Reflecting the Infrared
An Unoriented IR Duality?

Salvo Mancani

Università di Roma La Sapienza

21-22 December 2020

hep-th/2007.14749 Antinucci, SM, Riccioni

Contents

AdS/CFT correspondence

N D3-branes with near horizon geometry $\mathrm{AdS}_5 \times S^5$

(Supergravity in large N limit)

$\Leftrightarrow$

4d SU(N) $\mathcal{N} = 4$ SYM,

$$W = \epsilon_{ijk} \mathrm{Tr} \left(X^i X^j X^k \right)$$

AdS/CFT correspondence

N D3-branes with near horizon geometry $\mathrm{AdS}_5 \times S^5$

(Supergravity in large N limit)

$\Leftrightarrow$

4d SU(N) $\mathcal{N} = 4$ SYM,

$$W = \epsilon_{ijk} \mathrm{Tr}\left(X^i X^j X^k\right)$$

How to reduce $\mathcal{N}$? $\mathbb{Z}_n$ orbifold involution of the sphere S^5

$z^i \to e^{\frac{2\pi i}{n} w^i} z^i \,, \quad \sum_{i=1}^3 w^i = 0 \bmod n$

Chiral multiplets $X^i = \left(X^i\right)_{ab}$

$a, b = 0, \ldots, n-1$

The $\mathcal{N} = 4$ superpotential is projected into[a]

$W_{\mathbb{Z}_n} = \epsilon_{ijk} \sum_a X^i_{a, a-w_j-w_k} X^j_{a-w_k-w_j, a-w_k} X^k_{a-w_k, a}$

[a]Beasley, Greene, Lazaroiu, Plesser: 2000

AdS/CFT correspondence

N D3-branes with near horizon geometry $\mathrm{AdS}_5 \times S^5$

(Supergravity in large N limit)

$\Leftrightarrow$

4d SU(N) $\mathcal{N} = 4$ SYM,

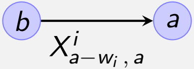
$$W = \epsilon_{ijk} \mathrm{Tr}\left(X^i X^j X^k \right)$$

How to reduce $\mathcal{N}$? $\mathbb{Z}_n$ orbifold involution of the sphere S^5

$z^i \to e^{\frac{2\pi i}{n} w^i} z^i \,, \quad \sum_{i=1}^{3} w^i = 0 \mod n$

Chiral multiplets $X^i = \left(X^i\right)_{ab}$
$a, b = 0, \ldots, n-1$

$b = a - w_i$



The $\mathcal{N} = 4$ superpotential is projected into[a]

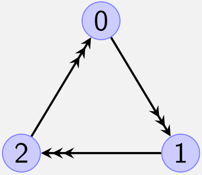
$$W_{\mathbb{Z}_n} = \epsilon_{ijk} \sum_a X^i_{a,\, a-w_j-w_k} X^j_{a-w_k-w_j,\, a-w_k} X^k_{a-w_k,\, a}$$

[a] Beasley, Greene, Lazaroiu, Plesser: 2000

Examples

$\mathcal{N} = 1$ example: $\mathbb{C}^3/\mathbb{Z}_3$, $(1,1,1)$

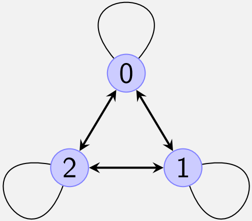
$$W_{\mathbb{Z}_3} = \epsilon_{ijk} \sum_a X^i_{a,\,a-2} X^j_{a-2,\,a-1} X^k_{a-1,\,a}$$
$$= 3\,\epsilon_{ijk}\, X^i_{12} X^j_{20} X^k_{01}$$



$\mathcal{N} = 2$ example: $\mathbb{C}^3/\mathbb{Z}'_3$, $(1,2,0)$

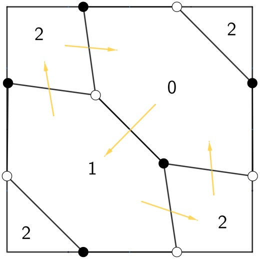
$$W_{\mathbb{Z}'_3} = \Big[\phi^3_0 \left(X^1_{01} X^2_{10} - X^2_{02} X^1_{20}\right) + \phi^3_1 \left(X^1_{12} X^2_{21} - X^2_{10} X^1_{01}\right) + \phi^3_2 \left(X^1_{20} X^2_{02} - X^2_{21} X^1_{12}\right)\Big] \,,$$

$$\phi^3_a = X^3_{aa}$$



A web of branes

- T-dual configuration to D3-branes on an Orbifold Singularity: D5 and NS5 branes on flat space;
- D5s wrapped along a torus. WV divided by NS5s;
- Bipartite graph on the torus[1]:



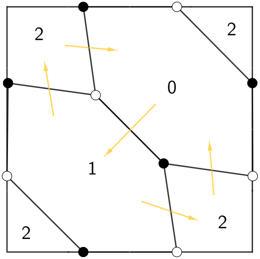
Dimer dictionary

- Face a of the tiling $\longrightarrow$ gauge groups $U(N_a)$
- Edges $\longrightarrow$ bi-fundamental fields X_{ab}
- White (Black) node $\longrightarrow$ interaction term $+(-)\mathrm{Tr}\prod X_{ab}$
- Example on the left: $\mathbb{C}^3/\mathbb{Z}_3$, $W = \epsilon_{ijk} X^i_{01} X^j_{12} X^k_{20}$

[1] Hanany, Kennaway: 2005

A web of branes

- T-dual configuration to D3-branes on an Orbifold Singularity: D5 and NS5 branes on flat space;
- D5s wrapped along a torus. WV divided by NS5s;
- Bipartite graph on the torus[1]:



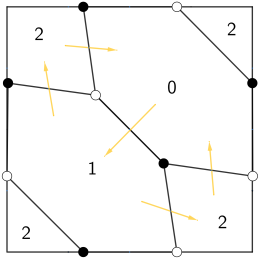
Dimer dictionary

- Face a of the tiling $\longrightarrow$ gauge groups $U(N_a)$
- Edges $\longrightarrow$ bi-fundamental fields X_{ab}
- White (Black) node $\longrightarrow$ interaction term $+(-)\mathrm{Tr}\prod X_{ab}$
- Example on the left: $\mathbb{C}^3/\mathbb{Z}_3$, $W = \epsilon_{ijk} X^i_{01} X^j_{12} X^k_{20}$

[1] Hanany, Kennaway: 2005

A web of branes

- T-dual configuration to D3-branes on an Orbifold Singularity: D5 and NS5 branes on flat space;
- D5s wrapped along a torus. WV divided by NS5s;
- Bipartite graph on the torus[1]:



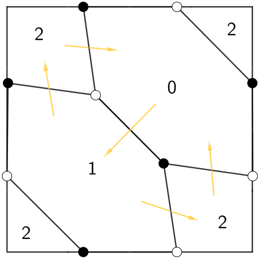
Dimer dictionary

- Face a of the tiling $\longrightarrow$ gauge groups $U(N_a)$
- Edges $\longrightarrow$ bi-fundamental fields X_{ab}
- White (Black) node $\longrightarrow$ interaction term $+(-)\mathrm{Tr}\prod X_{ab}$
- Example on the left: $\mathbb{C}^3/\mathbb{Z}_3$, $W = \epsilon_{ijk} X^i_{01} X^j_{12} X^k_{20}$

[1] Hanany, Kennaway: 2005

A web of branes

- T-dual configuration to D3-branes on an Orbifold Singularity: D5 and NS5 branes on flat space;
- D5s wrapped along a torus. WV divided by NS5s;
- Bipartite graph on the torus[1]:



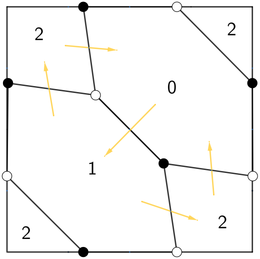
Dimer dictionary

- Face a of the tiling $\longrightarrow$ gauge groups $U(N_a)$
- Edges $\longrightarrow$ bi-fundamental fields X_{ab}
- White (Black) node $\longrightarrow$ interaction term $+(-)\mathrm{Tr}\prod X_{ab}$
- Example on the left: $\mathbb{C}^3/\mathbb{Z}_3$, $W = \epsilon_{ijk} X_{01}^i X_{12}^j X_{20}^k$

[1] Hanany, Kennaway: 2005

A web of branes

- T-dual configuration to D3-branes on an Orbifold Singularity: D5 and NS5 branes on flat space;
- D5s wrapped along a torus. WV divided by NS5s;
- Bipartite graph on the torus[1]:



Dimer dictionary

- Face a of the tiling $\longrightarrow$ gauge groups $U(N_a)$
- Edges $\longrightarrow$ bi-fundamental fields X_{ab}
- White (Black) node $\longrightarrow$ interaction term $+(-)\mathrm{Tr}\prod X_{ab}$
- Example on the left: $\mathbb{C}^3/\mathbb{Z}_3$, $W = \epsilon_{ijk} X^i_{01} X^j_{12} X^k_{20}$

[1] Hanany, Kennaway: 2005

Orientifold on brane tilings

$\mathbb{Z}_2$ involution of the torus[2]

- 4 fixed points $\longrightarrow$ orientifold plane at each point;
- (2) fixed line(s) $\longrightarrow$ orientifold plane wraps a cycle on the torus.

Fixed points

- black nodes mapped to white nodes w.r.t. fixed points;
- $+/-$ point on an edge $\longrightarrow$ symmetric/antisymmetric representation;
- $+/-$ point on a face $\longrightarrow$ SO/Sp gauge groups;
- constraint: $(-1)^{N_W/2} = \prod_{i=1}^{4} \tau_i$.

[2] Franco, Hanany, Krefl, Park, Uranga: 2007

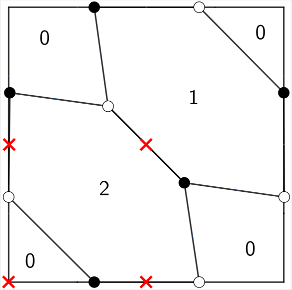
Orientifold on brane tilings

$\mathbb{Z}_2$ involution of the torus[2]

- 4 fixed points $\longrightarrow$ orientifold plane at each point;
- (2) fixed line(s) $\longrightarrow$ orientifold plane wraps a cycle on the torus.

Fixed points

- black nodes mapped to white nodes w.r.t. fixed points;
- $+/-$ point on an edge $\longrightarrow$ symmetric/antisymmetric representation;
- $+/-$ point on a face $\longrightarrow$ SO/Sp gauge groups;
- constraint: $(-1)^{N_W/2} = \prod_{i=1}^{4} \tau_i$.

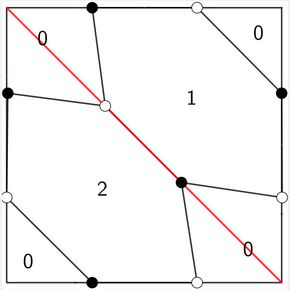


[2] Franco, Hanany, Krefl, Park, Uranga: 2007

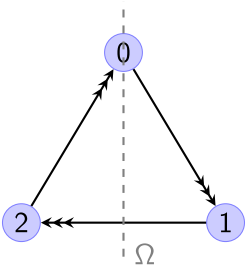
Orientifold: brane tiling

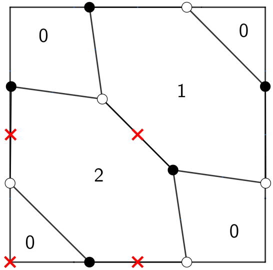
Fixed lines

- black/white nodes mapped to black/white nodes w.r.t. fxd lines;
- $+/-$ line crosses a face $\longrightarrow$ SO/Sp gauge groups;
- $+/-$ line lies on an edge $\longrightarrow$ symmetric/antisymmetric representation;



Example: $\left(\mathbb{C}^3/\mathbb{Z}_3\right)/\Omega\,(1,1,1)$





- clockwise order from bottom left: $(\tau_1, \tau_2, \tau_3, \tau_4)$;
- $N_W = 6$, $\prod_{i=1}^{4} \tau_i = -1 \implies (\mp, \pm, \pm, \pm)$;
- $N_0 = N_1 + \frac{4}{3}(\tau_2 + \tau_3 + \tau_4)$, $\quad W^\Omega = \epsilon_{ijk} X_{01}^i X_{12}^j (X_{01})^T$

$$\begin{cases} SO(N_0) \times SU(N_1)\,, & N_0 = N_1 - 4\,, \\ Sp(N_0) \times SU(N_1)\,, & N_0 = N_1 + 4\,. \end{cases}$$

Conformal invariance of $\mathcal{N} = 1$

- Conformal dimension of gauge-invariant operators determined by R-charges $\Delta = \frac{3}{2}R$;
- R-charges and flavour $U(1)^m$ mix $\Rightarrow$ R-charge not uniquely determined;
- At the conformal point, R-charges *maximizes*[3] central charge a, which is a count of d.o.f.

$$a = \frac{3}{32}\left(3\mathrm{Tr}R^3 - \mathrm{Tr}R\right) \ , \quad a_{IR} < a_{UV} \ ;$$

- The orientifold acts as a $\mathbb{Z}_2$ involution of spacetime. Matter fields are identified by $\mathbb{Z}_2$. We expect

$$a^{\Omega} = \frac{1}{2}a \ .$$

[3] Intriligator, Wecht: 2003

Conformal invariance of $\mathcal{N} = 1$

- Conformal dimension of gauge-invariant operators determined by R-charges $\Delta = \frac{3}{2}R$;
- R-charges and flavour $U(1)^m$ mix $\Rightarrow$ R-charge not uniquely determined;
- At the conformal point, R-charges *maximizes*[3] central charge a, which is a count of d.o.f.

$$a = \frac{3}{32}\left(3\mathrm{Tr}R^3 - \mathrm{Tr}R\right) \ , \quad a_{IR} < a_{UV} \ ;$$

- The orientifold acts as a $\mathbb{Z}_2$ involution of spacetime. Matter fields are identified by $\mathbb{Z}_2$. We expect

$$a^{\Omega} = \frac{1}{2}a \ .$$

[3] Intriligator, Wecht: 2003

Conformal invariance of $\mathcal{N} = 1$

- Conformal dimension of gauge-invariant operators determined by R-charges $\Delta = \frac{3}{2} R$;
- R-charges and flavour $U(1)^m$ mix $\Rightarrow$ R-charge not uniquely determined;
- At the conformal point, R-charges *maximizes*[3] central charge a, which is a count of d.o.f.

$$a = \frac{3}{32}\left(3\mathrm{Tr}R^3 - \mathrm{Tr}R\right) \ , \quad a_{IR} < a_{UV} \ ;$$

- The orientifold acts as a $\mathbb{Z}_2$ involution of spacetime. Matter fields are identified by $\mathbb{Z}_2$. We expect

$$a^{\Omega} = \frac{1}{2} a \ .$$

[3] Intriligator, Wecht: 2003

Conformal invariance of $\mathcal{N} = 1$

- Conformal dimension of gauge-invariant operators determined by R-charges $\Delta = \frac{3}{2}R$;
- R-charges and flavour $U(1)^m$ mix $\Rightarrow$ R-charge not uniquely determined;
- At the conformal point, R-charges *maximizes*[3] central charge a, which is a count of d.o.f.

$$a = \frac{3}{32}\left(3\mathrm{Tr}R^3 - \mathrm{Tr}R\right) \ , \quad a_{IR} < a_{UV} \ ;$$

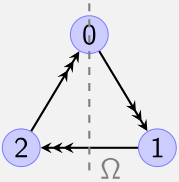
- The orientifold acts as a $\mathbb{Z}_2$ involution of spacetime. Matter fields are identified by $\mathbb{Z}_2$. We expect

$$a^{\Omega} = \frac{1}{2}a \ .$$

[3] Intriligator, Wecht: 2003

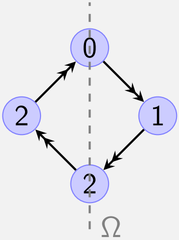
Possible scenarios

$O(1/N)$ corrections to observables, conformal point preserved (e.g. $\left(\mathbb{C}^3/\mathbb{Z}^3\right)/\Omega$):



$$a^{\Omega} = \frac{1}{2}\left(\frac{3}{4}N^2\right) = \frac{1}{2}a\ .$$

Conformal invariance is broken (e.g. $\left(\mathcal{C}/\mathbb{Z}_2\right)/\Omega$):

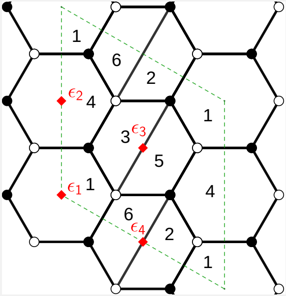


No local maximum for a.

$\mathrm{PdP}_{3c}/\Omega_1$

$$
\begin{aligned}
W_{3c} = & \; X_{12}X_{24}X_{41} + X_{45}X_{51}X_{14} - X_{13}X_{34}X_{41} \\
& - X_{46}X_{61}X_{14} + X_{13}X_{35}X_{56}X_{61} \\
& + X_{46}X_{62}X_{23}X_{34} - X_{12}X_{23}X_{35}X_{51} \\
& - X_{45}X_{56}X_{62}X_{24} \; ,
\end{aligned}
$$

$$
a_{3c} = \frac{3\sqrt{3}}{4} N^2 \; ,
$$



$$
\Omega_1 \; : (+,-,-,+,) \; ,
$$

$$
SO(N_1) \times U(N_2) \times U(N_3) \times Sp(N_4) \; , \quad X^A_{35} \; , \; X^S_{62}
$$

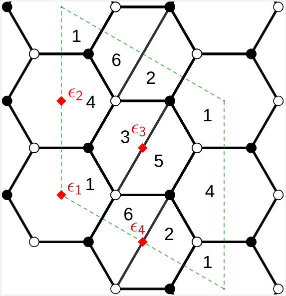
$$
a^{\Omega_1}_{3c} = \frac{1}{2} a_{3c} \; , \quad \text{Flavour U}(1)^3
$$

$$
N = N_1 = N_2 = N_3 - 2 = N_4 - 2 \; .
$$

PdP_{3c}/Ω_1

$$W_{3c} = X_{12}X_{24}X_{41} + X_{45}X_{51}X_{14} - X_{13}X_{34}X_{41} - X_{46}X_{61}X_{14} + X_{13}X_{35}X_{56}X_{61} + X_{46}X_{62}X_{23}X_{34} - X_{12}X_{23}X_{35}X_{51} - X_{45}X_{56}X_{62}X_{24} \, ,$$

$$a_{3c} = \frac{3\sqrt{3}}{4}N^2 \, ,$$



$$\Omega_1 \; : (+,-,-,+,) \, ,$$

$$SO(N_1) \times U(N_2) \times U(N_3) \times Sp(N_4) \, , \quad X^A_{35} \, , \, X^S_{62}$$

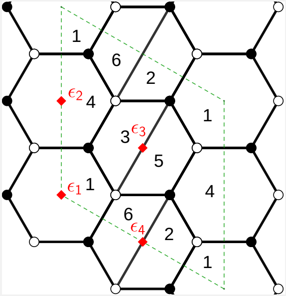
$$a^{\Omega_1}_{3c} = \frac{1}{2}a_{3c} \, , \quad \text{Flavour U}(1)^3$$

$$N = N_1 = N_2 = N_3 - 2 = N_4 - 2 \, .$$

PdP_{3c}/Ω_2

$$W_{3c} = X_{12}X_{24}X_{41} + X_{45}X_{51}X_{14} - X_{13}X_{34}X_{41} - X_{46}X_{61}X_{14} + X_{13}X_{35}X_{56}X_{61} + X_{46}X_{62}X_{23}X_{34} - X_{12}X_{23}X_{35}X_{51} - X_{45}X_{56}X_{62}X_{24} \, ,$$

$$a_{3c} = \frac{3\sqrt{3}}{4} N^2 \, ,$$



$$\Omega_2 \, : (-,+,-,+,) \, ,$$

$$Sp(N_1) \times U(N_2) \times U(N_3) \times SO(N_4) \, , \quad X_{35}^A \, , \, X_{62}^S$$

$$a_{3c}^{\Omega_2} = \frac{27}{8}\left(5\sqrt{5} - 11\right) \approx 0.47 a_{3c} \, , \quad \text{Flavour } \mathrm{U}(1)^2$$

$$N = N_2 = N_3 = N_1 + 2 = N_4 - 2 \, .$$

A third scenario

- $\Omega_1 \longrightarrow$ parent conformal point;
- $\Omega_2 \longrightarrow$ parent conformal point is broken, a flavour $U(1)$ is broken and a *new* fixed point developed in the IR.

A third scenario arises: the orientifold projection breaks the conformal symmetry and triggers an RG flow towards a *new* conformal point in the IR. Along the flow, flavour symmetry is broken.

That is not all...

A third scenario

- $\Omega_1 \longrightarrow$ parent conformal point;
- $\Omega_2 \longrightarrow$ parent conformal point is broken, a flavour $U(1)$ is broken and a *new* fixed point developed in the IR.

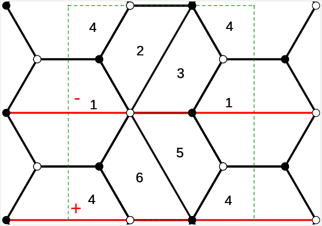
A third scenario arises: the orientifold projection breaks the conformal symmetry and triggers an RG flow towards a *new* conformal point in the IR. Along the flow, flavour symmetry is broken.

That is not all...

PdP_{3b}/Ω

$$W_{3b} = X_{13}X_{34}X_{41} - X_{46}X_{61}X_{14} + X_{45}X_{51}X_{14} - X_{24}X_{41}X_{12} + X_{62}X_{24}X_{46} - X_{35}X_{51}X_{13} + X_{23}X_{35}X_{56}X_{61}X_{12} - X_{23}X_{34}X_{45}X_{56}X_{62} \,,$$

$$a_{3b} = \frac{27}{4}N^2\left(5\sqrt{5} - 11\right)$$



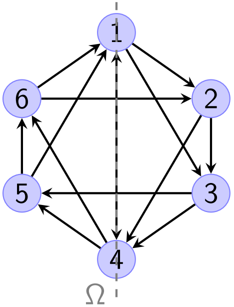
$$\Omega \; : (-,+) \,,$$

$$Sp(N_1) \times U(N_2) \times U(N_3) \times SO(N_4) \,, \quad X_{35}^A \,, \; X_{62}^S$$

$$a_{3b}^{\Omega} = \frac{1}{2}a_{3b} \,, \quad \text{Flavour } \mathrm{U}(1)^2$$

$$N = N_2 = N_3 = N_1 + 2 = N_4 - 2 \,.$$

Comparing PdP_{3b}/Ω and $\mathrm{PdP}_{3c}/\Omega_2$



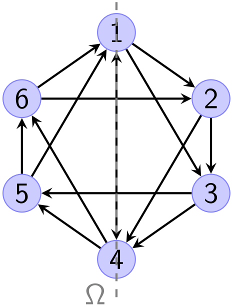
- Same matter content, same R-charges:

Ω_1	$\Omega_2 \vert \Omega$
$R^{\Omega_1}_{14} = 2 - \frac{2\sqrt{3}}{3}$	$R^{\Omega_2}_{23} = 7 - 3\sqrt{5}$
$R^{\Omega_1}_{23,\,35,\,62} = 1 - \frac{\sqrt{3}}{3}$	$R^{\Omega_2}_{13,\,14,\,24} = 3 - \sqrt{5}$
$R^{\Omega_1}_{12,\,13,\,24,\,34} = \frac{\sqrt{3}}{3}$	$R^{\Omega_2}_{12,\,34,\,35,\,62} = 2\sqrt{5} - 4$

$$a^{\Omega_2}_{3c} = a^{\Omega}_{3b} = \frac{1}{2} a_{3b}$$

- Same flavour symmetry;
- 't Hooft anomalies and Superconformal indices trivially match.

Comparing PdP_{3b}/Ω and $\mathrm{PdP}_{3c}/\Omega_2$



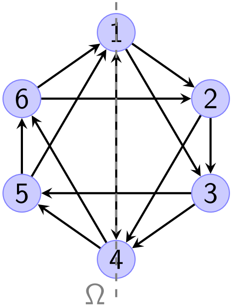
- Same matter content, same R-charges:

Ω_1	$\Omega_2 \vert \Omega$
$R^{\Omega_1}_{14} = 2 - \frac{2\sqrt{3}}{3}$	$R^{\Omega_2}_{23} = 7 - 3\sqrt{5}$
$R^{\Omega_1}_{23,\,35,\,62} = 1 - \frac{\sqrt{3}}{3}$	$R^{\Omega_2}_{13,\,14,\,24} = 3 - \sqrt{5}$
$R^{\Omega_1}_{12,\,13,\,24,\,34} = \frac{\sqrt{3}}{3}$	$R^{\Omega_2}_{12,\,34,\,35,\,62} = 2\sqrt{5} - 4$

$$a^{\Omega_2}_{3c} = a^{\Omega}_{3b} = \frac{1}{2} a_{3b}$$

- Same flavour symmetry;
- 't Hooft anomalies and Superconformal indices trivially match.

Comparing PdP_{3b}/Ω and PdP_{3c}/Ω_2



- Same matter content, same R-charges:

Ω_1	$\Omega_2 \vert \Omega$
$R^{\Omega_1}_{14} = 2 - \frac{2\sqrt{3}}{3}$	$R^{\Omega_2}_{23} = 7 - 3\sqrt{5}$
$R^{\Omega_1}_{23,\,35,\,62} = 1 - \frac{\sqrt{3}}{3}$	$R^{\Omega_2}_{13,\,14,\,24} = 3 - \sqrt{5}$
$R^{\Omega_1}_{12,\,13,\,24,\,34} = \frac{\sqrt{3}}{3}$	$R^{\Omega_2}_{12,\,34,\,35,\,62} = 2\sqrt{5} - 4$

$$a^{\Omega_2}_{3c} = a^{\Omega}_{3b} = \frac{1}{2} a_{3b}$$

- Same flavour symmetry;
- 't Hooft anomalies and Superconformal indices trivially match.

Claim: infrared duality for unoriented theories

$\mathrm{PdP}_{3c}^{\Omega_2}$ develops a new conformal point in the IR, where it is dual to $\mathrm{PdP}_{3b}^{\Omega}$.

Antinucci, SM, Riccioni: 2020

What is the origin of this duality?

- Parent theories have different conformal points.
- From Gubser formula[4] $\text{Vol}(H^5) \propto N^2\, a^{-1}$: geometry?
- S-duality?[5]
- Other examples could help in finding the origin: third scenario for SPP/Ω ($SPP/\mathbb{Z}_n$?)

[4] Gubser:1999

[5] Garcia-Etxebarria, Heidenreich, Timm Wrase: 2013; Garcia-Etxebarria, Heidenreich:2016

What is the origin of this duality?

- Parent theories have different conformal points.
- From Gubser formula[4] $\mathrm{Vol}(H^5) \propto N^2 a^{-1}$: geometry?
- S-duality?[5]
- Other examples could help in finding the origin: third scenario for SPP/Ω ($SPP/\mathbb{Z}_n$?)

[4] Gubser:1999

[5] Garcia-Etxebarria, Heidenreich, Timm Wrase: 2013; Garcia-Etxebarria, Heidenreich:2016

What is the origin of this duality?

- Parent theories have different conformal points.
- From Gubser formula[4] $\mathrm{Vol}(H^5) \propto N^2\, a^{-1}$: geometry?
- S-duality?[5]
- Other examples could help in finding the origin: third scenario for SPP/Ω ($\mathrm{SPP}/\mathbb{Z}_n$?)

[4] Gubser:1999

[5] Garcia-Etxebarria, Heidenreich, Timm Wrase: 2013; Garcia-Etxebarria, Heidenreich:2016

What is the origin of this duality?

- Parent theories have different conformal points.
- From Gubser formula[4] $\mathrm{Vol}(H^5) \propto N^2\, a^{-1}$: geometry?
- S-duality?[5]
- Other examples could help in finding the origin: third scenario for SPP/Ω ($SPP/\mathbb{Z}_n$?)

[4] Gubser:1999

[5] Garcia-Etxebarria, Heidenreich, Timm Wrase: 2013; Garcia-Etxebarria, Heidenreich:2016

What is the origin of this duality?

- Parent theories have different conformal points.
- From Gubser formula[4] $\mathrm{Vol}(H^5) \propto N^2\, a^{-1}$: geometry?
- S-duality?[5]
- Other examples could help in finding the origin: third scenario for SPP/Ω ($\mathrm{SPP}/\mathbb{Z}_n$?)

[4] Gubser:1999

[5] Garcia-Etxebarria, Heidenreich, Timm Wrase: 2013; Garcia-Etxebarria, Heidenreich:2016

Thanks
for your attention