Effective Field Theory approaches to Gravity

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GGI Training Lectures on
Gravitational scattering, inspiral, and radiation
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What is an EFT?

Top down

$$e^{\frac{i}{\hbar} S_W[\text{Light}]} = \int D\text{Heavy} \quad e^{\frac{i}{\hbar} S_{UV}[\text{Light,Heavy}]}$$

Bottom Up

Construct $S_W[Light]$ writing down every local operator consistent with symmetries of low energy theory, suppressed by cutoff scale to appropriate power

$$\Lambda^4 F\left(\frac{Boson}{\Lambda}, \frac{Fermion}{\Lambda^{3/2}}, \frac{\partial_\mu}{\Lambda}\right)$$

Tree level EFTs

Top down

$$e^{\frac{i}{\hbar} S_W[\text{Light}]} = \int D\text{Heavy} \quad e^{\frac{i}{\hbar} S_{UV}[\text{Light,Heavy}]}$$

at tree level ...

$$S_W^{\text{Tree}}[Light] = S_{UV}^{\text{Tree}}[Light, H[Light]]$$

where ...

$$\frac{\delta}{\delta Heavy} S_{UV}^{\text{Tree}}[Light, Heavy]\Big|_{Heavy=H[Light]} = 0$$

Heavy loop corrections

$$S_W[Light] = S_W^{\text{Tree}}[Light] + \Delta S_W[Light]$$

Split ..

$$Heavy = H[Light] + h$$

Heavy Loop contributions are

$$e^{i\Delta S_W[Light]} = \int Dh\, e^{iS_{\text{UV}}[Light, H[Light]+h] - S_{\text{UV}}[Light, H[Light]]}$$

$$\downarrow$$

$$\Delta S_W = \frac{1}{2} i \,\text{Tr}\ln\Big[-i \frac{\delta^2 S_{UV}}{\delta h(x)\delta h(y)}[Light, H[Light]]\Big] + \dots$$

Irrelevant Operators

- Generic operators in LEEFT are irrelevant and contain higher derivatives
- Neither is fundamentally a problem
- Naive additional states from higher derivatives have masses above cutoff of EFT, therefore do not exist! (no S-matrix)

E.g.

$$S = \int d^4x \frac{1}{2}\phi[\Box - \frac{1}{\Lambda^2}\Box^2 + \cdots - \frac{1}{4!}\lambda\phi^4]$$

sensitive to truncation!!!
do not take seriously!

$$G(k) \sim \frac{1}{k^2 + k^4/\Lambda^2 + \ldots} \sim \frac{1}{k^2} - \frac{1}{k^2 + \Lambda^2} + \ldots$$

Redundant Operators and Field redefinitions

- Frequently useful to perform field redefinitions of Light fields - scattering amplitudes invariant under invertible, local field redefinitions (that do not change asymptotic states)

E.g. $$Light' = Light + \frac{a}{\Lambda^5}(\Box Light)^2 + \ldots$$

$$Light = Light' - \frac{a}{\Lambda^5}(\Box Light')^2 + \ldots$$

invertible

- Operators which can be removed with a field redefinition are called redundant operators (N.B. Often carelessly and incorrectly referred to as using equations of motion)

UV MODIFICATIONS

Type I: UV Modifications:
eg. Quantum Gravity, string theory, extra dimensions, branes, supergravity

At energies well below the scale of new physics Λ , gravitational effects are well incorporated in the language of Effective Field Theories

Beyond Einstein Theories of Gravity

GR itself should be understood as an EFT with a cutoff at most at Planck scale physics - no problem quantizing gravity as a LEEFT,

Perturbative scattering breaks unitarity at Planck scale, known irrelevant operators can renormalize UV divergence

see e.g. reviews by Donoghue, Burgess

$$S = \int d^4x \sqrt{-g} \left[\frac{M_P^2}{2} R + \alpha R^2 + \beta R_{\mu\nu} R^{\mu\nu} + \dots + M_P^4 \left(\frac{\nabla}{M_P} \right)^{2N} \left(\frac{\text{Riemann}}{M_P^2} \right)^M \right]$$

Addition of Higher Dimension, (generally higher derivative operators), **no failure of well-posedness/ghosts** etc as all such operators should be treated perturbatively (rules of EFT)

GR+Light field EFT

Straight forward to extend to include low energy 'light' matter

$$e^{\frac{i}{\hbar} S_W[g_{\mu\nu}, \text{Light}]} = \int D\text{Heavy}\, e^{\frac{i}{\hbar} S_{UV}[g_{\mu\nu}, \text{Light}, \text{Heavy}]}$$

Include all local interaction consistent with symmetries:

$$S_W = \int d^4x \left[\frac{M_P^2}{2} R + \mathcal{L}(Light) + \ldots \Lambda^4 \left(\frac{\nabla}{\Lambda}\right)^{2N} \left(\frac{Riemann}{\Lambda^2}\right)^M \left(\frac{Light_B}{\Lambda}\right)^P \left(\frac{Light_F}{\Lambda^{3/2}}\right)^Q \right]$$

For example, we have no trouble computing loop corrections to scalar and tensor fluctuations produced during inflation

Example: Propagation of Gravitational Waves

Can the propagation of gravitational waves be different (e.g. around FRW?)

UV Completion/Quantum Effects: Yes!

Tree Level Effects: Addition of massive higher spin states with $s \geq 2$ will modify propagation provided their is kinetic mixing

Loop Level Effects: Loop effects from particles with all spin including $s < 2$ modify propagation speed

Curvature Squared Corrections

Generic EFT of gravity will include higher curvature operators. Leading curvature ones in four dimensions can be packaged into

$$S_W = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + C_{R^2}R^2 + C_{W^2}W^2_{\mu\nu\rho\sigma} + C_{GB}GB + \mathcal{L}_M + \dots\right]$$

only affects scalar sector

affects tensor (gravitational wave) sector

Curvature Squared Corrections

$$S_W = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + C_{R^2}R^2 + C_{W^2}W^2_{\mu\nu\rho\sigma} + C_{GB}GB + \mathcal{L}_M + \ldots\right]$$

Things to know:

- Can be removed with field redefinitions BUT will modify couplings to matter!!
- Loop contributions log divergent (in 4 dimensions), so values dependent on unknown UV boundary condition

‘Einstein/tensor frame picture’

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + C_{R^2}R^2 + C_{W^2}W^2_{\mu\nu\alpha\beta} + C_{\rm GB}{\rm GB} + \mathcal{L}_{\rm matter}(g,\psi)\right],$$

$$= \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + (C_{R^2} - \frac{2}{3}C_{W^2})R^2 + 2C_{W^2}R^2_{\mu\nu} + (C_{W^2} + C_{\rm GB}){\rm GB} + \mathcal{L}_{\rm matter}(g,\psi)\right]$$

Redundant

Perform the field redefinition:

$$g_{\mu\nu}^{\rm tensor} = g_{\mu\nu}^{\rm matter} - \frac{4C_{W^2}}{M_{\rm Pl}^2}(G_{\mu\nu}^{\rm matter} + \frac{1}{M_{\rm Pl}^2}T_{\mu\nu}) - \frac{2C_{R^2} - \frac{4}{3}C_{W^2}}{M_{\rm Pl}^2}(-R^{\rm matter} + \frac{1}{M_{\rm Pl}^2}T)g_{\mu\nu}^{\rm matter} + \ldots$$

‘Einstein/tensor frame picture’

Perform the field redefinition:

$$g_{\mu\nu}^{\text{tensor}} = g_{\mu\nu}^{\text{matter}} - \frac{4C_{W^2}}{M_{\text{Pl}}^2}\left(G_{\mu\nu}^{\text{matter}} + \frac{1}{M_{\text{Pl}}^2}T_{\mu\nu}\right) - \frac{2C_{R^2} - \frac{4}{3}C_{W^2}}{M_{\text{Pl}}^2}\left(-R^{\text{matter}} + \frac{1}{M_{\text{Pl}}^2}T\right)g_{\mu\nu}^{\text{matter}} + \dots$$

Field redefined Lagrangian

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_{\text{Pl}}^2}{2}R + C_{\text{GB}}\text{GB} + \mathcal{L}_{\text{matter}} + \frac{2C_{W^2}}{M_{\text{Pl}}^4}T_{\mu\nu}T^{\mu\nu} + \frac{(C_{R^2} - \frac{2}{3}C_{W^2})}{M_{\text{Pl}}^4}T^2 + \cdots\right]$$

Example: Gravitational Waves on FLRW

$$S_W = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + C_{R^2}R^2 + C_{W^2}W^2_{\mu\nu\rho\sigma} + C_{GB}GB + \mathcal{L}_M + \ldots\right]$$

Assume that matter is minimal (for simplicity)

Perturbing around a cosmological background:

$$ds^2 = a(\eta)^2(-d\eta^2 + d\vec{x}^2) + a(\eta)h_{ij}dx^i dx^j$$

$$[\Box + a''/a]h + \frac{4}{M_P^2}\left(\frac{1}{a^2}g_1\Box^2 + \frac{1}{a}g_3\Box\partial_\eta + g_4\Box + ag_5\partial_\eta + g_7\nabla^2 + g_8a^2\right)h + \mathcal{O}(1/M_P^4) = 0$$

$$\Box = -\partial_\eta^2 + \nabla^2$$

Equation for Gravitational Waves

$$S_W = \int d^4x\sqrt{-g}\left[\frac{M_P^2}{2}R + C_{R^2}R^2 + C_{W^2}W^2_{\mu\nu\rho\sigma} + C_{GB}GB + \mathcal{L}_M + \ldots\right]$$

$$ds^2 = a(\eta)^2(-d\eta^2 + d\vec{x}^2) + a(\eta)h_{ij}dx^i dx^j$$

Field redefining/Rearranging to remove higher time derivatives

$$\frac{1}{a^2}\left[-\partial_\eta^2 + \left(1 - \frac{16C_{W^2}\dot{H}}{M_{\rm Pl}^2}\right)\nabla^2 + \frac{4a\tilde{g}_5}{M_{\rm Pl}^2}\partial_\eta\right]h = m_0^2 h$$

‘Einstein/tensor frame picture’

$$\mathcal{L} = \sqrt{-g}\left[\frac{M_{\rm Pl}^2}{2}R + C_{\rm GB}{\rm GB} + \mathcal{L}_{\rm matter} + \frac{2C_{W^2}}{M_{\rm Pl}^4}T_{\mu\nu}T^{\mu\nu} + \frac{(C_{R^2} - \frac{2}{3}C_{W^2})}{M_{\rm Pl}^4}T^2 + \cdots\right]$$

Gravitational Waves Luminal:

$$c_s^2(tensor) = 1$$

Matter Fluctuations Subluminal if NEC satisfied and $C_{W^2} > 0$

$$c_s^2(matter) = 1 - \frac{16C_{W^2}(-\dot{H})}{M_{Pl}^2} + \mathcal{O}(1/M_{Pl}^4)$$

$\dfrac{c_s^2(\text{tensor})}{c_s^2(\text{matter})}$ is invariant under field redefinitions

Curvature Cubed Corrections

Even if we artificially tune curvature squared terms to zero

Finite Curvature Cubed corrections from lowest mass particles intgreated out in deriving EFT

$$\Gamma^{(1-\text{loop})}_{\text{dim}-6} = \frac{1}{12(2\pi)^4}\sqrt{-g} \sum_i \frac{1}{M_i^2}\Big[d_1^{(s_i)} R\Box R + d_2^{(s_i)} R_{\mu\nu}\Box R^{\mu\nu} + d_3^{(s_i)} R^3 + d_4^{(s_i)} R R^2_{\mu\nu} \\ + d_5^{(s_i)} R R^2_{\mu\nu\alpha\beta} + d_6^{(s_i)} R^3_{\mu\nu} + d_7^{(s_i)} R^{\mu\nu} R^{\alpha\beta} R_{\mu\alpha\nu\beta} + d_8^{(s_i)} R^{\mu\nu} R_{\mu\alpha\beta\gamma} R_\nu{}^{\alpha\beta\gamma} \\ + d_9^{(s_i)} R^{\mu\nu\alpha\beta} R_{\mu\nu\gamma\sigma} R_{\alpha\beta}{}^{\gamma\sigma} + d_{10}^{(s_i)} R^\mu{}_\alpha{}^\nu{}_\beta R^\alpha{}_\gamma{}^\beta{}_\sigma R^\gamma{}_\mu{}^\sigma{}_\nu\Big], \tag{5.1}$$

Pure Riemann terms cannot be removed with field redefinition

Scales

$$S = M_{\rm Planck}^2 \int d^4x \sqrt{-g}\left[\frac{1}{2}R + \frac{a}{\Lambda^2}R^2 + \frac{b}{\Lambda^2}R_{\mu\nu}^2 + \cdots + \frac{c}{\Lambda^4}R_{abcd}R^{cd}_{ef}R^{efab} + \cdots + \mathcal{L}_{\rm matter}\right] + \frac{d}{\Lambda^6}(R_{abcd}R^{abcd})^2 + \ldots$$

$$S_W = \int d^4x \left[\frac{M_P^2}{2}R + \mathcal{L}(Light) + \ldots \Lambda^4 \left(\frac{\nabla}{\Lambda}\right)^{2N} \left(\frac{Riemann}{\Lambda^2}\right)^M \left(\frac{Light_B}{\Lambda}\right)^P \left(\frac{Light_F}{\Lambda^{3/2}}\right)^Q\right]$$

Cutoff scale is scale of new gravitational physics - e.g. higher spin string states/KK modes

Except in early universe, realistic curvatures are far too small for these effects to be observable

To be interesting phenomenologically
- made artificially large... (ultimately inconsistent)
- or search elsewhere ... (IR modifications)

IR MODIFICATIONS

Type 2: IR Modifications:

Theorem: General Relativity is the Unique local and Lorentz invariant theory describing an interacting single massless spin two particle that couples
to matter

Weinberg, Deser, Wald, Feynman, ...

Locality

Massless ? Lorentz Invariant

Single Spin 2

IR extensions of gravity

If we want to preserve locality and Lorentz invariance

Extension or modification of gravity will either

I. Include new propagating degrees of freedom

E.g. spin 0 (Brans Dicke/f(R)/Galileon/Horndeski ....),
spin 2 (extra dimensions/Kaluza-Klein)

II. Render spin-2 graviton effectively massive (soft or hard)

Massive gravity/DGP/Cascading gravity/warped massive gravity

Explosion of models beyond GR+SM+standard extensions

Many models which attempt to solve various hierarchy problem or Dark Energy introduce new physics at lower scales

(eg. DBI, K-inflation, Brane-world models, DGP, Chameleon, Symmetron, ghost condensate, Massive Gravity, Galileon, Generalized galileon, Horndeski, beyond Horndeski, beyond beyond Horndeski, superfluid dark matter ...)

EFTs with cutoff typically lower than the Planck scale

e.g. even Higgs inflation breaks down at a scale parameterically well below Planck scale since $\xi R H^2$ is non-renormalizable

Explosion of models beyond GR+SM+standard extensions

Many models are non-traditional, in the sense that naive non-renormalizable operators play a significant role:

$$\mathcal{L} \sim \Lambda^4 \frac{\nabla^N \phi^M}{\Lambda^{N+M}}$$

Non-renormalizable/irrelevant if $N + M > 4$

Despite large irrelevant operators, EFT for fluctuations remains under control!

Poster child example 1: DBI

Example of P(X) model

DBI $$L = -\Lambda^4\sqrt{1+(\partial\phi)^2/\Lambda^2}$$

admits a weakly coupled UV completion by interpreting as a probe brane in an extra dimension

Poincare invariance in 5d implies global symmetry for DBI in 4d: $$\phi \to \phi + c + v_\mu\left(x^\mu + \phi\partial^\mu\phi\right)$$

$(\partial\phi)^2 \sim \Lambda^4$ as long as $\partial(\partial\phi)^2 \ll \Lambda^5$

Despite large irrelevant operators,
EFT for fluctuations remains under control!

Silverstein, Tong, PRD70, 2004
Alishahiha, Silverstein, Tong, PRD70, 2004

Poster child example 2: Galileon

Galileon $$L = -\frac{1}{2}(\partial\phi)^2 + \frac{1}{\Lambda^3}\Box\phi(\partial\phi)^2 + \dots$$

Galileon Symmetry $$\phi \to \phi + c + v_\mu x^\mu$$

Slightly optimistically

$\partial\partial\phi \sim \Lambda^3$ as long as $\partial\partial\partial\phi \ll \Lambda^4$

Very Optimistically in Vainshtein region ....

$Z \sim \frac{\partial\partial\phi}{\Lambda^3} \ll 1$ provided $\partial \ll \sqrt{Z}\Lambda$

EFTS FOR BROKEN SYMMETRIES

Breaking Symmetries

In SM, Electroweak symmetry
is spontaneously broken by the VEV of the Higgs field

$$SU(2) \times U(1)_Y \rightarrow U(1)_{\mathrm{EM}}$$

Result, W and Z bosons become massive

Goldstone Equivalence Theorem

At high energies, scattering of additional longitudinal modes of massive boson determined by Goldstone/Stuckelberg EFT

When symmetries are broken, frequently easier to work with EFT for broken symmetry

e.g. Abelian Higgs

$$D_\mu\Phi = \partial_\mu\Phi - iqA_\mu\Phi \qquad \Phi \to e^{iq\chi}\Phi \qquad A_\mu \to A_\mu + \partial_\mu\chi$$
$$D_\mu\Phi \to e^{iq\chi}D_\mu\Phi$$

Higgs vev

Higgs Boson

Goldstone/ Stuckelberg field

$$\Phi = (v+\rho)e^{i\pi}$$

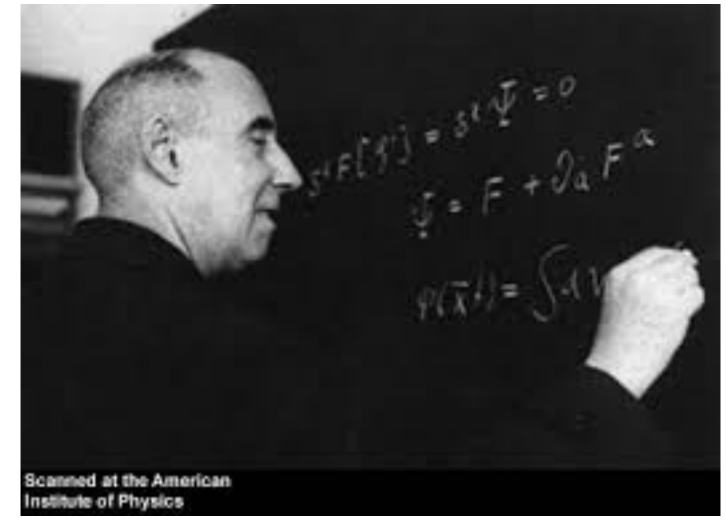
$$A_\mu \to A_\mu + \partial_\mu\chi \qquad \pi \to \pi + q\chi \qquad \rho \to \rho$$

Below the mass of the Higgs boson, integrate out ρ

In **unitary gauge** $\pi = 0$ after integrating out Higgs Boson ...

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2A_\mu A^\mu + \lambda(A_\mu A^\mu)^2 + \ldots$$

Stuckleberg Procedure



$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2 A_\mu A^\mu + \lambda(A_\mu A^\mu)^2 + \dots$$

Reintroduce Stuckelberg/Goldstone mode via a gauge transformation promoted to a field

$$A_\mu \to A_\mu - \frac{1}{m}\partial_\mu \pi$$

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2 (A_\mu - \frac{1}{m}\partial_\mu \pi)(A^\mu - \frac{1}{m}\partial^\mu \pi) + \lambda((A_\mu - \frac{1}{m}\partial_\mu \pi)(A^\mu - \frac{1}{m}\partial^\mu \pi))^2 + \dots$$

Massive theory is now gauge invariant under

$$A_\mu \to A_\mu + \partial_\mu \xi \qquad\qquad \pi \to \pi + m\xi$$

Therefore the number of degrees of freedom are

$$2 \quad A_\mu + 1 \quad \pi = 3$$

Decoupling Limit

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}m^2(A_\mu - \frac{1}{m}\partial_\mu\pi)(A^\mu - \frac{1}{m}\partial^\mu\pi) + \lambda((A_\mu - \frac{1}{m}\partial_\mu\pi)(A^\mu - \frac{1}{m}\partial^\mu\pi))^2 + \ldots$$

At high energies $E \gg m$

send $m \to 0$ $\lambda \to 0$ keeping $\Lambda = \frac{m}{\lambda^{1/4}}$ fixed

$$\mathcal{L}_{DL} = -\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}(\partial_\mu\pi\partial^\mu\pi) + \frac{1}{\Lambda^4}(\partial_\mu\pi\partial^\mu\pi)^2 + \ldots$$

Goldstone/Stuckelberg LEEFT!!!

EFTS FOR BROKEN SYMMETRIES - GRAVITY!

EFTS FOR BROKEN SYMMETRIES - GRAVITY!

Global Symmetry = Poincare Invariance

Local Symmetry = Diffeomorphism Invariance

Symmetries are spontaneously broken by Matter!

E.g. 1: Matter/Radiation/Inflation in Cosmology breaks time diffs

E.g. 2: A hairy black hole breaks radial diffs

EFTS FOR BROKEN TIME TRANSLATION

Unitary gauge

$$S = \int d^4x\,\sqrt{-g}\,\Big[\frac{1}{2}M_{\rm Pl}^2 R + M_{\rm Pl}^2 \dot{H} g^{00} - M_{\rm Pl}^2\left(3H^2+\dot{H}\right) + \frac{M_2(t)^4}{2!}(g^{00}+1)^2 + \frac{M_3(t)^4}{3!}(g^{00}+1)^3 + \ldots - \frac{\bar{M}_2(t)^2}{2}\delta K^{\mu}{}_{\mu}{}^2 + \ldots\Big]\,.$$

Stuckelberg

$$t \to t + \pi(x)$$

Goldstone LEEFT

$$S_\pi = \int d^4x\,\sqrt{-g}\left[M_{\rm Pl}^2\dot{H}\,(\partial_\mu\pi)^2 + 2M_2^4\left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi}\frac{1}{a^2}(\partial_i\pi)^2\right) - \frac{4}{3}M_3^4\dot{\pi}^3 - \frac{\bar{M}^2}{2}\,\frac{1}{a^4}(\partial_i^2\pi)^2 + \ldots\right]$$

EFT of inflation 0709.0293

/EFT of dark energy 1210.0201

EFTS FOR BROKEN RADIAL DIFFS

Unitary gauge

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\Big[\frac{1}{2}M_1^2(r)R - \Lambda(r) - f(r)g^{rr} - \alpha(r)\bar{K}_{\mu\nu}K^{\mu\nu}$$
$$+ M_2^4(r)(\delta g^{rr})^2 + M_3^3(r)\delta g^{rr}\delta K + M_4^2(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta K^{\mu\nu}$$
$$+ M_5^2(r)(\partial_r\delta g^{rr})^2 + M_6^2(r)(\partial_r\delta g^{rr})\delta K + M_7(r)\bar{K}_{\mu\nu}(\partial_r\delta g^{rr})\delta K^{\mu\nu} + M_8^2(r)(\partial_a\delta g^{rr})^2$$
$$+ M_9^2(r)(\delta K)^2 + M_{10}^2(r)\delta K_{\mu\nu}\delta K^{\mu\nu} + M_{11}(r)\bar{K}_{\mu\nu}\delta K\delta K^{\mu\nu} + M_{12}(r)\bar{K}_{\mu\nu}\delta K^{\mu\rho}\delta K^{\nu}{}_{\rho}$$
$$+ \lambda(r)\bar{K}_{\mu\rho}\bar{K}^{\rho}_{\nu}\delta K\delta K^{\mu\nu} + M_{13}^2(r)\delta g^{rr}\delta\hat{R} + M_{14}(r)\bar{K}_{\mu\nu}\delta g^{rr}\delta\hat{R}^{\mu\nu} + \dots\Big],$$

Stuckelberg

$$r \to r + \pi(r, x^a)$$

Goldstone LEEFT

$$S_\pi^{(2)} = \int \mathrm{d}t\mathrm{d}r\mathrm{d}\Omega\, ac^2 \left\{\left[\frac{\partial_r\left(ac^2 f'\right)}{ac^2} - \frac{(\Lambda'' + f'')}{2}\right]\pi^2 - \left(f - 4M_2^4\right)\pi'^2 - f(\partial_a\pi)(\partial^a\pi)\right\}$$

$$+ \dots$$

e.g. EFT of Black Hole Quasinormal Modes in Scalar-Tensor Theories 1810.07706

BREAK ALL DIFFS!: MASSIVE GRAVITY

Stuckelberg Formulation for Massive Gravity

Arkani-Hamed et al 2002
de Rham, Gabadadze 2009

Diffeomorphism invariance is spontaneously broken but maintained by introducing Stueckelberg fields

Vev of spin 2 Higgs field defines a 'reference metric'

$$f_{\mu\nu} = \langle \hat{O}_{\mu\nu} \rangle$$

reference metric

Stuckelberg fields

Dynamical Metric

$$g_{\mu\nu}(x)$$

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu \phi^A \partial_\nu \phi^B$$

helicity-1 mode of graviton

$$\phi^a = x^a + \frac{1}{mM_P}A^a + \frac{1}{\Lambda^3}\partial^a \pi$$

$$\Lambda^3 = m^2 M_P$$

helicity-0 mode of graviton

How to square root

$$F_{\mu\nu} = f_{AB}(\phi)\partial_\mu\phi^A\partial_\nu\phi^B \qquad \phi^a = x^a + \frac{1}{mM_P}A^a + \frac{1}{\Lambda^3}\partial^a\pi$$

Helicity zero mode enters reference metric **squared**

$$F_{\mu\nu} \approx \eta_{\mu\nu} + \frac{2}{\Lambda^3}\partial_\mu\partial_\nu\pi + \frac{1}{\Lambda^6}\partial_\mu\partial_\alpha\pi\partial^\alpha\partial_\nu\pi$$

To extract dominant helicity zero interactions we need to take a square root

$$\left[\sqrt{g^{-1}F}\right]_{\mu\nu} \approx \eta_{\mu\nu} + \frac{1}{\Lambda^3}\partial_\mu\partial_\nu\pi$$

Branch uniquely chosen to give rise to 1 when Minkowski

Helicity Zero mode = Galileon

The helicity zero mode $\pi(x)$ only enters in the combination

$$\Pi_{\mu\nu} = \partial_\mu \partial_\nu \pi(x)$$

This is invariant under the
global nonlinearly realized symmetry

$$\pi(x) \to \pi(x) + c + v_\mu x^\mu$$

$$\Pi_{\mu\nu} \to \Pi_{\mu\nu}$$

Galilean Operators

$$\mathcal{L}_2 = \Pi \, \varepsilon^{abcd} \, \varepsilon^{ABCD} \, \eta_{aA} \, \eta_{bB} \, \eta_{cC} \, \partial_d \partial_D \Pi$$

$$\mathcal{L}_3 = \Pi \, \varepsilon \, \varepsilon \, \eta \, \eta \, \partial\partial\Pi \, \partial\partial\Pi$$

$$\mathcal{L}_4 = \Pi \, \varepsilon \, \varepsilon \, \eta \, \partial\partial\Pi \, \partial\partial\Pi \, \partial\partial\Pi$$

$$\mathcal{L}_5 = \Pi \, \varepsilon \, \varepsilon \, \partial\partial\Pi \, \partial\partial\Pi \, \partial\partial\Pi \, \partial\partial\Pi$$

Galileon-helicity 2 interactions

$$\mathcal{L}_{2'} = \varepsilon^{abcd}\,\varepsilon^{ABCD}\,\eta_{aA}\,\eta_{bB}\,\partial\partial\pi_{cC}\,h_{dD}$$

$$\mathcal{L}_{3'} = \varepsilon\,\varepsilon\,\eta\,\partial\partial\pi\,\partial\partial\pi\,h$$

$$\mathcal{L}_{4'} = \varepsilon\,\varepsilon\,\partial\partial\pi\,\partial\partial\pi\,\partial\partial\pi\,h$$

'Characteristic polynomials'

$$\det\left(\alpha h + \beta\,\partial\partial\pi + \gamma\,\eta\right)$$

de Rham, Gabadadze, AJT 2010

Λ_3 Massive Gravity

Unitary gauge

$$\mathcal{L} = \frac{1}{2}\sqrt{-g}\left(M_P^2 R[g] - m^2 \sum_{n=0}^{4} \beta_n \mathcal{U}_n\right) + \mathcal{L}_M$$

$$K = 1 - \sqrt{g^{-1} f}$$

$$\mathrm{Det}[1 + \lambda K] = \sum_{n=0}^{d} \lambda^n \mathcal{U}_n(K)$$

Characteristic Polynomials

Double epsilon structure!!!!!

Unique low energy EFT where the strong coupling scale is

$$\Lambda_3 = (m^2 M_P)^{1/3}$$

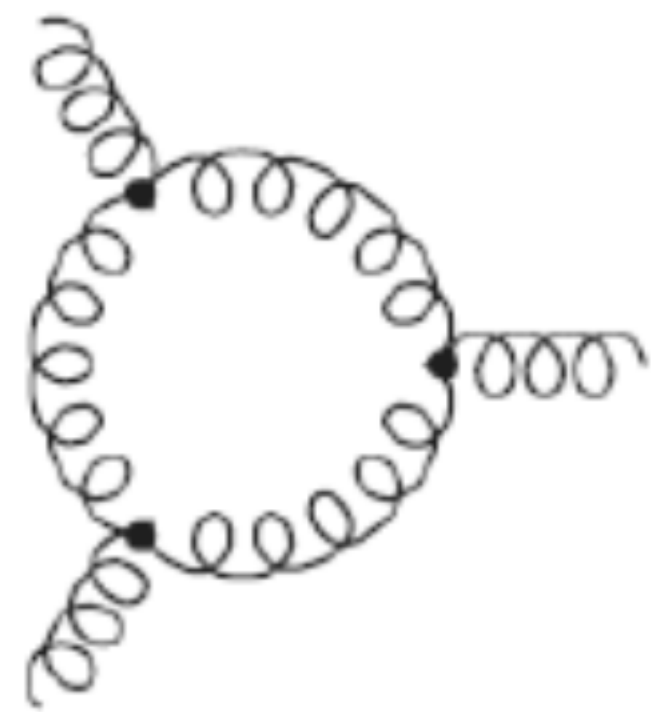
Massive Gravity as an EFT

At energies $m \ll E \ll M_{\rm PLanck}$ $\Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$

All Lorentz invariant Hard and Soft and Multi-graviton theories look like **Galileon theories** (plus massless spin 2 plus Maxwell)

$\pi \to \pi + v_\mu x^\mu + c$ $K_{\mu\nu} = \frac{\partial_\mu \partial_\nu \pi}{\Lambda_3^3}$

Generic one-loop Graviton diagram needs counter-terms at the scale (principally due to helicity zero mode interactions)



$$\Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$$

Counter-terms which are <u>not needed in GR</u>!

Massive Gravity as an EFT

$$K = 1 - \sqrt{g^{-1} f}$$

$$\Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$$

In decoupling limit: $M_{\rm PLanck} \to \infty, m \to 0$

$$K_{\mu\nu} \to \frac{\partial_\mu \partial_\nu \pi}{\Lambda_3^3}$$

de Rham, Melville, Tolley 2017

EFT corrections then take the form:

$$\mathcal{L} = \left[\frac{M_P^2}{2} R - M_P \Lambda_3^4 \sum_n \alpha_n \mathcal{E}\mathcal{E} g^{4-n} K^n \right] + \Lambda_3^4 F\left[\frac{\nabla_\mu}{\Lambda_3}, K_{\mu\nu}, \frac{R_{\mu\nu\rho\sigma}}{\Lambda_3^2} \right]$$

Infinite number of derivative suppressed operators

Generic feature of IR modifications

Cosmologically motivated models desire to change physics at scale

$$m \sim H_{\rm today}$$

Decoupling limit EFTs generally indicate new physics at scale

$$\Lambda_n = (m^{n-1} M_{\rm pl})^{1/n}$$

Intermediate scale new physics!!!!!

WHAT DISTINGUISHES EFTS?

How do we distinguish between different EFTs?

Remain agnostic and wait for observations to decide:

Right approach in principle but # theories > observations

Work with a given UV completion like String theory:

Subject to limited current understanding

Look at low energy consistency questions (causality, caustics, ...):

Not always insensitive to field redefinitions, or may only indicate break down of LEEFT and not fundamental problem

How do we distinguish between different EFTs?

Look at low energy consistency questions (causality, caustics, ...):

Not always insensitive to field redefinitions, or may only indicate break down of LEEFT and not fundamental problem

e.g. $g_{\mu\nu} \to g_{\mu\nu} + f(\phi, X)\partial_\mu\phi\partial_\nu\phi$ e.g. in b. Horndeski

or $g_{\mu\nu} \to g_{\mu\nu} + \alpha R_{\mu\nu}$

Clearly do not preserve speed of light of gravitons, rendering meaningless the requirement

$$c_T^2 \leq 1$$

Solution: Remove field redefinition ambiguities by looking at the S-matrix

Look for asymptotic superluminalities:

We cannot send signals faster than what is allowed by asymptotic causal structure of the spacetime Gao and Wald 2000

e.g. Camanho, Edelstein, Maldacena, Zhiboedov, `Causality Constraints on Corrections to the Graviton Three-Point Coupling," arXiv:1407.5597
Massive Spin-2 Scattering and Asymptotic Superluminality
Hinterbichler. Joyce, Rosen arXiv:1708.05716

Amounts to demanding that the Eisenbud-Wigner scattering time delay is positive

$$T \sim \frac{d\delta(E)}{dE} \quad > 0$$

$\delta(E)$ is phase shift between scattered wave and unscattered

Solution: Remove field redefinition ambiguities by looking at the S-matrix

Closely related are the requirements that the S-matrix is

1. Local (Polynomially *(or exponentially)* bounded in momentum space) and
2. Causal (Analytic as a function of Mandelstam variables)

A precise definition of analyticity for the 2-2 scattering amplitude at fixed momentum transfer t was rigourously proven in the 50's and 60's

although not too much progress ever made beyond this

Positivity Constraints

S-Matrix lore

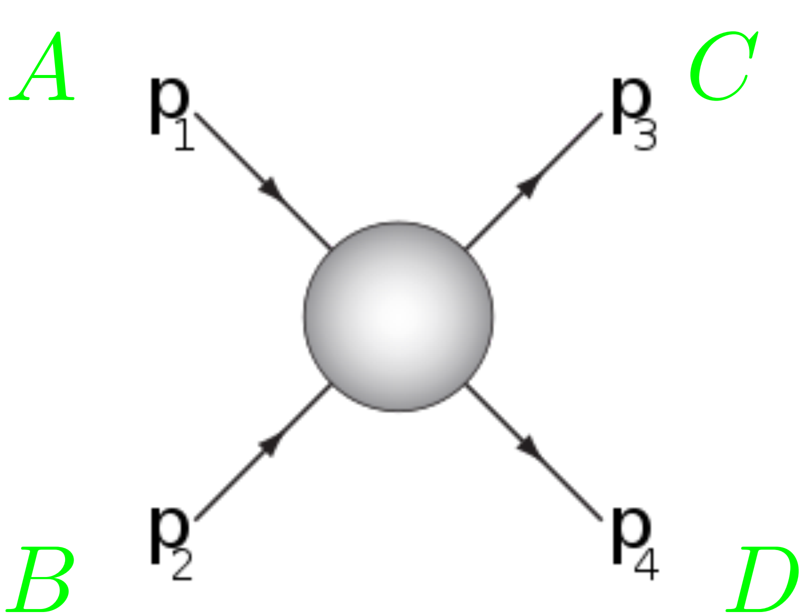
$|A(k)| < \alpha e^{\beta|k|}$

1. Unitarity $S^\dagger S = 1$
2. Locality: Scattering Amplitude Polynomially (Exponentially) Bounded
3. Causality: Analytic Function of Mandelstam variables (modulo poles+cuts)
4. Poincare Invariance
5. Crossing Symmetry: Follows from above assumptions
6. Mass Gap: Existence of Mandelstam Triangle and Validity of Froissart Bound

Added Ingredient: Crossing Symmetry

s-channel

$$A + B \to C + D$$



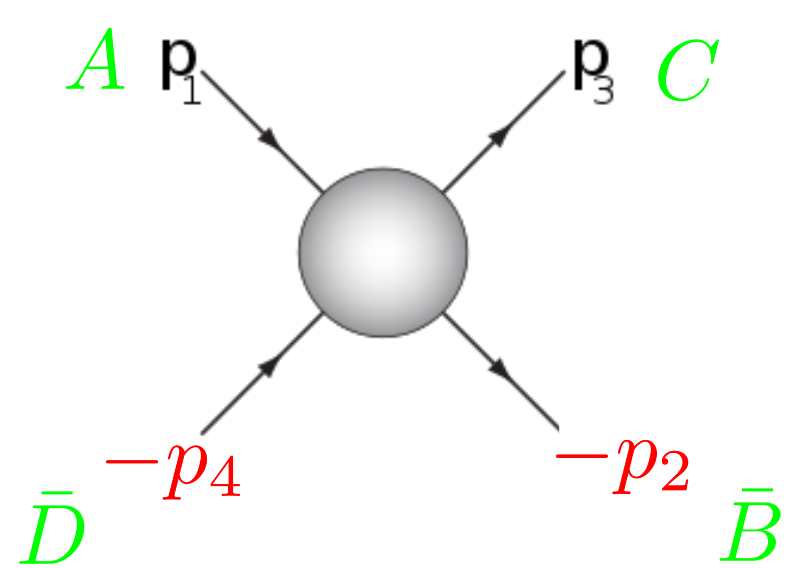
$$s = -(p_1 + p_2)^2$$

$$t = -(p_1 + p_3)^2$$

$$u = -(p_1 + p_4)^2$$

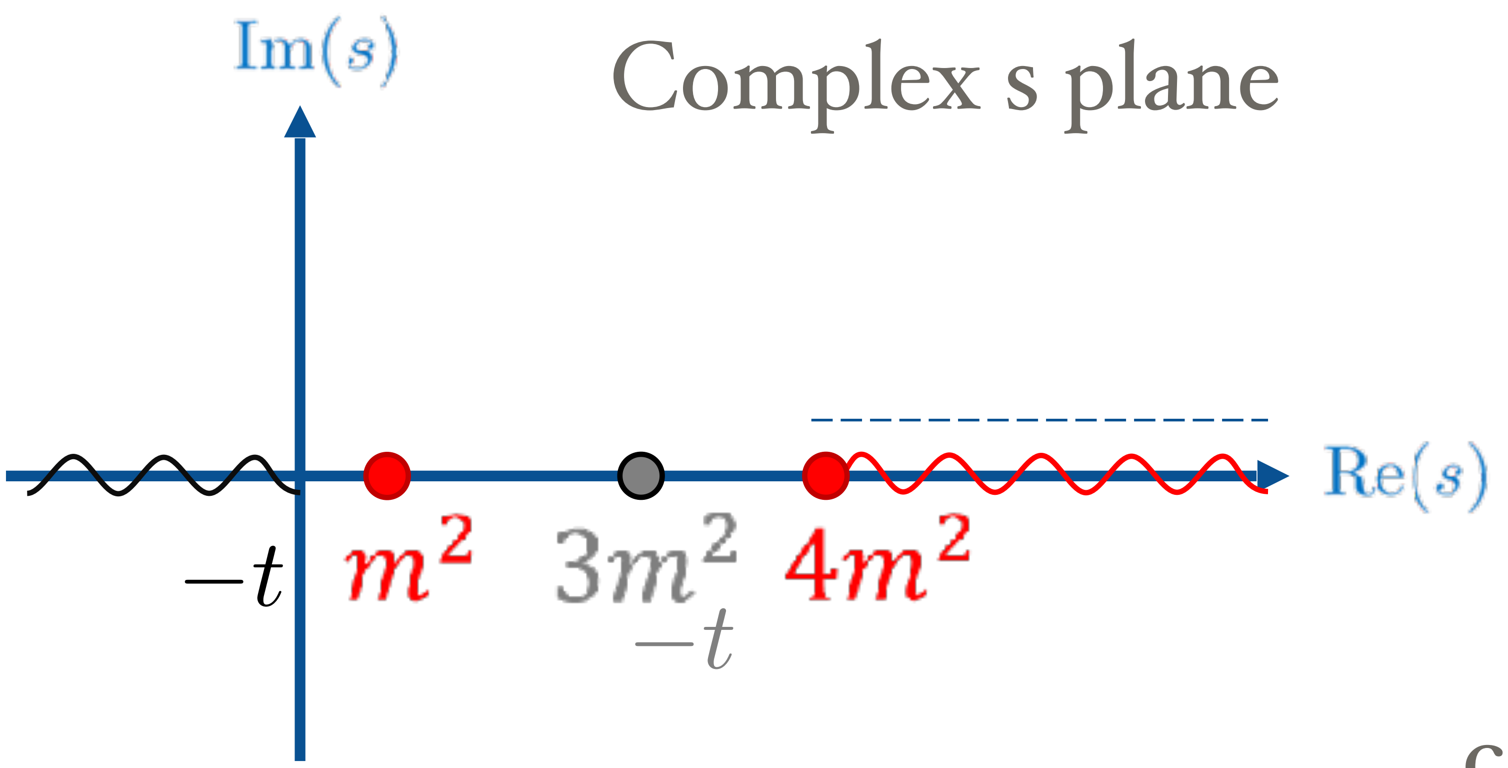
u-channel

$$A + \bar{D} \to C + \bar{B}$$



Scattering Amplitude Analyticity

Complex s plane



Physical scattering region is $s \geq 4m^2$

crossing: $u = 4m^2 - s - t$

$$\mathcal{A}_s(s,t) = \frac{\lambda_s(t)}{m^2 - s} + \frac{\lambda_u(t)}{m^2 - u} + (c_0(t) + c_1(t)s) + \frac{s^2}{\pi}\int_{4m^2}^{\infty} d\mu \frac{Im(A_s(\mu,t))}{\mu^2(\mu - s)} + \frac{u^2}{\pi}\int_{4m^2}^{\infty} d\mu \frac{Im(A_u(\mu,t))}{\mu^2(\mu - u)}$$

Poles Subtractions Branch cuts

Forward Limit Positivity Bounds

$$\mathcal{A}'_s(s,t) = c_0(t) + c_1(t)s + \frac{s^2}{\pi}\int_{\Lambda^2}^{\infty} d\mu \frac{Im(\mathcal{A}_s(\mu,t))}{\mu^2(\mu-s)} + \frac{u^2}{\pi}\int_{\Lambda^2}^{\infty} d\mu \frac{Im(\mathcal{A}_u(\mu,t))}{\mu^2(\mu-u)}$$

↓

$$\frac{1}{M!}\frac{d^M}{ds^M}\mathcal{A}'_s(2m^2,0) = \frac{1}{\pi}\int_{\Lambda^2}^{\infty} d\mu \frac{Im(\mathcal{A}_s(\mu,t)) + Im(\mathcal{A}_u(\mu,t))}{(\mu-2m^2)^{M+1}} > 0$$

$M \geq 2$

RH Cut

LH Cut

Positivity Bounds = (Sub)luminality

For example: $$\mathcal{L} = -\frac{1}{2}(\partial\phi)^2 + \frac{c}{\Lambda^4}(\partial\phi)^4$$

Adams et. al. 2006

Positivity implies: $$\partial_s^2 A_s' = c - \alpha c^2 + \cdots > 0$$

$$c > \alpha c^2 > 0$$

Causality implies: $$c_s^2 = 1 - \frac{c}{\Lambda^4}\dot{\phi}^2 < 1$$

Makes sense since positivity derivation relies on Analyticity=Causality

DBI versus anti-DBI

$$\mathcal{L}_{\mathrm{DBI}} \sim -\sqrt{1+(\partial\phi)^2}$$

Model relevant for inflation

$$(\partial\phi)^2 = -\dot{\phi}^2 \to -1$$

Model that naturally emerges as probe brane in extra dimension

No obstructions to standard UV completion (known so far)

$$\mathcal{L}_{\overline{\mathrm{DBI}}} \sim \sqrt{1-(\partial\phi)^2}$$

Model relevant for dark energy with screening in dense environments

$$(\partial\phi)^2 = \phi'(r)^2 \to 1$$

Model that naturally emerges as probe brane in extra *time* dimension…

Known obstructions to standard UV completion

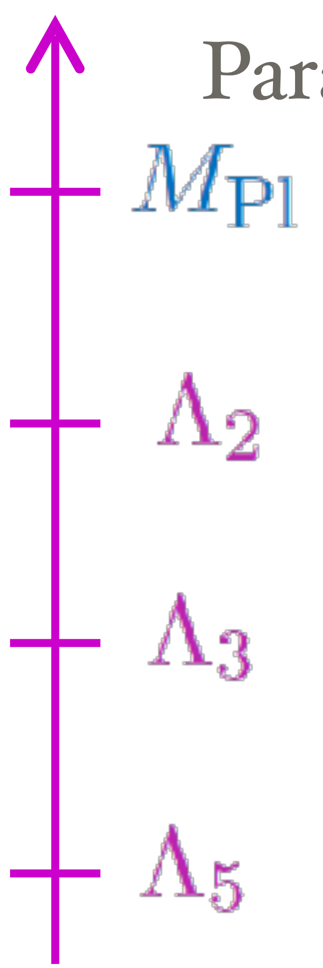
Example: Positivity constraints in interacting massive spin-2 theories

Application to EFT of Interacting Spin 2 (aka Massive Gravity)

Unitary Gauge Massive Gravity

Einstein-Hilbert   Mass Term

$$\mathcal{L} \supset \frac{M_{\rm Pl}^2}{2}\left(R\left[g\right] - \frac{m^2}{4}V(g,h)\right)$$



Parameterize generic mass term (without dRGT tuning) as

$$V(g,h) \supset [h^2] - [h]^2 + (c_1 - 2)[h^3] + (c_2 + \frac{5}{2})[h^2][h]$$
$$+ (d_1 + 3 - 3c_1)[h^4] + (d_3 - \frac{5}{4} - c_2)[h^2]^2 + \ldots .$$

$\Lambda_5 = (M_{\rm Pl} m^4)^{1/5} \ll M_{\rm Pl}$

where $\quad [h] = \eta^{\mu\nu} h_{\mu\nu},\ [h^2] = \eta^{\mu\nu} h_{\mu\alpha}\eta^{\alpha\beta}h_{\beta\nu},$ $\quad \Lambda_3 = (M_{\rm Pl} m^2)^{1/3} \gg \Lambda_5$

$$d_3 = -d_1/2 + 3/32 + \Delta d, \quad c_2 = -3c_1/2 + 1/4 + \Delta c$$

Application to Massive Gravity

Forward Limit positivity bounds

$$2M_{\rm Pl}^2 m^6 \frac{\partial^2}{\partial v^2} f_{\alpha\beta}|_{t=0} = \frac{352}{9}|\alpha_S\beta_S|^2\left(\Delta c\left(-6+9c_1-4\Delta c\right)-6\Delta d\right) + \frac{176}{3}\alpha_S^*\beta_S^*(\alpha_{V_1}\beta_{V_1}-\alpha_{V_2}\beta_{V_2})\,\Delta c\left(3-3c_1+4\Delta c\right)$$

Positivity for general helicity implies: $\Delta c = 0$

Application to Massive Gravity

Melville, Tolley, Zhou, CdR 1804.10624

Beyond forward

$$\frac{\partial}{\partial t} f_{\tau_1 \tau_2}(v,t) \;\propto\; \frac{v}{\Lambda_5^{10}} \Delta d + \mathcal{O}\left(\frac{m^2}{\Lambda_5^{10}}\right) \;>\; 0$$

for either sign of $v = s - 2m^2 + t/2$

hence $\Delta d = 0$

Together positivity bounds fix

$$\Delta c = \Delta d = 0$$

These are precisely the tunings that raise the cutoff from

$$\Lambda_5 = (m^4 M_{\rm Planck})^{1/5} \longrightarrow \Lambda_3 = (m^2 M_{\rm Planck})^{1/3}$$

Λ_5 Massive Gravity

$$\mathcal{L} = \left[\frac{M_P^2}{2} R - M_P^2 m^2 \sum_n [K^2] - [K]^2\right] + \Lambda_5^4 \tilde{F}\left[\frac{\nabla_\mu}{\Lambda_5}, \frac{\Lambda_5^2}{m^2} K_{\mu\nu}, \frac{R_{\mu\nu\rho\sigma}}{\Lambda_5^2}\right]$$

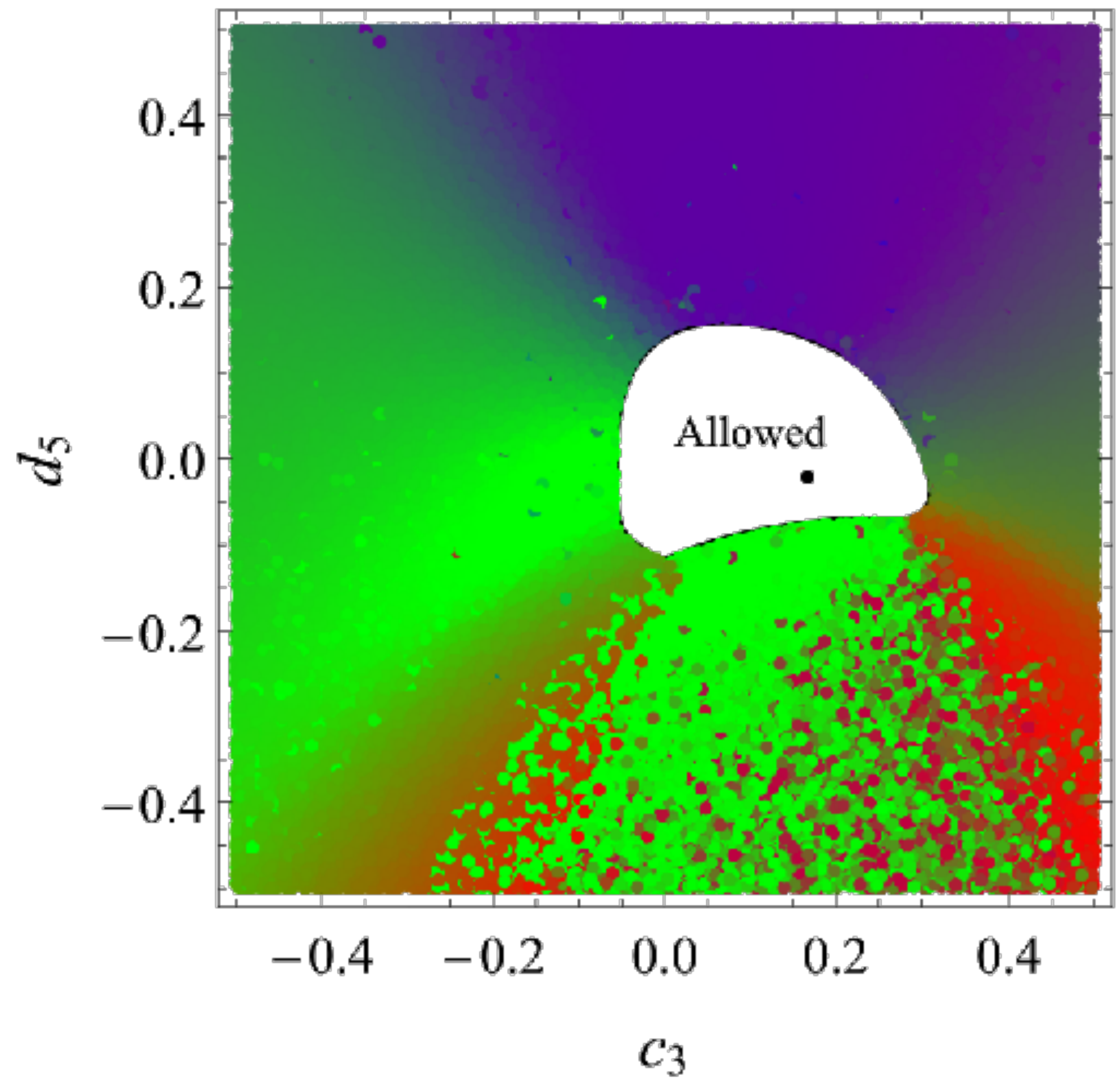
Λ_3 Massive Gravity

$$K = 1 - \sqrt{g^{-1}\eta}$$

$$\mathcal{L} = \left[\frac{M_P^2}{2} R - M_P \Lambda_3^4 \sum_n \alpha_n \mathcal{E}\mathcal{E} g^{4-n} K^n\right] + \Lambda_3^4 F\left[\frac{\nabla_\mu}{\Lambda_3}, K_{\mu\nu}, \frac{R_{\mu\nu\rho\sigma}}{\Lambda_3^2}\right]$$

Positivity Constraints on Λ_3 massive gravity

Cheung-Remmen 1601.04068

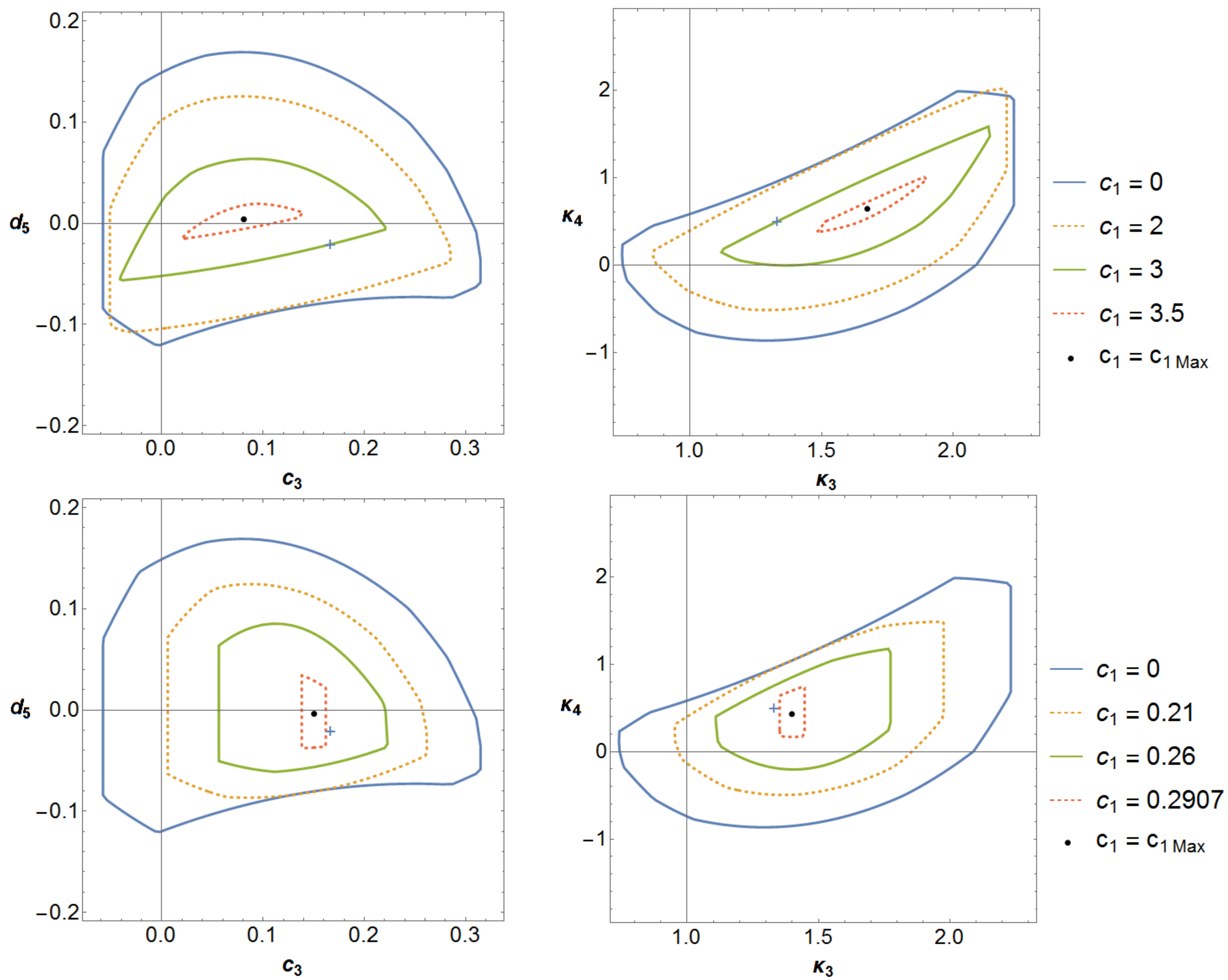


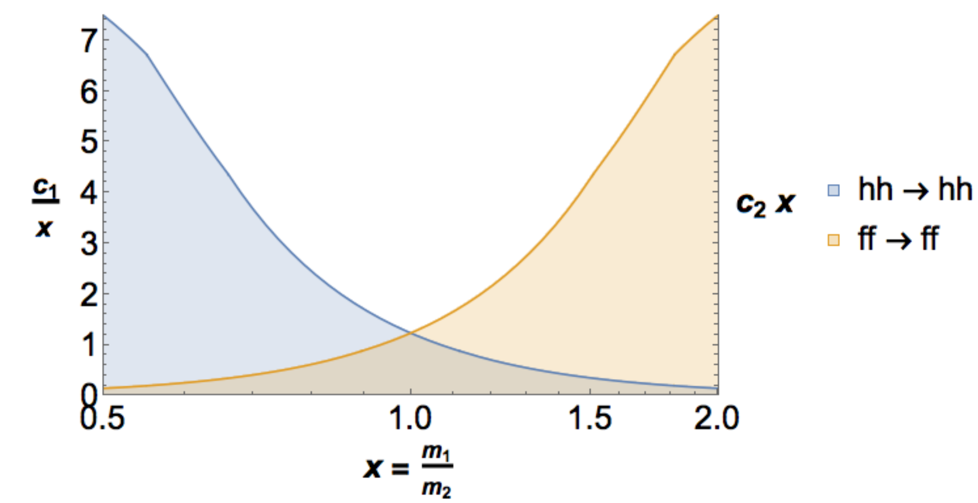
Multiple Interacting Spin 2

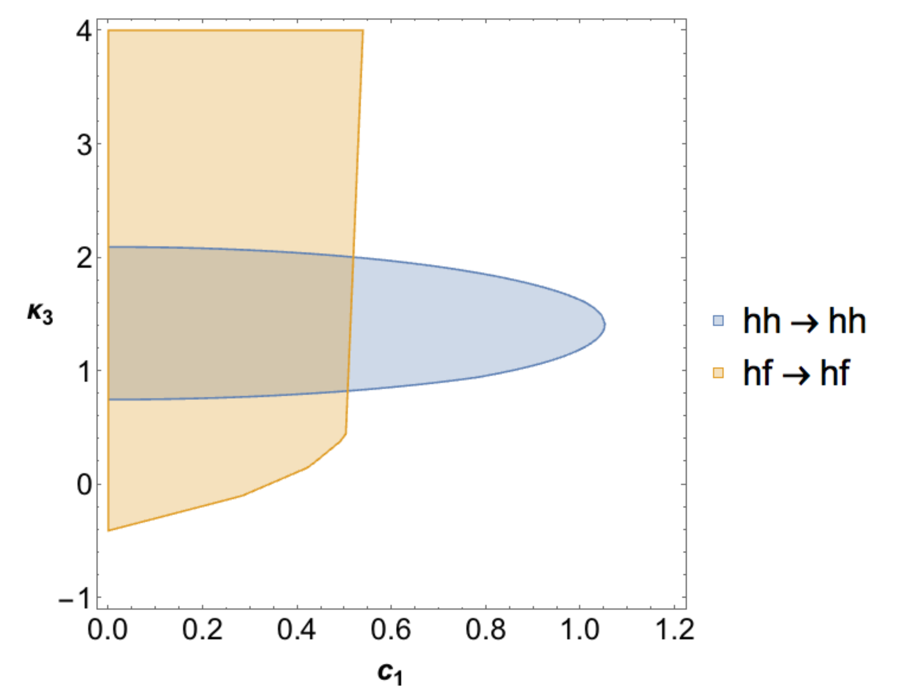
Alberte, de Rham, Momeni, Rumbutis, AJT 2020

$$g^{(1)}_{\mu\nu} = (\eta_{\mu\nu} + h_{\mu\nu})^2 \, , \qquad g^{(2)}_{\mu\nu} = (\eta_{\mu\nu} + f_{\mu\nu})^2$$

$$\mathcal{L}_{\rm int} = \frac{\gamma m^2 M_{Pl}^2}{2} c_1 \mathcal{L}_{hhf} + \frac{\gamma m^2 M_{Pl}^2}{2} c_2 \mathcal{L}_{hff} + \frac{\gamma m^2 M_{Pl}^2}{4} \lambda \mathcal{L}_{hhff}$$







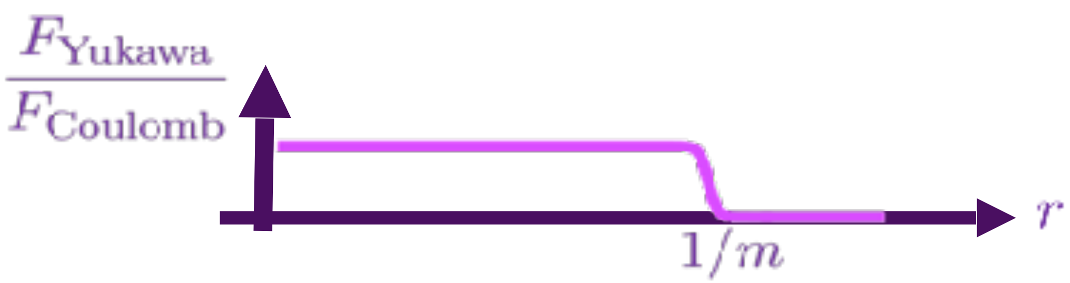
OBSERVATIONAL CONSEQUENCES

Constraints on the Graviton Mass

de Rham, Deskins, AJT, Zhou, Reviews of Modern Physics

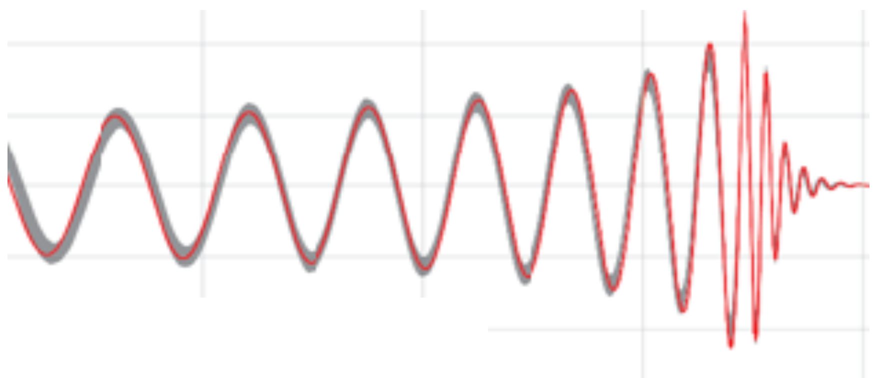
Yukawa

m_g (eV)	λ_g (km)	
10^{-23}	10^{12}	Solar System tests
10^{-32}	10^{21}	Weak lensing
10^{-29}	10^{19}	Bound clusters



Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB



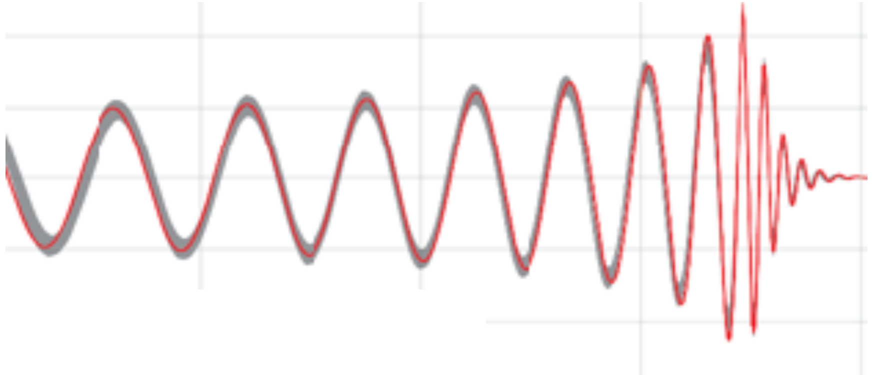
Fifth Force

m_g (eV)	λ_g (km)	
10^{-32}	10^{22}	Lunar Laser Ranging
10^{-27}	10^{17}	Binary pulsar
10^{-32}	10^{22}	Structure formation

Dispersion Relation

m_g (eV)	λ_g (km)	
10^{-22}	10^{11}	aLIGO bound
10^{-20}	10^{9}	Pulsar timing
10^{-30}	10^{20}	B–mode's in CMB

Direct Detection of GW

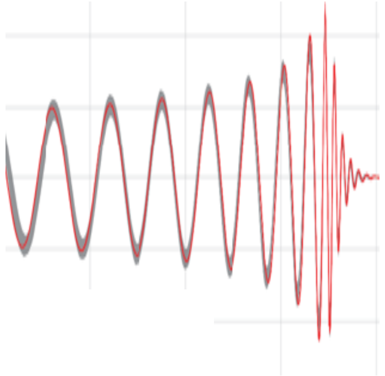


Constraints modifications of the dispersion relation

$$E^2 = \mathbf{k}^2 + m_g^2$$

Generic for the helicity-2 modes of any Lorentz invariant model of massive gravity

GW signal would be more squeezed than in GR



Speed increases with frequency

$$v_g/c \approx 1 - \frac{1}{2}(c/\Lambda_g f)^2$$

$$1 - \frac{v_g}{c} = 5 \times 10^{-17} \left(\frac{200\text{Mpc}}{D}\right) \left(\frac{\Delta t}{1\text{s}}\right)$$

$$m_g \lesssim 4 \times 10^{-22}\text{eV} \left(f \Delta t \frac{f}{100\text{Hz}} \frac{200\text{Mpc}}{D} \right)^{1/2}$$

For GW150914,

$$D \sim 400\text{Mpc}, \;\; f \sim 100\text{Hz}, \;\; \rho \sim 23 \quad \Rightarrow \quad m_g \lesssim 10^{-22}\text{eV}$$

Will 1998
Abbott et al., 2016

Massive Gravity leads a scalar (helicity zero) field

Massive spin-2 field, has 5 degrees of freedom

$$h_{\mu\nu} \sim \frac{G_N}{\Box_4 - m^2}\left(T_{\mu\nu} - \frac{1}{3}g_{\mu\nu}T\right)$$

$$\left(\text{in GR its } h_{\mu\nu} \sim \frac{G_N}{\Box_4}\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right)\right)$$

$$\frac{1}{3} \neq \frac{1}{2}$$

van Dam & Veltman, Nucl.Phys.B 22, 397 (1970)
Zakharov, JETP Lett.12 (1970) 312

Why?

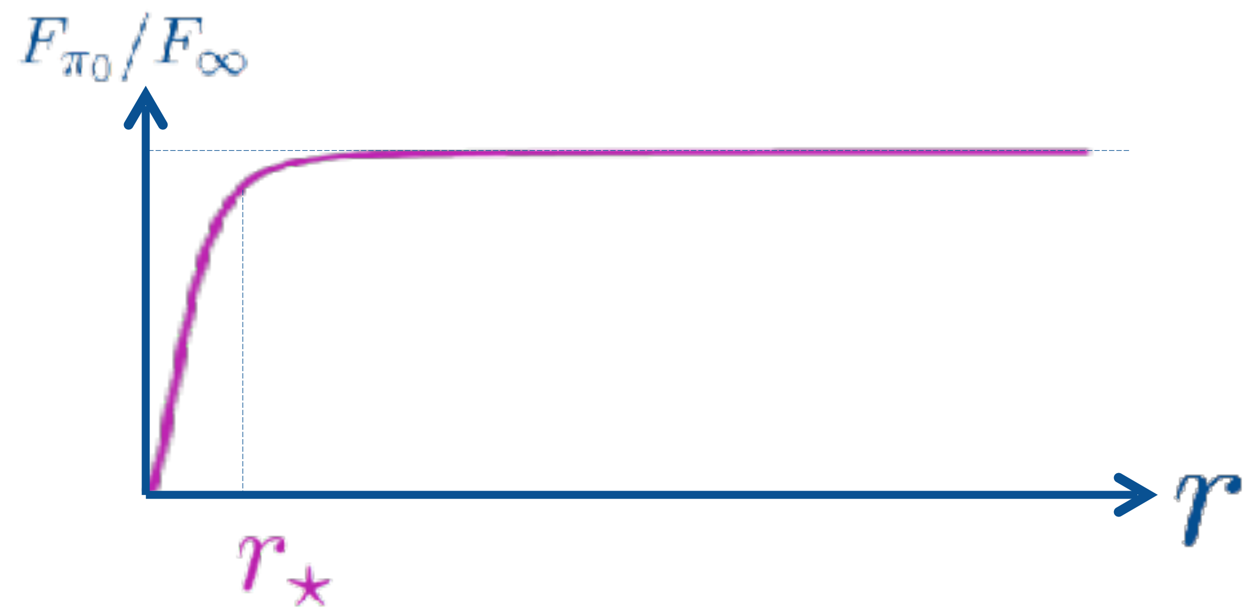
$$h_{\mu\nu} = h'_{\mu\nu} + \eta_{\mu\nu}\pi$$

← New scalar degree of freedom that couples to the trace of the stress energy momentum tensor

Vainshtein, PLB**39**, 393 (1972)

Vainshtein mechanism

Well understood for Static & Spherically Symmetric configurations



$$\frac{F_{r\ll r_\star}}{F_{r\gg r_\star}} = \left(\frac{r}{r_\star}\right)^{3/2}$$

For Sun

$r_* \sim 250pc$

$m^{-1} \sim 4000Mpc$

$r_s \sim 3km$

$$\frac{F_{r\ll r_\star}}{F_{r\gg r_\star}} = \left(\frac{r}{r_\star}\right)^{3/2} \sim 10^{-12}$$ For Sun-Earth System

$\sim 10^{-13}$ For Earth-Moon System

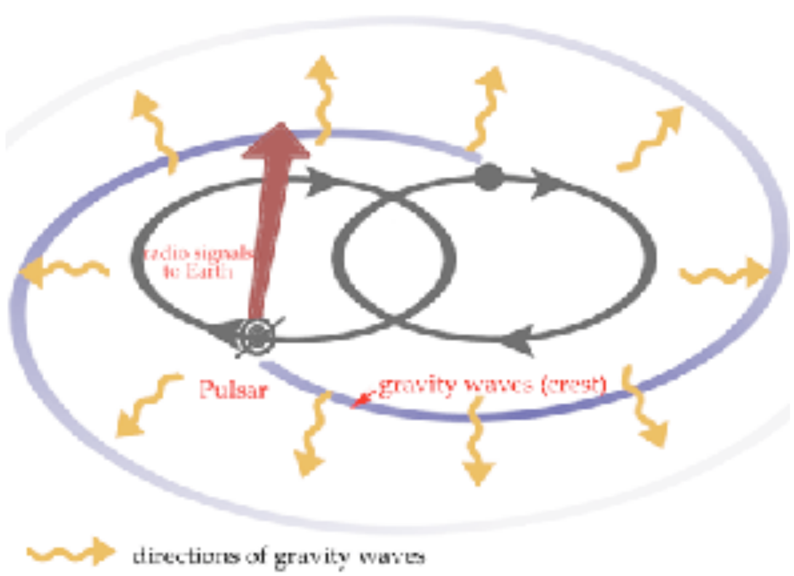
$r_\star = \frac{1}{\Lambda}\left(\frac{M_\odot}{M_{\rm Pl}}\right)^{1/3}$ $\Lambda = (m^2 M_{\rm Pl})^{1/3}$ $\sim 10^{-15}$ For Hulse-Taylor Pulsar

Vainshtein mechanism in a Binary System

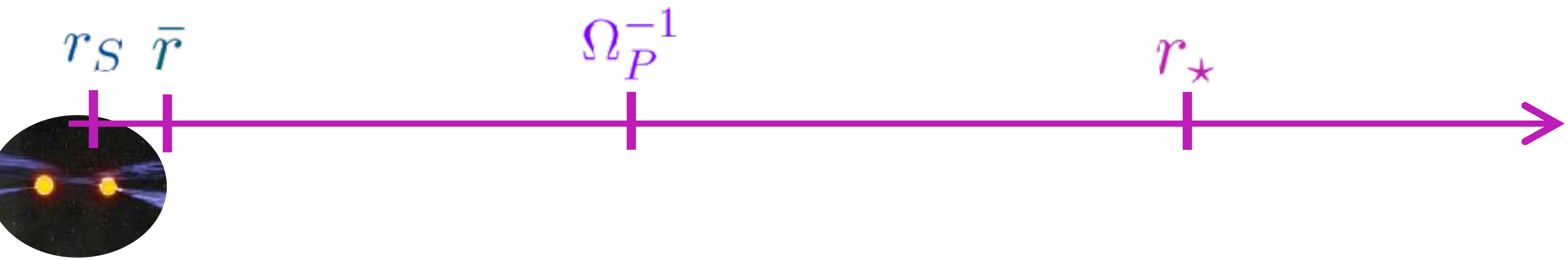
de Rham, AJT, Wesley 2012
de Rham, Matas, AJT 2013
Dar, de Rham, Deskins, Giblin, AJT 2018

Extra polarizations of graviton = extra modes of gravitational wave

Binary pulsars lose energy faster than in GR so the orbit slows down more rapidly



Hierarchy of Scales



$r_S \sim 10$ km

$\bar{r} \sim 10^6$ km

$\Omega_P^{-1} \sim 10^9$ km

$r_\star \sim 10^{15}$ km

$$r_S \ll \bar{r} \ll \Omega_P^{-1} \ll r_\star \ll m^{-1}$$

Slide thanks to C. de Rham

Cubic Galileon

$$S = \int \mathrm{d}^4x \left(-\frac{3}{4}(\partial\pi)^2 - \frac{1}{4\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{2M_{\rm Pl}}\pi T \right)$$

$$T^\mu_\nu = -M \sum_{i=1,2} \delta^{(3)}(\vec{x} - \vec{x}_i(t))\delta^\mu_0 \delta^0_\nu .$$

$$T^\mu_\nu = T_0{}^\mu_\nu + \delta T^\mu_\nu \qquad T_0{}^\mu_\nu = -2M\delta^{(3)}(\vec{x})\delta^\mu_0\delta^0_\nu$$

Background/Perturbation Split

$$S = \int \mathrm{d}^4x \left(-\frac{3}{4}(\partial\pi)^2 - \frac{1}{4\Lambda^3}(\partial\pi)^2\Box\pi + \frac{1}{2M_{\mathrm{Pl}}}\pi T \right)$$

$$\pi(t,\vec{x}) = \pi_0(r) + \sqrt{2/3}\,\phi(t,\vec{x})$$

Background due to centre of gas

Radiation emitted by that scalar

$$\frac{E(r)}{r} + \frac{2}{3\Lambda^3}\left(\frac{E(r)}{r}\right)^2 = \frac{1}{12\pi r^3}\frac{M}{M_{\mathrm{Pl}}}$$

$$E(r) = \partial_r \pi_0(r)$$

Perturbation Action

$$\mathcal{S}_\phi = \int \mathrm{d}^4x \left(-\frac{1}{2} Z^{\mu\nu} \partial_\mu\phi\partial_\nu\phi + \frac{\phi\delta T}{\sqrt{6}M_{\mathrm{Pl}}} \right)$$

Vainshtein effect

$$Z^{tt}(r) = -\left[1 + \frac{2}{3\Lambda^3}\left(2\frac{E(r)}{r} + E'(r)\right)\right]$$
$$Z^{rr}(r) = 1 + \frac{4}{3\Lambda^3}\frac{E(r)}{r},$$
$$Z^{\Omega\Omega}(r) = 1 + \frac{2}{3\Lambda^3}\left(\frac{E(r)}{r} + E'(r)\right).$$

$$\phi_{lm\omega}(t, r, \theta, \phi) = u_{l\omega}(r) Y_{lm}(\theta, \phi) e^{-i\omega t}$$

Vainshtein region $Z \gg 1$ fifth force suppressed by $\frac{1}{Z}$

Effective Action

Goldberger+ Rothstein hep-th/0409156

$$\mathcal{S}_\phi = \int \mathrm{d}^4x \left(-\frac{1}{2} Z^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{\phi \delta T}{\sqrt{6} M_{\mathrm{Pl}}} \right)$$

$$\phi_{\mathrm{F}}(x) = \frac{i}{\sqrt{6} M_{\mathrm{Pl}}} \int \mathrm{d}^4y \, G_{\mathrm{F}}(x,y) \delta T(y)$$

$$\partial_\mu \left(Z^{\mu\nu} \partial_\nu \right) \quad G_{\mathrm{F}}(x,y) = i\delta^4(x-y)$$

$$S_{\mathrm{eff}} = \frac{i}{12 M_{\mathrm{Pl}}^2} \int \mathrm{d}^4x \, \mathrm{d}^4y \, \delta T(x) G_{\mathrm{F}} \delta T(y)$$

Power emitted

Goldberger+ Rothstein hep-th/0409156

$$\frac{2\mathrm{Im}(S_{\mathrm{eff}})}{T_{\mathrm{P}}} = \int_0^\infty \mathrm{d}\omega\, f(\omega) \qquad P = \int_0^\infty \mathrm{d}\omega\, \omega f(\omega)$$

Radiated power is

$$P = \frac{\pi}{3M_{\mathrm{Pl}}^2} \sum_{n=0}^{\infty} \sum_{lm} n\Omega_{\mathrm{p}} |M_{lmn}|^2$$

where

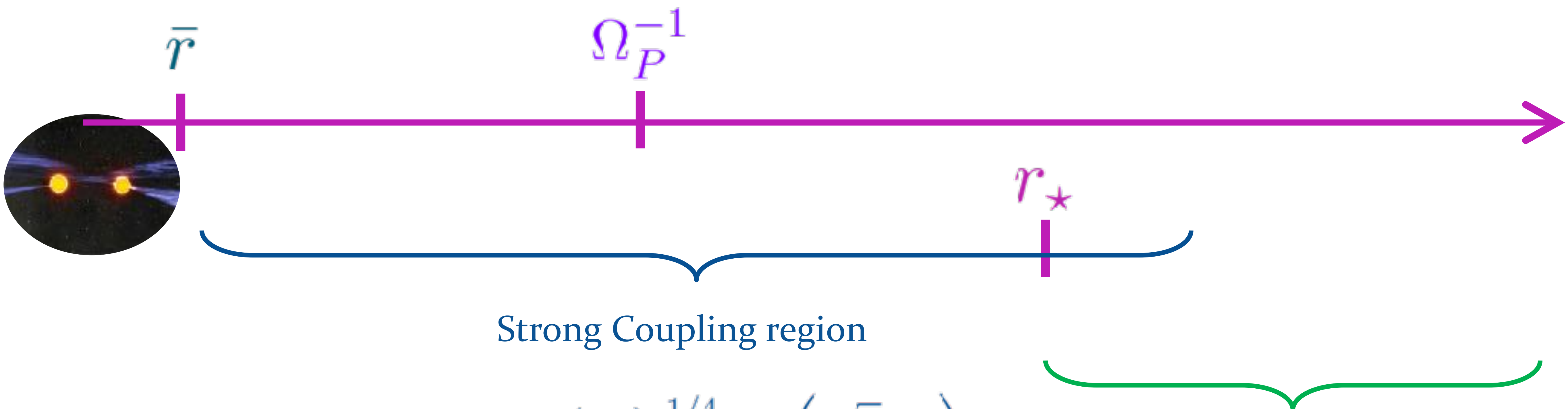
$$\mathcal{M}_{lmn} = \frac{1}{T_P} \int_0^{T_p} dt \int d^3x u_{ln}(r) Y_{lm}(\theta,\phi) e^{-int/T_p} \delta T(x,t)$$

and modes satisfy

$$\partial_\mu (Z^{\mu\nu}(\pi_0) \partial_\nu \left[u_\ell(r) Y_{\ell m}(\Omega) e^{-i\omega t} \right]) = 0$$

WKB Matching

$$\partial_\mu(Z^{\mu\nu}(\pi_0)\partial_\nu\left[u_\ell(r)Y_{\ell m}(\Omega)e^{-i\omega t}\right]) = 0$$



$$u_\ell = \bar{u}\left(\frac{r}{r_\star}\right)^{1/4} J_\nu\left(\frac{\sqrt{3}}{2}\omega r\right)$$

$$\nu = \begin{cases} (2\ell+1)/4 & \text{for} \quad \ell > 0 \\ -1/4 & \text{for} \quad \ell = 0 \end{cases}$$

$$u_\ell = \frac{1}{\sqrt{\pi\omega r}}\cos\left(\omega r\right)$$

Scalar Gravitational Waves: Power Radiated

Dominated by Quadrupole Radiation:

$$P_{\rm quadrupole} = 2^{7/2}\frac{5\lambda_1^2}{32}\frac{(\Omega_P\bar{r})^3}{(\Omega_P r_*)^{3/2}}\frac{M_Q^2}{M_{\rm pl}^2}\Omega_P^2$$

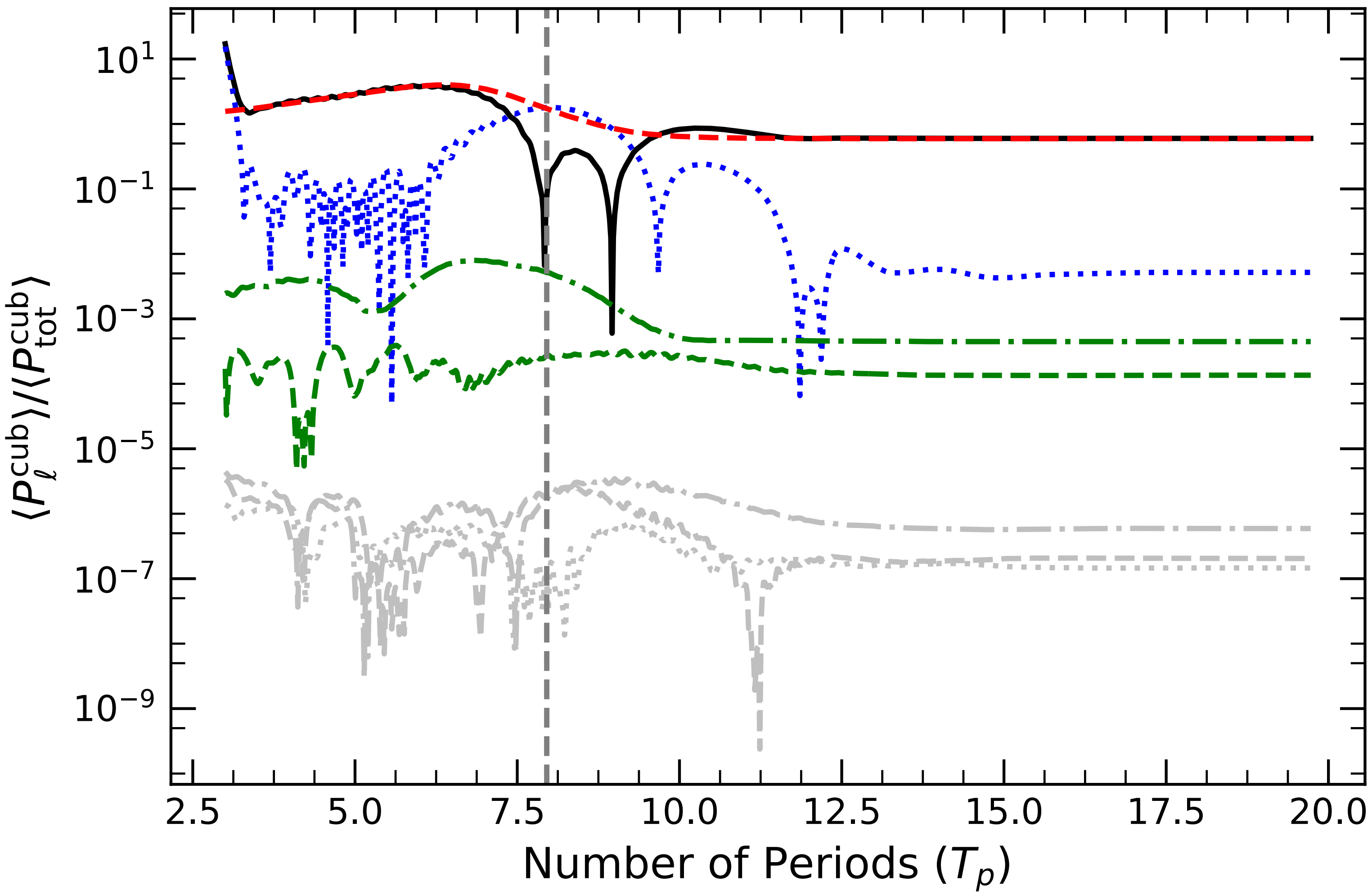
relative to GR result:

$$\frac{P^{\rm Galileon}_{\rm quadrupole}}{P^{\rm GR}_{\rm quadrupole}} = q(\Omega_P r_*)^{-3/2}(\Omega_P\bar{r})^{-1}$$

For realistic binary pulsars suppressed by 10^{-9}-10^{-7}


Movie

Power per multipole (numerics)



Black: Total power
Dotted Blue: Monopole
Dotted Grey: Dipole
Dashed Red: Quadrupole

Consistent with analytic estimate:

$$\left.\frac{P_2^{\rm cub}}{P_2^{\rm KG}}\right|_{\rm numeric} \propto \Omega_{\rm p}^{-2.49}$$

$$\left.\frac{P_2^{\rm cub}}{P_2^{\rm KG}}\right|_{\rm analytic} \propto \Omega_{\rm p}^{-5/2}$$

Summary

- EFT methods are well established for UV modifications of gravity
- Unfortunately largely uninteresting phenomenologically except in early universe
- IR EFTs are more interesting - new physics in the IR but also at intermediate scales $\Lambda_n = (m^{n-1} M_{\rm pl})^{1/n}$
- Emerge quite generally in EFTs for broken symmetries
- Common features: additional light states - significant irrelevant operators - emergence of approximate global symmetries - significance of decoupling limit EFT
- IR EFTs are very testable gravitationally - new polarisations - fifth forces - new gravitational waves
- Nonlinear screening mechanisms (e.g. Vainshtein) more poorly understood, don't fit well into PN parameterisations - as yet not well understood how affects e.g. black hole physics