The Entropy of Black Holes from the Superconformal Index

Ziruo Zhang (SISSA)

Joint work with Francesco Benini, Edoardo Colombo, Saman Soltani, Alberto Zaffaroni (2005.12308)

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- Successfully compared it to the near-horizon geometry of black holes in $\mathrm{AdS}_5 \times T^{1,1}$

Black hole entropy from field theory

- In $\mathrm{AdS}_5/\mathrm{CFT}_4$, supersymmetric black hole $\leftrightarrow$ ensemble of BPS states with $\{J_{1,2}, Q_{a=1,\cdots n}\}$. We study the superconformal index

$$\mathcal{I}(p,q,y) = \mathrm{Tr}(-1)^F p^{J_1+\frac{r}{2}} q^{J_2+\frac{r}{2}} \prod_{a=1}^{n-1} y_a^{Q_a} \ .$$

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$$e^{S(J,Q)} = d(J,Q) = \int d\tau d\sigma \prod_{a=1}^{n-1} \left(d\Delta_a \ y_a^{-Q_a}\right) p^{-J_1} q^{-J_2} \mathcal{I}(p,q,y) \ ,$$

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$$2\pi i\tau = \log p,\ 2\pi i\sigma = \log q,\ 2\pi i\Delta_a = \log y_a \ .$$

- $S(J,Q)$ at $\mathcal{O}(N^2)$ is the integrand at the saddle point, i.e. the extremal value wrt. τ, σ, Δ_a of the entropy function

$$\mathcal{S} = \log \mathcal{I}(\tau,\sigma,\Delta) - 2\pi i \left(\sum_{a=1}^{n} \Delta_a Q_a + \tau J_1 + \sigma J_2\right) \ .$$

Bethe ansatz formula

- For angular fugacities $p = e^{2\pi i a\omega}$, $q = e^{2\pi i b\omega}$, $\gcd\{a,b\} = 1$, the superconformal index can be expressed as [Benini and Milan, 2018]

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\begin{aligned}
\mathcal{I}(p,q,y) &= \kappa \oint_{\mathbb{T}^{\mathrm{rk}(G)}} \mathcal{Z}(u;p,q,y) \\
&= \kappa \sum_{\hat{u}\in \mathrm{BAEs}} \sum_{\{m_i\}=1}^{ab} \mathcal{Z}(\hat{u}+m\omega;p,q,y) H^{-1} \,.
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- The Bethe ansatz equations are defined on a torus with modulus ω and have the form

$$Q_i(u;\Delta,\omega) = 1, \quad i = 1,\cdots,\mathrm{rk}(G) \, .$$

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- For this contribution, we computed $\log \mathcal{Z}$ at leading order for toric quivers and found

$$\log \mathcal{Z} = -i\pi N^2 \sum_{a,b,c=1}^{n} \frac{C_{abc}\Delta_a \Delta_b \Delta_c}{6\tau\sigma}$$

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  C_{abc} are 't Hooft anomaly coefficients.
- Reproduces the entropy function for known black holes [Gutowski and Reall, 2004] in $\mathrm{AdS}_5 \times S^5$ [Hosseini et al., 2017] but does this work in other cases?

Supergravity checks

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- Otherwise, try to apply the strategy [Hosseini et al., 2017] .

The strategy

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- Reducing along the Hopf fiber of S^3, generically get rotating charged black hole in 4d with the same entropy.
- $J_1 - J_2$ is the 4d angular momentum $\implies$ black holes with $J_1 = J_2$ become spherically symmetric in 4d.
- Restricting to this case, the entropy is determined by horizon data, via the attractor mechanism, which is also an extremization problem.

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- Charges were matched using the $\text{AdS}_5/\text{CFT}_4$ dictionary and then reduced.

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- Is there an attractor mechanism in 5d? Or a rotating attractor mechanism in 4d?
- Can we compute subleading corrections to the Bekenstein-Hawking entropy?
- Can we go beyond extremality and SUSY?