Muon g–2: the showdown

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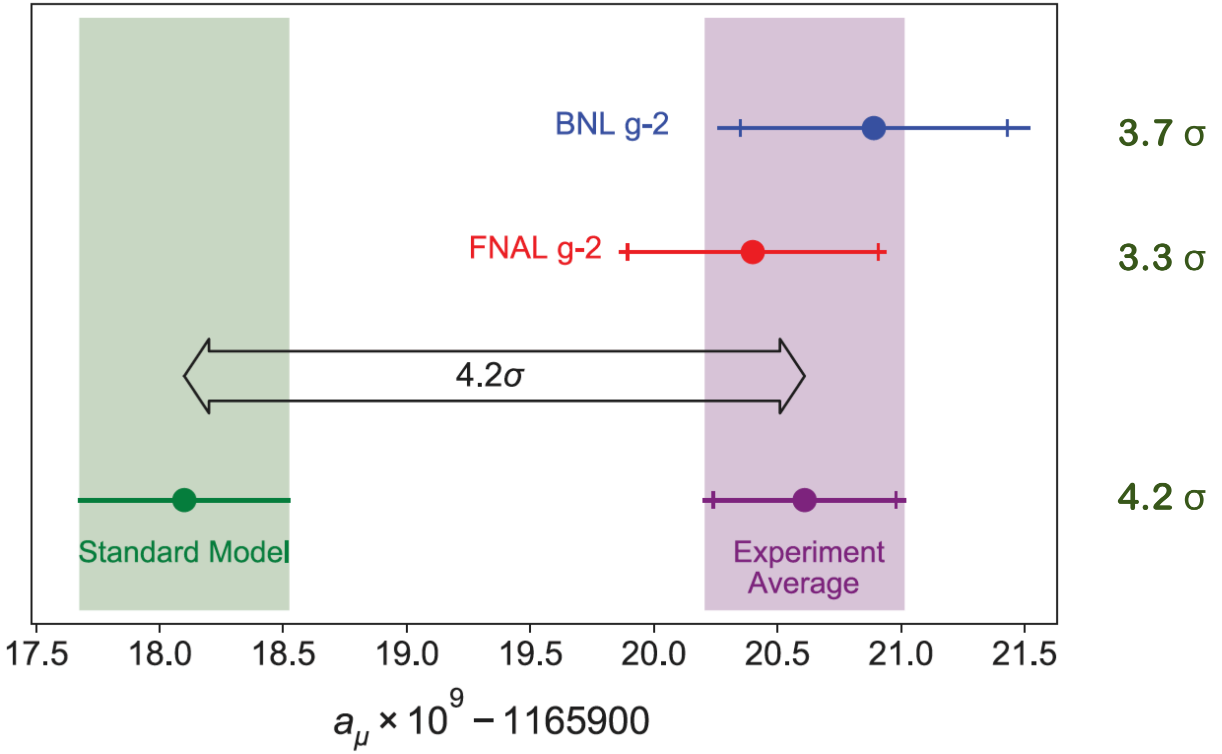
GGI Tea Breaks
Firenze - April 21st 2021

Muon g-2: FNAL confirms BNL



$a_\mu^{EXP} = (116592089 \pm 63) \times 10^{-11}$ [0.54ppm] BNL E821

$a_\mu^{EXP} = (116592040 \pm 54) \times 10^{-11}$ [0.46ppm] FNAL E989 Run 1

$a_\mu^{EXP} = (116592061 \pm 41) \times 10^{-11}$ [0.35ppm] WA

- **FNAL aims at 16×10^{-11}. First 3 runs completed, 4th in progress.**
- **Muon g-2 proposal at J-PARC: Phase-1 with ~ BNL precision.**

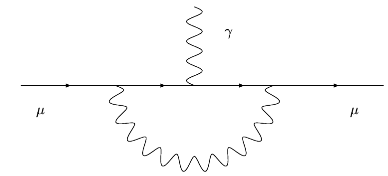
- **Muon g-2: the Standard Model prediction**
- **Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection**
- **The MUonE project**

Muon g-2: the Standard Model prediction

WP20 = White Paper of the Muon g-2 Theory Initiative: arXiv:2006.04822

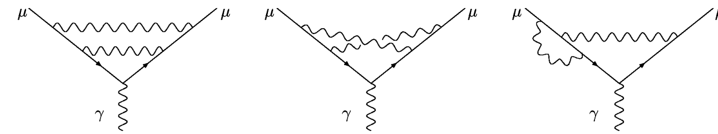
Muon g-2: the QED contribution

a_μ^{QED} = $(1/2)(\alpha/\pi)$ Schwinger 1948



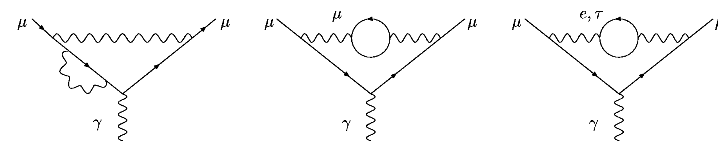
+ $0.765857426\ (16)\ (\alpha/\pi)^2$

Sommerfield; Petermann; Suura&Wichmann '57; Elend '66; MP '04



+ $24.05050988\ (28)\ (\alpha/\pi)^3$

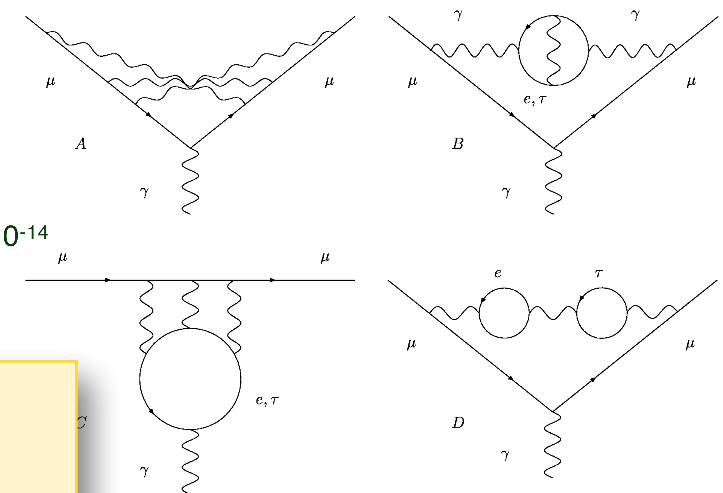
Remiddi, Laporta, Barbieri … ; Czarnecki, Skrzypek '99; MP '04;
Friot, Greynat & de Rafael '05, Ananthanarayan, Friot, Ghosh 2020



+ $130.8780\ (60)\ (\alpha/\pi)^4$

Kinoshita & Lindquist '81, … , Kinoshita & Nio '04, '05;
Aoyama, Hayakawa,Kinoshita & Nio, 2007, Kinoshita et al. 2012 & 2015;
Steinhauser et al. 2013, 2015 & 2016 (all electron & τ loops, analytic);
Laporta, PLB 2017 (mass independent term) COMPLETED[2]!

+ $750.86\ (88)\ (\alpha/\pi)^5$ COMPLETED!



Kinoshita et al. '90, Yelkhovsky, Milstein, Starshenko, Laporta,…
Aoyama, Hayakawa, Kinoshita, Nio 2012, 2015, 2017 & 2019.
Volkov 1909.08015: $A_1^{(10)}$[no lept loops] at variance, but negligible $\delta a_\mu \sim 6\times10^{-14}$

Adding up, we get:

$a_\mu^{QED} = 116584718.931\ (19)(100)(23) \times 10^{-11}$

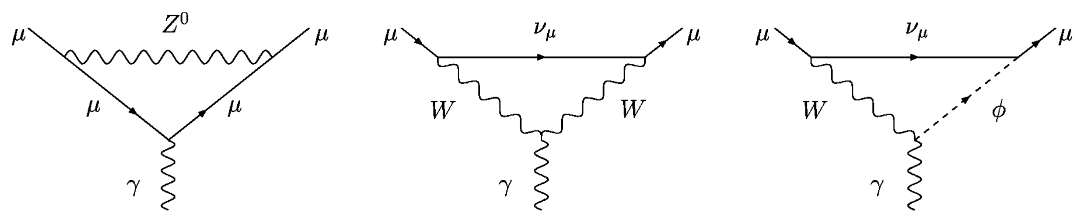
mainly from 4-loop coeff. unc. 6-loop from α(Cs)

α = 1/137.035999046(27) [0.2ppb] Parker et al 2018 WP20 value

Shift down in $a_\mu^{QED} \sim 1.4 \times 10^{-12}$ with new LKB Paris α(Rb) value (Morel et al 2020)

The electroweak contribution

- **One-loop term:**



$$a_\mu^{\mathrm{EW}}(\text{1-loop}) = \frac{5G_\mu m_\mu^2}{24\sqrt{2}\pi^2}\left[1+\frac{1}{5}\left(1-4\sin^2\theta_W\right)^2+O\left(\frac{m_\mu^2}{M_{Z,W,H}^2}\right)\right] \approx 195\times 10^{-11}$$

1972: Jackiv, Weinberg; Bars, Yoshimura; Altarelli, Cabibbo, Maiani; Bardeen, Gastmans, Lautrup; Fujikawa, Lee, Sanda;
Studenikin et al. '80s

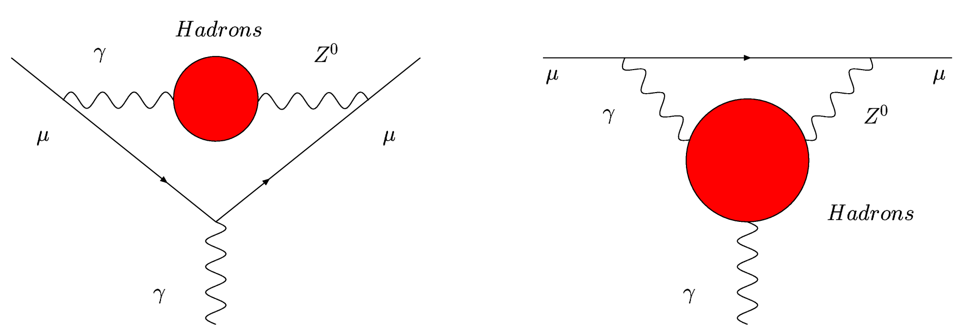
- **One-loop plus higher-order terms:**

$\boxed{a_\mu^{\mathrm{EW}} = 153.6\ (1.0) \times 10^{-11}}$

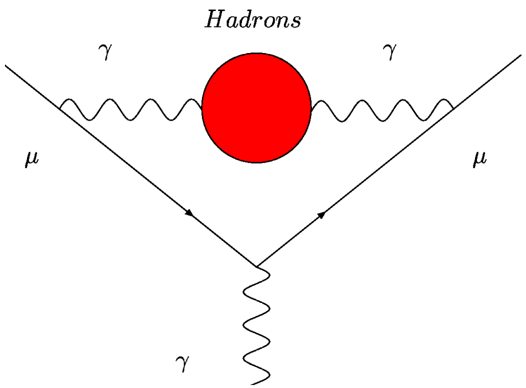
Kukhto et al. '92; Czarnecki, Krause, Marciano '95; Knecht, Peris, Perrottet, de Rafael '02; Czarnecki, Marciano and Vainshtein '02; Degrassi and Giudice '98; Heinemeyer, Stockinger, Weiglein '04; Gribouk and Czarnecki '05; Vainshtein '03; Gnendiger, Stockinger, Stockinger-Kim 2013, Ishikawa, Nakazawa, Yasui, 2019.

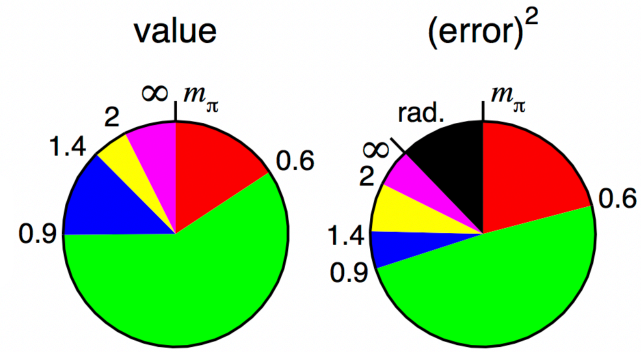
Hadronic loop uncertainties (and 3-loop nonleading logs).

WP20 value



The hadronic LO contribution





Keshavarzi, Nomura, Teubner 2018

$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\, \sigma^{(0)}_{\rm had}(s) \qquad K(s) = \int_0^1 dx \frac{x^2\,(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

a_μ^{HLO} = 6895 (33) x 10^{-11} F. Jegerlehner, arXiv:1711.06089

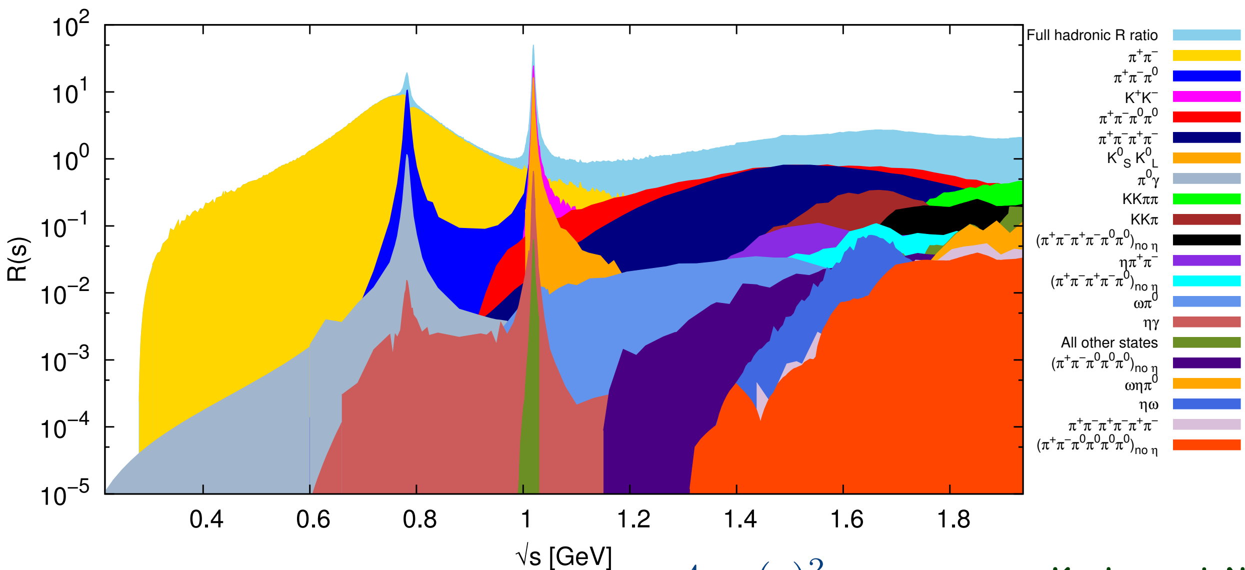
= 6939 (40) x 10^{-11} Davier, Hoecker, Malaescu, Zhang, arXiv:1908.00921

= 6928 (24) x 10^{-11} Keshavarzi, Nomura, Teubner, arXiv:1911.00367

= 6931 (40) x 10^{-11} (0.6%) WP20 value

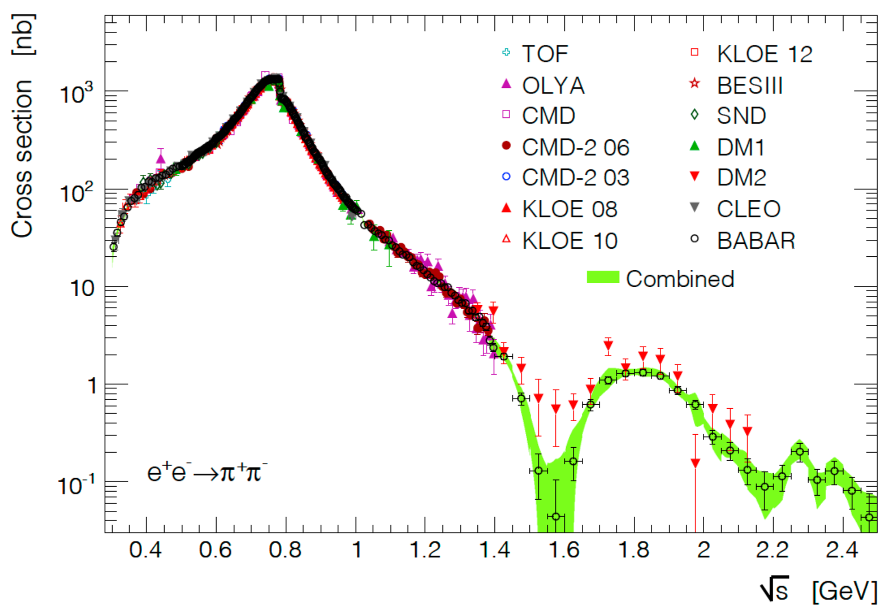
- **WP20 value obtained merging conservatively DHMZ + KNT + constraints from CHHKS**
  Colangelo, Hoferichter, Hoid, Kubis, Stoffer 2018-19
- **Radiative Corrections to σ(s) are crucial.** S. Actis et al, Eur. Phys. J. C66 (2010) 585

The low-energy hadronic cross section



$$R(s) = \sigma(e^+e^- \to \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$

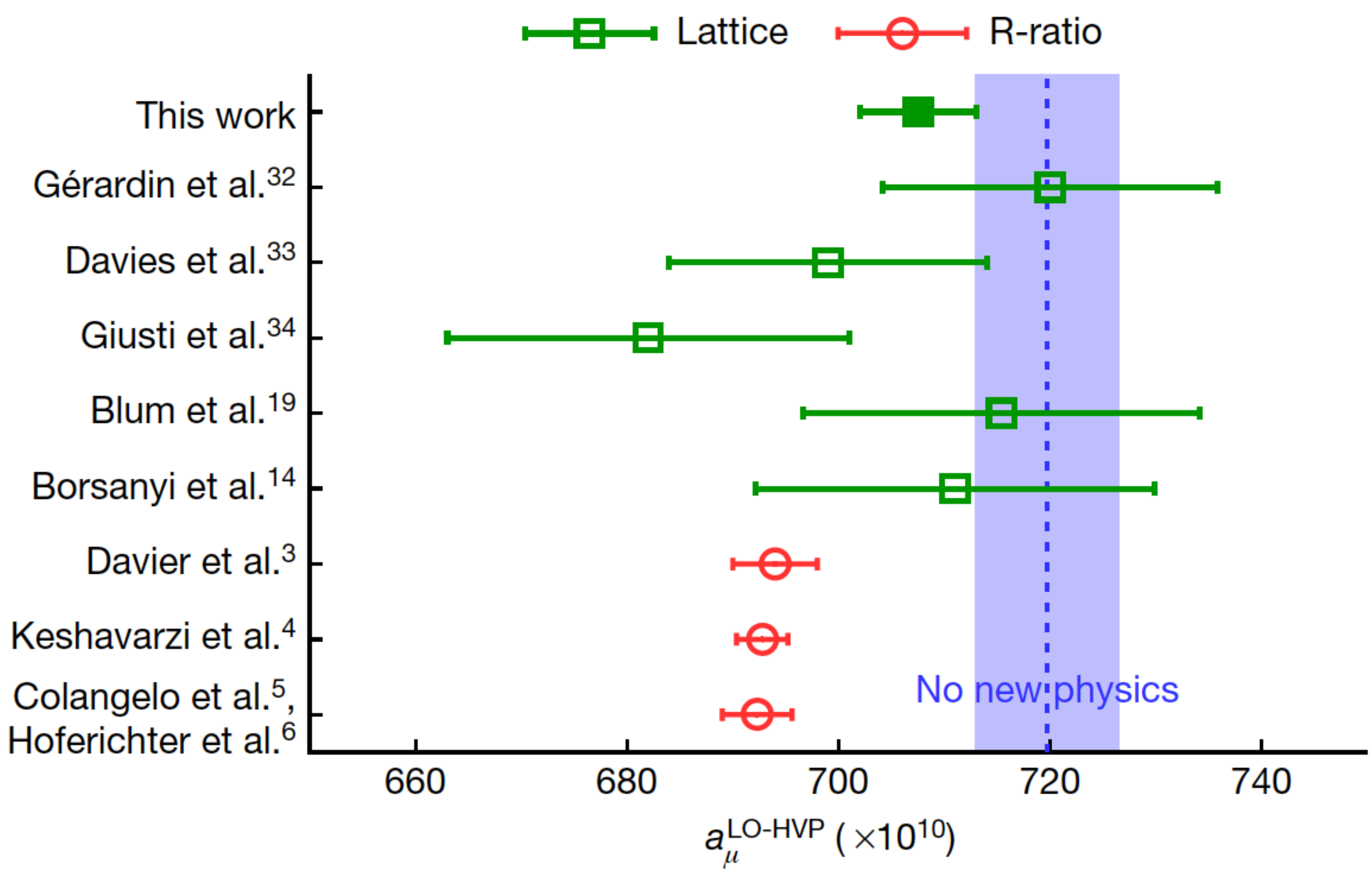
Keshavarzi, Nomura Teubner
PRD 2018



Davier, Hoecker, Malaescu, Zhang
EPJC 2020

The hadronic LO contribution from lattice QCD

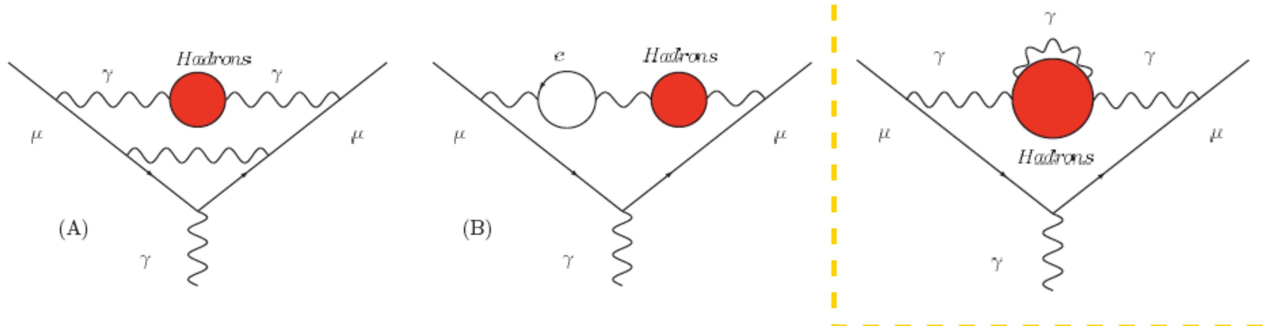
- **Great progress in lattice QCD results. The BMW collaboration reached 0.8% precision:** $a_\mu^{HLO} = 7075\underbrace{(23)_{stat}(50)_{syst}}_{55} \times 10^{-11}$**. Some tension with dispersive evaluations.** BMWc 2021



Borsanyi et al (BMWc), Nature 2021

- **O(α³) contributions of diagrams containing HVP insertions:**

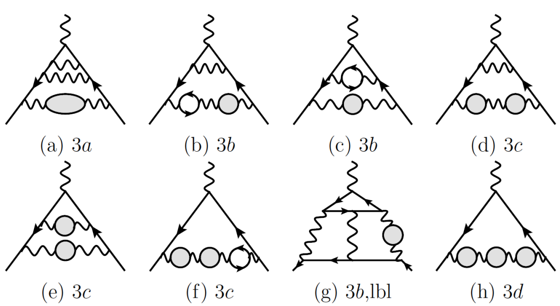
Already included in a_μ^{HLO}



$$a_\mu^{HNLO}(vp) = -98.3\ (7) \times 10^{-11}$$

Krause ’96; Keshavarzi, Nomura, Teubner 2019; WP20.

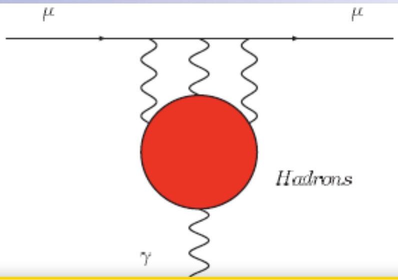
- **O(α⁴) contributions of diagrams containing HVP insertions:**



$$a_\mu^{HNNLO}(vp) = 12.4\ (1) \times 10^{-11}$$

Kurz, Liu, Marquard, Steinhauser 2014

- **Hadronic light-by-light at $O(\alpha^3)$**



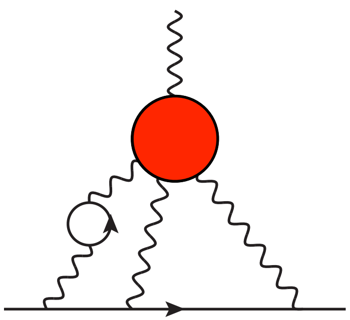
  - **This term had a troubled life! But nowadays:**

$$a_\mu^{HNLO}(lbl) = 80\ (40) \times 10^{-11}$$ Knecht & Nyffeler ’02
$$= 136\ (25) \times 10^{-11}$$ Melnikov & Vainshtein ’03
$$= 105\ (26) \times 10^{-11}$$ Prades, de Rafael, Vainshtein ’09
$$= 100\ (29) \times 10^{-11}$$ Jegerlehner, arXiv:1705.00263
$$= 92\ (19) \times 10^{-11}$$ WP20 (phenomenology)

  - **Significant improvements due to data-driven dispersive approach.**
    Colangelo, Hoferichter, Procura, Stoffer, 2014–17; Pauk, Vanderhaeghen 2014.
  - **Lattice: RBC: $82(35)\times10^{-11}$** 1911.08123 **Mainz: $110(15)\times10^{-11}$** 2104.02632

- **Hadronic light-by-light at $O(\alpha^4)$**

$$\boxed{a_\mu^{HNNLO}(lbl) = 2\ (1) \times 10^{-11}}$$



Colangelo, Hoferichter, Nyffeler, MP, Stoffer 2014; WP20

Comparing the SM prediction with the measured muon g-2 value:

$$a_\mu^{EXP} = 116592061\ (41) \times 10^{-11}$$ BNL+FNAL

$$a_\mu^{SM} = 116591810\ (43) \times 10^{-11}$$ WP20

$$\Delta a_\mu = a_\mu^{EXP} - a_\mu^{SM} = 251\ (59) \times 10^{-11}$$ 4.2 σ

Is Δa_μ due to new physics beyond the SM? Could be due to:

- NP at the weak scale and weakly coupled to SM particles
- NP very heavy and strongly coupled to SM particles
- NP very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles

Muon g-2 $\Longleftrightarrow$ $\Delta\alpha$ connection

Marciano, MP, Sirlin 2008 & 2010

Keshavarzi, Marciano, MP, Sirlin 2020

- Can Δa_μ be due to missing contributions in the hadronic $\sigma(s)$?
- An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.
- Consider:

$a_\mu^{HLO} \rightarrow$
$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\, \sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$
$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\, \sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

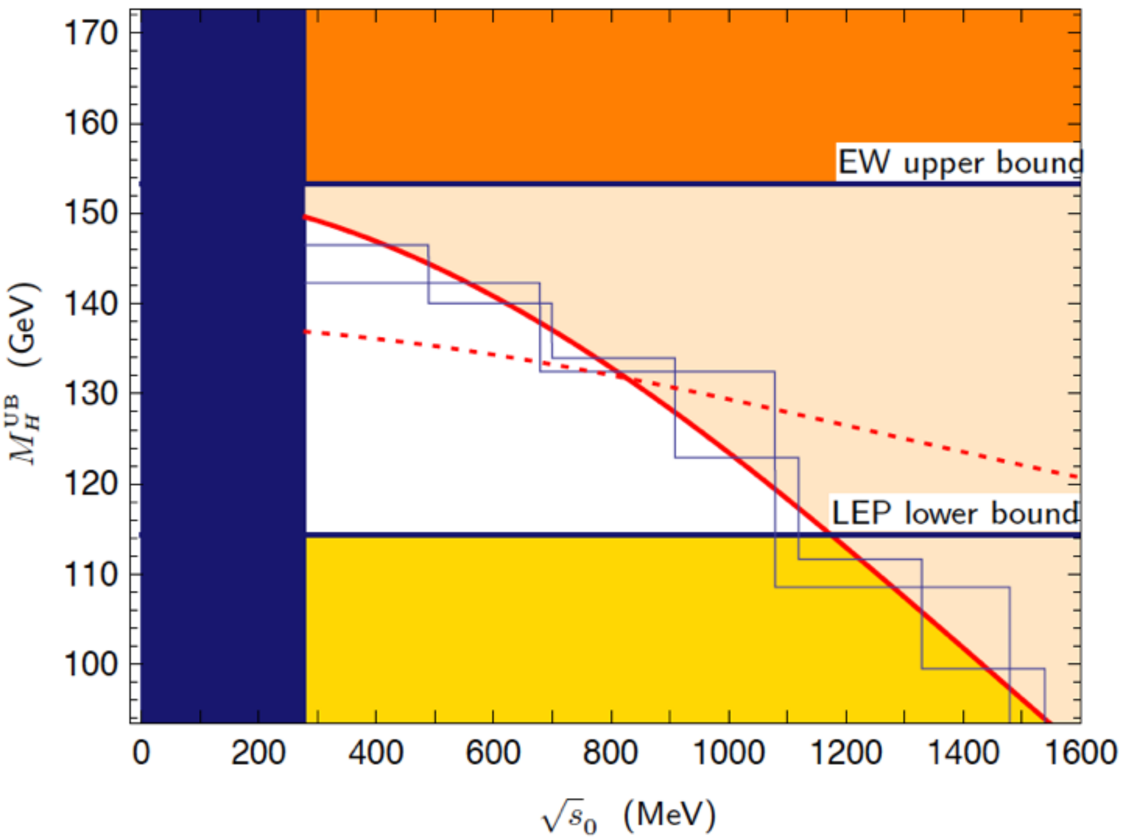
and the increase

$$\Delta\sigma(s) = \epsilon\sigma(s)$$

$\epsilon>0$, in the range:

$$\sqrt{s} \in [\sqrt{s_0} - \delta/2, \sqrt{s_0} + \delta/2] \longrightarrow$$

How much does the M_H upper bound from the EW fit change when we shift up $\sigma(s)$ by $\Delta\sigma(s)$ [and thus $\Delta\alpha_{had}^{(5)}(M_Z)$] to fix Δa_μ ?



Marciano, MP, Sirlin, PRD 2008

Muon g-2: connection with the SM Higgs mass (update)

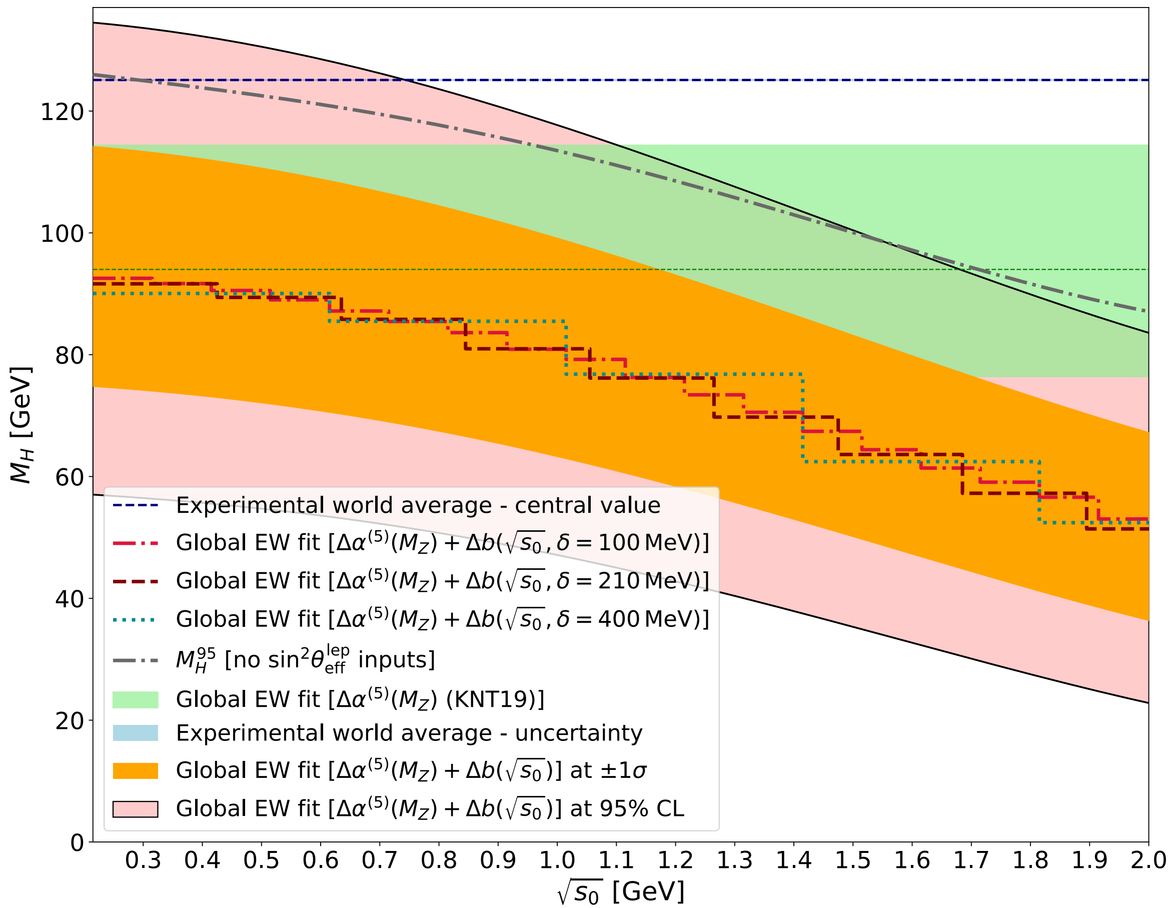
Major update: Higgs discovered, improved EW observables (M_W, $\sin^2\theta$, M_{top}, …), updates to σ(s), theory improvements, global fit, …

Parameter	Input value	Reference	Fit result	Result w/o input value
M_W (GeV)	80.379(12)	[5]	80.359(3)	80.357(4)(5)
M_H (GeV)	125.10(14)	[5]	125.10(14)	94^{+20+6}_{-18-6}
$\Delta\alpha^{(5)}_{\rm had}(M_Z^2) \times 10^4$	276.1(1.1)	[23]	275.8(1.1)	272.2(3.9)(1.2)
m_t (GeV)	172.9(4)	[5]	173.0(4)	⋯
$\alpha_s(M_Z^2)$	0.1179(10)	[5]	0.1180(7)	⋯
M_Z (GeV)	91.1876(21)	[5]	91.1883(20)	⋯
Γ_Z (GeV)	2.4952(23)	[5]	2.4940(4)	⋯
Γ_W (GeV)	2.085(42)	[5]	2.0903(4)	⋯
$\sigma^0_{\rm had}$ (nb)	41.541(37)	[108]	41.490(4)	⋯
R^0_l	20.767(25)	[108]	20.732(4)	⋯
R^0_c	0.1721(30)	[108]	0.17222(8)	⋯
R^0_b	0.21629(66)	[108]	0.21581(8)	⋯
$\bar{m}_c$ (GeV)	1.27(2)	[5]	1.27(2)	⋯
$\bar{m}_b$ (GeV)	$4.18^{+0.03}_{-0.02}$	[5]	$4.18^{+0.03}_{-0.02}$	⋯
$A^{0,l}_{\rm FB}$	0.0171(10)	[108]	0.01622(7)	⋯
$A^{0,c}_{\rm FB}$	0.0707(35)	[108]	0.0737(2)	⋯
$A^{0,b}_{\rm FB}$	0.0992(16)	[108]	0.1031(2)	⋯
A_ℓ	0.1499(18)	[75,108]	0.1471(3)	⋯
A_c	0.670(27)	[108]	0.6679(2)	⋯
A_b	0.923(20)	[108]	0.93462(7)	⋯
$\sin^2\theta^{\rm lep}_{\rm eff}(Q_{\rm FB})$	0.2324(12)	[108]	0.23152(4)	0.23152(4)(4)
$\sin^2\theta^{\rm lep}_{\rm eff}$(Had Coll)	0.23140(23)	[100]	0.23152(4)	0.23152(4)(4)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (using Gfitter)

Muon g-2: connection with the SM Higgs mass (2020)

$\Delta\alpha$

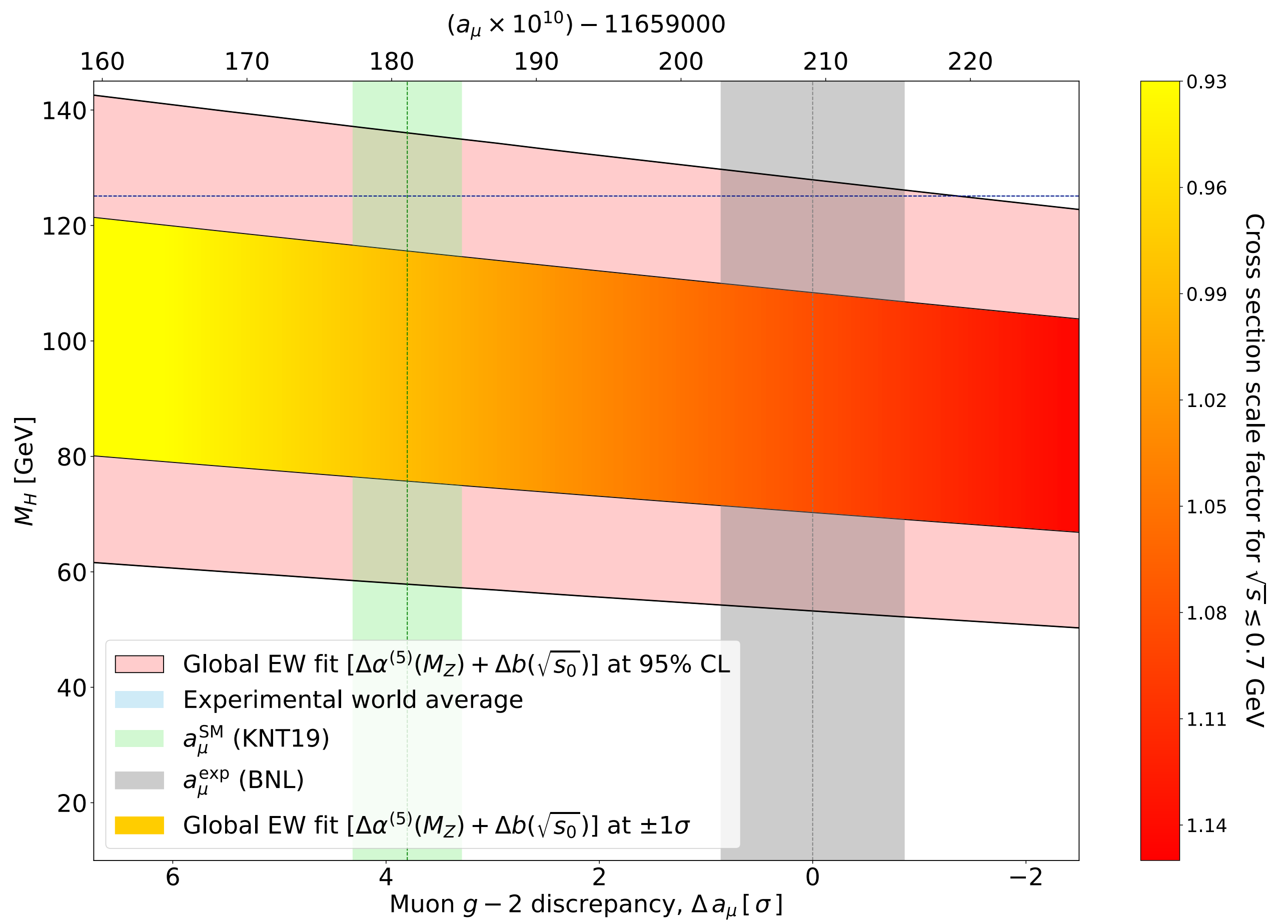


Shifts $\Delta\sigma(s)$ to fix Δa_μ are possible, but conflict with the EW fit if they occur above ~1 GeV

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

The muon g-2: connection with the SM Higgs mass (2020) - II

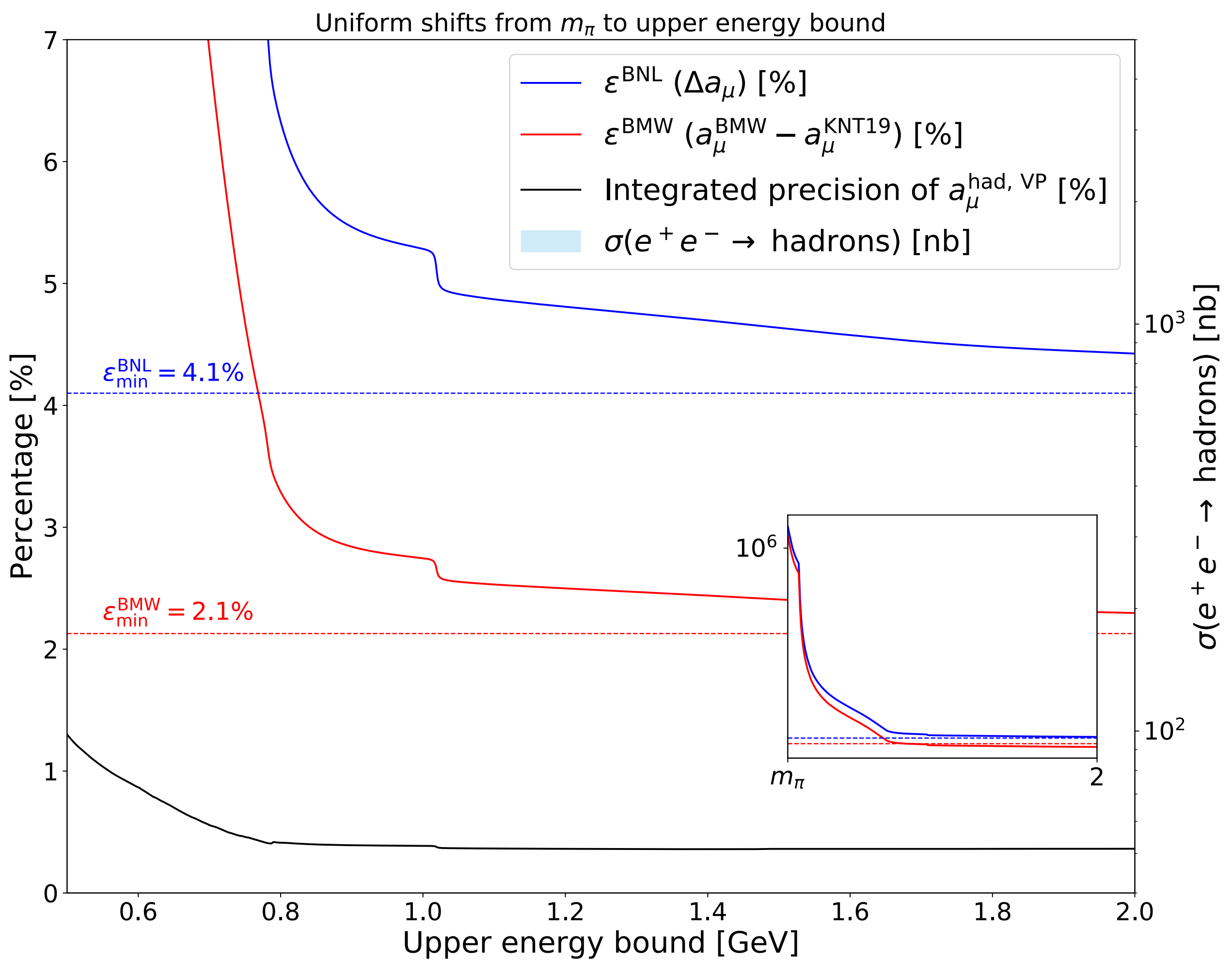
Δα



Uniform scaling of σ(s) below ~0.7 GeV? +9% required!

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

How large are the required shifts Δσ(s) ?



Shifts below ~1 GeV conflict with the quoted exp. precision of σ(s)

Keshavarzi, Marciano, MP, Sirlin, PRD 2020 (updated 2021)

What happens to the electron g-2?

The electron g-2 no longer provides the best value of α

- The 2008 measurement of the electron g-2 is:

$a_e^{EXP} = 11596521807.3\,(2.8) \times 10^{-13}$ Hanneke et al, PRL100 (2008) 120801

vs. old (factor of 15 improvement, 1.8σ difference):

$a_e^{EXP} = 11596521883\,(42) \times 10^{-13}$ Van Dyck et al, PRL59 (1987) 26

- Equate $a_e^{SM}(\alpha) = a_e^{EXP}$ → "g_e-2" determination of alpha:

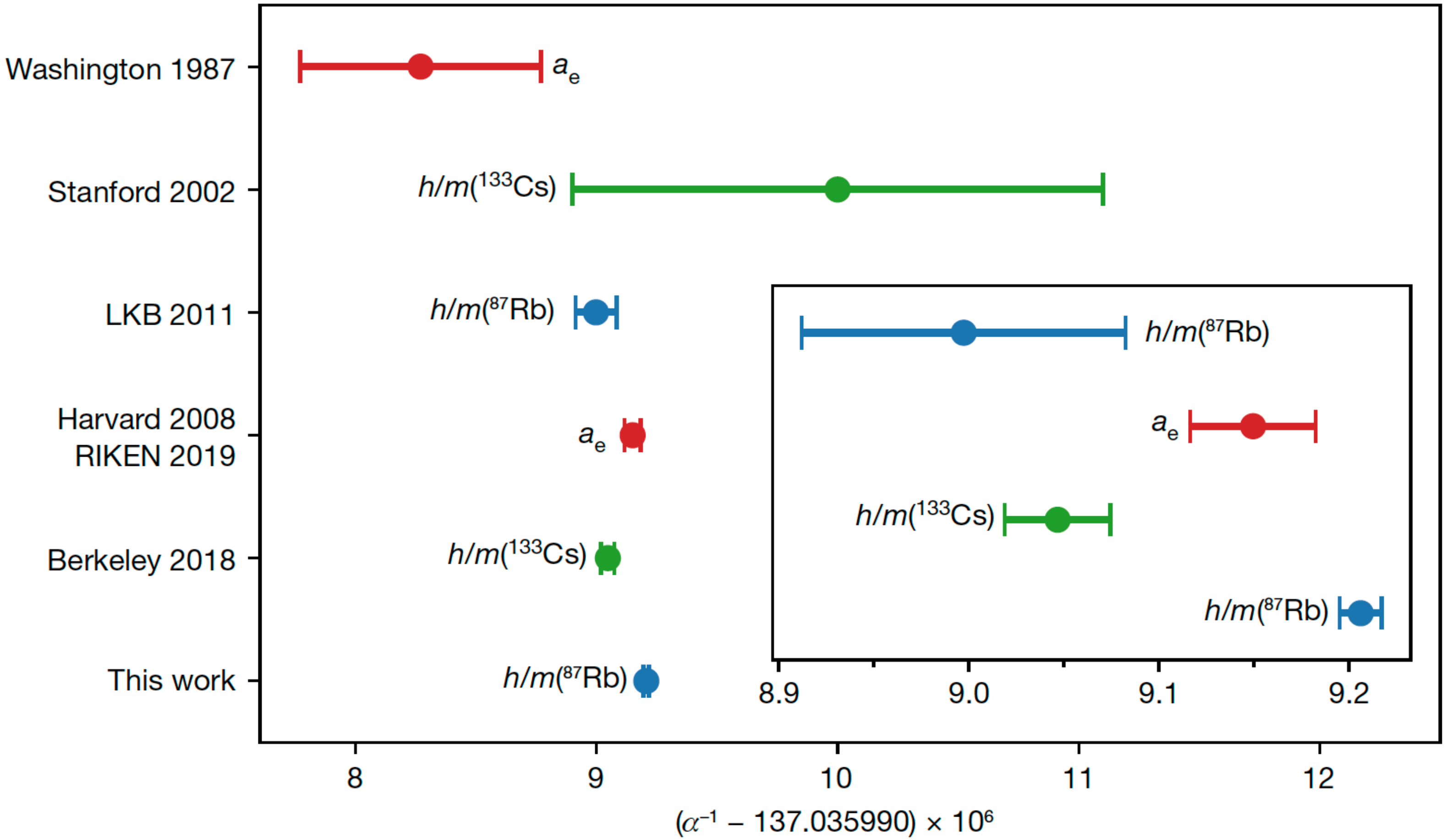
$$\alpha^{-1} = 137.035\,999\,151\,(33) \quad [0.24 \text{ ppb}]$$

- The best determination of α is obtained via atomic interferometry:

$\alpha^{-1} = 137.035\,999\,046\,(27)$ [0.20 ppb] Parker et al, Science 360 (2018) 192 (Cs)

$\alpha^{-1} = 137.035\,999\,206\,(11)$ [0.08 ppb] Morel et al, Nature 588 (2020) 61 (Rb)

2018→2020: improvement in precision, but 5.4σ difference!



Morel et al, Nature 588 (2020) 61

Electron g-2: SM vs experiment

e

- Using the best determinations of α (which differ by 5.4σ!):

$$\alpha = 1/137.035\,999\,046\,(27) \quad \text{[Cs 2018]}$$
$$\alpha = 1/137.035\,999\,206\,(11) \quad \text{[Rb 2020]}$$

$$a_e^{SM} = 115\,965\,218\,16.16\,(0.11)\,(0.08)\,(2.28) \times 10^{-13} \text{ [Cs18]}$$
$$= 115\,965\,218\,02.64\,(0.11)\,(0.08)\,(0.93) \times 10^{-13} \text{ [Rb20]}$$

δC_5^{qed} δa_e^{had} from $\delta\alpha$

$a_e^{EXP} = 115\,965\,218\,07.3\,(2.8) \times 10^{-13}$ Hanneke et al, PRL 2008

- The (EXP – SM) difference is:

$$\Delta a_e = a_e^{EXP} - a_e^{SM} = -\,8.9\,(3.6) \times 10^{-13} \text{ [}2.5\sigma\text{] [Cs18]}$$
$$= +\,4.7\,(3.0) \times 10^{-13} \text{ [}1.6\sigma\text{] [Rb20]}$$

QED 5-loop: $a_e^{QED5} = 4.6 \times 10^{-13}$

- NP sensitivity limited only by the experimental errors in α and a_e. May soon play a pivotal role in probing NP in the leptonic sector.

Giudice, Paradisi, MP 2012

Testing new physics with the electron g-2

- Using α(Rb2020), the sensitivity is $\delta\Delta a_e = 3.0 \times 10^{-13}$, ie ($\times 10^{-13}$):

$$\underbrace{(0.1)_{\mathrm{QED5}}, \quad (0.1)_{\mathrm{HAD}},}_{(0.2)_{\mathrm{TH}}} \quad (0.9)_{\delta\alpha}, \quad (2.8)_{\delta a_e^{\mathrm{EXP}}}$$

- The $(g\text{-}2)_e$ experimental error may soon drop below 10^{-13} → a_e sensitivity below 10^{-13} may soon be reached!

- In a broad class of BSM theories, contributions to a_l scale as

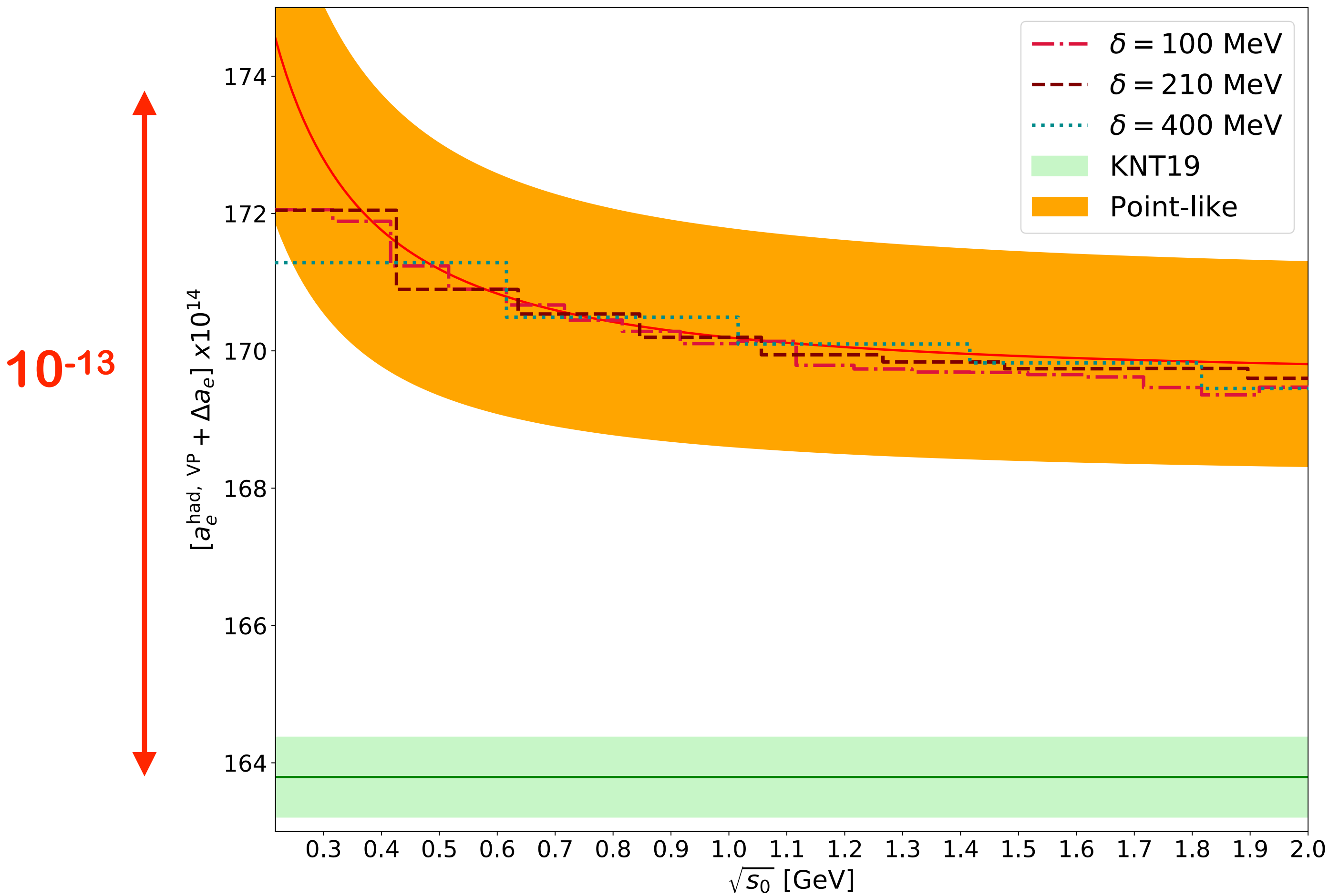
$$\frac{\Delta a_{\ell_i}}{\Delta a_{\ell_j}} = \left(\frac{m_{\ell_i}}{m_{\ell_j}}\right)^2$$

This Naive Scaling leads to:

$$\boxed{\Delta a_e = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.7 \times 10^{-13}; \qquad \Delta a_\tau = \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 0.8 \times 10^{-6}}$$

Giudice, Paradisi & MP, JHEP 2012

Shift of the electron g-2

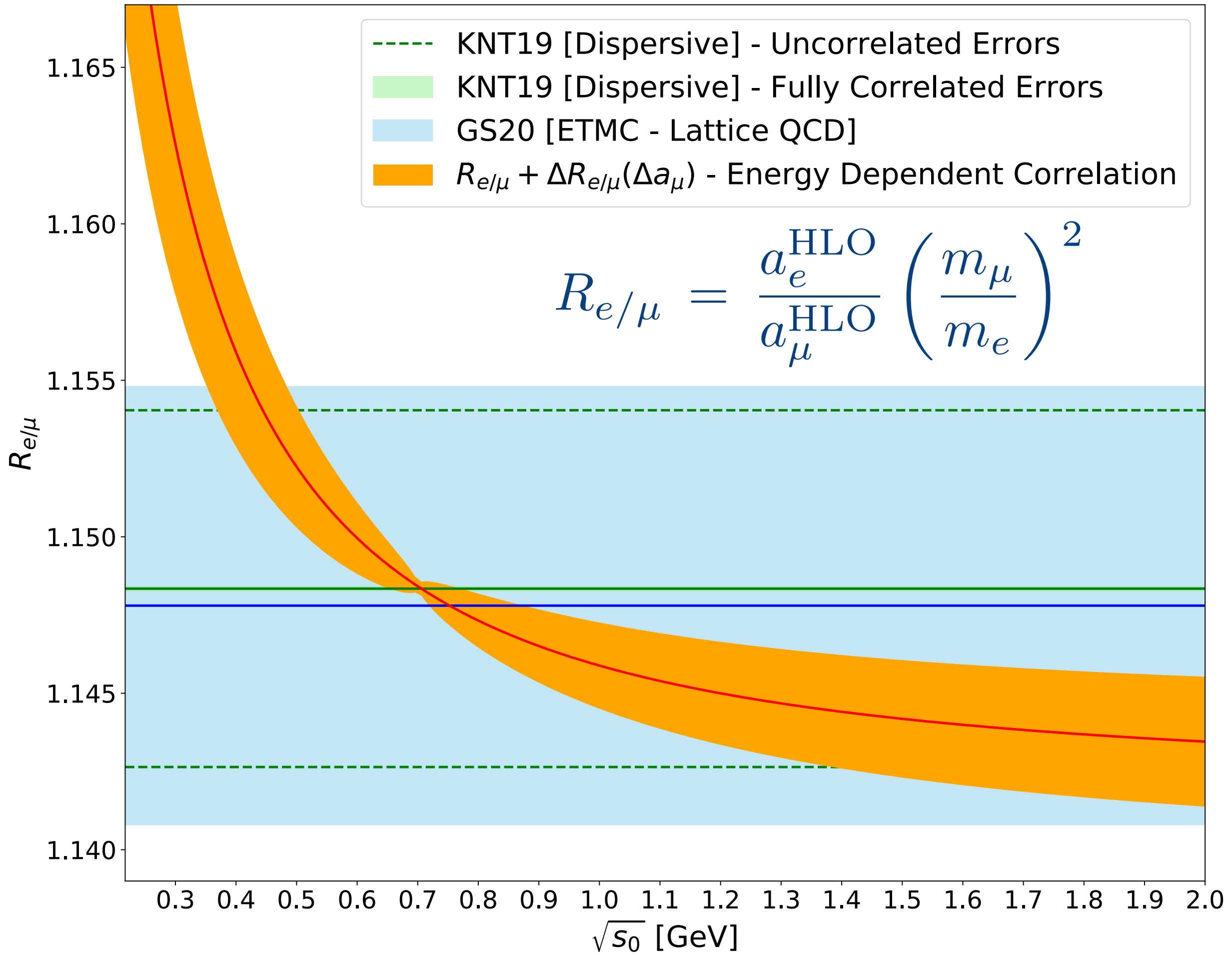


Shifts $\Delta\sigma(s)$ to fix Δa_μ only slightly change Δa_e

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

Shift of the e/μ g-2 scaled HLO ratio

e/μ



Good agreement between lattice [Giusti & Simula 2020] and KNT19.
Possible future bounds on very low energy shifts Δσ(s)?

Keshavarzi, Marciano, MP, Sirlin, PRD 2020

$a_\mu \Longleftrightarrow \Delta\alpha$: recent related literature

- Crivellin, Hoferichter, Manzari and Montull, “Hadronic vacuum polarization: $(g\text{-}2)_\mu$ versus global electroweak fits,” arXiv:2003.04886.

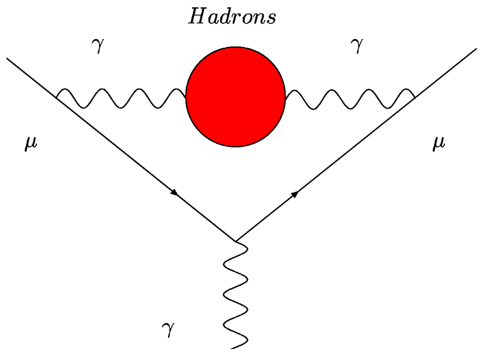
- Eduardo de Rafael, “On Constraints Between $\Delta\alpha_{had}(M_Z^2)$ and $(g_\mu\text{-}2)_{HVP}$,’’ arXiv:2006.13880.

- Malaescu and Schott, “Impact of correlations between a_μ and α_{QED} on the EW fit,” arXiv:2008.08107.

- Colangelo, Hoferichter and Stoffer, “Constraints on the two-pion contribution to hadronic vacuum polarization,” arXiv:2010.07943.

The MUonE project

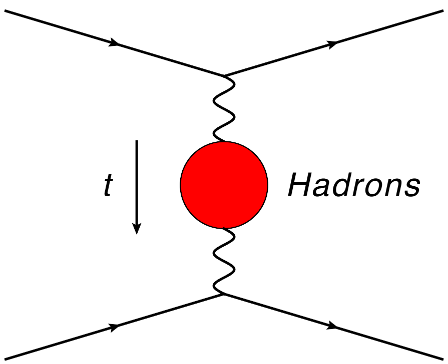
- The leading hadronic contribution a_μ^{HLO} computed via the timelike formula:



$$a_\mu^{\rm HLO} = \frac{1}{4\pi^3}\int_{m_\pi^2}^{\infty} ds\, K(s)\, \sigma_{\rm had}^{(0)}(s)$$

$$K(s) = \int_0^1 dx\, \frac{x^2\,(1-x)}{x^2 + (1-x)\,(s/m_\mu^2)}$$

- Alternatively, simply exchanging the x and s integrations:



$$a_\mu^{\rm HLO} = \frac{\alpha}{\pi}\int_0^1 dx\,(1-x)\,\Delta\alpha_{\rm had}[t(x)]$$

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0$$

Lautrup, Peterman, de Rafael, 1972

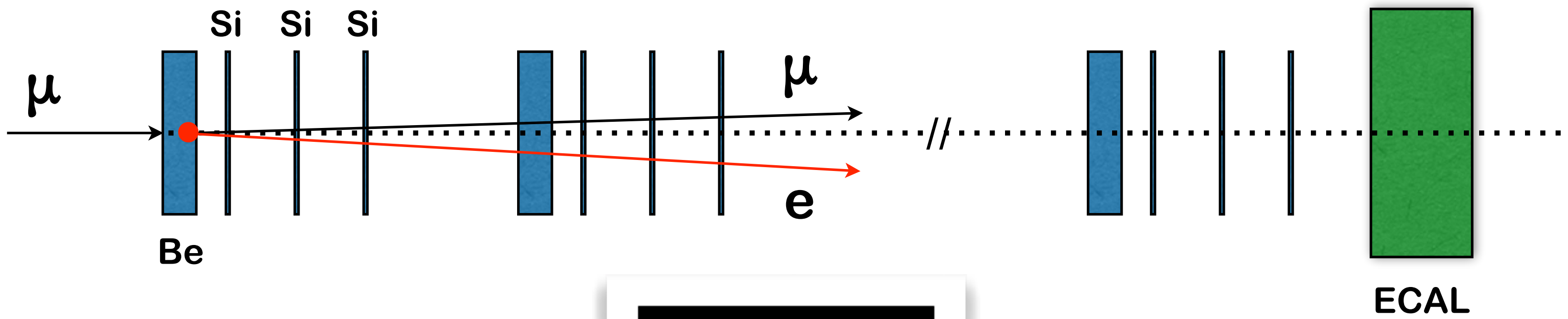
$\Delta\alpha_{had}(t)$ is the hadronic contribution to the running of α in the spacelike region: a_μ^{HLO} can be extracted from scattering data!

Carloni Calame, MP, Trentadue, Venanzoni, 2015

MUonE: Muon-electron scattering @ CERN

- $\Delta\alpha_{had}(t)$ can be measured via the elastic scattering $\mu\, e \to \mu\, e$.

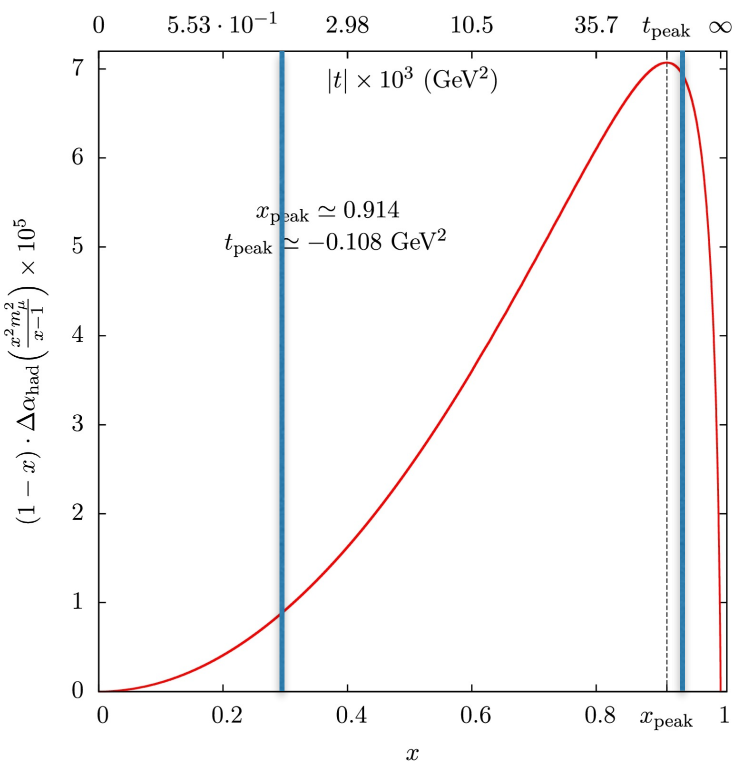
- We propose to scatter a 150 GeV muon beam, available at CERN's North Area, on a fixed electron target (Beryllium). Modular apparatus: each station has one layer of Beryllium (target) followed by several thin Silicon strip detectors.

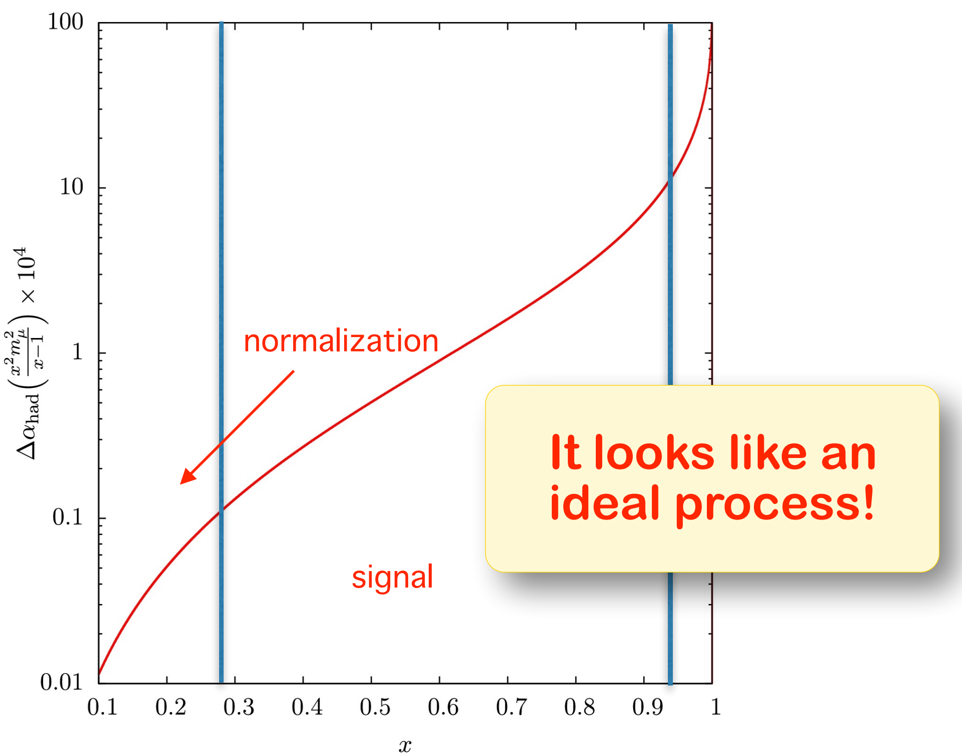


Abbiendi, Carloni Calame, Marconi, Matteuzzi, Montagna, Nicrosini, MP, Piccinini, Tenchini, Trentadue, Venanzoni
EPJC 2017 - arXiv:1609.08987

- For a 150 GeV muon beam ($\sqrt{s}$~400 MeV), MUonE's scan region extends up to x=0.932, ie beyond the peak! (the peak is at x=0.914)

- The high-energy region inaccessible to MUonE contributes only 13% of a_μ^{HLO} integral. It can be determined with timelike data and/or lattice QCD results. Already obtained via lattice QCD! Giusti&Simula and Marinkovic´&Cardoso 2019





MUonE — Status

- **Statistics**: With CERN's 150 GeV muon beam M2 (1.3×10^7 μ/s), incident on 40 15mm Be targets (total thickness 60cm), 2-3 years of data taking ($2{\times}10^7$ s/yr) → $\mathscr{L}_{\rm int} \sim 1.5 \times 10^7\ {\rm nb}^{-1}$.

- With this $\mathscr{L}_{\rm int}$ we estimate that measuring the shape of dσ/dt we can reach a statistical sensitivity of ~0.3% on a_μ^{HLO}, ie $\sim 20 \times 10^{-11}$.

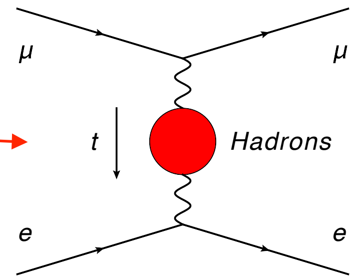
- **Systematic** effects must be known at ≲ 10ppm!

- Test beams performed at CERN in 2017 & 2018 arXiv:1905.11677, 2102.11111

- LoI submitted to CERN SPSC in 2019: Test run approved for 2021.

- Full-statistics run hopefully in 2022–24.

- To extract $\Delta\alpha_{had}(t)$ from MUonE's measurement, the ratio of the SM cross sections in the signal and normalisation regions must be known at ≲ 10ppm!



- **Fully differential fixed-order MC @ NLO ready** Pavia and PSI 2018-19
- **NNLO QED: Master Integrals for 2-loop box diagrams computed. Full 2-loop amplitude close to completion.** Padova 2017 - present
- **Two MC built including partial subsets of the NNLO QED corrections due to electron and muon radiation** Pavia and PSI 2020
- **NNLO hadronic effects computed** Padova and KIT 2019
- **Extraction of the leading electron mass effects from the massless muon-electron scattering amplitudes** PSI 2019-present
- **New Physics extracting $\Delta\alpha_{had}(t)$ at MUonE?** Padova and Heidelberg 2020
- …

Theory for muon-electron scattering @ 10 ppm:
A report of the MUonE theory initiative. arXiv:2004.13663

Muon-electron scattering: Theory kickoff workshop

4-5 September 2017

Padova
Europe/Rome timezone

- **Overview**
- Venue
- Timetable
- Logistic
- Map

Support

MUonE theory workshops: Padova 2017, Mainz 2018, Zurich 2019
Next MUonE theory workshop: MITP Mainz 2020-21 postponed to 2022

Conclusions

- Fermilab's Muon g-2 experiment confirms BNL's result.
- The discrepancy between experiment and SM increases to 4.2σ.
- The BMWc lattice QCD result weakens the exp-SM discrepancy. It must be confirmed or refuted by other lattice calculations.
- Is the present Δa_μ discrepancy due to missed contributions in the hadronic $\sigma(s)$?

  Shifts $\Delta\sigma(s)$ to fix Δa_μ conflict with the global EW fit above ~1 GeV
  Shifts below ~1 GeV conflict with the quoted exp. error of $\sigma(s)$.

- Leading hadronic contribution to a_μ: dispersive vs lattice? MUonE will provide a new independent determination alternative to both.