat{n})\right]:,$$

$$\mathcal{N}^{\text{spin}}_{z,\text{gluon}}(\hat{n}) = i \int \widetilde{\mathrm{d}^3 p}\, \partial_{z_{\hat{n}}} \delta^2(z_{\hat{n}} - z_{\hat{p}}) \sum_{\sigma=\pm} \sigma :\left[a^{\dagger a}_{\sigma}(\omega_p \hat{n}) a^{a}_{\sigma}(\omega_p \hat{n})\right]: .$$

Consistent with the point particle particle structure of the stress tensor!

Linearized gravity light-ray operators

The light-ray definition in linearized gravity comes from Einstein equations solved near null infinity and in the PSZ convention are

$$\mathcal{E}_{\mathrm{GR}}(\hat{\mathsf{n}}) = \int \widetilde{\mathrm{d}^3 p}\, \omega_p\, \delta^2(z_{\hat{\mathsf{n}}} - z_{\hat{\mathsf{p}}}) \sum_{\sigma=\pm} : \left[a^\dagger_\sigma(\omega_p \hat{\mathsf{n}}) a_\sigma(\omega_p \hat{\mathsf{n}}) \right] :,$$

$$\mathcal{K}_{\mathrm{GR}}(\hat{\mathsf{n}}) = (-i) \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega_-}{2(2\pi)^3} \delta^{(1)}(\omega_-) \int_{\max\{-\omega_-,\omega_-\}}^{+\infty} \mathrm{d}\omega_+ \left((\omega_+)^2 - (\omega_-)^2 \right)$$
$$\times \sum_{\sigma=\pm} : \left[a^\dagger_\sigma\left((\omega_+ + \omega_-)\hat{\mathsf{n}} \right) a_\sigma\left((\omega_+ - \omega_-)\hat{\mathsf{n}} \right) \right] :,$$

$$\mathcal{N}^{\mathrm{PSZ,orb}}_{z,\mathrm{GR}}(\hat{\mathsf{n}}) = i \int \widetilde{\mathrm{d}^3 p}\, \delta^2(z_{\hat{\mathsf{n}}} - z_{\hat{\mathsf{p}}}) \sum_{\sigma=\pm} : \left[a^\dagger_\sigma(\omega_p \hat{\mathsf{n}}) \overleftrightarrow{\partial}_{z_{\hat{\mathsf{n}}}} a_\sigma(\omega_p \hat{\mathsf{n}}) \right] :,$$

$$\mathcal{N}^{\mathrm{PSZ,spin}}_{z,\mathrm{GR}}(\hat{\mathsf{n}}) = i \int \widetilde{\mathrm{d}^3 p}\, \partial_{z_{\hat{\mathsf{n}}}} \delta^2(z_{\hat{\mathsf{n}}} - z_{\hat{\mathsf{p}}}) \sum_{\sigma=\pm} \sigma : \left[a^\dagger_\sigma(\omega_p \hat{\mathsf{n}}) a_\sigma(\omega_p \hat{\mathsf{n}}) \right] : .$$

Linearized gravity light-ray operators

The light-ray definition in linearized gravity comes from Einstein equations solved near null infinity and in the PSZ convention are

$$\mathcal{E}_{\mathrm{GR}}(\hat{\mathsf{n}}) = \int \widetilde{\mathrm{d}^3 p}\, \omega_p\, \delta^2(z_{\hat{\mathsf{n}}} - z_{\hat{\mathsf{p}}}) \sum_{\sigma=\pm} : \left[a^\dagger_\sigma(\omega_p \hat{\mathsf{n}}) a_\sigma(\omega_p \hat{\mathsf{n}}) \right] :,$$

$$\mathcal{K}_{\mathrm{GR}}(\hat{\mathsf{n}}) = (-i) \int_{-\infty}^{+\infty} \frac{\mathrm{d}\omega_-}{2(2\pi)^3} \delta^{(1)}(\omega_-) \int_{\max\{-\omega_-,\omega_-\}}^{+\infty} \mathrm{d}\omega_+ \left((\omega_+)^2 - (\omega_-)^2\right) \times \sum_{\sigma=\pm} : \left[a^\dagger_\sigma \left((\omega_+ + \omega_-)\hat{\mathsf{n}}\right) a_\sigma \left((\omega_+ - \omega_-)\hat{\mathsf{n}}\right) \right] :,$$

$$\mathcal{N}^{\mathrm{PSZ,orb}}_{z,\mathrm{GR}}(\hat{\mathsf{n}}) = i \int \widetilde{\mathrm{d}^3 p}\, \delta^2(z_{\hat{\mathsf{n}}} - z_{\hat{\mathsf{p}}}) \sum_{\sigma=\pm} : \left[a^\dagger_\sigma(\omega_p \hat{\mathsf{n}}) \overleftrightarrow{\partial}_{z_{\hat{\mathsf{n}}}} a_\sigma(\omega_p \hat{\mathsf{n}}) \right] :,$$

$$\mathcal{N}^{\mathrm{PSZ,spin}}_{z,\mathrm{GR}}(\hat{\mathsf{n}}) = i \int \widetilde{\mathrm{d}^3 p}\, \partial_{z_{\hat{\mathsf{n}}}} \delta^2(z_{\hat{\mathsf{n}}} - z_{\hat{\mathsf{p}}}) \sum_{\sigma=\pm} \sigma : \left[a^\dagger_\sigma(\omega_p \hat{\mathsf{n}}) a_\sigma(\omega_p \hat{\mathsf{n}}) \right] : .$$

Curious fact: the orbital-spin algebra seems problematic!

Summary

- By interpreting light-ray operators as localized detectors, we have provided explicit expressions for the hard mode contributions as operators acting on on-shell particle states
- The algebra we found is consistent with the complexified Cordova-Shao algebra, but more analysis is needed on the orbital-spin contribution
- We have defined gravitational energy event shapes from the saddle point estimate of the gravitational ANEC operator at infinity $\mathcal{E}_{\mathrm{GR}}(\hat{n})$