Inception Neural Networks for Complete Intersection Calabi-Yau Manifolds

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String Theory Compactification

Superstrings defined in $D = 10 \quad \Rightarrow \quad \mathcal{M}^{1,9} = \mathcal{M}^{1,3} \otimes X_6$

Requirements

- X_6 is a compact manifold (M, g)
- $N = 1$ **SUSY** in 4D
- **SM** $\subset$ arising gauge algebra

Solution

- $\dim_{\mathbb{C}} M = 3$
- $\mathrm{Hol}(g) \subseteq \mathrm{SU}(3)$
- $\mathrm{Ric}(g) \equiv 0$ or $c_1(M) \equiv 0$

Calabi-Yau Manifolds

- no known **metric** for compact CY
- need to study **topology** (*Hodge numbers*) to infer **4D properties**

$$h^{r,s} = \dim_{\mathbb{C}} \mathrm{H}^{r,s}_{\bar{\partial}}(M, \mathbb{C})$$

Complete Intersection Calabi-Yau Manifolds

Systems of k **homogeneous equations** from products of m **projective spaces**

$$\sum_{r=1}^{m} p_{\alpha}^{i_r} \left(z_{i_r} \right)^{a_{\alpha}^{r}} = 0 \quad \rightarrow \quad X = \begin{bmatrix} \mathbb{P}^{n_1} & | & a_1^1 & \cdots & a_k^1 \\ \vdots & & \vdots & \ddots & \vdots \\ \mathbb{P}^{n_m} & | & a_1^m & \cdots & a_k^m \end{bmatrix}$$

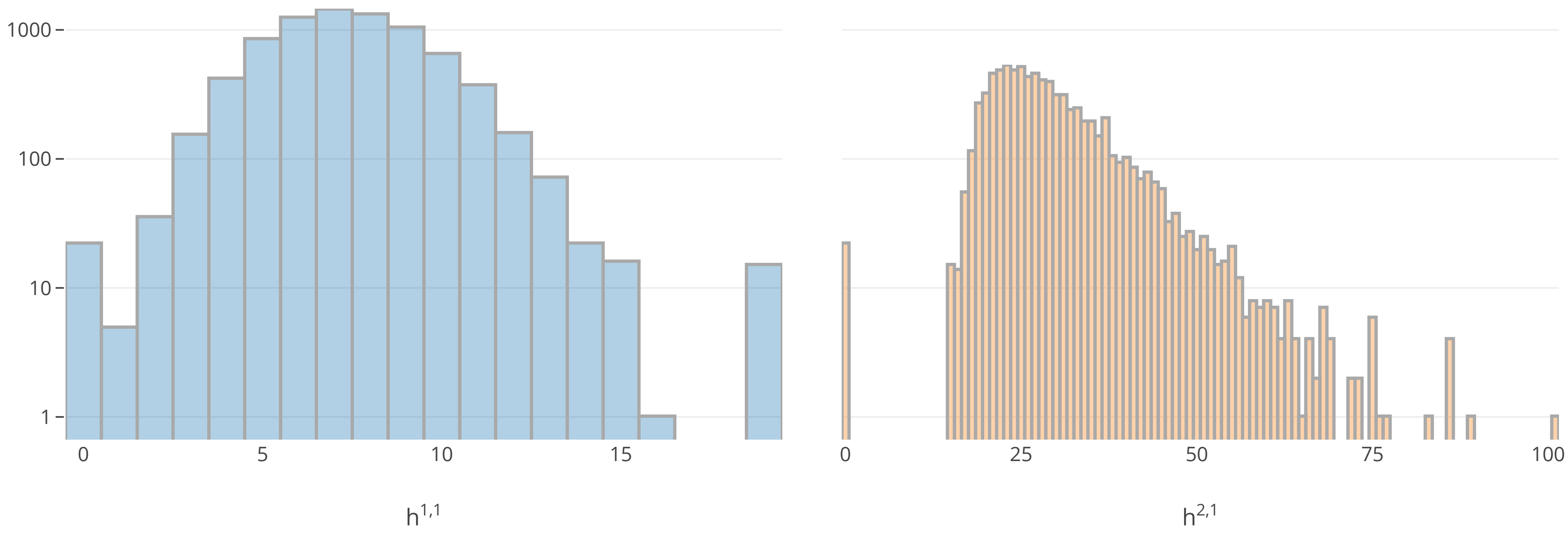
such that

$$\begin{array}{ccc} \text{degree of eq.} & \text{dim. of CY} & c_1 \equiv 0 \\ \Downarrow & \Downarrow & \Downarrow \\ a_{\alpha}^{r} \in \mathbb{N} & \dim_{\mathbb{C}} X = \sum\limits_{r=1}^{m} n_r - k = 3 & n_r + 1 = \sum\limits_{\alpha=1}^{k} a_{\alpha}^{r} \end{array}$$

where a_{α}^{r} are **powers** of coordinates on $\mathbb{P}^{n_r}$ in equation α.

Available Data

∃ compiled datasets of **7890** CICY 3-folds with all **Hodge numbers** [Green et al. (1987)]



Supervision and Function Approximation

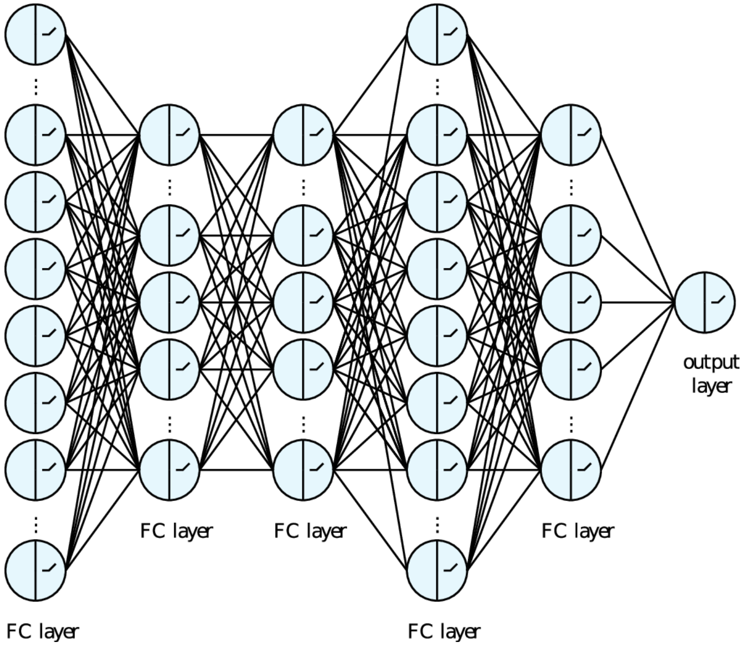
$$\begin{array}{lccc} \mathcal{R}\colon & \mathbb{N}^{m \times k} & \longrightarrow & \mathbb{N}^2 \\ & X & \mapsto & (h^{1,1},\, h^{2,1}) \\ & \text{(conf. matrix)} & & \text{(Hodge no.)} \end{array}$$

SUPERVISED LEARNING

- replace $\mathcal{R}(X)$ with $\mathcal{R}(X;\, W)$ (W **weights**)
- feed the algorithms X and $h^{p,q}$ (**true values**)
- $$W = \arg\min_{w} \mathcal{L}(h^{p,q},\, \mathcal{R}(X;\, w))$$
- follow **gradient descent** of $\mathcal{L} \rightarrow$ tune W

Neural Networks as Function Approximators

FULLY CONNECTED NETWORKS

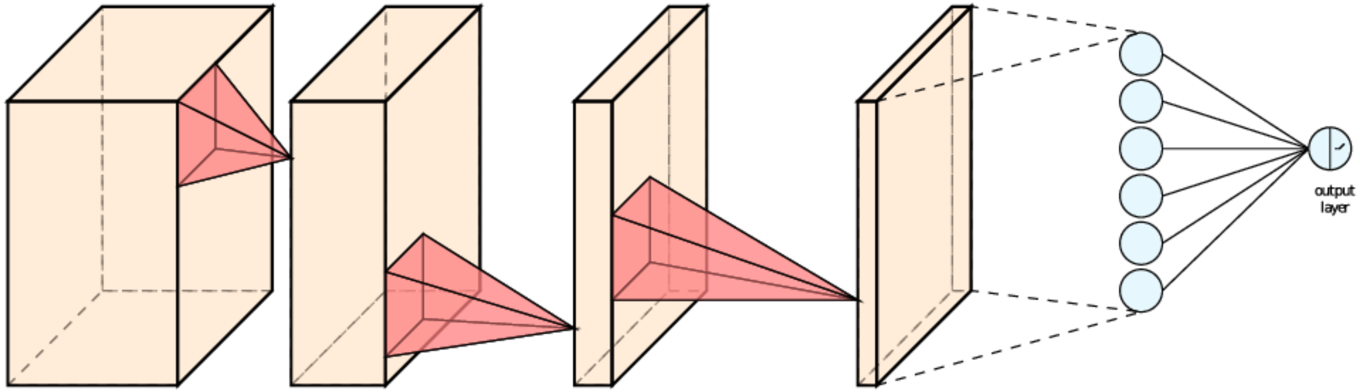


- older in design [Rosenblatt (1958)]
- analogy *neuron* - *"perceptron"*
- rose to fame **in the 70s/80s**
- simple **matrix multiplications** + **non linearity**

$$a^{\{l+1\}} = \phi\left(w^{\{l\}} a^{\{l\}} + b^{\{l\}} \mathbb{I} \right) \in \mathbb{R}^p$$

$$\phi(z) = \mathrm{ReLU}(z) = \max(0, z)$$

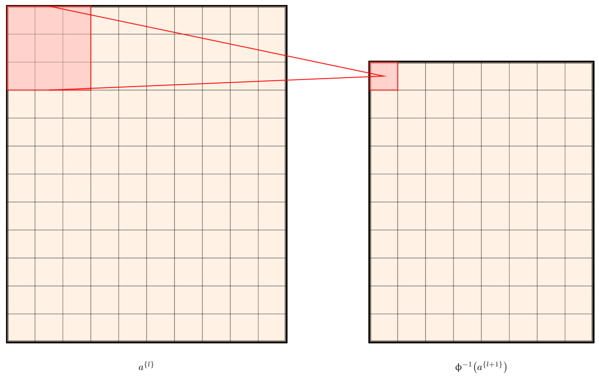
CONVOLUTIONAL NEURAL NETWORKS



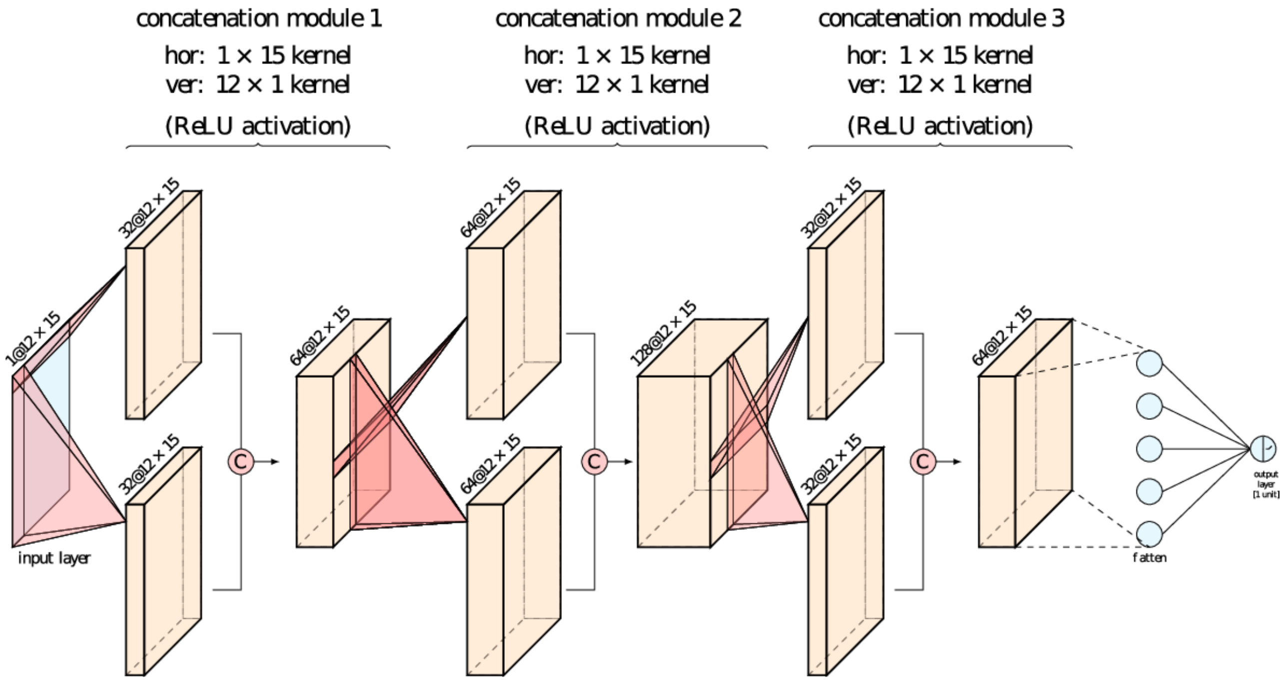
- newer in conception [LeCun et al. (1989)]
- based on **sliding windows** (aka *convolutions*)

$$a^{\{l+1\}} = \phi\left(w^{\{l\}} * a^{\{l\}} + b^{\{l\}}\mathbb{I}\right) \in \mathbb{R}^{p\times q}$$

- **less parameters** to *isolate features*

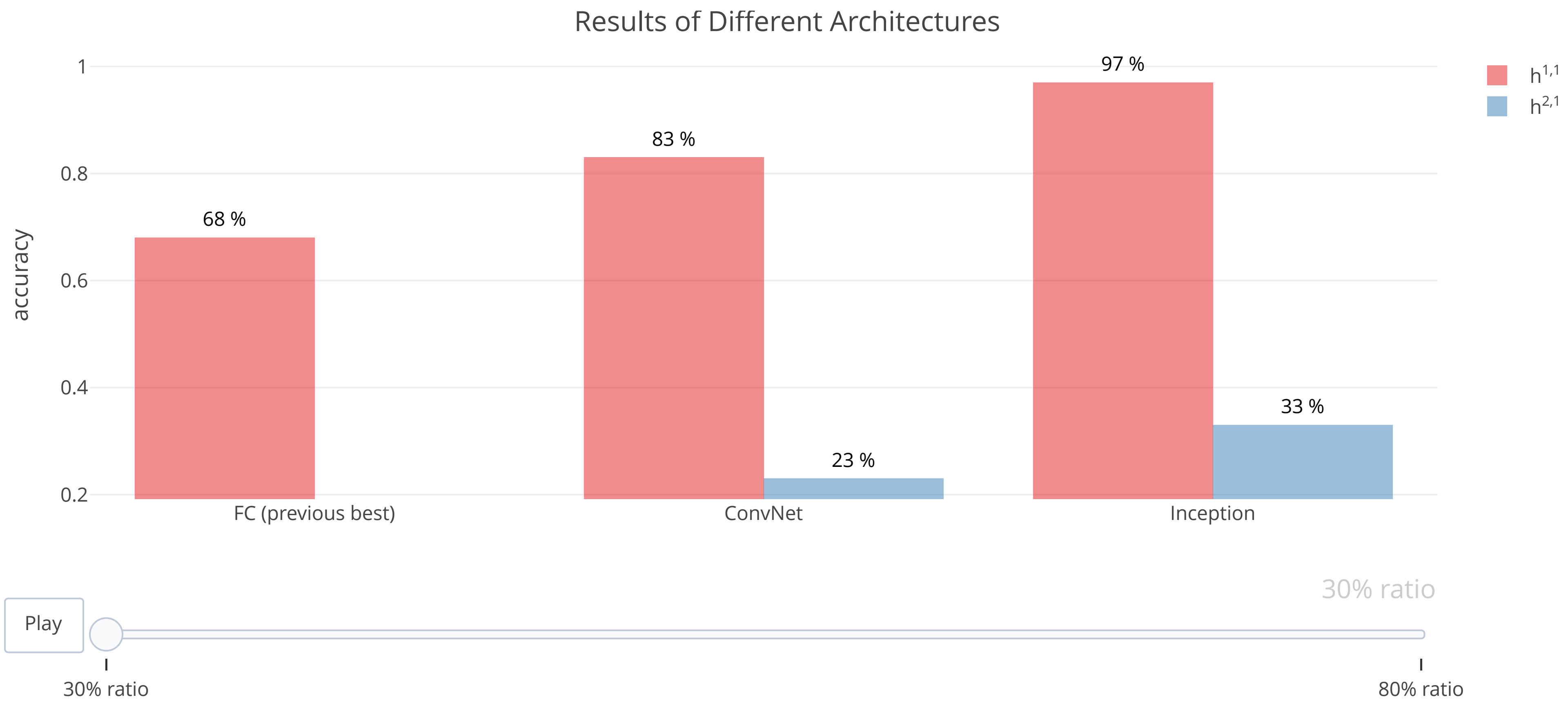


Inception Neural Networks



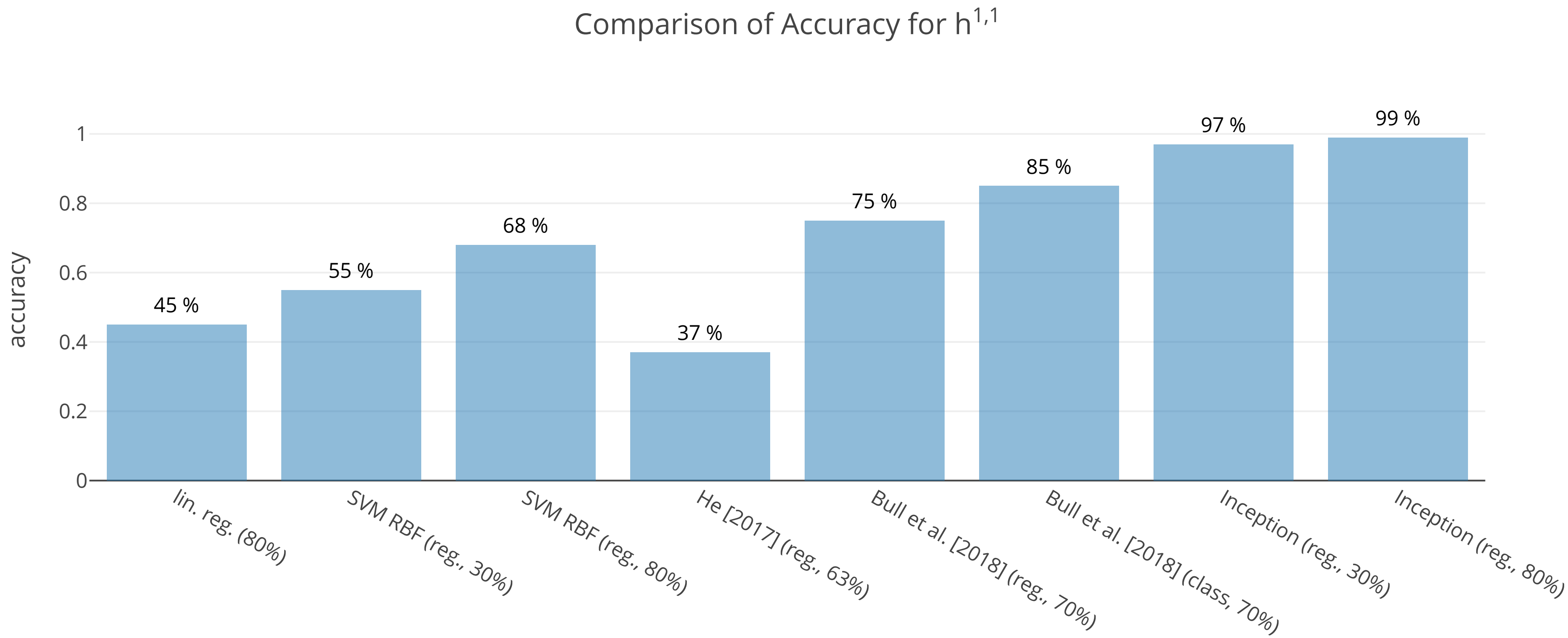
- inspired by **Google** [Szegedy et al. (2014)]
- created for **computer vision**
- 2×10^5 parameters (vs $\geq 2 \times 10^6$)
- **concurrent kernels** $\Rightarrow$ shared parameters
- retains **"spatial awareness"**
- improved **generalisation ability**

Results



[Erbin, RF (2020)]

Comparison



[Erbin, RF (2020)]

Conclusion

[Machine learning is]

the field of study that gives computers the ability to learn without being explicitly programmed.

A. Samuel (1959)

- **deep learning** can be a **reliable predictive method**
- it can be used as **source of inspiration** for **inference and generalisation**
- **CNN**s have a lot of **unexpressed potential** in physics (*first time?*)
- the approach intersects **mathematics**, **physics** and **computer science**

What Lies Ahead?

- improve $h^{2,1}$ and **"un-blackbox" the model** (SHAP, filter analysis, etc.)
- exploration for CICY 4-folds and **representation learning** (DGAN, VAE, RL, etc.)
- study **symmetries** (GNNs, transformers architectures, etc.) and explore the **string landscape**

THANK YOU