Protected states from AdS_3 integrability

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Highlights:

- Find correct protected spectrum using ABA, which matches with the $\mathrm{Sym}^N(T^4)$ protected spectrum (extension of earlier work by Baggio *et al*, 2017)
- Protected spectrum for K3, realised as T^4 orbifolds
- Generalisation to mixed flux backgrounds
- Each background has a moduli space(20 D) and our results hold across full moduli space

AdS_3/CFT_2: a lightning intro

- Type IIB superstrings in $AdS_3 \times S_3 \times M_4$ dual to CFT_2 with $\mathcal{N} = (4, 4)$ SUSY
- Two geometries (preserving max SUSY): $M_4 = T^4$, and $M_4 = S^3 \times S^1$
- Background supported by non-trivial R-R, and NS-NS 3-form fluxes
- Integrable spin chain descriptions include massive and massless modes
- Integrable S matrix is exact in α' (reason behind non-renormalisation claims)
- Nice reviews for elaborate reference list: arXiv:1406.2971,1605.03173

Setup: massless ABA in $AdS_3 \times S^3 \times T^4$

- $\mathrm{psu}(1|1)^4_{c.e.}$: tensor product of two commuting copies of $\mathrm{psu}(1|1)^2_{c.e.}$ and has 8 supercharges S^i_I, Q^i_I (with $I = L, R$ and $i = 1, 2$), which satisfy

$$\{\mathrm{Q}^i_L, \mathrm{S}^i_L\} = \mathrm{H}^i_L \quad \{\mathrm{Q}^i_R, \mathrm{S}^i_R\} = \mathrm{H}^i_R \quad ,$$
$$\{\mathrm{Q}^i_L, \mathrm{Q}^i_R\} = \mathrm{P}^i \quad \{\mathrm{S}^i_L, \mathrm{S}^i_R\} = \mathrm{K}^i$$

  where $\mathrm{H}^i_L, \mathrm{H}^i_R, \mathrm{P}^i, \mathrm{K}^i$ are central elements
- $\mathrm{su}(2)_\circ \subset so(4)$: useful to organise the spectrum
- The massless modes are in a tensor product representation

$$\rho_{\mathrm{psu}(1|1)^4} = \rho_L \otimes \tilde{\rho}_L$$

- Short reps. of two $\mathrm{psu}(1|1)^2_{c.e.}$: graded (1|1)D vector spaces

$$\mathcal{V}_{\rho_L} = \{|\phi\rangle, |\psi\rangle\}, \qquad \mathcal{V}_{\tilde{\rho}_L} = \{|\tilde{\psi}\rangle, |\tilde{\phi}\rangle\}$$

$$\Rightarrow |\chi\rangle \equiv |\phi\rangle |\tilde{\psi}\rangle , \quad |T^2\rangle \equiv |\phi\rangle |\tilde{\phi}\rangle , \quad |T^1\rangle \equiv |\psi\rangle |\tilde{\psi}\rangle , \quad |\tilde{\chi}\rangle \equiv |\psi\rangle |\tilde{\phi}\rangle$$

$\mathrm{psu}(1|1)^4_{\mathrm{c.e.}}$ ABA contd.

- $\mathrm{psu}(1|1)^4$ R-matrix

$$R_{\mathrm{psu}(1|1)^4} = R^{LL}_{\mathrm{psu}(1|1)^2} \hat{\otimes} R^{\tilde{L}\tilde{L}}_{\mathrm{psu}(1|1)^2}$$

- Constituent $\mathrm{psu}(1|1)^2$ R-matrices

$$R^{LL}_{\mathrm{psu}(1|1)^2}(\gamma_1, \gamma_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -b & a & 0 \\ 0 & a & b & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$R^{\tilde{L}\tilde{L}}_{\mathrm{psu}(1|1)^2}(\gamma_1, \gamma_2) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -b & -a & 0 \\ 0 & -a & b & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

where $a(\gamma_1, \gamma_2) = \operatorname{sech} \frac{\gamma_1 - \gamma_2}{2}$, $b(\gamma_1, \gamma_2) = \tanh \frac{\gamma_1 - \gamma_2}{2}$, $\gamma = \log \tan \frac{p}{4}$

$\mathrm{psu}(1|1)^4_{\text{c.e.}}$ ABA contd.

- Monodromy matrix $\mathcal{M}_{\mathrm{psu}(1|1)^4}$

$$\mathcal{M}_{\mathrm{psu}(1|1)^4}(\gamma_0|\vec{\gamma}) = \mathcal{M}_L \hat{\otimes} \mathcal{M}_{\tilde{L}}$$

with component $\mathrm{psu}(1|1)^2$ monodoromy matrices

$$\mathcal{M}_L = \begin{pmatrix} a^1 & b^1 \\ c^1 & d^1 \end{pmatrix} , \qquad \mathcal{M}_{\tilde{L}} = \begin{pmatrix} a^2 & b^2 \\ c^2 & d^2 \end{pmatrix}$$

- Transfer matrix $\mathcal{T}_{\mathrm{psu}(1|1)^4}$

$$\mathcal{T}_{\mathrm{psu}(1|1)^4}(\gamma_0|\vec{\gamma}) = \mathrm{str}_0\, \mathcal{M}_{\mathrm{psu}(1|1)^4}(\gamma_0|\vec{\gamma})$$

- B operators

$$B_1 \equiv b^1 \hat{\otimes} 1^2 , \qquad B_3 \equiv 1^1 \hat{\otimes} b^2$$

$\mathrm{psu}(1|1)^4_{\mathrm{c.e.}}$ ABA contd.

- Algebraic Bethe ansatz

$$|\Psi\rangle = \prod_{i=1}^{N_3} B_3(\beta_{3,i}) \prod_{j=1}^{N_1} B_1(\beta_{1,j}) \, |\chi_{\gamma_1} \ldots \chi_{\gamma_{N_0}}\rangle$$

- Bethe equations

$$e^{-iLp_k} = (-1)^{N_0-1} \prod_{i \neq k}^{N_0} S^2(\gamma_{kj}) \prod_{j=1}^{N_1} \coth \frac{\beta_{1,jk}}{2} \prod_{l=1}^{N_3} \coth \frac{\beta_{3,lk}}{2}$$

$$1 = \prod_{i=1}^{N_0} \tanh \frac{\beta_{I,ki}}{2} \,, \qquad\qquad I = 1, 3 \,, \quad k = 1, \ldots, N_1$$

where $\beta_{I,jk} \equiv \beta_{I,j} - \gamma_k$, $I = 1, 3$, and $S(\gamma)$ is the Zamolodchikov sine-Gordon scalar factor

T^4 protected states from $\mathrm{psu}(1|1)^4_{\mathrm{c.e.}}$ ABA

Protected states(multiplets) $\Leftrightarrow$ Bethe states with ONLY fermionic zero-momentum massless excitations $\chi, \tilde{\chi}$

Why: they are annihilated by $\mathrm{psu}(1|1)^4_{\mathrm{c.e.}}$

- Including the $\mathrm{su}(2)_\circ$ symmetry:

$$\{\chi\,,\,T^a\,,\tilde{\chi}\} \quad \Rightarrow \quad \{\chi^\pm\,,\,T^{a\pm}\,,\tilde{\chi}^\pm\}$$

- ABA states in zero-momentum limit, c.f. $N_0 = 0, 1, 2, 3, 4$
- Protected states organise as a T^4 Hodge diamond (for every set of global charges) with $h^{0,0} = h^{2,2} = h^{2,0} = h^{0,2} = 1$, $h^{1,1} = 4$, $h^{0,1} = h^{1,0} = h^{2,1} = h^{1,2} = 2$

Orbifolds

- Orbifold group $\Gamma = \mathbb{Z}_n$, $n = 2, 3, 4, 6$
- Non-trivial action only on $\mathrm{su}(2)_\circ$
- $\mathbb{Z}_n$ action on the 1-magnon fermionic ABA states

$$g \left|\chi^{+}_{\gamma_1\to-\infty}\right\rangle = e^{\frac{2\pi i}{n}} \left|\chi^{+}_{\gamma_1\to-\infty}\right\rangle , \qquad g \left|\chi^{-}_{\gamma_1\to-\infty}\right\rangle = e^{-\frac{2\pi i}{n}} \left|\chi^{-}_{\gamma_1\to-\infty}\right\rangle ,$$
$$g \left|\tilde{\chi}^{+}_{\gamma_1\to-\infty}\right\rangle = e^{\frac{2\pi i}{n}} \left|\tilde{\chi}^{+}_{\gamma_1\to-\infty}\right\rangle , \qquad g \left|\tilde{\chi}^{-}_{\gamma_1\to-\infty}\right\rangle = e^{-\frac{2\pi i}{n}} \left|\tilde{\chi}^{-}_{\gamma_1\to-\infty}\right\rangle .$$

- Untwisted sector: Orbifold invariant states

$$n \geq 2 : \quad \epsilon^{ab} \left|\chi^{+}_{\gamma_1\to-\infty}\, \chi^{-}_{\gamma_2\to+\infty}\right\rangle , \quad \left|\chi^{\pm}_{\gamma_1\to-\infty}\, \tilde{\chi}^{\mp}_{\gamma_2\to+\infty}\right\rangle ,$$
$$\epsilon^{ab} \left|\tilde{\chi}^{+}_{\gamma_1\to-\infty}\, \tilde{\chi}^{-}_{\gamma_2\to+\infty}\right\rangle , \quad \left|\tilde{\chi}^{\pm}_{\gamma_1\to-\infty}\, \chi^{\mp}_{\gamma_2\to+\infty}\right\rangle$$
$$n = 2 : \quad \left|\chi^{\pm}_{\gamma_1\to-\infty}\, \tilde{\chi}^{\pm}_{\gamma_2\to+\infty}\right\rangle , \quad \left|\tilde{\chi}^{\pm}_{\gamma_1\to-\infty}\, \chi^{\pm}_{\gamma_2\to+\infty}\right\rangle$$

- Twisted sectors: twisted boundary conditions at level-0 with twist angle $\phi_0 = \frac{2\pi}{n}$. Only ground state survives as $\gamma_k \to -\infty$

Future Directions

- Identify the protected states in the massless thermodynamic Bethe ansatz by putting some suitable chemical potential that can disentangle ground-state degeneracy.
- Prove that wrapping is not contributing at any order actually for generic mixed flux backgrounds (using the massless TBA)

Thank You!

References

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Extra slide: Orbifolds contd. and mixed-flux

- For $AdS_3 \times S^3 \times T^4/\mathbb{Z}_2$:

$$\begin{array}{ccccc} & & h^{0,0} & & \\ & h^{1,0} & h^{\frac{1}{2},\frac{1}{2}} & h^{0,1} & \\ h^{2,0} & h^{\frac{3}{2},\frac{1}{2}} & h^{1,1} & h^{\frac{1}{2},\frac{3}{2}} & h^{0,2} \\ & h^{2,1} & h^{\frac{3}{2},\frac{3}{2}} & h^{1,2} & \\ & & h^{2,2} & & \end{array} = \begin{array}{ccccc} & & 1 & & \\ & 0 & 0 & 0 & \\ 1 & 0 & 4 & 0 & 1 \\ & 0 & 0 & 0 & \\ & & 1 & & \end{array} \oplus 16 \times \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} = \begin{array}{ccccc} & & 1 & & \\ & 0 & 0 & 0 & \\ 1 & 0 & 20 & 0 & 1 \\ & 0 & 0 & 0 & \\ & & 1 & & \end{array}$$

- For $AdS_3 \times S^3 \times T^4/\mathbb{Z}_n, n = 3, 4, 6$:

$$\begin{array}{ccccc} & & h^{0,0} & & \\ & h^{1,0} & h^{\frac{1}{2},\frac{1}{2}} & h^{0,1} & \\ h^{2,0} & h^{\frac{3}{2},\frac{1}{2}} & h^{1,1} & h^{\frac{1}{2},\frac{3}{2}} & h^{0,2} \\ & h^{2,1} & h^{\frac{3}{2},\frac{3}{2}} & h^{1,2} & \\ & & h^{2,2} & & \end{array} = \begin{array}{ccccc} & & 1 & & \\ & 0 & 0 & 0 & \\ 1 & 0 & 2 & 0 & 1 \\ & 0 & 0 & 0 & \\ & & 1 & & \end{array} \oplus 18 \times \begin{array}{c} 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{array} = \begin{array}{ccccc} & & 1 & & \\ & 0 & 0 & 0 & \\ 1 & 0 & 20 & 0 & 1 \\ & 0 & 0 & 0 & \\ & & 1 & & \end{array}$$

- Mixed-flux backgrounds: same analysis, but in terms of Zhukovskis