Electromagnetic Quasitopological Gravities

Ángel Jesús Murcia Gil

Instituto de Física Teórica UAM/CSIC, Madrid.

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with Pablo A. Cano (KU Leuven, Belgium)

Instituto de Física Teórica UAM-CSIC

Introduction

- The gravitational effective action expected to have infinite tower higher-derivative terms (String Theory).
- However, recently interest in studying higher-order gravities by themselves and take an EFT approach.
- Examples of higher-curvature gravities:
  - Lovelock gravities [Lovelock]:

$$\mathcal{L} = \frac{1}{16\pi G}\left(R + \sum_{n=2}^{[(d-1)/2]} \lambda_n \, \ell^{2n-2} \, \chi_{2n}\right),$$

where $\chi_{2n} = \frac{(2n)!}{2^n} \delta^{[\mu_1}_{\nu_1} \dots \delta^{\mu_{2n}]}_{\nu_{2n}} R_{\mu_1\mu_2}{}^{\nu_1\nu_2} \dots R_{\mu_{2n-1}\mu_{2n}}{}^{\nu_{2n-1}\nu_{2n}}$.

  - $f(R)$ theories [Sotiriou, Faraoni]:

$$\mathcal{L} = \frac{1}{16\pi G} f(R)\,.$$

Generalized Quasitopological Gravities (GQs).

- Important examples of higher-curvature gravities: Generalized Quasitopological Gravities[1] (GQs) [Bueno, Cano, Hennigar, Kubizňak, Mann,...].
- GQs admit static, spherically-symmetric (SSS) solutions of the form:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_{d-2}^2 \,.$$

- Properties of GQs:
  - The EoM for $f(r)$ is at most[2] of order 2.
  - Linearized EoM on constant-curvature backgrounds are second-order.
  - BH thermodynamics can be computed analytically.
  - Any higher-derivative theory can be mapped via field redefinitions to a GQ [Bueno, Cano, Moreno, ÁM].
- Most paradigmatic example of GQ: Einsteinian Cubic Gravity [Bueno, Cano, 2016].

[1] Lovelock gravities are a subclass of GQs.

[2] We assume no explicit covariant derivatives of the curvature appear in the action.

Motivation for Electromagnetic Quasitopological Gravities

- GQs are purely gravitational theories, no coupling to matter.
- Desirable to extend definition of GQs to include matter. Simple and relevant example: an Abelian gauge field.
- GQs with minimally-coupled vector field have been considered [Bueno, Cano, Frassino, Hennigar, Rocha...]. But this is very restrictive. In general, higher-derivative actions may contain all possible couplings.
- Question: is it possible theories analogous to GQs with non-minimal couplings between curvature and gauge field?
- Answer: Yes! Electromagnetic Quasitopological Gravities (EQs).

Electromagnetic Quasitopological Gravities (EQs)

- We will fix from now on the space-time dimension to 4.
- We search for higher-order theories of gravity with a non-minimally coupled vector field satisfying (EQ conditions):
  1. Diffeomorphism- and gauge-invariance.
  2. Assuming electric or magnetic ansatz for vector field, existence of SSS solutions characterised by single function $f(r)$:

$$\mathrm{d}s^2 = -f(r)\mathrm{d}t^2 + \frac{1}{f(r)}\mathrm{d}r^2 + r^2\mathrm{d}\Omega^2_{d-2} \quad \text{and} \quad \begin{cases} F^{\mathrm{e}} = -\Phi'(r)\mathrm{d}t \wedge \mathrm{d}r\,. \\ F^{\mathrm{m}} = \chi'(\theta)\mathrm{d}\theta \wedge \mathrm{d}\varphi\,. \end{cases}$$

  3. The equation of motion for $f(r)$ is at most second-order[3].

[3] We shall not consider any explicit covariant derivative on the curvature or the field strength in the action.

Electromagnetic Quasitopological Gravities (EQs)

- A clever way to implement EQ conditions:
  1. Set $F = -\Phi'(r)\mathrm{d}t \wedge \mathrm{d}r$ or $F = \chi'(\theta)\mathrm{d}\theta \wedge \mathrm{d}\varphi$.
  2. Search for some choice of $\Phi_{\mathsf{sol}}(r)$ or $\chi_{\mathsf{sol}}(\theta)$ solving Maxwell equation for any higher-order theory (under SSS ansatz).
  3. Determine which theories become GQs after imposing F_{sol}.
- For magnetic vector fields, the previous programme can be carried out, because

$$F^{\mathrm{m}} = P \sin\theta \mathrm{d}\theta \wedge \mathrm{d}\varphi$$

  always solves the Maxwell equation for an SSS metric.
- For electric vector fields, the programme does not work: if $F^{\mathrm{e}} = -\Phi'(r)\mathrm{d}t \wedge \mathrm{d}r$, then $\Phi(r)$ depends on the theory.
- How to define theories canonically admitting electric solutions? $\rightarrow$ Dualizing theory with magnetic solutions!

Electromagnetic Quasitopological Gravities (EQs)

- Dualization: A map between two theories:

$$\begin{pmatrix} g_{\mu\nu} \\ \\ F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} \\ \\ \mathcal{L}(R,F) \end{pmatrix} \longrightarrow \begin{pmatrix} g_{\mu\nu} \\ \\ G_{\mu\nu} = 2\partial_{[\mu}B_{\nu]} \\ \\ \mathcal{L}'(R,G) = \mathcal{L}(R,F(G)) - 2F(G)^{\mu\nu}(\star G)_{\mu\nu} \end{pmatrix}$$

where $F_{\mu\nu}(G_{\rho\sigma})$ is obtained by inverting

$$\frac{\partial \mathcal{L}}{\partial F^{\mu\nu}} = 2(\star G)_{\mu\nu} \tag{1}$$

- Imposing (1) on the EoMs and Bianchi identity of dual theory, one recovers the set of EoMs and Bianchi of original theory[4].
- If we have a theory with an SSS magnetic solution, the dual theory will have SSS electric solutions!

[4]Net effect is exchange of Bianchi identities and Maxwell equations.

Electromagnetic Quasitopological Gravities (EQs)

Definition 1 (Electromagnetic Quasitopological Gravities)

A given theory $\mathcal{L}(R, F)$ is an Electromagnetic Quasitopological Gravity iff its Lagrangian or the Lagrangian of its dual theory admits SSS magnetic solutions characterised by a single metric function $f(r)$.

Examples:

- Any GQ with a minimally-coupled Abelian vector field is an EQ.
- Non-trivial examples:

$$\mathcal{L}^{(a)}_{n,m} = \left(2nR_\mu{}^\alpha \delta_\nu{}^\beta - (3n-3+4m)R^{\alpha\beta}{}_{\mu\nu}\right)\left(R^{n-1}\right)^{\mu\nu}{}_{\rho\sigma} F^{\rho\sigma} F_{\alpha\beta}\left(F^2\right)^{m-1} ,$$

$$\mathcal{L}^{(b)}_{n,m} = \left(F^2\right)^{m-1} F_{\mu\nu}F^{\rho\sigma}\left(\frac{n}{2}R\left(R^{n-1}\right)^{\mu\nu}{}_{\rho\sigma} + \right.$$
$$\left. + \frac{(n+4-4m)}{4}(3n-3+4m)\left(R^n\right)^{\mu\nu}{}_{\rho\sigma}\right) - n\left(F^2\right)^{m-1} F_{\alpha\nu}F^{\rho\sigma}R_\mu{}^\alpha \Big\{$$
$$(1+2n)\left(R^{n-1}\right)^{\mu\nu}{}_{\rho\sigma} - (n-1)R^\beta{}_\rho\left(R^{n-2}\right)^{\mu\nu}{}_{\beta\sigma}\Big\} ,$$

$$\mathcal{L}^{(c)}_{n,m} = \left(R^{n-1}\right)^{\mu\nu}\left[nRg^{\alpha\beta} - (4n+4m-3)R^{\alpha\beta}\right]F_{\mu\alpha}F_{\nu\beta}\left(F^2\right)^{m-1} .$$

Properties of EQs (I)

Most relevant properties of EQs:

1. They admit electrically/magnetically charged solutions (by def.).

2. The EoM for $f(r)$ is at most second-order.

3. The only gravitational mode propagated on maximally-symmetric backgrounds is a spin 2-massless graviton.

Properties of EQs (II)

4. BH thermodynamics can be computed analytically. For all solutions we have studied, the following first law of BH thermodynamics holds:

$$\mathrm{d}M = T\mathrm{d}S + \Psi_h \mathrm{d}P\,,$$

where M mass, T temperature, S entropy, Ψ_h the *electric* potential and P the (electric or magnetic) charge.

5. A subset of EQs admit completely regular (magnetic or electric) SSS solutions: regular in both electromagnetic and gravitational fields. Focusing on electric solutions, it was found the first explicit theory resolving the Reissner-Nordström singularities for any value of mass and electric charge. [Cano, ÁM]:

Conclusions and Future Directions

- Message to take Home: A new type of gravitational theories with a non-minimally coupled Maxwell field has been identified. These theories are defined by admitting electrically/magnetically charged SSS solutions with nice and reasonable properties.

Future directions:

- How general EQs? Can every theory be mapped via field redefinitions to an EQ? (this happens for GQs...)
- Holographic dual of these theories? What type of CFT duals have these theories?
- Higher-dimensional generalizations of EQs?

Grazie mille per la vostra attenzione

"Piano piano si arriva lontano"

Regular solutions in EQs

- Regular electric solutions are specially relevant → built by dualization of theory admitting regular magnetic ones. First explicit theory regularizing gravitational and EM fields for any M and Q [Cano, ÁM]:

$$\mathcal{L}(R,F) = R - F_{\mu\nu}F_{\rho\sigma}\chi^{\mu\nu\rho\sigma}\,, \quad \chi^{\mu\nu}{}_{\rho\sigma} = 6\delta^{[\mu\nu}{}_{\rho\sigma}\left(\mathcal{Q}^{-1}\right)^{\alpha\beta]}{}_{\alpha\beta}\,,$$

where $\left(\mathcal{Q}^{-1}\right)^{\alpha\beta}{}_{\mu\nu}\mathcal{Q}^{\mu\nu}{}_{\rho\sigma} = \delta^{\alpha\beta}{}_{\rho\sigma}$ and

$$\begin{aligned}\mathcal{Q}^{\mu\nu}{}_{\rho\sigma} = \delta^{\mu\nu}{}_{\rho\sigma} &+ \alpha\left(6R^{[\mu}{}_{[\sigma}\delta^{\nu]}{}_{\rho]} + 7R^{\mu\nu}{}_{\rho\sigma} + \frac{1}{2}R\delta^{\mu\nu}{}_{\rho\sigma}\right)\\ &+ \alpha^2\left(\frac{9}{4}R_{\alpha}{}^{[\mu}R^{\nu]\alpha}{}_{\rho\sigma} + \frac{9}{4}R^{\alpha}{}_{[\rho}R^{\mu\nu}{}_{\sigma]\alpha} + \frac{1}{4}RR^{\mu\nu}{}_{\rho\sigma}\right.\\ &\left.+\frac{35}{8}R^{\mu\nu\alpha\beta}R_{\alpha\beta\rho\sigma} + \frac{1}{2}R_{\lambda}{}^{[\mu}\delta^{\nu]\lambda}{}_{\beta[\rho}R_{\sigma]}{}^{\beta}\right)\,,\end{aligned}$$

- No need to know $\left(\mathcal{Q}^{-1}\right)^{\alpha\beta}{}_{\rho\sigma}$ to solve the EoMs!