Pietro Ferrero

University of Oxford

M2- and D3-branes wrapped on spindles

Based on work with

JP Gauntlett, JM Perez Ipina, D Martelli, JF Sparks

arXiv: **2011.10579** [hep-th]

arXiv: **2012.08530** [hep-th]

Review: black branes and RG flows in AdS/CFT

Black branes as RG flows

Maldacena, Nunez 0007018 [hep-th]

Flat (d+1)-branes at CY singularity ⟷ **$SCFT_{d+2}$ on $\mathbb{R}^{d+2}$**

(Al)AdS_{d+3}

↑ Boundary

Supergravity solution: (d-1) black brane

↓ Horizon

$AdS_{d+1} \times \Sigma$

(S)CFT_{d+2} on $\mathbb{R}^d \times \Sigma$

UV

↓ RG flow

IR

(S)CFT_d on $\mathbb{R}^d$

$AdS_{d+1} \times \Sigma$ ⟷ (S)CFT_d on $\mathbb{R}^d$

Microstates counting in $\mathrm{AdS}_{4,5}$

See Zaffaroni 1902.07176 [hep-th] for a review and references

$3d$ $\mathcal{N}=2$ SCFTs: **M2-branes**

↕ **UV**

AdS_4 black holes

↓ **Horizon**

$\mathrm{AdS}_2 \times \Sigma$

↕

$1d$ $\mathcal{N}=2$ SCQM

***I*-extremization**

Benini, Hristov, Zaffaroni 1511.04085 [hep-th]

$4d$ $\mathcal{N}=1$ SCFTs: **D3-branes**

↕ **UV**

AdS_5 black strings

↓ **Horizon**

$\mathrm{AdS}_3 \times \Sigma$

↕

$2d$ $\mathcal{N}=(0,2)$ SCFT*s*

***c*-extremization**

Benini, Bobev 1211.4030 [hep-th]

Plebanski-Demianski black holes in AdS_4

Plebanski-Demianski black holes

Most general solution to **4d Einstein-Maxwell*** theory (no NUT charge)

$$ds_4^2 = \frac{1}{\Omega^2}\left\{-\frac{Q}{\Sigma}\left[dt - a\sin^2\theta d\phi\right]^2 + \frac{\Sigma}{Q}dr^2 + \frac{\Sigma}{P}d\theta^2 + \frac{P}{\Sigma}\sin^2\theta\left[a dt - \left(r^2 + a^2\right)d\phi\right]^2\right\}$$

$$A = -e\frac{r}{\Sigma}\left(dt - a\sin^2\theta d\phi\right) + g\frac{\cos\theta}{\Sigma}\left(a dt - \left(r^2 + a^2\right)d\phi\right)$$

$$\Omega = 1 - \alpha\, r\, \cos\theta$$

$$\Sigma = r^2 + a^2\cos^2\theta$$

$$P = 1 - 2\,\alpha\, m\,\cos\theta + \left(\alpha^2\left(a^2 + e^2 + g^2\right) - a^2\right)\cos^2\theta$$

$$Q = \left(r^2 - 2\,m\,r + a^2 + e^2 + g^2\right)\left(1 - \alpha^2 r^2\right) + \left(a^2 + r^2\right)r^2$$

$$\Lambda = -3$$

$\alpha \leftrightarrow$ acceleration

$a \leftrightarrow$ rotation

$m \leftrightarrow$ mass

$e \leftrightarrow$ electric charge

$g \leftrightarrow$ magnetic charge

*bosonic sector of 4d $\mathcal{N} = 2$ minimal gauged supergravity

Accelerating black holes have
conical singularities at $\theta = 0, \pi$!
Should we throw them away?

No: uplift to 11d, constrain the
parameters and regularize!

For alternative approaches:
Ernst '75,
Achúcarro, Gregory, Kuijken '95,
Eardley, Horowitz, Kastor, Taschen '95,
Dowker, Gauntlett, Gibbons, Horowitz '95,
...

The uplift to 11d

Gauntlett, Varela 0707.2315 [hep-th]

4d metric

Sasaki-Einstein manifold (regular class)

Kahler-Einstein manifold (positive curvature)

$$\mathrm{d}s_{11}^2 = \frac{1}{4}\,\mathrm{d}s_4^2 + ds^2(SE_7) = \frac{1}{4}\,\mathrm{d}s_4^2 + \left(\eta + \frac{1}{2}A\right)^2 + \mathrm{d}s^2(KE_6)$$

$$G = \frac{3}{8}\mathrm{vol}_4 - \frac{1}{2} *_4 F \wedge J$$

Fibration term:

$$\eta = \frac{1}{4}d\psi + \sigma\,, \quad d\sigma = 2J$$

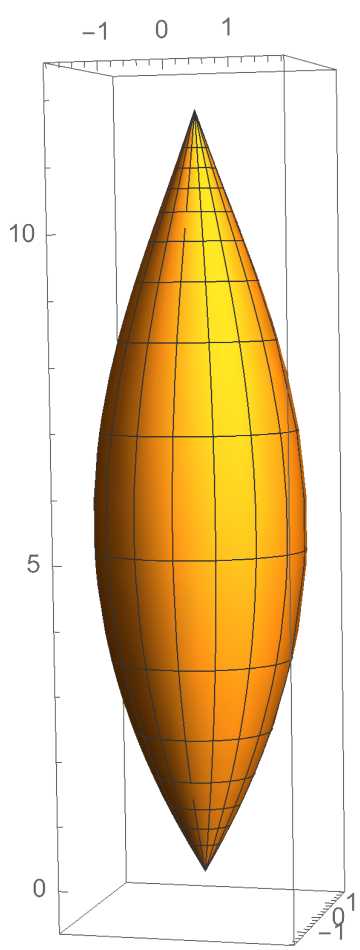
This solves the eom of 11d supergravity and preserves SUSY

Regularize in 11d: the spindle

Near the poles $\theta_- = 0\,,\ \theta_+ = \pi\,,$ we find:

$$ds^2_{\theta,\phi} \approx \left[\frac{\Sigma}{P\Omega^2}\right]_{\theta=\theta_\pm} \left[d\theta^2 + P^2_\pm \left(\theta - \theta_\pm\right)^2 d\phi^2\right]$$

Due to the acceleration, $P_- \neq P_+$ we cannot make it regular!
Then quantize the **conical deficits**: make it into a **spindle** $\mathbb{WCP}^1_{[n_-,n_+]}$



$$\frac{P_+}{P_-} = \frac{n_-}{n_+} \quad \Rightarrow \quad \Delta\phi = \frac{2\pi}{n_+ P_+} = \frac{2\pi}{n_- P_-}$$

Spindle

Regularize in 11d: the Lens space

Now look at space in the θ, ϕ, ψ directions: we can make it into a **Lens space** $S^3/\mathbb{Z}_\mathrm{q}$, seen as a Hopf-like fibration over $\mathbb{WCP}^1_{[n_-,n_+]}$. This requires

$$m = \frac{g}{\alpha}$$

Compatible with susy

→ At fixed t, r we have a Riemannian manifold Y_9 : completely **smooth** $S^3/\mathbb{Z}_\mathrm{q}$ **fibration** over KE_6 !

Near-horizon limit of BPS & extremal BHs

The BPS & extremal horizon

$$ds^2 = \frac{1}{4}\left(y^2 + \mathtt{j}^2\right)\left(-\rho^2 d\tau^2 + \frac{d\rho^2}{\rho^2}\right) + \frac{y^2+\mathtt{j}^2}{q(y)}dy^2 + \frac{q(y)}{4\left(y^2+\mathtt{j}^2\right)}(dz + \mathtt{j}\,\rho\, d\tau)^2$$

AdS_2

(Spinning) spindle

$y \leftrightarrow \theta\,, \quad z \leftrightarrow \phi$

$$A = h(y)(dz + \mathtt{j}\,\rho\, d\tau)$$

Two parameters:

- $\mathtt{j} \in (0, 1/\sqrt{2})$ is continuous → **rotation**
- $\mathtt{a}$ is fixed by $n_\pm$ and $\mathtt{j}$ → **acceleration**

$$\mathtt{a} = \frac{\left(1 - 2\,\mathtt{j}^2\right)\left(n_-^2 - n_+^2\right)}{n_+^2 + n_-^2}$$

$$Q_e = \frac{\sqrt{2}\,\mathtt{j}}{\sqrt{1-2\mathtt{j}^2}}\,\frac{\sqrt{n_-^2+n_+^2}}{4\,n_-n_+}$$

$$Q_m = \frac{n_- - n_+}{4\,n_-n_+}$$

$$J = Q_e\frac{\sqrt{2}\sqrt{8\,n_-^2n_+^2Q_e^2 + n_-^2 + n_+^2} - (n_- + n_+)}{4\,n_-n_+}$$

The entropy

Euler number of the spindle

In the supersymmetric and extremal case, we have

$$S_{BH} = \frac{\pi}{G_{(4)}} \frac{J}{Q_e} = \frac{\pi}{4\,G_{(4)}} \left(\sqrt{\chi(\Sigma)^2 + 16\,\left(Q_e^2 + Q_m^2\right)} - \chi(\Sigma) \right)$$

Related to known formulas in the literature for AdS_4 black holes

Hristov, Katmadas, Toldo 1907.05192 [hep-th]
Cassani, Papini 1906.10148 [hep-th]

Can we reproduce it from the Field Theory?
Done for D3-branes: $AdS_3 \times \mathbb{WCP}^1_{[n_-,n_+]}$ solutions!

The spinors

In these constructions, supersymmetry is typically preserved with a topological twist: $\omega_\mu \sim A_\mu$, in such a way that

$$\nabla_\mu \sim \partial_\mu + \omega_\mu + A_\mu \sim \partial_\mu$$

To check if this is the case, compare

Euler number

$$\chi(\Sigma) = \frac{1}{4\pi}\int_\Sigma R_2 \mathrm{vol}_2 = \frac{n_- + n_+}{n_+ n_-}$$

Magnetic flux

$$Q_m = \frac{1}{4\pi}\int_\Sigma dA = \frac{n_- - n_+}{4 n_+ n_-}$$

They are not proportional: **no topological twist!**

How is supersymmetry realized in the dual Field Theory?

The R-symmetry

From the spinor bilinears and the $\mathcal{N} = (0,2)$ superconformal algebra:

$$R_{1d} = R_{3d} + 2\sqrt{1 - \mathrm{j}^2}\,\partial_z$$

Generator of U(1) isometry of the spindle

First example of mixing between higher and lower dimensional R-symmetry in supergravity (with no rotation)!

D3-branes on spindles

The type IIB picture

$AdS_3 \times \mathbb{WCP}^1_{[n_-,n_+]}$ solutions of 5d minimal gauged supergravity

Uplift to 10d ↓ **Regularization of conical deficits**

Completely regular $AdS_3 \times Y_7$ solutions of type IIB supergravity

Gauntlett, Kim, Waldram 0612253 [hep-th]

With analogies…

- **Interpretation as (D3-)branes wrapped on spindles**

- **No topological twist**

- **Mixing of R_{4d} and $U(1)_{spindle}$ in R_{2d}**

...and differences

- **Lack the full geometry:** we conjecture the AdS_3 solutions to arise as near-horizon geometry of a 5d black string

- **No rotation:** no continuous parameters

- Entropy and R-symmetry mixing reproduced with a **Field Theory computation**

→ **Confirms our AdS/CFT interpretation!**

Thank you
for the attention!