Entanglement and symmetry resolution in two dimensional free quantum field theories

XVI Avogadro Meeting, 22 December 2020

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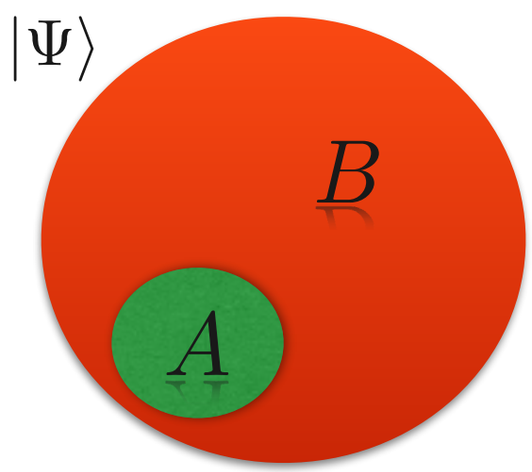
Based on: S.M., Giuseppe Di Giulio, Pasquale Calabrese,
JHEP 2008 (2020) 073

Reduced Density Matrix (RDM) and useful definitions

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B, \qquad \rho = |\Psi\rangle\langle\Psi| \qquad \rho_A = \mathrm{Tr}_B \rho$$



A measure of the entanglement between A and B are the Rényi entropies

$$S_n \equiv \frac{1}{1-n} \log \mathrm{Tr} \rho_A^n$$

Q: charge operator generating the $U(1)$ symmetry

$$[\rho, Q] = 0 \xrightarrow{\mathrm{Tr}_B} [\rho_A, Q_A] = 0$$

Reduced Density Matrix (RDM) and useful definitions

$$\rho_A = \begin{pmatrix} q_1 & & & \\ & q_2 & & \\ & & q_3 & \\ & & & \ddots \end{pmatrix}$$

$$\mathrm{Tr}\rho_A(q_1) = p(q_1) \equiv \mathcal{Z}_1(q_1)$$

q-moments of the RDM: $\quad \mathcal{Z}_n(q) \equiv \mathrm{Tr}(\Pi_q \, \rho_A^n)$

Symmetry Resolved Rényi entropies (SREE): $\quad S_n(q) \equiv \dfrac{1}{1-n} \log \mathrm{Tr}\rho_A^n(q)$

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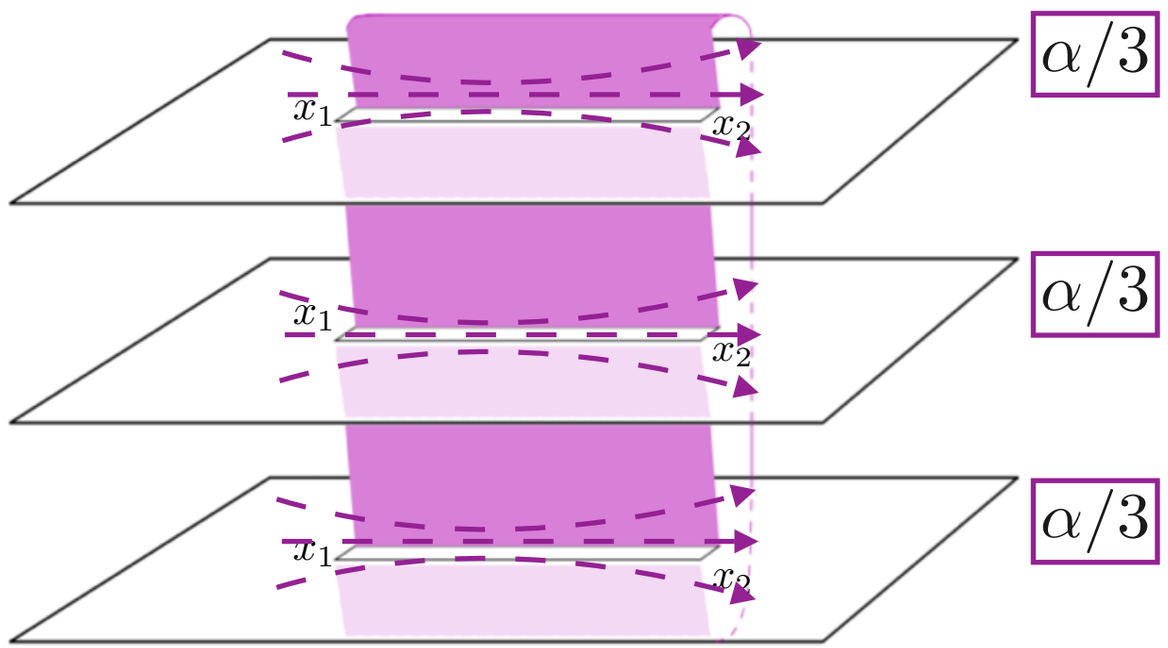


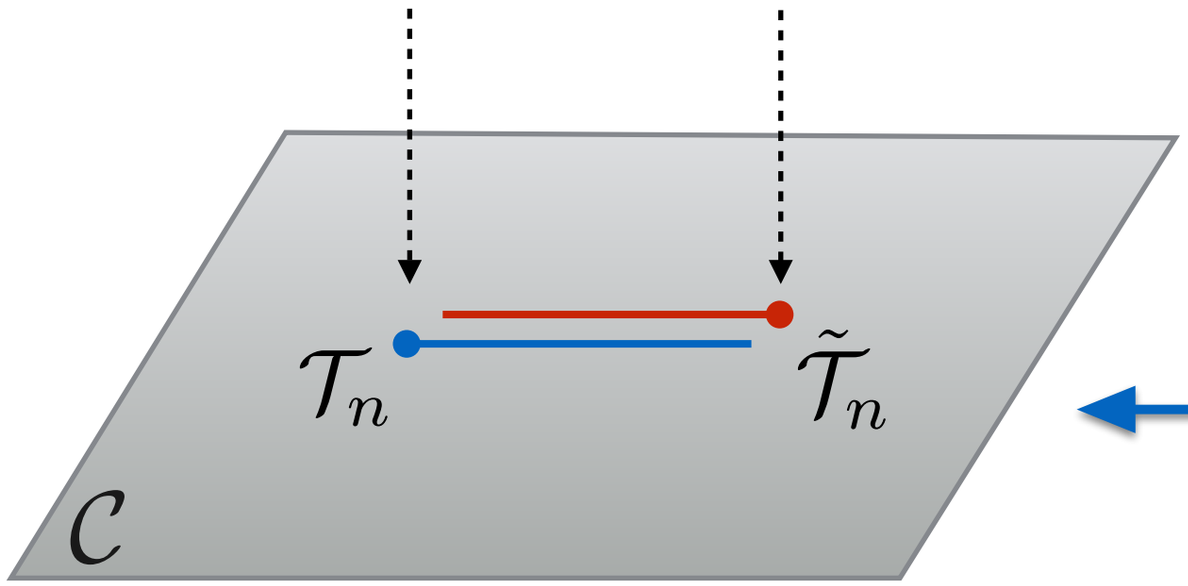
q-moments of the RDM: $$\mathcal{Z}_n(q) \equiv \mathrm{Tr}(\Pi_q \rho_A^n) = \int_{-\pi}^{\pi} \frac{d\alpha}{2\pi} e^{-iq\alpha} Z_n(\alpha)$$

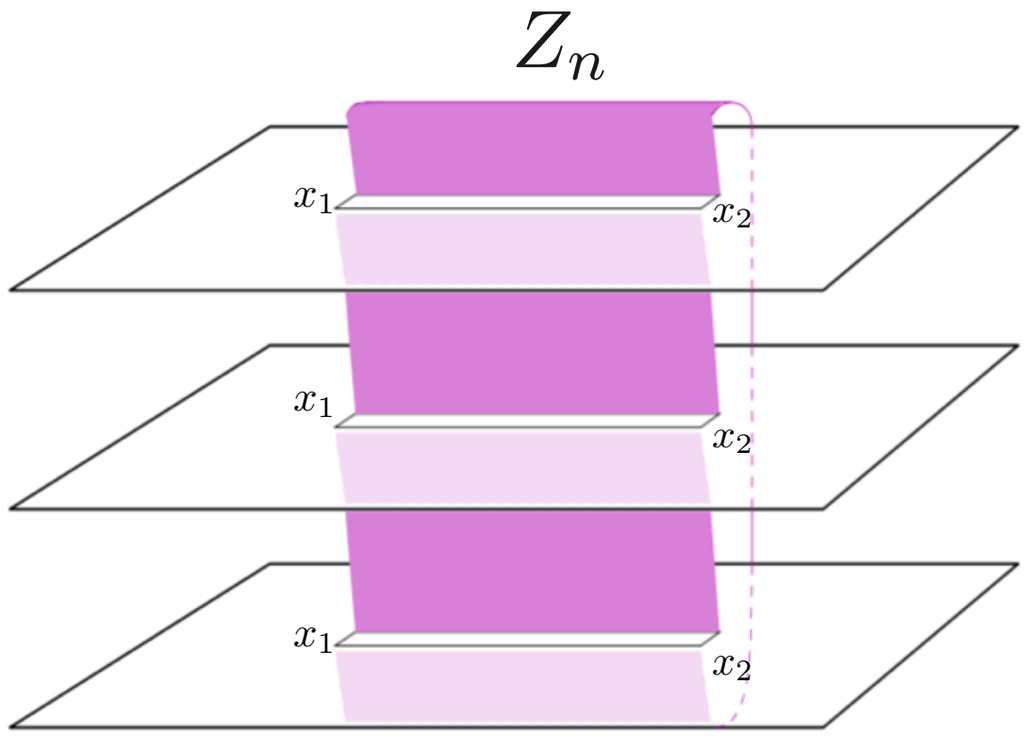
Symmetry Resolved Rényi entropies (SREE): $$S_n(q) \equiv \frac{1}{1-n} \log \mathrm{Tr}\rho_A^n(q)$$

Charged moments of the RDM: $$Z_n(\alpha) \equiv \mathrm{Tr}\rho_A^n e^{iQ_A\alpha}$$

M. Goldstein and E. Sela, Phys. Rev. Lett. 120, 200602 (2018).

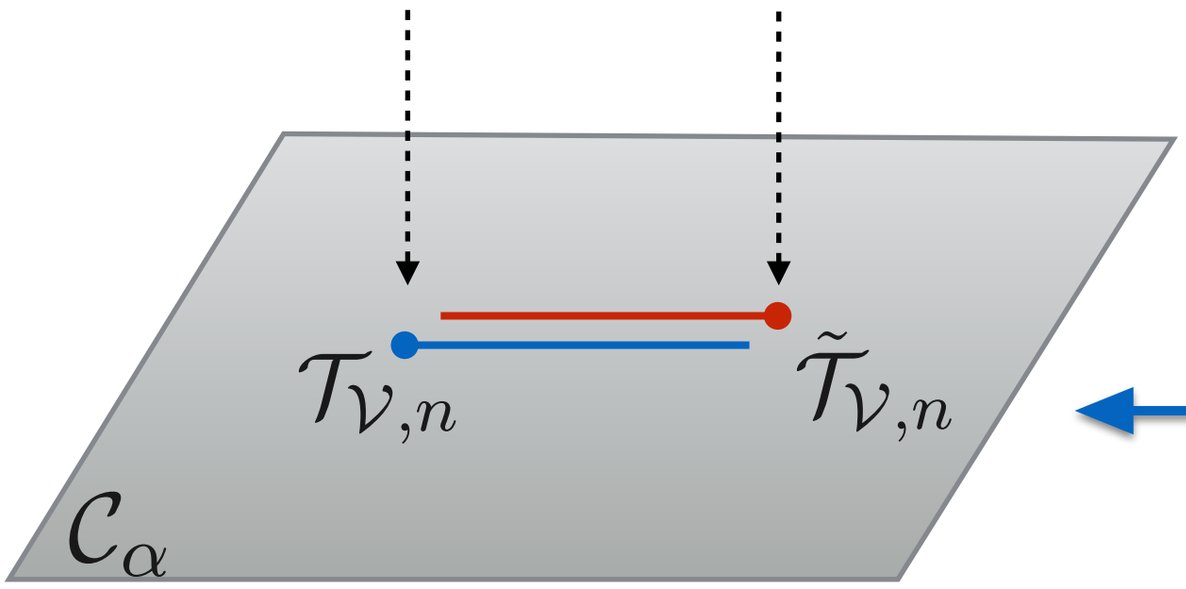
Replica method and results from QFT

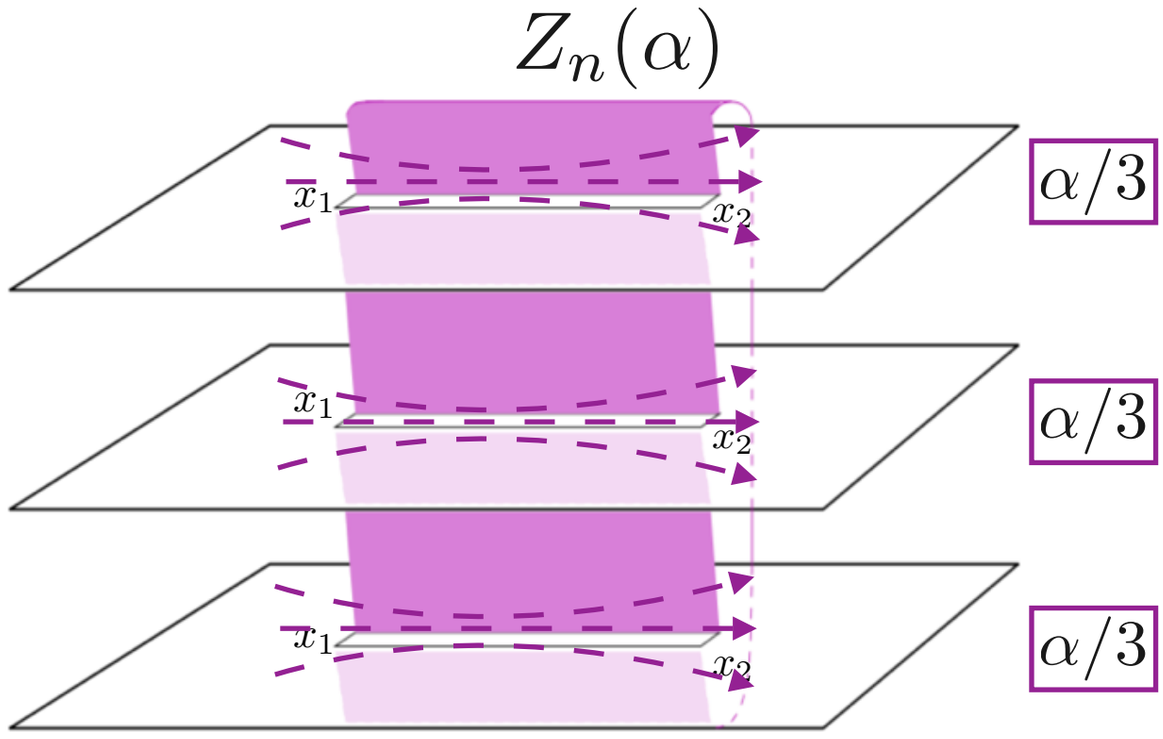




$\langle \mathcal{T}_n(x_1,0)\tilde{\mathcal{T}}_n(x_2,0)\rangle_{\mathcal{C}}$ ← $\mathrm{Tr}\rho_A^n$

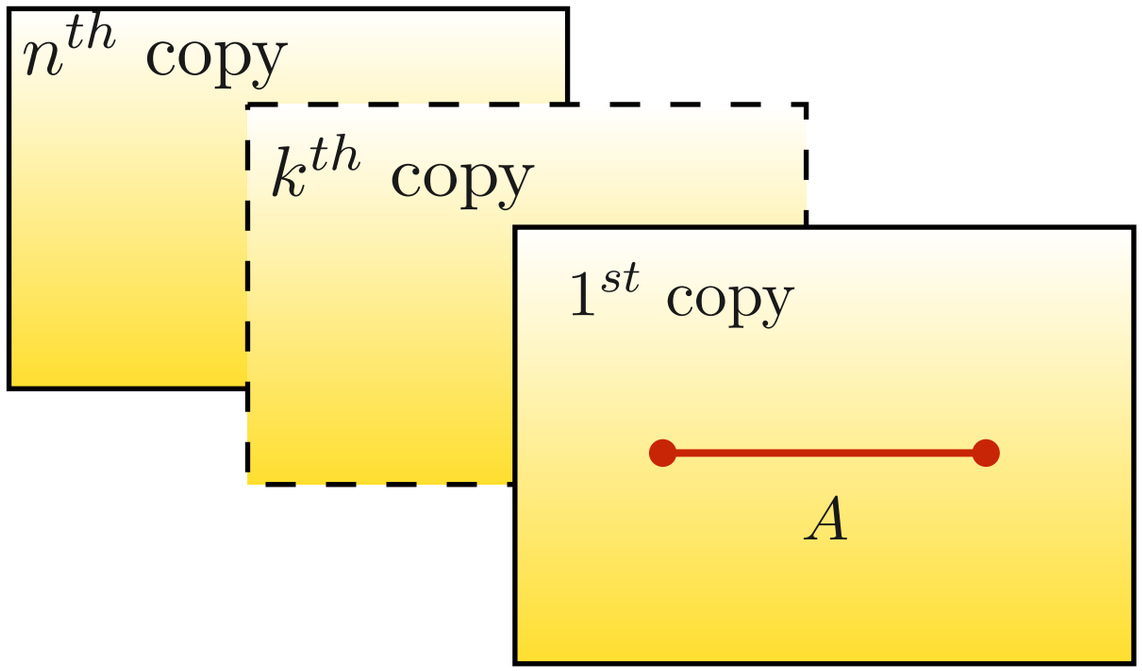
Replica method and results from QFT

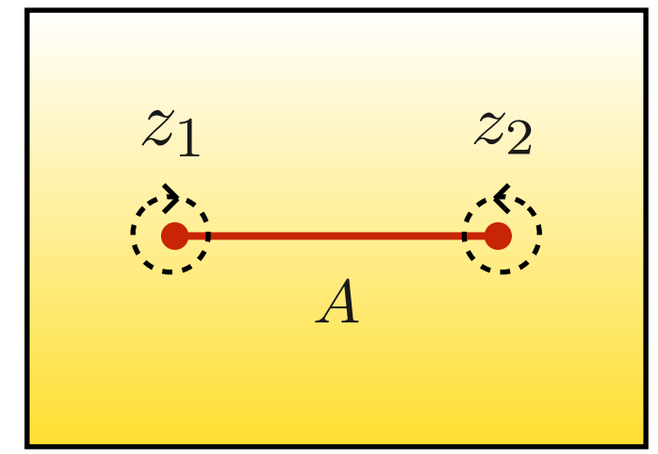


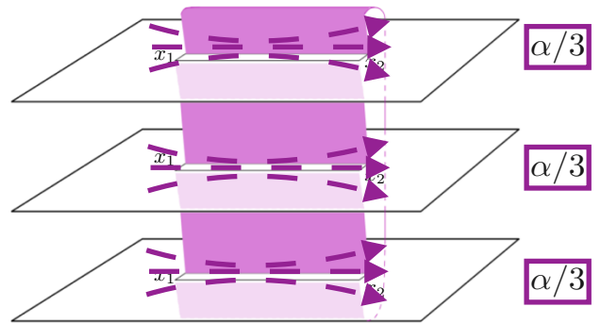


$$\langle \mathcal{T}_{\mathcal{V},n}(x_1,0)\tilde{\mathcal{T}}_{\mathcal{V},n}(x_2,0)\rangle_{\mathcal{C}_\alpha} \longleftarrow \mathrm{Tr}\rho_A^n e^{iQ_A\alpha}$$

Diagonalisation in replica space: modified twist fields





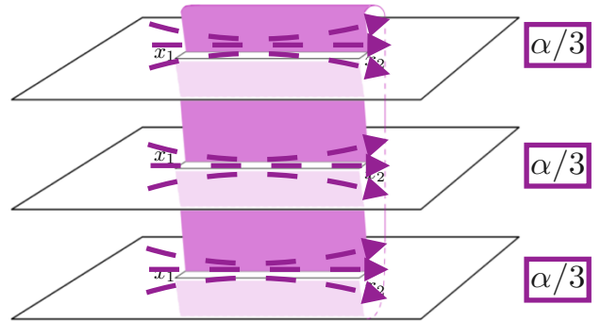


$\mathrm{Tr}\rho_A^n e^{iQ_A\alpha}$

$$\tilde{\varphi}_k(e^{2\pi i}z, e^{-2\pi i}\bar{z}) = e^{2\pi i a}\tilde{\varphi}_k(z,\bar{z}), \qquad a = \frac{k}{n} + \frac{\alpha}{2\pi n} \qquad \zeta_{k,\alpha} \propto \langle \mathcal{T}_{n,k,\alpha}(z_1)\tilde{\mathcal{T}}_{n,k,\alpha}(z_2)\rangle$$

The partition function $\zeta_{k,\alpha}$ is computed through the two-point function of twist fields, which in conformal invariant theories is fixed by their scaling dimension.

Diagonalisation in replica space: Green's function approach



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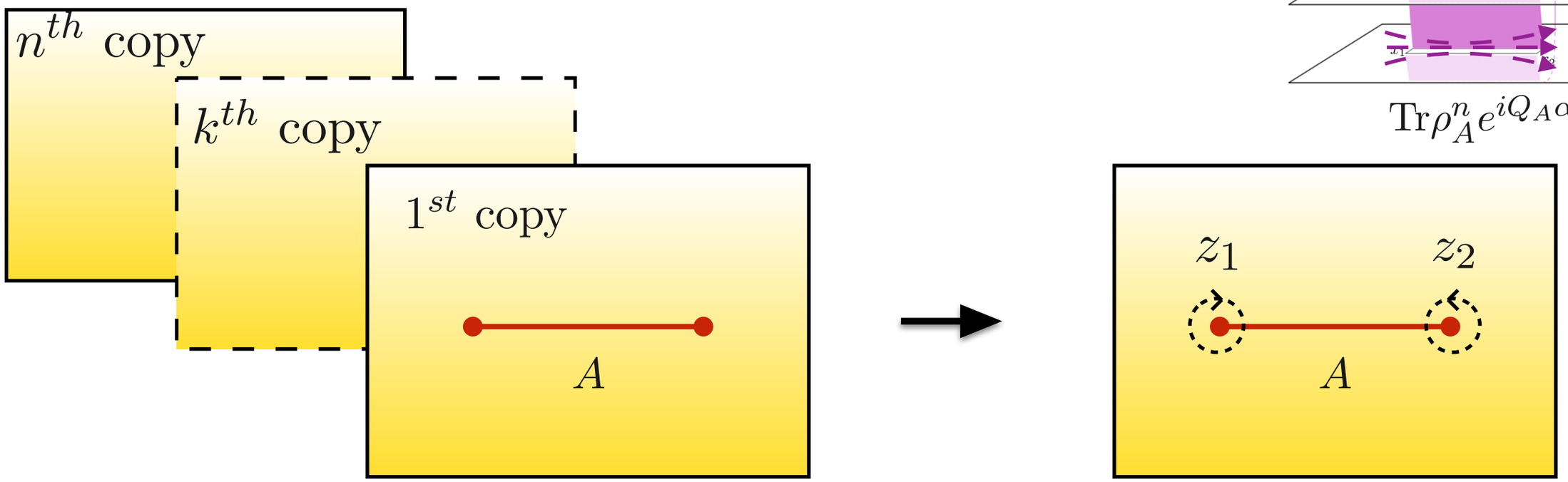


Dirac field

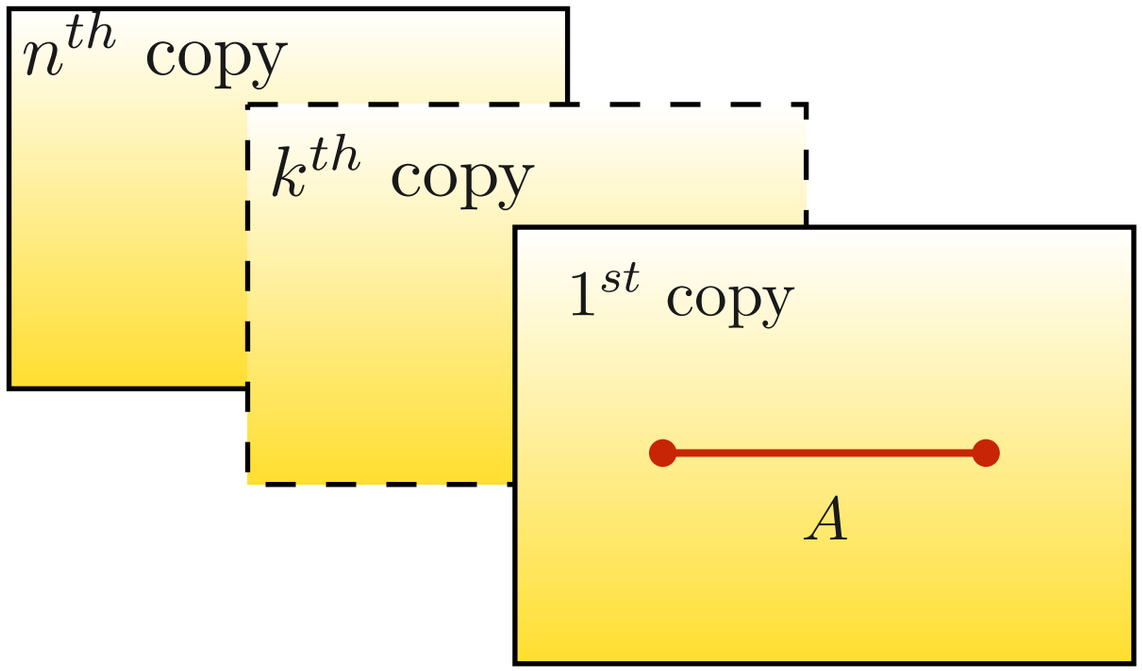
$$\partial_m \ln \zeta_k = \mathrm{tr}\, G_D,$$

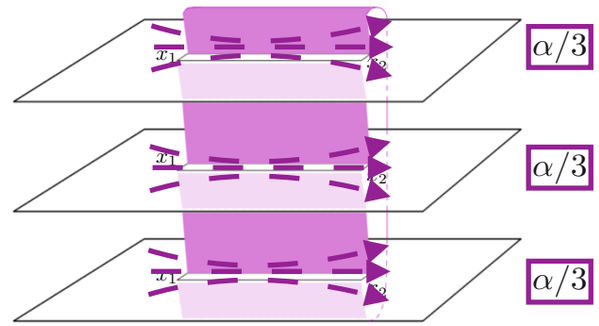
scalar field

$$\partial_{m^2} \ln \zeta_k = -\int dr^2\, G_S(\mathbf{r}, \mathbf{r}')$$

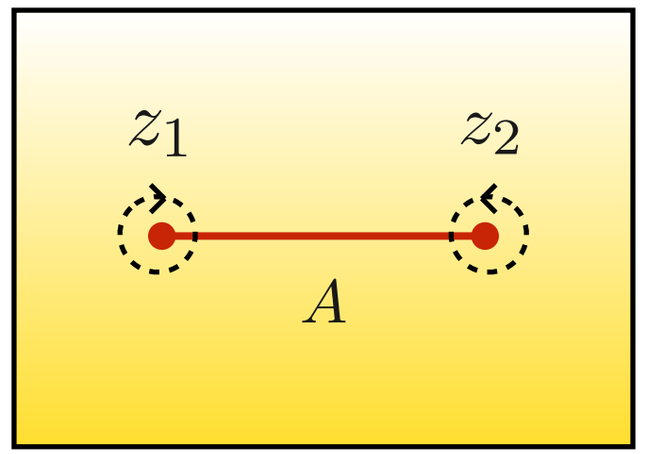
Equivalence between the Green's function $G_{D/S}$ in the k-th copy and the partition function $\zeta_{k,\alpha}$ in the same geometry.

Diagonalisation in replica space: Green's function approach





$$\mathrm{Tr}\rho_A^n e^{iQ_A\alpha}$$



Dirac field

$$\partial_m \ln \zeta_k = \mathrm{tr}\, G_D,$$

scalar field

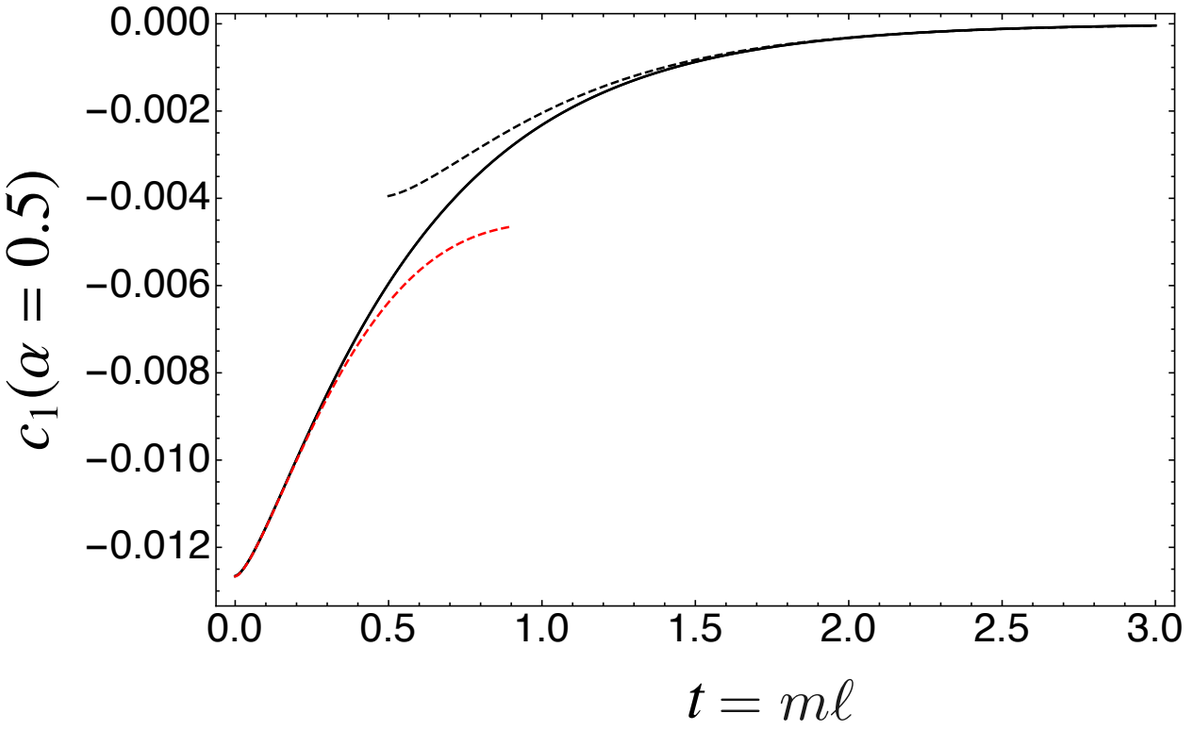
$$\partial_{m^2} \ln \zeta_k = -\int dr^2\, G_S(\mathbf{r}, \mathbf{r}')$$

$$w_a \equiv \ell \partial_\ell \ln \zeta_a, \qquad c_n(\alpha) \equiv \sum_k w_{\frac{k}{n}+\frac{\alpha}{2\pi n}}$$

$$c_n(\alpha) = \ell \frac{\partial \ln Z_n(\alpha)}{\partial \ell} \quad \Rightarrow \quad \ln Z_n(\alpha) = \int_\epsilon^\ell \frac{c_n(\alpha)}{\ell'} d\ell'$$

Dirac field

$$S_D = \frac{1}{4\pi}\int dz d\bar{z}\,(\psi_R^* \partial_z \psi_R + \psi_L^* \partial_{\bar{z}} \psi_L + m(\psi_L^* \psi_R + \psi_R^* \psi_L)$$



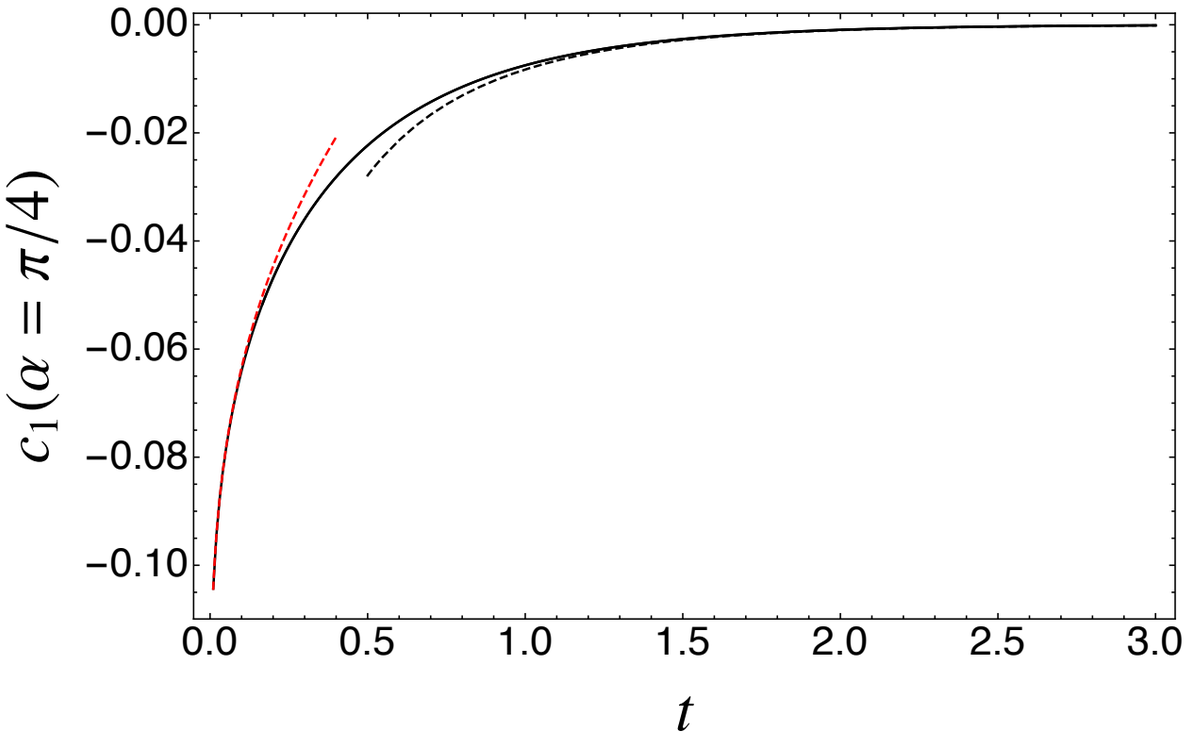
Entanglement equipartition at leading-order

$$S(q) = S^D - \frac{1}{2}\ln\left(\frac{2}{\pi}\ln\delta_1\ell\right) - \frac{1}{2} + q^2\pi^4\frac{h_1 + h_1'}{2(\ln\ell\kappa_1)^2} + o(\ln\ell^{-2}) \qquad m\ell \to 0$$

$$S(q) = -\frac{1}{3}\ln(m\epsilon) - \frac{1}{4}K_0(2m\ell) - \frac{1}{2}\ln\left(\frac{2|\ln(m\epsilon)|}{\pi}\right) - \frac{1}{2} + O((\ln m\epsilon)^{-1}) \qquad m\ell \to \infty$$

Complex scalar field

$$S_S = \frac{1}{4\pi} \int dz d\bar{z} \left(\partial_z \varphi^* \partial_{\bar{z}} \varphi + \partial_{\bar{z}} \varphi^* \partial_z \varphi + m^2 \varphi^* \varphi \right)$$



Entanglement equipartition at leading-order

Towards d>2:

Same dependence on α of the charged moments of a free massive scalar theory across a hyperplane in d Euclidean dimensions and hence the symmetry resolved entropies are also the same in any dimension.

Entanglement and symmetry resolution in two dimensional free quantum field theories

Entanglement entropy in each sector for the ground state of conformal invariant critical one-dimensional systems? ✓

What happens away from criticality? ✓

What about higher-dimensional systems? ✓

Non-abelian symmetries? 

Other measures of resolved-entanglement?