Protected states from $\text{AdS}_3$ integrability

Suvajit Majumder

City, University of London

in collaboration with: B. Stefanski, A. Torrielli, and O.O. Sax

Dec 22, 2020
Highlights:

- Find correct protected spectrum using ABA, which matches with the $\text{Sym}^N(T^4)$ protected spectrum (extension of earlier work by Baggio et al, 2017)
- Protected spectrum for K3, realised as $T^4$ orbifolds
- Generalisation to mixed flux backgrounds
- Each background has a moduli space (20 D) and our results hold across full moduli space
Type IIB superstrings in $AdS_3 \times S_3 \times M_4$ dual to $CFT_2$ with $\mathcal{N} = (4, 4)$ SUSY

Two geometries (preserving max SUSY): $M_4 = T^4$, and $M_4 = S^3 \times S^1$

Background supported by non-trivial R-R, and NS-NS 3-form fluxes

Integrable spin chain descriptions include massive and massless modes

Integrable S matrix is exact in $\alpha'$ (reason behind non-renormalisation claims)

Nice reviews for elaborate reference list: arXiv:1406.2971,1605.03173
Setup: massless ABA in $AdS_3 \times S^3 \times T^4$

- $\text{psu}(1|1)^4_{\text{c.e.}}$: tensor product of two commuting copies of $\text{psu}(1|1)^2_{\text{c.e.}}$ and has 8 supercharges $S^i_j$, $Q^i_j$ (with $I = L, R$ and $i = 1, 2$), which satisfy

\[
\{Q^i_L, S^i_L\} = H^i_L \quad \{Q^i_R, S^i_R\} = H^i_R, \\
\{Q^i_L, Q^i_R\} = P^i \quad \{S^i_L, S^i_R\} = K^i
\]

where $H^i_L, H^i_R, P^i, K^i$ are central elements.

- $\text{su}(2) \subset \text{so}(4)$: useful to organise the spectrum.

- The massless modes are in a tensor product representation

\[
\rho_{\text{psu}(1|1)^4} = \rho_L \otimes \tilde{\rho}_L
\]

- Short reps. of two $\text{psu}(1|1)^2_{\text{c.e.}}$: graded $(1|1)$D vector spaces

\[
\mathcal{V}_{\rho_L} = \{|\phi\rangle, |\psi\rangle\}, \quad \mathcal{V}_{\tilde{\rho}_L} = \{|\tilde{\phi}\rangle, |\tilde{\psi}\rangle\}
\]

\[
|\chi\rangle \equiv |\phi\rangle |\tilde{\psi}\rangle, \quad |T^2\rangle \equiv |\phi\rangle |\tilde{\phi}\rangle, \quad |T^1\rangle \equiv |\psi\rangle |\tilde{\psi}\rangle, \quad |\tilde{\chi}\rangle \equiv |\psi\rangle |\tilde{\phi}\rangle
\]
\(\text{psu}(1|1)^4\) \text{c.e. ABA contd.}

- \(\text{psu}(1|1)^4\) R-matrix

\[
R_{\text{psu}(1|1)^4} = R_{\text{psu}(1|1)^2}^{LL} \otimes R_{\text{psu}(1|1)^2}^{\tilde{L}\tilde{L}}
\]

- Constituent \(\text{psu}(1|1)^2\) R-matrices

\[
R_{\text{psu}(1|1)^2}^{LL}(\gamma_1, \gamma_2) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -b & a & 0 \\
0 & a & b & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

\[
R_{\text{psu}(1|1)^2}^{\tilde{L}\tilde{L}}(\gamma_1, \gamma_2) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & -b & -a & 0 \\
0 & -a & b & 0 \\
0 & 0 & 0 & -1
\end{pmatrix}
\]

where \(a(\gamma_1, \gamma_2) = \text{sech} \frac{\gamma_1 - \gamma_2}{2}\), \(b(\gamma_1, \gamma_2) = \text{tanh} \frac{\gamma_1 - \gamma_2}{2}\), \(\gamma = \log \tan \frac{p}{4}\)
psu(1|1)\textsuperscript{c.e.} \textsuperscript{4} ABA contd.

- Monodromy matrix $\mathcal{M}_{\text{psu}(1|1)^4}$
  $$
  \mathcal{M}_{\text{psu}(1|1)^4}(\gamma_0|\vec{\gamma}) = \mathcal{M}_L \hat{\otimes} \mathcal{M}_{\bar{L}}
  $$
  with component $\text{psu}(1|1)^2$ monodromy matrices
  $$
  \mathcal{M}_L = \begin{pmatrix}
  a^1 & b^1 \\
  c^1 & d^1
  \end{pmatrix}, \quad \mathcal{M}_{\bar{L}} = \begin{pmatrix}
  a^2 & b^2 \\
  c^2 & d^2
  \end{pmatrix}
  $$

- Transfer matrix $\mathcal{T}_{\text{psu}(1|1)^4}$
  $$
  \mathcal{T}_{\text{psu}(1|1)^4}(\gamma_0|\vec{\gamma}) = \text{str}_0 \mathcal{M}_{\text{psu}(1|1)^4}(\gamma_0|\vec{\gamma})
  $$

- B operators
  $$
  B_1 \equiv b^1 \hat{\otimes} 1^2, \quad B_3 \equiv 1^1 \hat{\otimes} b^2
  $$
psu(1|1)_{c.e.}^4 \text{ ABA contd.}

- **Algebraic Bethe ansatz**

\[ |\Psi\rangle = \prod_{i=1}^{N_3} B_3(\beta_{3,i}) \prod_{j=1}^{N_1} B_1(\beta_{1,j}) |\chi_{\gamma_1} \cdots \chi_{\gamma_{N_0}}\rangle \]

- **Bethe equations**

\[ e^{-iLp_k} = (-1)^{N_0-1} \prod_{i \neq k}^{N_0} S^2(\gamma_{kj}) \prod_{j=1}^{N_1} \coth \frac{\beta_{1,jk}}{2} \prod_{l=1}^{N_3} \coth \frac{\beta_{3,lk}}{2} \]

\[ 1 = \prod_{i=1}^{N_0} \tanh \frac{\beta_{l,ki}}{2}, \quad I = 1, 3, \quad k = 1, \ldots, N_1 \]

where \( \beta_{l,jk} \equiv \beta_{l,j} - \gamma_k, I = 1, 3, \) and \( S(\gamma) \) is the Zamolodchikov sine-Gordon scalar factor
$T^4$ protected states from $\text{psu}(1|1)^4_{\text{c.e.}}$ ABA

Protected states(multiplets) $\Leftrightarrow$ Bethe states with ONLY fermionic zero-momentum massless excitations $\chi, \tilde{\chi}$

Why: they are annihilated by $\text{psu}(1|1)^4_{\text{c.e.}}$.

- Including the $\text{su}(2)_g$ symmetry:

$$\{\chi, T^a, \tilde{\chi}\} \Rightarrow \{\chi^\pm, T^{a\pm}, \tilde{\chi}^\pm\}$$

- ABA states in zero-momentum limit, c.f. $N_0 = 0, 1, 2, 3, 4$

- Protected states organise as a $T^4$ Hodge diamond (for every set of global charges) with $h^{0,0} = h^{2,2} = h^{2,0} = h^{0,2} = 1$, $h^{1,1} = 4$, $h^{0,1} = h^{1,0} = h^{2,1} = h^{1,2} = 2$
Orbifolds

- Orbifold group $\Gamma = \mathbb{Z}_n$, $n = 2, 3, 4, 6$
- Non-trivial action only on $su(2)$.
- $\mathbb{Z}_n$ action on the 1-magnon fermionic ABA states

\[
g | \chi^{+}_{\gamma_1 \to -\infty} \rangle = e^{\frac{2\pi i}{n}} | \chi^{+}_{\gamma_1 \to -\infty} \rangle, \quad g | \chi^{-}_{\gamma_1 \to -\infty} \rangle = e^{-\frac{2\pi i}{n}} | \chi^{-}_{\gamma_1 \to -\infty} \rangle, \quad g | \tilde{\chi}^{+}_{\gamma_1 \to -\infty} \rangle = e^{\frac{2\pi i}{n}} | \tilde{\chi}^{+}_{\gamma_1 \to -\infty} \rangle, \quad g | \tilde{\chi}^{-}_{\gamma_1 \to -\infty} \rangle = e^{-\frac{2\pi i}{n}} | \tilde{\chi}^{-}_{\gamma_1 \to -\infty} \rangle.
\]

- Untwisted sector: Orbifold invariant states

\[
n \geq 2 : \quad \epsilon^{ab} | \chi^{+}_{\gamma_1 \to -\infty} \chi^{-}_{\gamma_2 \to +\infty} \rangle, \quad | \chi^{\pm}_{\gamma_1 \to -\infty} \tilde{\chi}^{\mp}_{\gamma_2 \to +\infty} \rangle, \quad \epsilon^{ab} | \tilde{\chi}^{+}_{\gamma_1 \to -\infty} \tilde{\chi}^{-}_{\gamma_2 \to +\infty} \rangle, \quad | \tilde{\chi}^{\pm}_{\gamma_1 \to -\infty} \chi^{\mp}_{\gamma_2 \to +\infty} \rangle,
\]

\[
n = 2 : \quad | \chi^{\pm}_{\gamma_1 \to -\infty} \tilde{\chi}^{\mp}_{\gamma_2 \to +\infty} \rangle, \quad | \tilde{\chi}^{\pm}_{\gamma_1 \to -\infty} \chi^{\mp}_{\gamma_2 \to +\infty} \rangle,
\]

- Twisted sectors: twisted boundary conditions at level-0 with twist angle $\phi_0 = \frac{2\pi}{n}$. Only ground state survives as $\gamma_k \to -\infty$
Future Directions

- Identify the protected states in the massless thermodynamic Bethe ansatz by putting some suitable chemical potential that can disentangle ground-state degeneracy.
- Prove that wrapping is not contributing at any order actually for generic mixed flux backgrounds (using the massless TBA)
Thank You!


For $AdS_3 \times S^3 \times T^4/\mathbb{Z}_2$:

\[
\begin{array}{cccc}
  h^{0,0} & h^{1,0} & h^{0,1} & 1 \\
  h^{1,0} & h^{1,1} & h^{0,1} & 0 \\
  h^{2,0} & h^{1,1} & h^{0,2} & = 1 \\
  & h^{3,1} & h^{1,2} & 0 \\
  & h^{2,1} & h^{3,2} & 0 \\
  & h^{2,2} & & 1 \\
\end{array}
\oplus
\begin{array}{cccc}
  16 \times & 1 & 0 & 4 \\
  0 & 0 & 0 & 1 \\
\end{array}
= 1 \oplus 16 \times 1 = 1 \oplus 16 \times 1 = 1 \oplus 20 \times 1

For $AdS_3 \times S^3 \times T^4/\mathbb{Z}_n$, $n = 3, 4, 6$:

\[
\begin{array}{cccc}
  h^{0,0} & h^{1,0} & h^{0,1} & 1 \\
  h^{1,0} & h^{1,1} & h^{0,1} & 0 \\
  h^{2,0} & h^{1,1} & h^{0,2} & = 1 \\
  & h^{3,1} & h^{1,2} & 0 \\
  & h^{2,1} & h^{3,2} & 0 \\
  & h^{2,2} & & 1 \\
\end{array}
\oplus
\begin{array}{cccc}
  18 \times & 1 & 0 & 2 \\
  0 & 0 & 0 & 1 \\
\end{array}
= 1 \oplus 18 \times 1 = 1 \oplus 20 \times 1

Mixed-flux backgrounds: same analysis, but in terms of Zhukovskis.