All loop structures in SUGRA Amplitudes from CFT

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Based on 2002.04604 & 2010.12557
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Overview & Motivations

- We will to study $AdS_5 \times S^5$ **SUGRA amplitudes** from **CFT correlation functions**
- We will see how CFT tree level data can fix part of them to all orders in a perturbative expansion

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Motivations

- Understand better how **unitarity** translates in the AdS/CFT scenario Aharony, Alday, Bissi, Caron-Huot, Meltzer, Perlmutter, Sivaramakrishnan
- We will consider flat space amplitude, but our results should extend to $AdS_5 \times S^5$ background $\Rightarrow$ Apply **Bootstrap techniques** to get information on amplitudes in curved space-times (hard to access with other methods)

Setup & Strategy

4d $SU(N)\,\mathcal{N}=4$ SYM

at large N and large $\lambda = g_{YM}^2 N$

Type IIB SUGRA

on $\mathrm{AdS}_5 \times S^5$ with g_s coupling

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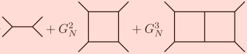
CFT

$\langle \mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\mathcal{O}_2\rangle = \mathcal{G}^{(0)}(U,V) + \frac{1}{N^2}\mathcal{G}^{(1)}(U,V) + \frac{1}{N^4}\mathcal{G}^{(2)}(U,V) + \frac{1}{N^6}\mathcal{G}^{(3)}(U,V) + \dots$

Flat Space ↔

Amplitude

$\mathcal{A}_{10}^{sugra} = G_N$ (tree) $+ G_N^2$ (box) $+ G_N^3$ (double box) $+ \dots$



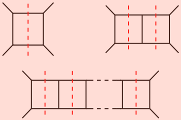
↕

dDisc

$\{\gamma_{n,l}^{(1)}, a_{n,l}^{(0)}\}$

↕

Cuts & Discontinuities



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4d $SU(N)\,\mathcal{N}=4$ SYM

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Type IIB SUGRA

on $\mathbb{R}^{10}$ with G_N coupling

$\frac{1}{2}$BPS scalar
$[0,2,0]_R$ $\Delta = 2$

$T_{\mu\nu}$ supermultiplet

$G_N = \frac{\pi^4 L^8}{2(N^2-1)}$

Graviton multiplet

CFT

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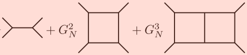

dDisc

$\{\gamma^{(1)}_{n,l}, a^{(0)}_{n,l}\}$

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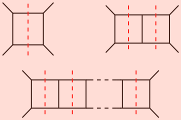

Setup & Strategy

Mellin Space

$$\widetilde{\mathcal{M}}(s,t)=\frac{1}{N^2}\frac{8}{(s-2)(t-2)(u-2)}+\sum_{m,n=2}\frac{1}{N^4}\frac{c^{(2)}_{mn}}{(s-2m)(t-2n)}+\frac{1}{N^6}\frac{c^{(3)}_{mn}}{(s-2m)^2(t-2n)}\ \dots$$

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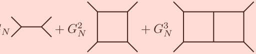
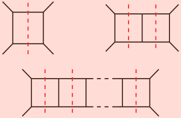


dDisc

$$\{\gamma^{(1)}_{n,l},a^{(0)}_{n,l}\}$$

Cuts & Discontinuities



4-point function

$$\langle \mathcal{O}_2(x_1)\mathcal{O}_2(x_2)\mathcal{O}_2(x_3)\mathcal{O}_2(x_4)\rangle \sim \frac{\mathcal{G}(u,v)}{x_{12}^4 x_{34}^4}$$

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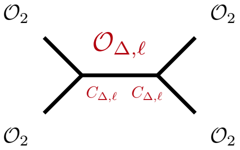

$$\mathcal{O}_{\Delta,\ell} \in \begin{cases} \text{(semi-) short multiplets} \\ \text{long multiplets} \end{cases}$$

$$\mathcal{G}(u,v) = \underbrace{\mathcal{G}^{short}(u,v,N)}_{\text{known, } N^{-2} \text{ exact}} + \underbrace{\mathcal{H}(u,v,N,\lambda)}_{\text{dynamical part}}$$

Dolan, Osborn

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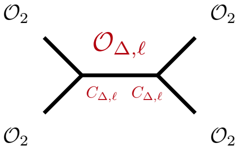

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$$\mathcal{H}(u,v,N,\lambda) = \sum_{\Delta,\ell} a_{\Delta,\ell}\, u^{\frac{\Delta-\ell}{2}} g_{\Delta+4,\ell}(u,v)$$

↱ leading order

↳ expand

$$a_{\Delta,\ell} = a^{(0)}_{\Delta,\ell} + \frac{a^{(\kappa)}_{\Delta,\ell}}{N^{2\kappa}} \qquad \Delta = 4 + 2n + \ell + \frac{\gamma^{(\kappa)}_{\Delta,\ell}}{N^{2\kappa}}$$

Double trace ops $[\mathcal{O}_p\mathcal{O}_p]_{n,\ell}$ at $O(N^{-4})$

Higher trace ops at higher orders

One-loop example

- **Mixing problem**: degeneracy between operators with the same bare dimension

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Aprile, Drummond, Heslop, Paul   Alday, Bissi

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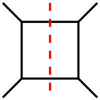
At one loop

- Possible to fully reconstruct the correlator from its dDisc

$$\mathcal{H}^{(2)}(u,v) \supset \log^2 u \, u^{n+2} a^{(0)}_{n,\ell,I} \left(\gamma^{(1)}_{n,\ell,I}\right)^2 g_{2n+8,\ell}(u,v)$$

- Establish a connection between dDisc of the correlator and discontinuity of the corresponding 10d flat space amplitude

$$\mathrm{dDisc}\,\mathcal{H}^{(2)}(u,v) \xleftrightarrow[\text{limit}]{\text{Bulk point}} G_N^2$$



Alday, Caron-Huot

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CAVEATS

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Which term in the dual amplitude it maps to?

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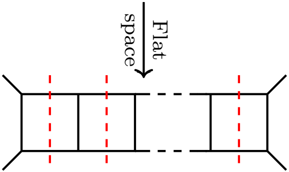

Iterated s-cuts

Conclusions

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Thank you!