$T\overline{T}$-deformation of q-Yang-Mills theory

Joint work with Richard J. Szabo and Miguel Tierz:

JHEP (2019) [1810.05404], JHEP (2020) [2009.00657](today)

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XVI Avogadro Meeting, GGI via Zoom
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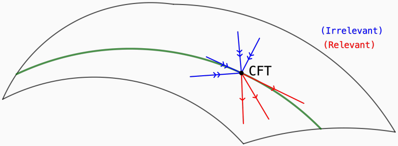
Overview

- Introduction: perturbing a QFT with the operator $T\overline{T}$.
- Lightening review: The family of two-dimensional Yang-Mills theories.
- $T\overline{T}$-deformed q-Yang-Mills.
- Outlook.

Perturbing QFTs

Understanding all possible deformations of a CFT allows to explore its neighbourhood in the space of theories.



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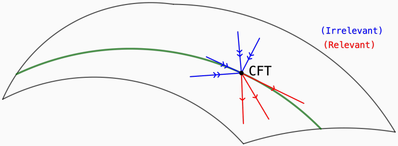

Eventually, we want to chart the space of all consistent QFTs. *Vaste programme!*

More instructive approach: characterize deformations that preserve a distinguished property, e.g. integrability.

Perturbing with the $T\overline{T}$ operator (I)

Irrelevant deformations of two-dimensional QFTs by the composite operator $T\overline{T}$ *[Cavaglià-Negro-Szécsényi-Tateo,Smirnov-Zamolodchikov]* have attracted considerable attention in recent years.

$$T = -2\pi T_{zz}, \ \overline{T} = -2\pi T_{\bar{z}\bar{z}}, \ \Theta = 2\pi T_{z\bar{z}}$$

$$T\overline{T} = \lim_{(z',\bar{z}') \to (z,\bar{z})} \big(T(z,\bar{z})\overline{T}(z',\bar{z}') - \Theta(z,\bar{z})\Theta(z',\bar{z}') \big) + \text{(total deriv.)}$$

The deformation is solvable: allows to reconstruct the irrelevant flow. (Regardless of the original QFT being integrable, $T, \overline{T}, \Theta$ are conserved).

Perturbing with the $T\overline{T}$ operator (II)

Broad variety of applications in 2d QFT, including

- integrability,
- holography (both $\mathrm{AdS}_3/\mathrm{CFT}_2^{T\overline{T}}$ and $\mathrm{AdS}_2^{T\overline{T}}/\mathrm{CFT}_1$),
- SCFTs,
- perturbation by generalized irrelevant operators, etc.

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Powerful insight *[Dubovsky-Gorbenko,Mirbabayi,HernándezChifflet]*:

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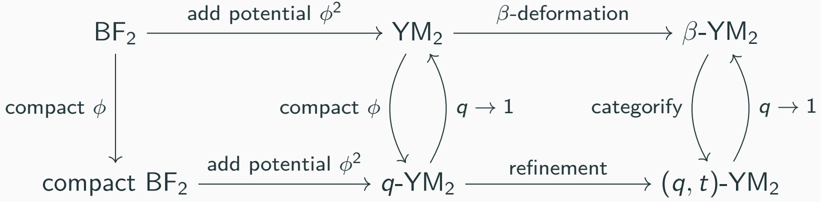
We use Euclidean formulation *[Coleman-AguileraDamia-Freedman-Sony]*.

Two-dimensional Yang-Mills and its deformations

$$S_{\mathrm{YM}} = \frac{1}{2g_{\mathrm{YM}}^2} \int_\Sigma \mathrm{Tr}\, F^A * F^A = \int_\Sigma \mathrm{Tr}\Big(\mathrm{i}\, \phi\, F^A + \frac{g_{\mathrm{YM}}^2}{2}\, \phi^2\, \omega \Big) \ ,$$

YM_2 (or deformations) $\cong$ topological BF_2 (or deformations) $+ V(\phi)$.



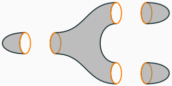
Abelianization *[Blau-Thompson]* $\Rightarrow$ path integral reduces to finite-dimensional expression:

$$\mathcal{Z}_{q\text{-}\mathrm{YM}}[\Sigma] = \sum_R \left(\dim_q R\right)^{\chi(\Sigma)}\, q^{\frac{p}{2} C_2(R)}.$$

$T\overline{T}$-deformation of two-dimensional Yang-Mills (I)

$T\overline{T}$-deform YM_2 (and its relatives):

- start with BF_2 and use TFT axioms to cut Σ in elementary pieces;



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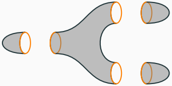

- turn on $V(\phi)$ (and q-deformation) on each component;

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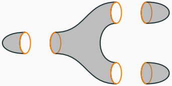

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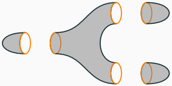

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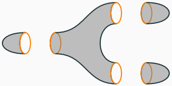

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We have shown that

- solving the gravitational path integral gives $V(\phi) \mapsto \frac{V(\phi)}{1-\frac{\tau}{N^3}V(\phi)}$,
- Abelianization persists, and
- we can glue back the pieces, obtaining the $T\overline{T}$-def. theory on Σ.

$T\overline{T}$-deformation of two-dimensional Yang-Mills (II)

For closed Σ,

$$\mathcal{Z}_{q\text{-YM}}^{T\overline{T}}[\Sigma] = \sum_{R} \left(\dim_q(R)\right)^{\chi(\Sigma)} \ q^{\frac{p}{2}\, C_2^{T\overline{T}}(R,\tau)}$$

with $C_2^{T\overline{T}}(R,\tau) = \frac{C_2(R)}{1-\frac{\tau}{N^3}C_2(R)}$. Similar expressions can be obtained for: boundaries, punctures, Wilson loops, symmetry defects.

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- Factorization of the partition function *[Aganagic,Caporaso,Cirafici,Griguolo, Ooguri,Pasquetti,Saulina,Seminara,Szabo,Tanzini,Vafa...]*. In the free fermion formalism *[Douglas]* $T\overline{T}$ introduces non-local interactions.

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- Truncation of sum to integrable R at $q \to 1$ and connection with Chern-Simons theory.

Large N phase transition (I)

U(N) YM_2 on $\Sigma = \mathbb{S}^2$ undergoes a 3rd order phase transition *[Douglas-Kazakov]* induced by instantons *[Gross-Matytsin]*.

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The same is true for q- and (q,t)-YM_2 and $p > 2$ *[Caporaso-Cirafici-Griguolo-Pasquetti-Seminara-Szabo,Arsiwalla-Boels-Mariño-Sinkovics,Jafferis-Marsano,Kokenyesi-Sinkovics-Szabo]*. The q-deformation enhances the volume of the weak coupling phase in parameter space.

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q- and (q,t)-$\mathrm{YM}_2^{T\overline{T}}$ experience again a instanton-induced 3rd order phase transition. Volume of weak coupling phase is

- reduced, compared to q-YM_2
- enhanced, compared to $\mathrm{YM}_2^{T\overline{T}}$

Transition extends below $p=2$ *[LS-Szabo-Tierz]* all the way down to $p=1$ *[Gorsky-Pavshinkin-Tyutyakina]*.

Outlook

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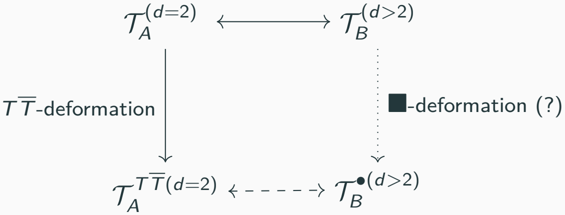
Eventually, we would like to gain analytic control on flows by irrelevant operators in higher dimensions.



Outlook

YM_2 is tightly related to four-dimensional SUSY theories:

- q-YM_2 descends from $\mathcal{N} = 4$ SYM_4 in the total space of a line bundle over Σ *[Aganagic-Ooguri-Saulina-Vafa]*;
- Class S *[Gaiotto,Gadde-Rastelli-Razamat-Yan]*;
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What happens if we couple the 2d sector to JT gravity? Can we pull the $T\overline{T}$-deformation back to 4d?

This is the end of the presentation.

Thank you for your attention.