All loop structures in SUGRA Amplitudes from CFT

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Based on 2002.04604 & 2010.12557
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Overview & Motivations

- We will to study $\text{AdS}_5 \times S^5$ SUGRA amplitudes from CFT correlation functions
- We will see how CFT tree level data can fix part of them to all orders in a perturbative expansion
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• We will study AdS$_5 \times S^5$ SUGRA amplitudes from CFT correlation functions
• We will see how CFT tree level data can fix part of them to all orders in a perturbative expansion

Motivations

• Understand better how unitarity translates in the AdS/CFT scenario
  Aharony, Alday, Bissi, Caron-Huot, Meltzer, Perlmutter, Sivaramakrishnan
• We will consider flat space amplitude, but our results should extend to AdS$_5 \times S^5$ background ⇒ Apply Bootstrap techniques to get information on amplitudes in curved space-times (hard to access with other methods)
4d $SU(N) \mathcal{N} = 4$ SYM
at large $N$ and large $\lambda = g_{YM}^2 N$

Type IIB SUGRA
on $AdS_5 \times S^5$ with $g_s$ coupling
Setup & Strategy

4d $SU(N) \mathcal{N} = 4$ SYM
at large $N$ and large $\lambda = g_{YM}^2 N$

Type IIB SUGRA
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\begin{align*}
\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle &= \mathcal{G}^{(0)}(U,V) + \frac{1}{N^2} \mathcal{G}^{(1)}(U,V) + \frac{1}{N^4} \mathcal{G}^{(2)}(U,V) + \frac{1}{N^6} \mathcal{G}^{(3)}(U,V) + \ldots \\
A_{10}^{\text{sugra}} &= G_N + G_N^3 + G_N^3 + \ldots
\end{align*}

CFT
\[ \{ \gamma_{n,i}^{(1)}, a_{n,i}^{(0)} \} \]

Flat Space

Amplitude

Cuts & Discontinuities
**Setup & Strategy**

**4d \( SU(N) \mathcal{N} = 4 \) SYM**

at large \( N \) and large \( \lambda = g_{YM}^2 N \)

\( \frac{1}{2} \) BPS scalar

\([0, 2, 0]_R \ \Delta = 2 \)

\( T_{\mu\nu} \) supermultiplet

**Type IIB SUGRA**

on \( \mathbb{R}^{10} \) with \( G_N \) coupling

\[
G_N = \frac{\pi^4 L^8}{2(N^2 - 1)}
\]

Graviton multiplet

\[
A_{10}^{\text{agra}} = G_N + G_N^2 + G_N^3 + \ldots
\]
Setup & Strategy

Mellin Space

\[ \tilde{M}(s, t) = \frac{1}{N^2} \left( s-2 \right) \left( t-2 \right) + \sum_{m,n=2}^{\infty} \frac{c^{(2)}_{mn}}{(s-2m)(t-2n)} + \frac{c^{(3)}_{mn}}{(s-2m)^2(t-2n)} \ldots \]

CFT

\[ \langle O_2 O_2 O_2 O_2 \rangle = G^{(0)}(U, V) + \frac{1}{N^2} G^{(1)}(U, V) + \frac{1}{N^4} G^{(2)}(U, V) + \frac{1}{N^6} G^{(3)}(U, V) + \ldots \]

dDisc

\[ \{ \gamma_n^{(1)}, \alpha_n^{(0)} \} \]

Flat Space

Amplitude

\[ A_{10}^{a\bar{a}r\bar{a}} = G_N + G_N^3 + G_N^3 + \ldots \]

Cuts & Discontinuities
4-point function

\[ \langle \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4) \rangle \sim \frac{G(u,v)}{x_{12}^4 x_{34}^4} \]
4-point function

\[ \langle \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4) \rangle \sim \frac{\mathcal{G}(u,v)}{x_{12}^4 x_{34}^4} \]

\[ \mathcal{G}(u,v) = \mathcal{G}^{\text{short}}(u,v,N) + \mathcal{H}(u,v,N,\lambda) \]

\( \mathcal{O}_{\Delta,\ell} \in \begin{cases} \text{(semi-) short multiplets} \\ \text{long multiplets} \end{cases} \)

\( \mathcal{G}^{\text{short}}(u,v,N) \) known, \( N^{-2} \) exact

Dolan, Osborn
4-point function

\[ \langle O_2(x_1)O_2(x_2)O_2(x_3)O_2(x_4) \rangle \sim \frac{G(u,v)}{x_{12}^4 x_{34}^4} \]

\[ G(u,v) = \sum a_{\Delta,\ell} u^{\frac{\Delta-\ell}{2}} g_{\Delta+4,\ell}(u,v) \]

\[ a_{\Delta,\ell} = a^{(0)}_{\Delta,\ell} + \frac{a^{(\kappa)}_{\Delta,\ell}}{N^{2\kappa}} \]

\[ \Delta = 4 + 2n + \ell + \frac{\gamma_{\Delta,\ell}}{N^{2\kappa}} \]

\( G(u,v) = \sum \frac{G^{\text{short}}(u,v,N)}{N^2} + \mathcal{H}(u,v,N,\lambda) \)

\( \mathcal{H}(u,v,N,\lambda) \) known, \( N^{-2} \) exact

Dolan, Osborn

Double trace ops \([O_pO_p]_{n,\ell} \) at \( O(N^{-4}) \)

Higher trace ops at higher orders
One-loop example

- **Mixing problem**: degeneracy between operators with the same bare dimension
One-loop example

- **Mixing problem:** degeneracy between operators with the same bare dimension
- **Solved up to** $O(N^{-2})$: $a_{n,\ell,I}^{(0)}$ and $\gamma_{n,\ell,I}^{(1)}$ known for each degeneracy index $I$

Aprile, Drummond, Heslop, Paul Alday, Bissi
One-loop example

- **Mixing problem**: degeneracy between operators with the same bare dimension

- **Solved up to $O(N^{-2})$**: $a_{n,\ell,I}^{(0)}$ and $\gamma_{n,\ell,I}^{(1)}$ known for each degeneracy index $I$

- At one loop
  - Possible to fully reconstruct the correlator from its $d\text{Disc}$
  
  $\mathcal{H}^{(2)}(u, v) \supset \log^2 u u^{n+2} a_{n,\ell,I}^{(0)} \left(\gamma_{n,\ell,I}^{(1)}\right)^2 g_{2n+8,\ell}(u, v)$

  - Establish a connection between $d\text{Disc}$ of the correlator and discontinuity of the corresponding 10d flat space amplitude
Higher loops

- **Mixing problem**: degeneracy between operators with the same bare dimension
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Higher loops

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\[ \downarrow \]
\[ \text{At higher loops} \]

- Completely fix **leading logs** in $\mathcal{H}^{(\kappa)}(u,v)$, $\kappa \geq 3$

\[
\log^\kappa u u^{n+2} a^{(0)}_{n,\ell,I} \left( \gamma^{(1)}_{n,\ell,I} \right)^\kappa g_{2n+8,\ell}(u,v)
\]

**Caveats**

- Not full dDisc
- Higher trace ops
Higher loops

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\[ \log^\kappa u u^{n+2} a^{(0)}_{n,\ell,I} \left( \gamma^{(1)}_{n,\ell,I} \right)^\kappa g_{2n+8,\ell}(u,v) \]

Which term in the dual amplitude it maps to? 

**CAVEATS**

Not full dDisc
Higher trace ops

Caron-Huot, Trinh
Higher loops

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- **Solved up to $O(N^{-2})$**: $a^{(0)}_{n,\ell,I}$ and $\gamma^{(1)}_{n,\ell,I}$ known for each degeneracy index $I$

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CAVEATS
Not full dDisc
Higher trace ops

\[ \text{Iterated } s\text{-cuts} \]
Conclusions

• All CFT loop structures dual to iterated s-cuts in the 10d amplitude ⇒ possible extension to curved space-time

Open problems:
▶ Include stringy corrections
▶ Perform a similar analysis in AdS$_5 \times$S$_5$
▶ Understand better the singularity structure

Thank you!
Conclusions

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• Similar results hold in Mellin space
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