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\begin{aligned}
\partial_t \tilde{\Gamma}^i = & -2\,\tilde{A}^{ij}\,\partial_j \alpha + 2\,\alpha \Big[\tilde{\Gamma}^i{}_{jk}\,\tilde{A}^{jk} - \frac{3}{2}\,\tilde{A}^{ij}\,\partial_j \ln(\chi) \\
& - \frac{1}{3}\,\tilde{\gamma}^{ij}\,\partial_j (2\,\hat{K} + \Theta) - 8\,\pi\,\tilde{\gamma}^{ij}\,S_j \Big] + \tilde{\gamma}^{jk}\,\partial_j \partial_k \beta^i \\
& + \frac{1}{3}\,\tilde{\gamma}^{ij}\,\partial_j \partial_k \beta^k + \beta^j\,\partial_j \tilde{\Gamma}^i - (\tilde{\Gamma}_{\rm d})^j\,\partial_j \beta^i \\
& + \frac{2}{3}\,(\tilde{\Gamma}_{\rm d})^i\,\partial_j \beta^j - 2\,\alpha\,\kappa_1\,[\tilde{\Gamma}^i - (\tilde{\Gamma}_{\rm d})^i]\,, \\
\partial_t \Theta = & \frac{1}{2}\,\alpha\,[R - \tilde{A}_{ij}\,\tilde{A}^{ij} + \frac{2}{3}\,(\hat{K} + 2\,\Theta)^2] \\
& - \alpha\,[8\,\pi\,\rho + \kappa_1\,(2 + \kappa_2)\,\Theta] + \beta^i \partial_i \Theta\,,
\end{aligned}
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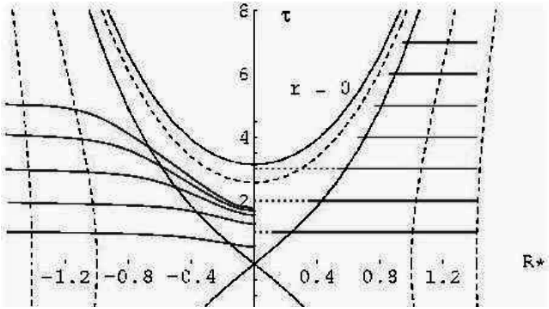

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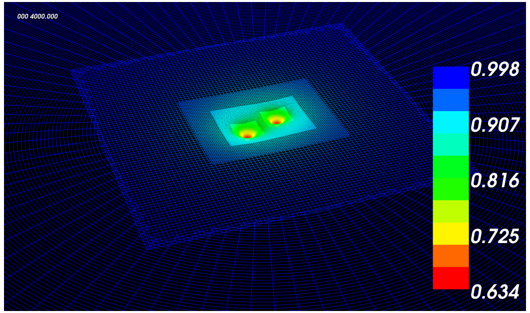

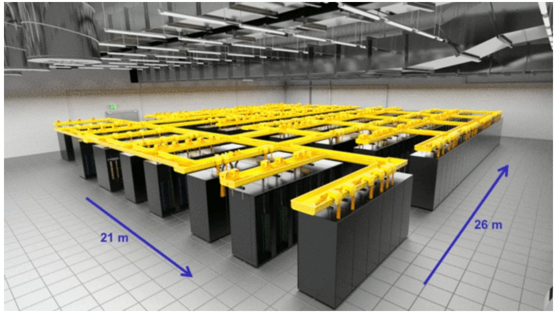

PRL 95, 121101 (2005) PHYSICAL REVIEW LETTERS week ending 16 SEPTEMBER 2005

Evolution of Binary Black-Hole Spacetimes

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We describe early success in the evolution of binary black-hole spacetimes with a numerical code based on a generalization of harmonic coordinates. Indications are that with sufficient resolution this scheme is capable of evolving binary systems for enough time to extract information about the orbit, merger, and gravitational waves emitted during the event. As an example we show results from the evolution of a binary composed of two equal mass, nonspinning black holes, through a single plunge orbit, merger, and ringdown. The resultant black hole is estimated to be a Kerr black hole with angular momentum parameter $a \simeq 0.70$. At present, lack of resolution far from the binary prevents an accurate estimate of the energy emitted, though a rough calculation suggests on the order of 5% of the initial rest mass of the system is radiated as gravitational waves during the final orbit and ringdown.

DOI: 10.1103/PhysRevLett.95.121101 PACS numbers: 04.25.Dm, 04.30.Db, 04.70.Bw

I. Introduction.—One of the more pressing, unsolved problems in general relativity today is to understand the structure of spacetime describing the evolution and merger of binary black-hole systems. Binary black holes are thought to exist in the Universe, and the gravitational waves emitted during a merger event are expected to be one of the most promising sources for detection by gravitational wave observatories (LIGO, VIRGO, TAMA, GEO 600, etc.). Detection of such an event would be an unprecedented test of general relativity in the strong-field regime, and could shed light on many issues related to the formation and evolution of black holes and their environments within the Universe. Given the design-goal sensitivities of current gravitational wave detectors, matched filtering may be essential to detect the waves from a merger and extract information about the astrophysical source. During the early stages of a merger, and the later stages of the ringdown, perturbative analytic methods should give a good approximation to the waveform [1,2]; however, during the last several orbits, plunge, and early stages of the ringdown, it is thought a numerical solution of the full problem will be needed to provide an accurate waveform.

Smarr [3] pioneered the numerical study of binary black-hole spacetimes in the mid-1970s, where he considered the head-on collision process in axisymmetry. The full 3D problem has, for many reasons, proven to be a more challenging undertaking, and only recently has progress been made in the ability of numerical codes to evolve binary systems [4–8]. However, until now no code has been able to simulate a nonaxisymmetric collision through coalescence and ringdown. The purpose of this Letter is to report on a recently introduced numerical method based on generalized harmonic coordinates [9] that *can* evolve a binary black hole during these crucial stages of a merger. At a given resolution the code will not run "forever," though convergence tests suggest that with sufficient resolution the code can evolve the system for as long as needed to extract the desired physics from the problem. As an example we describe an evolution that completes approximately one orbit before coalescence, and runs for long enough afterwards to extract a waveform at large distances from the black hole.

The code has several features of note, some or all of which may be responsible for its stability properties: (1) a formulation of the field equations based on harmonic coordinates as first suggested in [10], (2) a discretization scheme where the only evolved quantities are the covariant metric elements, harmonic source, and matter functions, thus minimizing the number of constraint equations that need to be solved [which is similar to the discretization scheme used in [11]], (3) the use of a compactified coordinate system where the outer boundaries of the grid are at spatial infinity, hence the physically correct boundary conditions can be placed there, (4) the use of adaptive mesh refinement to adequately resolve the relevant length scales in the problem, (5) dynamical excision that tracks the motion of the black holes through the grid, (6) addition of numerical dissipation to control high-frequency instabilities, (7) a time slicing that slows down the "collapse" of the lapse that would otherwise occur in pure harmonic time slicing, and (8) the addition of "constraint-damping" terms to the field equations [12,13]. This final element was not present in the version of the code discussed in [9], and though these terms seem to have little effect when black holes are not present in the numerical domain, they have a significant effect on how long a simulation with black holes can run with reasonable accuracy at a given resolution.

An outline of the rest of the Letter is as follows. In Sec. II we give a brief overview of the numerical method, focusing on details not present in [9]. Section III gives results from the simulation of one such orbital configuration. We conclude in Sec. IV with a summary of future work. More details, including convergence tests, the effect of constraint damping, and a thorough description of the initial data calculation, will be presented elsewhere.

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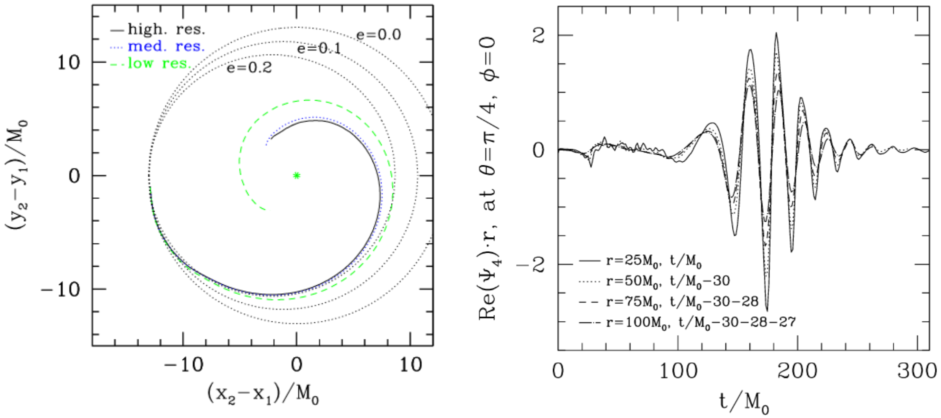

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PHYSICAL REVIEW D VOLUME 14, NUMBER 10 15 NOVEMBER 1976

Collision of two black holes: Theoretical framework*

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(Received 21 June 1976)

Highly nonspherical time-dependent collisions between black holes may be powerful sources of gravitational radiation. We consider various attempts at estimating the efficiency of the generation of radiation by such collisions. To determine the actual efficiency as well as to understand the details of the dynamical coalescence of black-hole event horizons, we have developed a numerical method for solving the Einstein gravitational field equations in these high-velocity strong-field regions. The head-on collision of two nonrotating vacuum black holes is chosen as an example of our technique. We use the geometrodynamical model of a black hole as an Einstein-Rosen bridge. The initial data to be evolved are the time-symmetric conformally flat data discovered by Misner. A new set of spatial coordinates for these data is derived. Then the general space plus time decomposition of Einstein's equations is presented and specialized to the axisymmetric nonrotating case. Details of the evolution will be given in later papers.

I. INTRODUCTION

Because of the rapid increase in the sensitivity of Weber-type resonant-bar gravitational wave antennas,[1] we may soon be capable of detecting[2] bursts of gravitational radiation emitted in our galaxy by collisions of black holes or by the nonspherical final collapse of stars. Furthermore, new techniques, such as Doppler tracking of interplanetary spacecraft,[3] may let us observe these violent events in the nuclei of distant quasars, galaxies, or globular clusters.[4] Therefore, it is important to calculate the details of the expected waves. Unfortunately, it is precisely in these situations of strong gravitational field ($R \sim R_{Sch} = 2GM/c^2$) and fast motion ($v \sim c$) that all known approximation schemes in general relativity fail (see Thorne and Kovács[5] for a review of existing techniques). What is needed is a method for obtaining highly nonspherical, time-dependent, and physically realistic solutions of the full nonlinear Einstein equations which describe gravitational fields produced by collapse or collision.

Since we could not solve this problem analytically, we developed a numerical method using digital computer programs to integrate the axisymmetric, finite-differenced, Einstein equations. (The restriction of axisymmetry was imposed solely because of limited computer memory and speed. Our techniques can be generalized to the generic case of no spatial symmetry.) In this procedure, we start with some initial data specified on a spacelike hypersurface. Using the 3+1 (space+time) decomposition of spacetime[6] we then build up both the four-dimensional coordinate system and the Cauchy evolution of the initial data simultaneously. The general theory of how to build a "good" spacetime coordinate system slice by slice will be described elsewhere.[7] The present paper is the first in a series describing the calculation of a specific spacetime representing the head-on collision of two black holes.

We chose the collision problem as a first "test case" for several reasons. First, the spacetime can be purely vacuum by using Einstein-Rosen bridges[8] to represent the two black holes. This means we can avoid all the messy hydrodynamics which is needed even in spherical stellar collapse.[9] Second, an initial data set is known analytically[10] and has been exhaustively analyzed (see Sec. III for references). Third, the spacetime involves many unexplored aspects of highly nonspherical black holes, generation of gravitational waves in time-dependent strong-field regions, and propagation of the waves outward into the wave zone where they can be measured. Fourth, the results of this calculation will be relevant for astrophysics since they will test whether the high efficiencies (~10%) which are usually assumed[4,11] for conversion of rest mass to gravitational radiation actually occur.

As the computational geometrodynamical tech-

VOLUME 71, NUMBER 18 PHYSICAL REVIEW LETTERS 1 NOVEMBER 1993

Collision of Two Black Holes

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(Received 13 August 1993)

We study the head-on collision of two equal-mass, nonrotating black holes. We consider various cases, from holes surrounded by a common horizon to holes separated by about $20M$, where M is the mass of each hole. The wave forms and energy output are computed, showing that normal modes of the final black hole are clearly excited. We also estimate analytically the total gravitational radiation emitted, considering tidal heating of horizons and other effects. The analytic calculations, perturbation theory, and strong-field, nonlinear numerical calculations agree very well with each other.

PACS numbers: 04.30.+x, 04.20.Jb, 95.30.Sf, 97.60.Lf

The collision of two black holes is considered to be one of the most promising and important astrophysical sources of detectable gravitational radiation in our Universe [1]. Since LIGO [1] and VIRGO [1] are expected to begin taking data during this decade, it is important to perform accurate calculations detailing the shape and strength of the wave forms generated during such events. The information gained from the detected wave forms should allow one to reconstruct the astrophysical parameters of the system, and will provide the first direct and unambiguous evidence for the existence of black holes if the unique signature of the quasinormal modes [2] of the hole is excited.

In this paper we present results of numerical studies of time symmetric, axisymmetric head-on collisions of equal-mass black holes. We have been able to extract the wave forms and the total energy emission resulting from the collision. Analysis of the wave forms reveals clearly for the first time that the quasinormal modes of the final black hole are strongly excited from the collision. Although the term quasinormal mode refers specifically to the response of a black hole to infinitesimal perturbations, we also use it here to describe the finite amplitude oscillations of the final black hole. As we will see in the next section, the finite amplitude oscillations are described very well by the true quasinormal modes of the final black hole. This work extends and refines the early work of Smarr and Eppley [3,4] that suggested that the normal modes of the final hole were excited, but the resolution and wave form extraction techniques available at that time did not permit a clean and unambiguous matching to black hole normal modes. The numerical difficulties inherent in this problem also led to fairly large uncertainties in the total energy radiated (Smarr quotes a probable uncertainty factor of 2 [4]). Because of the importance of this fundamental physical problem, we have revisited this calculation with the benefit of more powerful computers and improved analytic and numerical techniques developed over the intervening 15 years to calculate unambiguous wave forms and energy fluxes resulting from the collision.

The initial data sets that we adopt are analytic solutions originally discovered by Misner [5] and subsequently analyzed and evolved by Smarr and Eppley. They are characterized by a parameter μ determining the mass M of a hole and the proper separation L/M of the two holes (see Table I), and consist of two throats connecting identical asymptotically flat spacetimes [6]. In this paper we apply the code described in Ref. [7] to compute evolutions for a family of Misner spacetimes representing equal-mass black holes colliding from distances of between $4M$ and $20M$. If the throats are close enough together (for $\mu < 1.36$) [8] a common apparent horizon surrounds them both, so that the system really represents a single, highly perturbed black hole. For throats separated by more than about $8M$ we are confident that there is no common event horizon surrounding them, as shown by directly integrating light rays (see also a hoop conjecture argument [9]).

Numerically generated wave form templates will be essential for analysis of data collected by future gravitational wave detectors [1]. The method we use to calculate wave forms is based on the gauge invariant extraction technique developed by Abrahams and Evans [10] and applied in Ref. [11] to black hole spacetimes.

For all of the cases studied in this paper we have extracted both the $l=2$ and $l=4$ wave forms at radii of $30M$, $40M$, $50M$, $60M$, and $70M$. By comparing results

TABLE I. The physical parameters of the six initial data sets studied are summarized. M is the mass parameter defined in the text, L/M is the proper distance between the throats, and we note whether or not one apparent horizon surrounds both holes.

μ	M	L/M	Apparent horizon
1.2	1.85	4.46	global
1.8	0.81	6.76	separate
2.2	0.50	8.92	separate
2.7	0.29	12.7	separate
3.0	0.21	15.8	separate
3.25	0.16	19.1	separate

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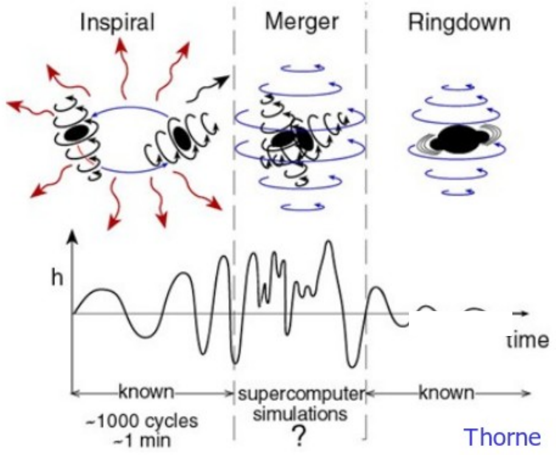

Inspiral
Merger
Ringdown
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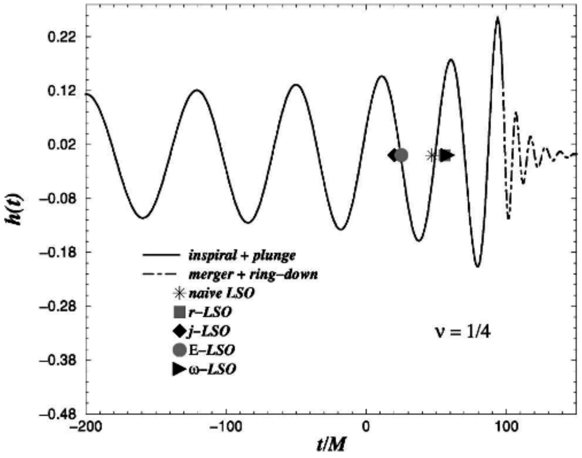

0.22
0.12
0.02
−0.08
−0.18
−0.28
−0.38
−0.48
h(t)
inspiral + plunge
merger + ring–down
naive LSO
r–LSO
j–LSO
E–LSO
ω–LSO
ν = 1/4
−200
−100
0
100
t/M

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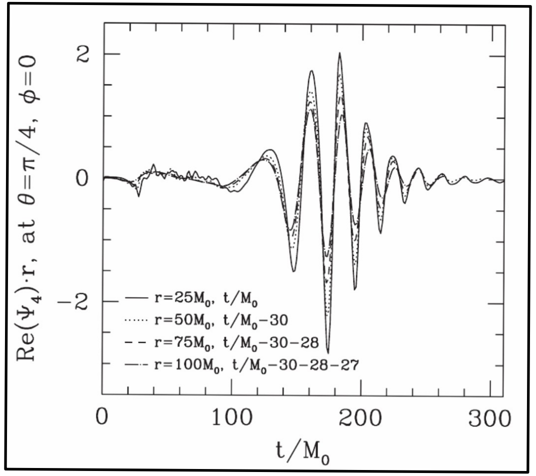
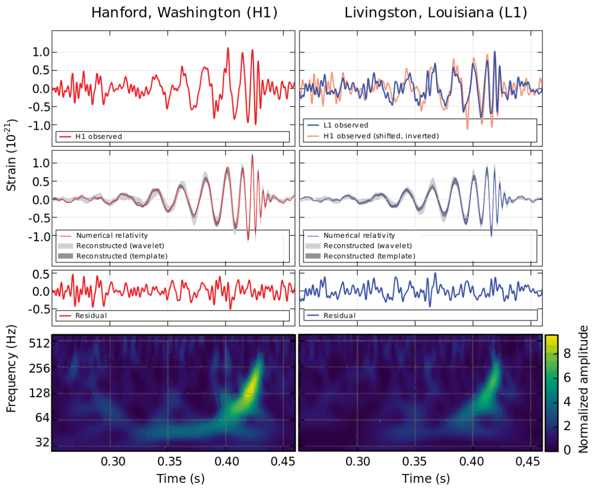


Re(Ψ4)·r, at θ=π/4, ϕ=0
2
0
−2
r=25M0, t/M0
r=50M0, t/M0−30
r=75M0, t/M0−30−28
r=100M0, t/M0−30−28−27
0
100
200
300
t/M0



Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^-21)
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
512
256
128
64
32
Normalized amplitude
Time (s)
0.30
0.35
0.40
0.45

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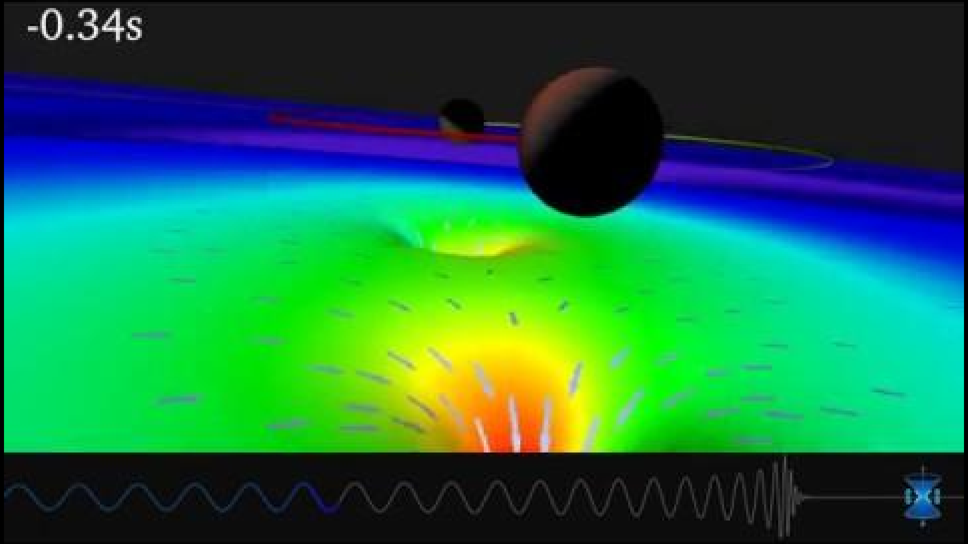

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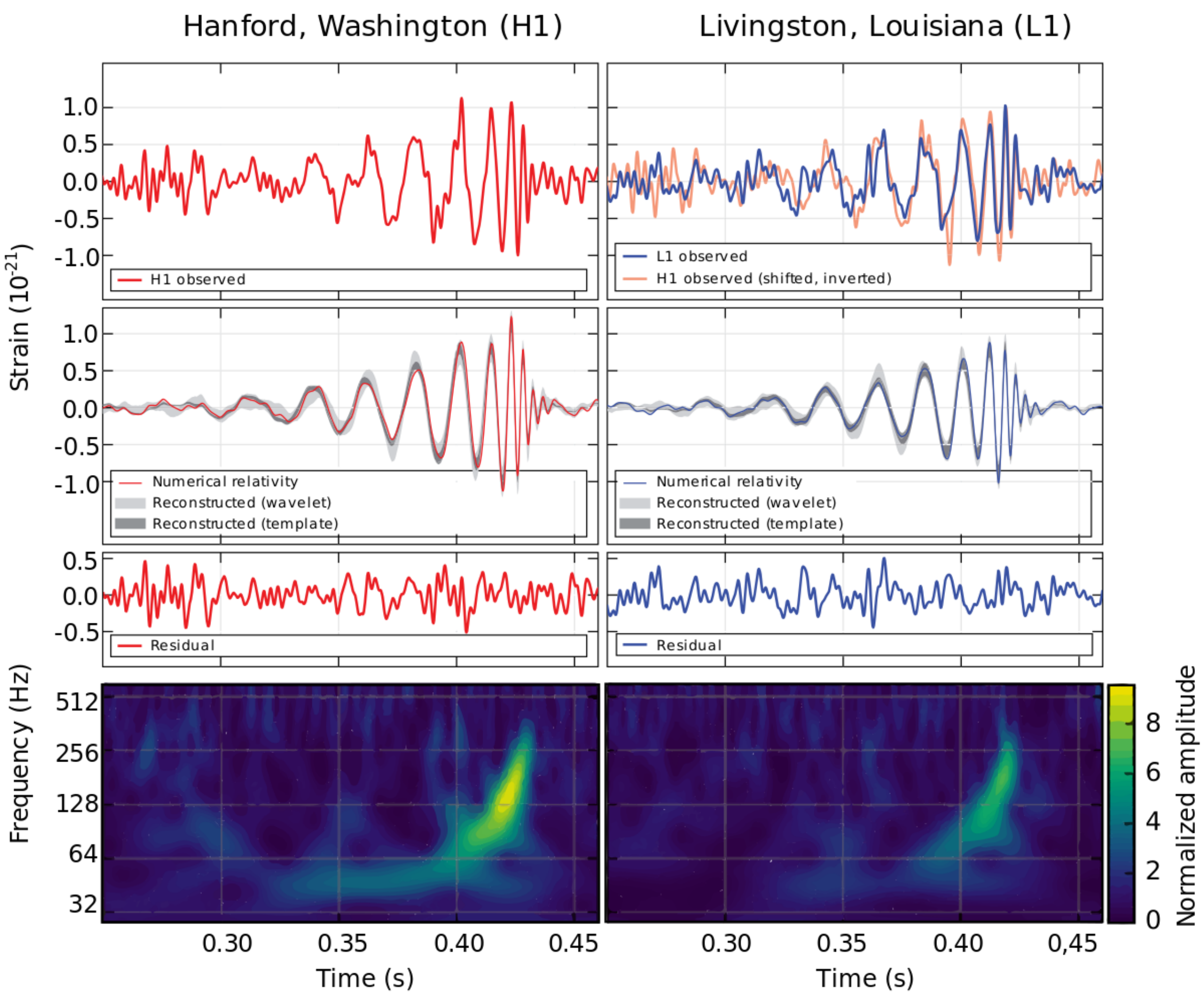

Hanford, Washington (H1)
Livingston, Louisiana (L1)
Strain (10^{-21})
H1 observed
L1 observed
H1 observed (shifted, inverted)
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
Normalized amplitude
Time (s)

$$h_+(t) \simeq \frac{4}{r}\left[\frac{GM_c}{c^2}\right]^{5/3}\left[\frac{\pi f_{\rm gw}(t)}{c}\right]^{2/3}\underbrace{\cos\left[2\pi f_{\rm gw}(t)t\right]}_{:=\cos[\phi_{\rm gw}(t)]}$$

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$$M_c := \mu^{3/5}M^{2/5} = \left(\frac{\mu}{M}\right)^{3/5} M = \nu^{3/5} M$$

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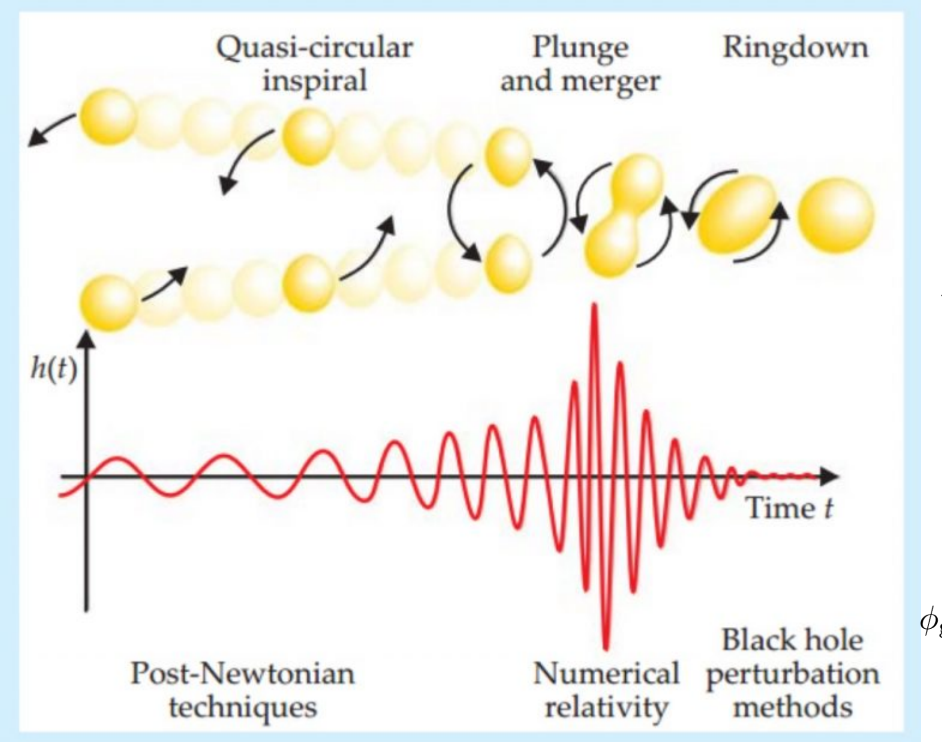
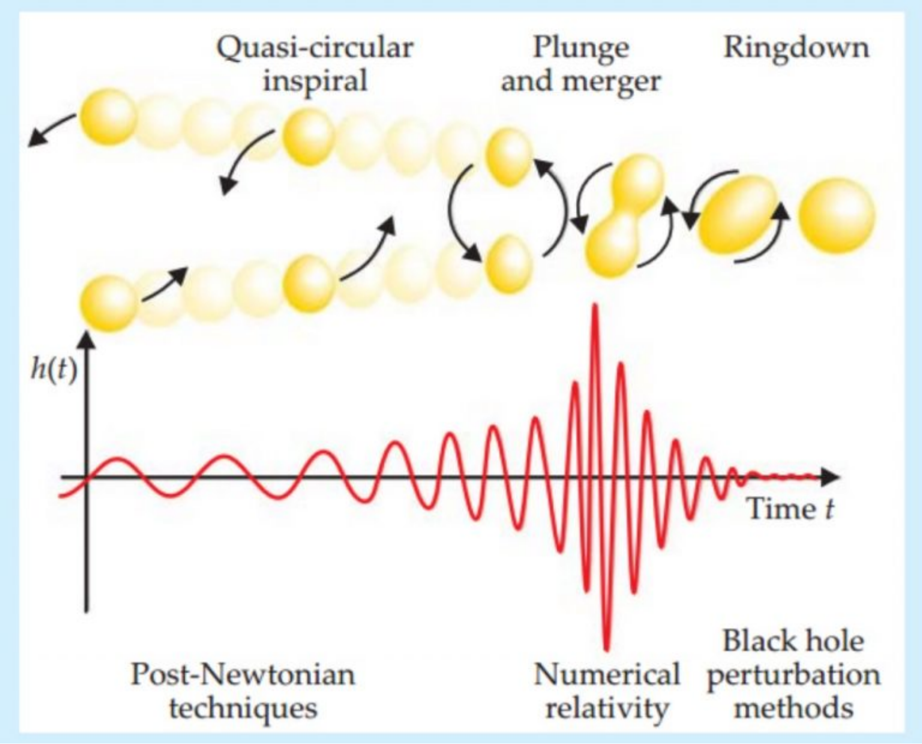
$$h(t) \sim \frac{1}{r}M_c^{5/3} f_{\rm gw}^{2/3}(t) = \nu\frac{1}{r}M^{5/3} f_{\rm gw}^{2/3} = \nu\frac{1}{(r/M)}(M f_{\rm gw}(t))^{2/3}$$

$$\phi_{\rm gw}(t) \sim 2\phi_{\rm orb}(t) = 2M_c^{-5/8}t^{5/8} = 2\nu^{-3/8}\left(\frac{t}{M}\right)^{5/8}$$





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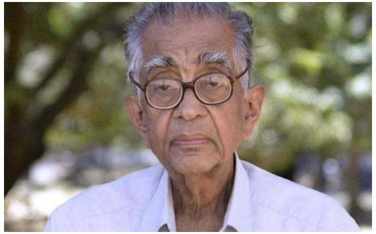
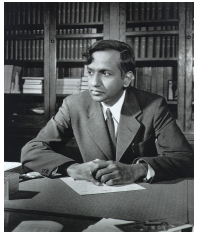

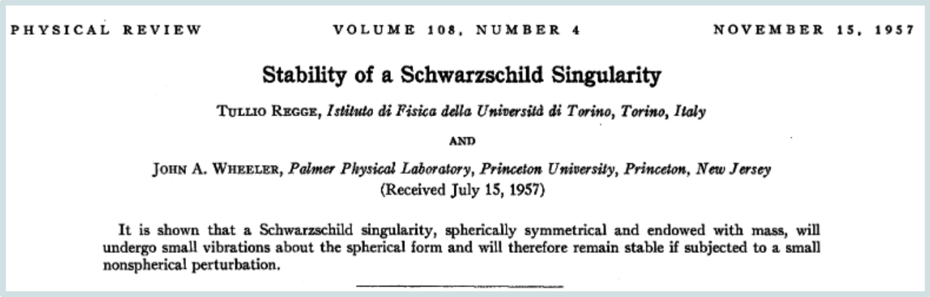
PHYSICAL REVIEW VOLUME 108, NUMBER 4 NOVEMBER 15, 1957

Stability of a Schwarzschild Singularity

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AND

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(Received July 15, 1957)

It is shown that a Schwarzschild singularity, spherically symmetrical and endowed with mass, will undergo small vibrations about the spherical form and will therefore remain stable if subjected to a small nonspherical perturbation.

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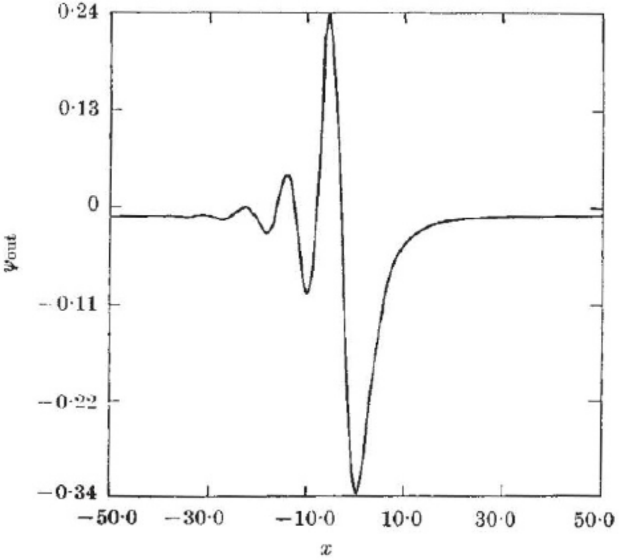

Fig. 3. The outgoing wave packet $\psi_{out}(x)$ at spatial infinity corresponding to the incident Gaussian wave packet $\psi_{in}(x) = e^{-ax^2}$ with $a = 1$.

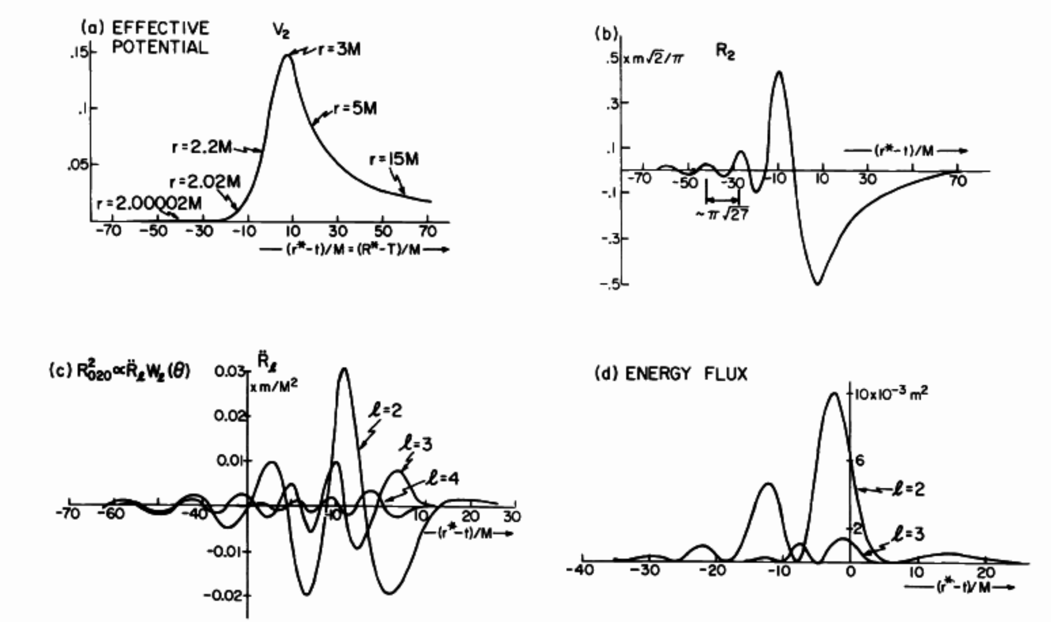

FIG. 1. Asymptotic behavior of the outgoing burst of gravitational radiation compared with the effective potential, as a function of the retarded time $(t - r^*)/M$. (a) Effective potential for $l = 2$ in units of M^2 as a function of the retarded time $(t - r^*)/M = (T - R^*)/M$. For selected points the value of the Schwarzschild coordinate r is also given. (b) Radial dependence of the outgoing field $R_l(r,t)$ as a function of the retarded time for $l = 2$. (c) $\ddot{R}_l(r^*,t)$ factors of the Riemann tensor components (see text) given as a function of the retarded time for $l = 2, 3, 4$. (d) Energy flux integrated over angles for $l = 2, 3$; the contributions of higher l are negligible.

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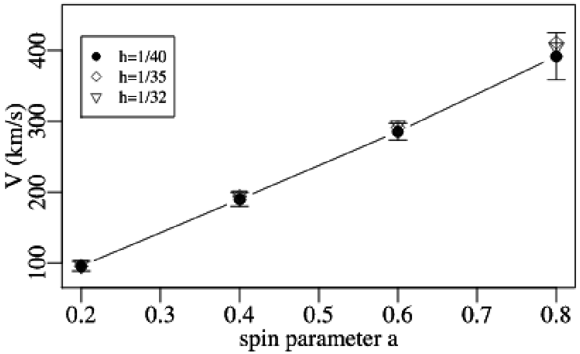

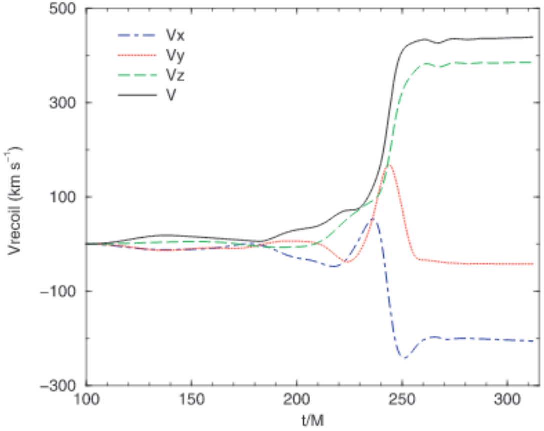

FIG. 1.—Recoil velocities for the SP6 configurations as measured for an observed at $r = 30M$.

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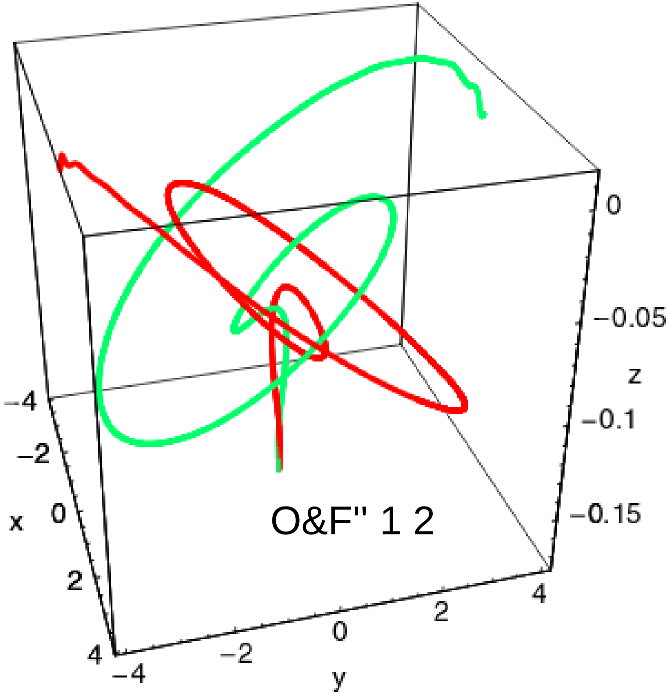

FIG. 3 (color online). Coordinate positions of the black-hole punctures for model MII up to $t = 180$. The black holes move out of the original plane and after merger the final black hole receives a kick in the negative z direction.

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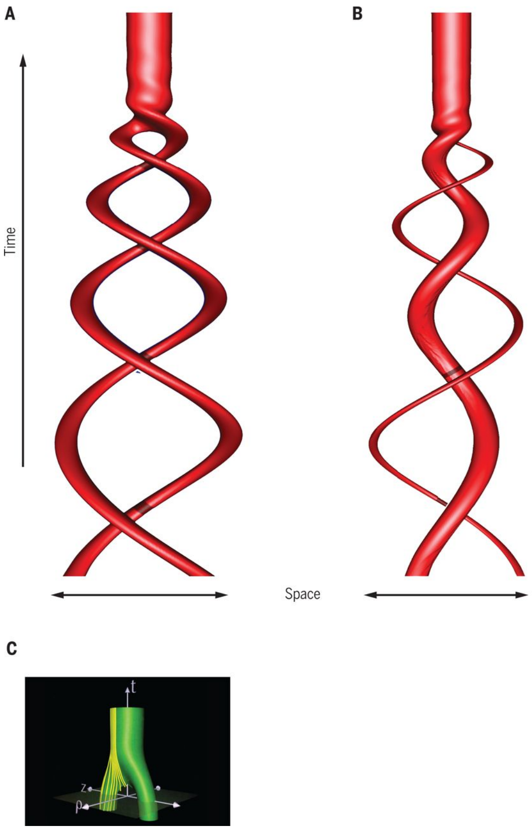

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Simulation of merging binary neutron stars in full general relativity: $\Gamma=2$ case

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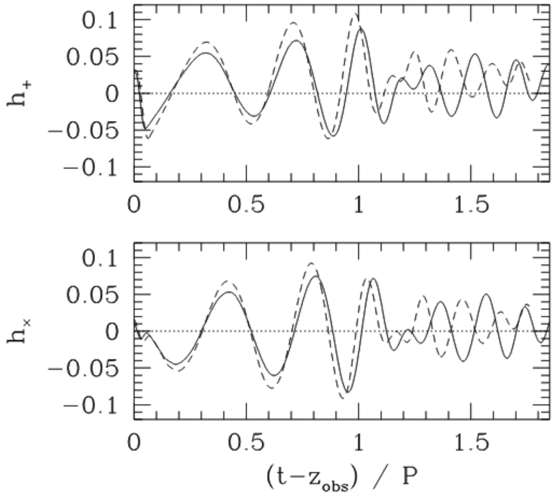
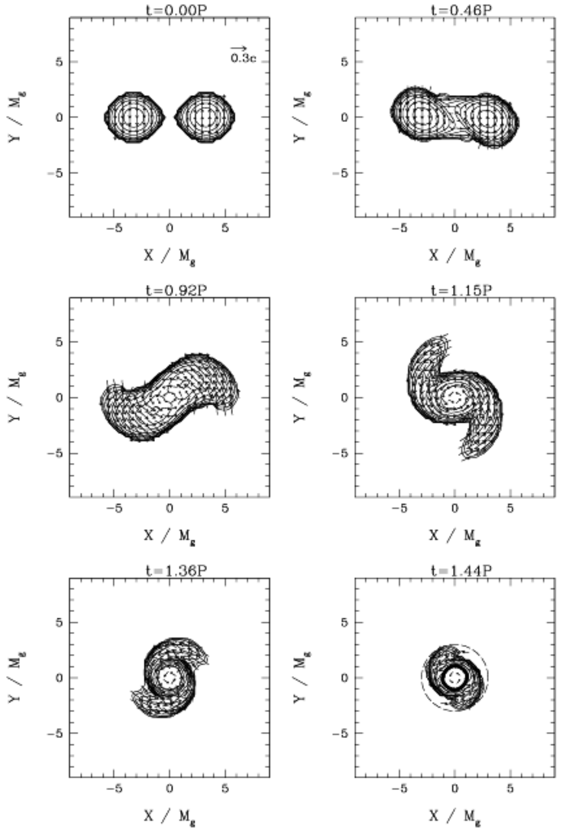
We perform 3D numerical simulations for the merger of equal mass binary neutron stars in full general relativity. We adopt a Γ-law equation of state in the form $P=(\Gamma-1)\rho\varepsilon$ where P, ρ, ε and Γ are the pressure, rest mass density, specific internal energy, and the adiabatic constant with $\Gamma=2$. As initial conditions, we adopt models of corotational and irrotational binary neutron stars in a quasiequilibrium state which are obtained using the conformal flatness approximation for the three geometry as well as the assumption that a helicoidal Killing vector exists. In this paper, we pay particular attention to the final product of the coalescence. We find that the final product depends sensitively on the initial compactness parameter of the neutron stars: In a merger between sufficiently compact neutron stars, a black hole is formed in a dynamical time scale. As the compactness is decreased, the formation time scale becomes longer and longer. It is also found that a differentially rotating massive neutron star is formed instead of a black hole for less compact binary cases, in which the rest mass of each star is less than 70–80 % of the maximum allowed mass of a spherical star. In the case of black hole formation, we roughly evaluate the mass of the disk around the black hole. For the merger of corotational binaries, a disk of mass $\sim 0.05-0.1M_*$ may be formed, where M_* is the total rest mass of the system. On the other hand, for the merger of irrotational binaries, the disk mass appears to be very small: $<0.01M$





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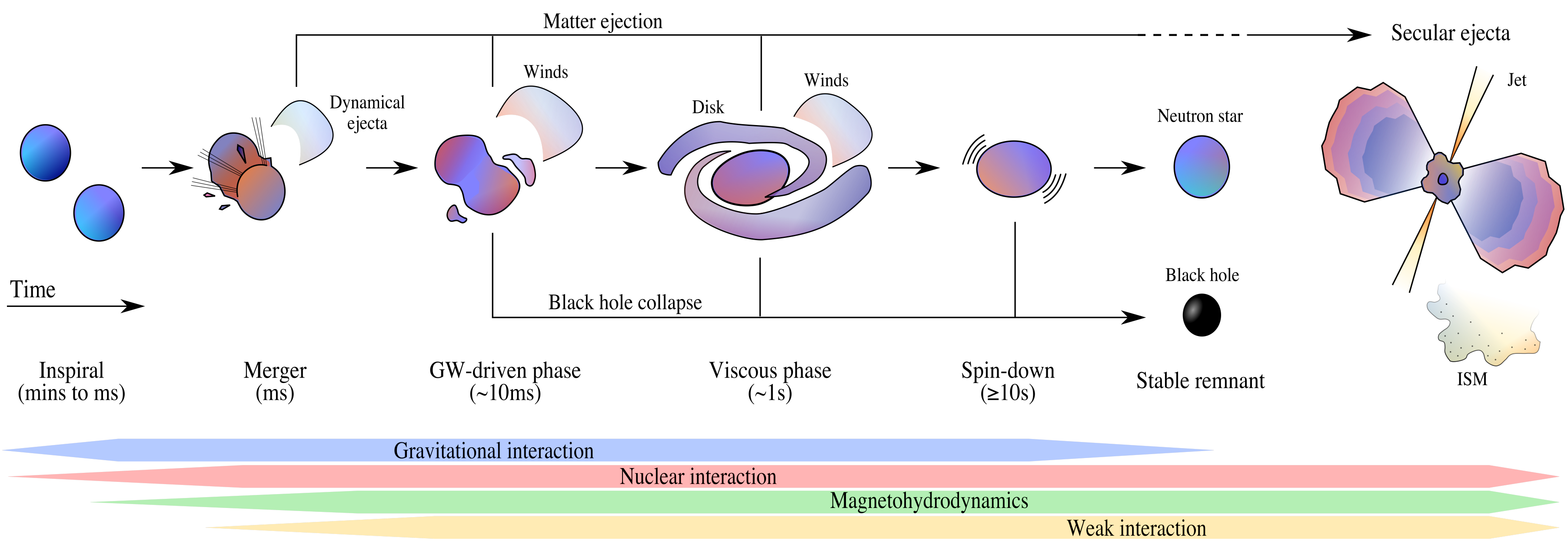

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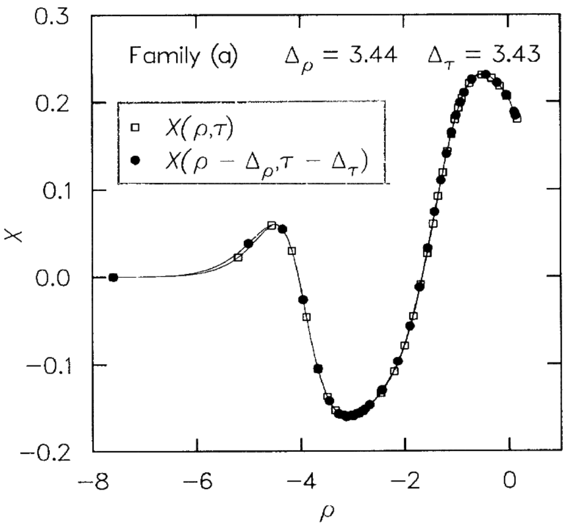

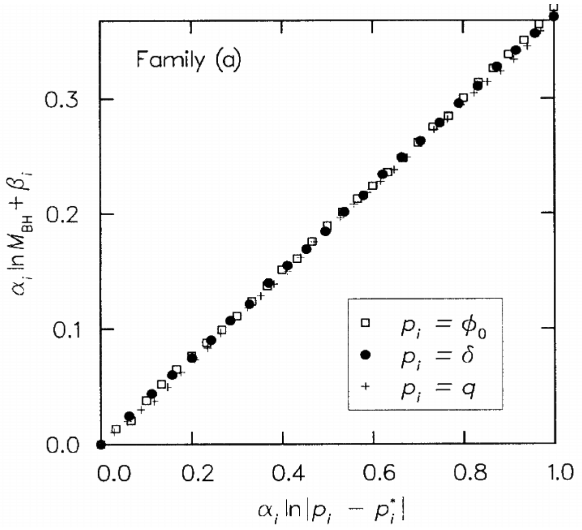

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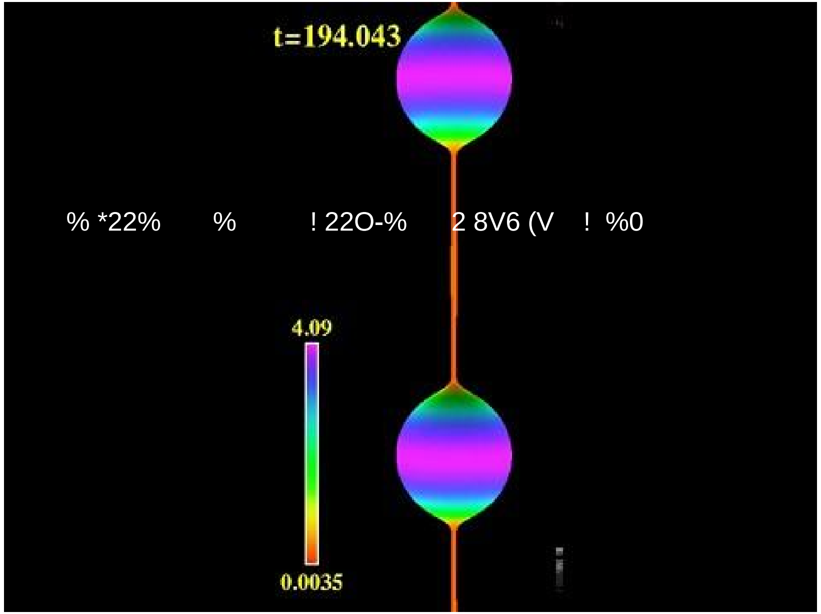

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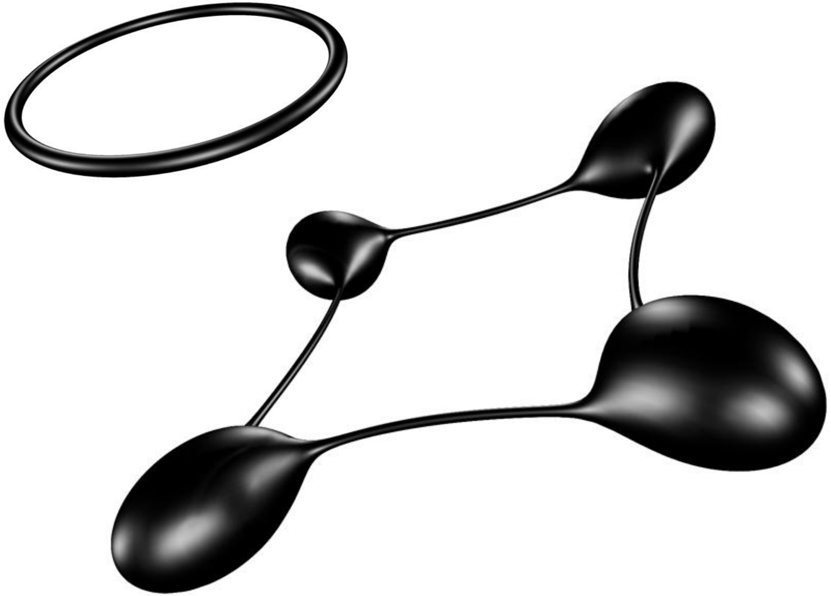

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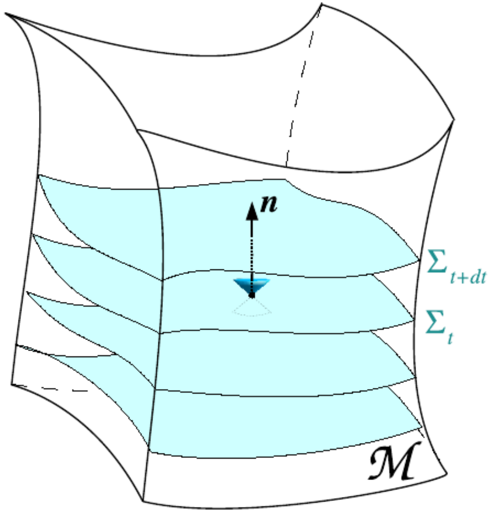

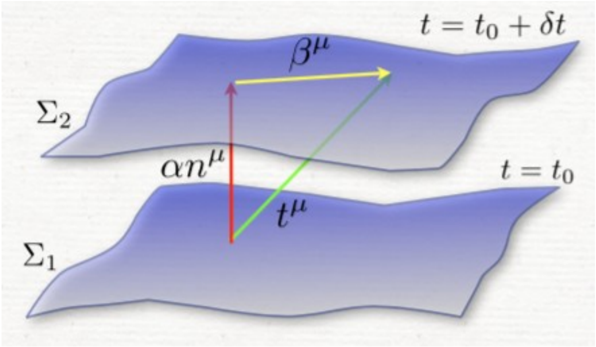

$$(\Sigma_t, \gamma_{ij}) \quad D_i, R_{ij}, \ldots$$

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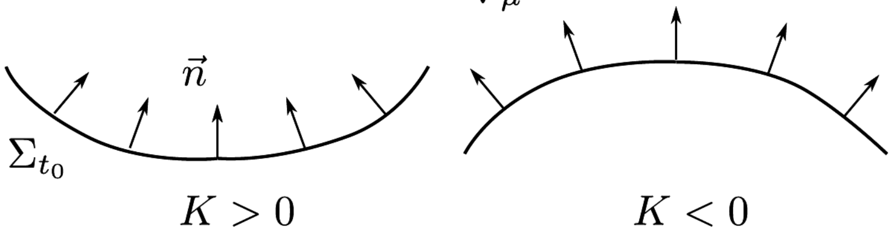
$$K_{\mu\nu} = -\gamma^{\alpha}{}_{\mu} \nabla_{\alpha} n_{\nu}$$

Tangent projection of the gradient of the normal vector

The trace of the extrinsic curvature is associated with the expansion of the world lines of the Eulerian observers:

$$K = -\nabla_{\mu} n^{\mu}$$



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$$\gamma_{ij}\,, K_{ij}\,; \alpha, \beta^i; E, j_i, S_{ij}$$

$$(\partial_t - \mathcal{L}_{\vec{\beta}})\gamma_{ik} = -2\alpha K_{ik}$$ # 2

$$(\partial_t - \mathcal{L}_{\vec{\beta}})K_{ik} = -D_i D_k \alpha + \alpha\left({}^{(3)}R_{ik} - 2K_{ij}K^j{}_k + KK_{ik}\right) - 8\pi\alpha\left(S_{ik} - \frac{1}{2}\gamma_{ik}(S - E)\right)$$

$${}^{(3)}R + K^2 - K_{ik}K^{ik} = 16\pi E$$

$$D_k(K\gamma^k{}_i - K^k{}_i) = 8\pi j_i$$

$S_{ik} = \gamma^\mu{}_i \gamma^\nu{}_k T_{\mu\nu}$ $S = S^i{}_i$ $j_i = -\gamma^\mu{}_i n^\nu T_{\mu\nu}$ $E = T^{\mu\nu} n_\mu n_\nu$

$$\nabla_\mu T^{\nu\mu} = 0 \rightarrow \partial_t q_\mu + \partial_i F^i_\mu(q) = s_\mu$$: 2

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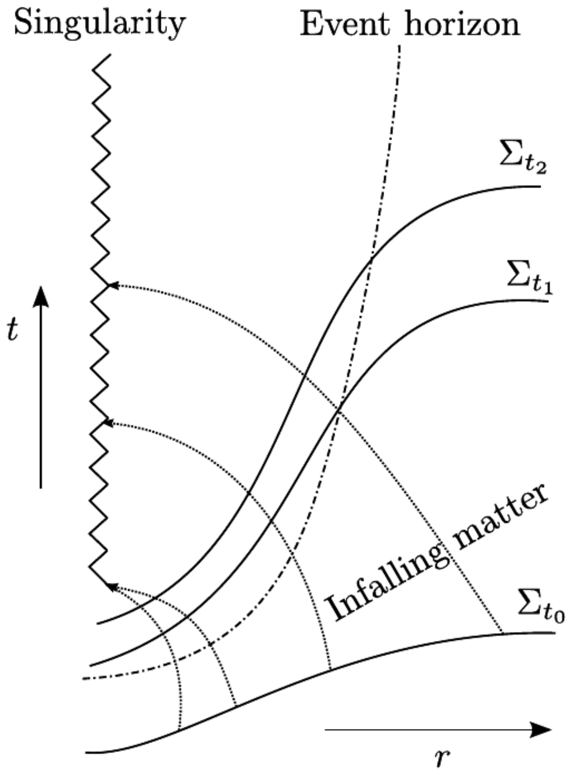

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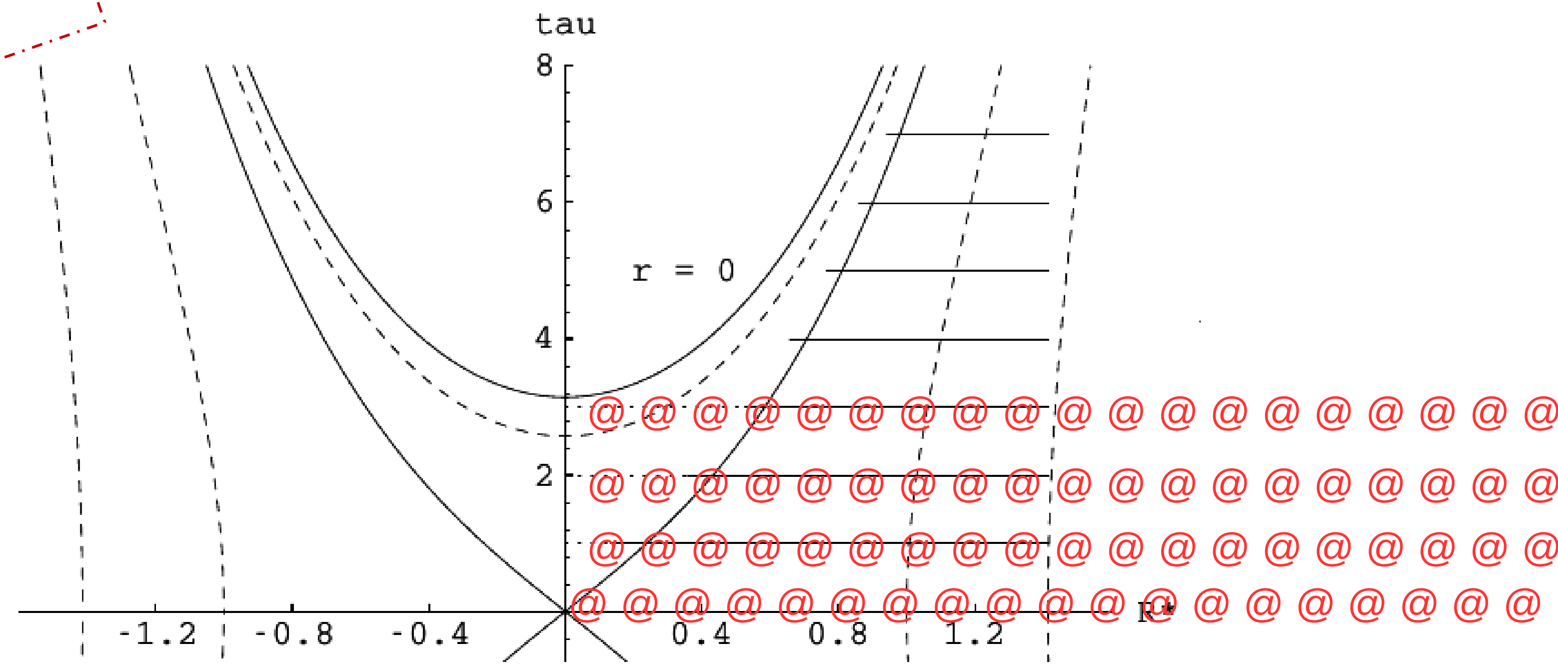

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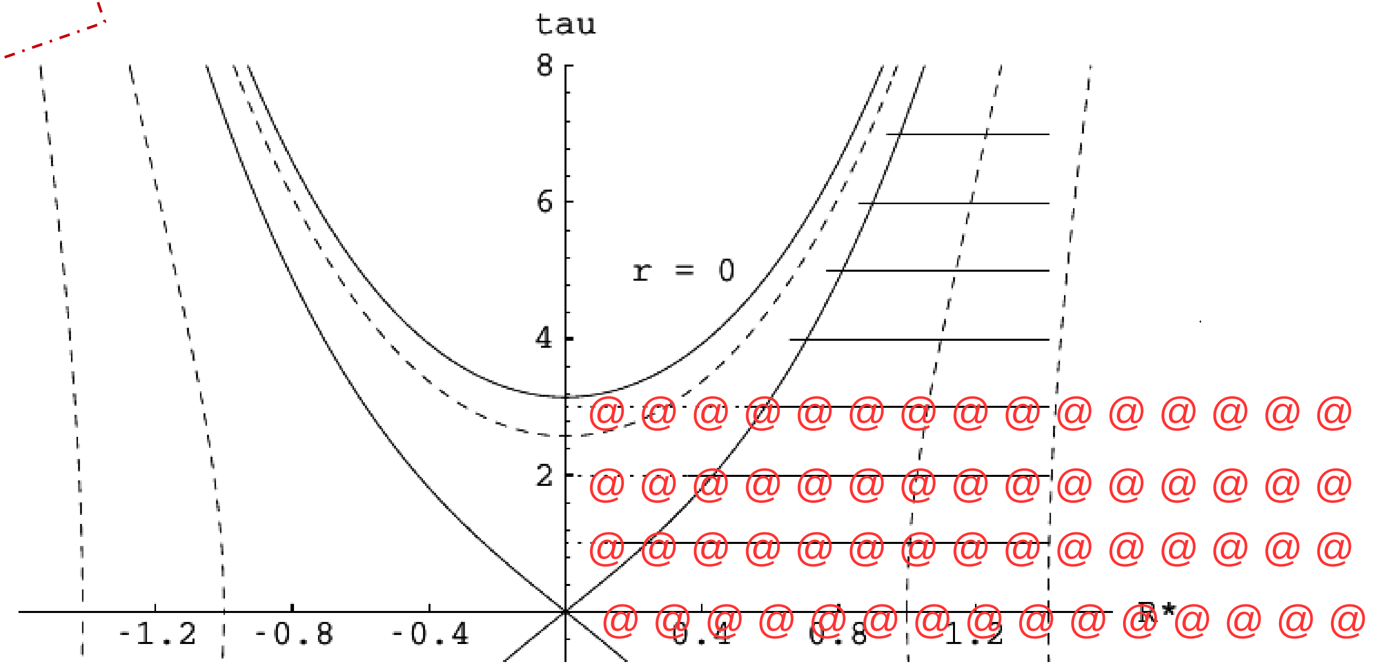

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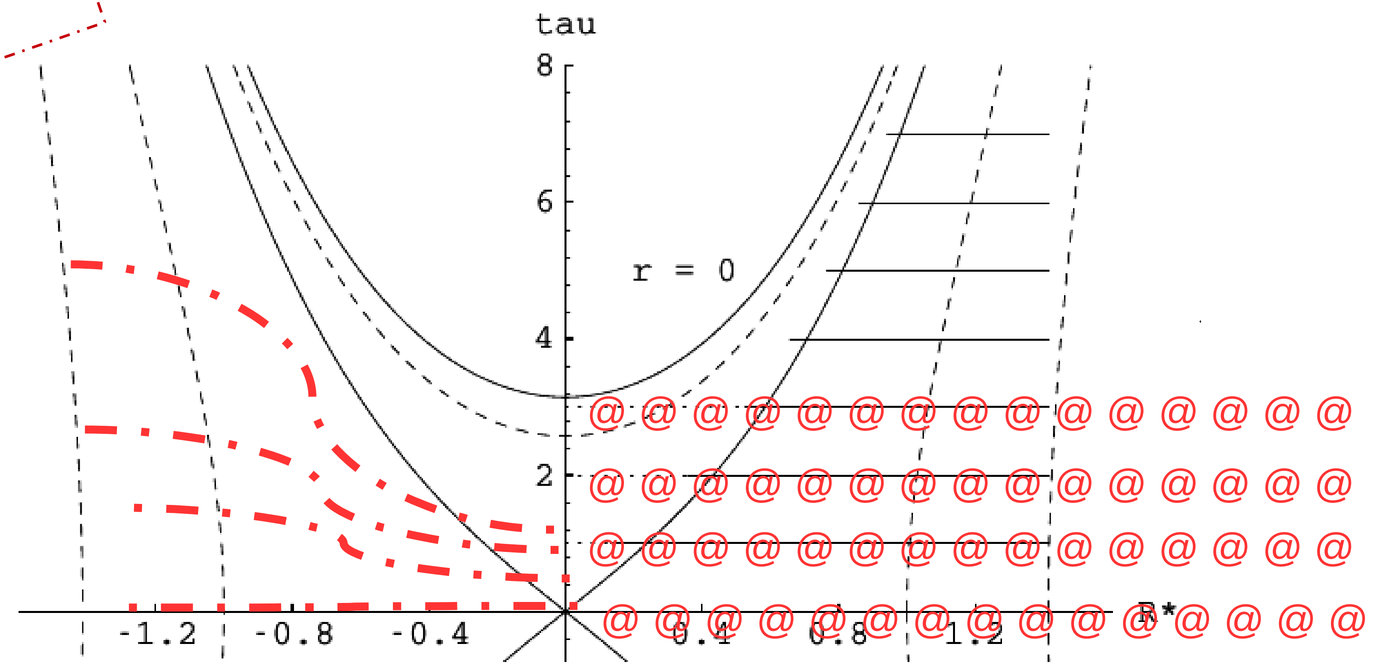

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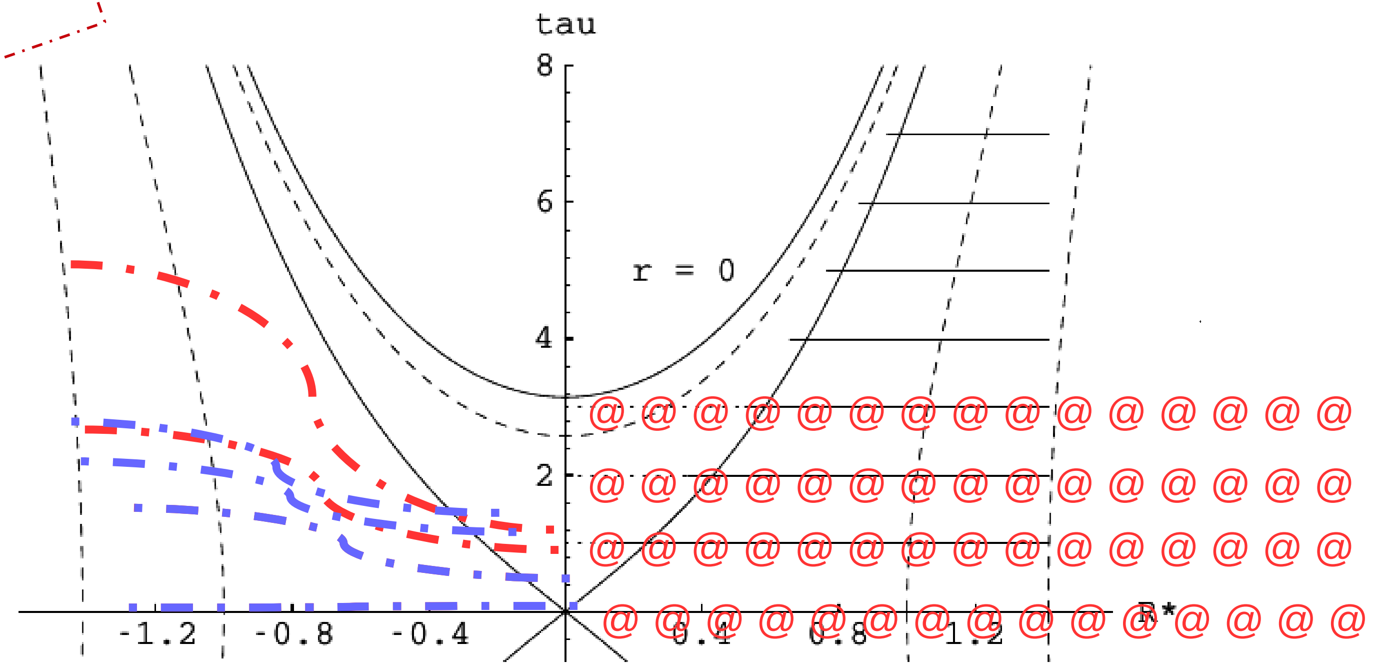

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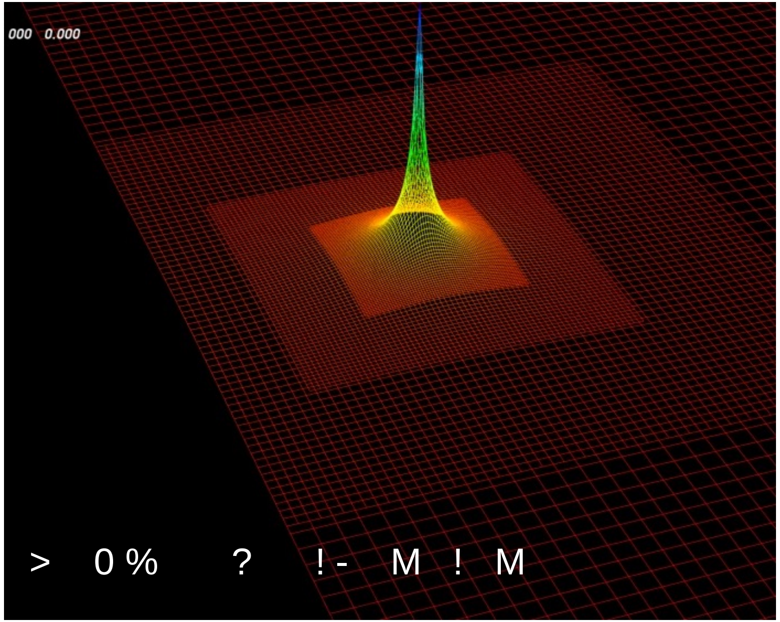

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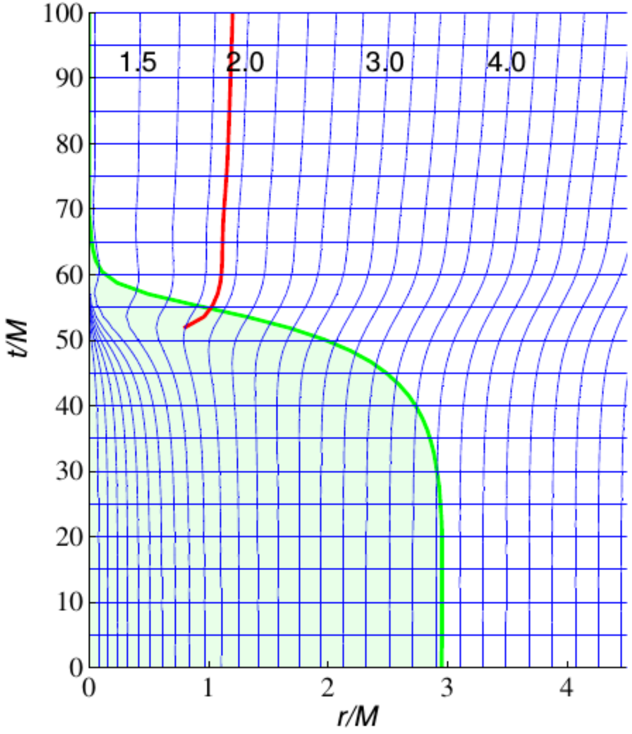

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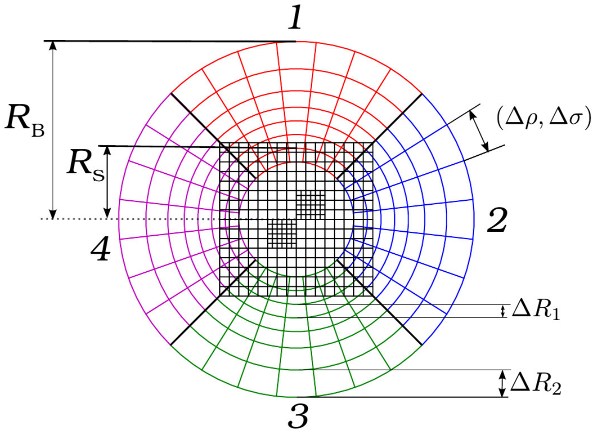

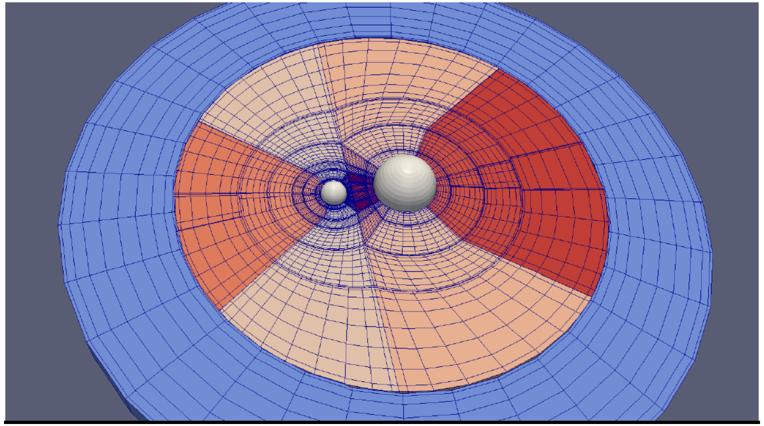

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