Status & challenges of theoretical & experimental gravitational-wave physics

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Gravitational Waves Ushered in New Era of Astrophysics

- **Discovery** of GW from a binary black-hole merger by LIGO

- **Since GW150914** was observed, 47 more binary black holes (BBH) and two binary neutron stars (BNS) discovered by LIGO/Virgo.

![Masses in the Stellar Graveyard](chart.png)
• With respect to O2, median BNS range increased by a factor 1.64 and 1.53 for LIGOs, and 1.73 for Virgo.
Gravitational-Wave Landscape until ~2030

- From **several tens to hundreds** of binary detections per year.

- **Inference of astrophysical properties of** BHBs, NSBHs and BNSs **in local Universe.**

![Gravitational-Wave Landscape until ~2030](image-url)

(Aasi et al. Living Rev. Rel. 21, 2020)

<table>
<thead>
<tr>
<th>Observation run</th>
<th>Network</th>
<th>Expected BNS detections</th>
<th>Expected NSBH detections</th>
<th>Expected BBH detections</th>
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<td>O3</td>
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<td>$10^{+52}_{-10}$</td>
<td>$1^{+91}_{-1}$</td>
<td>$79^{+89}_{-44}$</td>
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</tbody>
</table>

(Aasi et al. Living Rev. Rel. 21, 2020)
Gravitational-Wave Landscape after 2030 on the Ground and in Space

Einstein Telescope

Cosmic Explorer

Laser Interferometer Space Antenna (LISA)

(3G Science-Case Report 21)

(audley et al. 17)
Outline

- Highlights on science (astrophysical-source properties, tests of General Relativity) from the latest observing run of LIGO and Virgo.

- What have we learned from the “exceptional” GW events of the latest observing run?

- Is a clear picture of the population properties (masses and spins) of compact-object binaries emerging?

- What has been the role of theoretical predictions for the two-body dynamics and gravitational radiation in interpreting signals and unveiling their properties?

- What to do (theoretically) to keep up with the pace at which GW detectors will be improving their sensitivity in the next years and decades?
Gravitational Waves are Fingerprints of Sources and Gravity Theory

- At fixed binary's mass, the lower the GW frequency, the larger the binary's separation, and the earlier the inspiral stage

\[ \omega = \sqrt{\frac{GM}{r^3}} \]

- Binary black holes merge at \( f_{GW} \sim \frac{4400 \text{ Hz}}{M/M_{\odot}} \)

- By comparing to waveforms with deviations from GR, we can probe gravity
Solving Two-Body Problem in General Relativity

- **GR is non-linear theory.**
  \[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]

- Einstein’s field equations can be solved:
  - approximately, but **analytically** (fast way)
  - “exactly”, but **numerically** on supercomputers (slow way)

- **Synergy** between **analytical** and **numerical relativity** is crucial to provide GW detectors with templates to use for searches and inference analyses.

- **Post-Newtonian (PN)** (large separation, and slow motion, bound motion, i.e., early inspiral)
  - expansion in
  \[ \frac{v^2}{c^2} \sim \frac{GM}{rc^2} \]

- **Post-Minkowskian (PM)** (large separation, unbound motion, i.e., scattering)
  - expansion in \( G \)

- **Small mass-ratio** (gravitational self-force, GSF, i.e., early to late inspiral)
  - expansion in \( \frac{m_2}{m_1} \)
Highly Accurate Waveform Models for GW Observations

- **GR is non-linear theory.**
  \[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]

- Einstein’s field equations can be solved:
  - approximately, but analytically (fast way)
  - “exactly”, but numerically on supercomputers (slow way)

- **Synergy** between analytical and numerical relativity is crucial to provide GW detectors with templates to use for searches and inference analyses.

- **Effective-one-body (EOB)** (combines results from all methods, i.e., entire coalescence)

- **Key ideas** of EOB theory inspired by quantum field theory.

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bound orbits: \( v^2/c^2 \sim GM/rc^2 \)

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GW time

- EOB theory
- numerical simulation

inspiral merger ringdown

---

calibration of EOB theory to numerical simulations
Numerical Relativity

- Einstein’s equations solved numerically
  \[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} \]

- 376 GW cycles, zero spins & mass-ratio 7 (8 months, few millions CPU-h)
  (Szilagyi, Blackman, AB, Taracchini et al. 15)

- Public Simulating eXtreme Spacetimes (SXS) NR catalog.
  (Boyle et al. 19, Ossokine et al. 20)

- Other public NR catalogs.
  (Husa et al. 15, Jani et al. 17, Healy et al. 17, 19, 20)
EOB Hamiltonian: Resummed Conservative Dynamics

- **Real Hamiltonian**

\[ H_{\text{real}}^{\text{PN}} = H_{\text{Newt}} + H_{1\text{PN}} + H_{2\text{PN}} + \cdots \]

- **Effective Hamiltonian**

\[ H_{\text{eff}}^{\nu} = \mu \sqrt{A_\nu(r) \left[ 1 + \frac{p^2}{\mu^2} + \left( \frac{1}{B_\nu(r)} - 1 \right) \frac{p_r^2}{\mu^2} \right]} \]

- **EOB Hamiltonian**

\[ H_{\text{real}}^{\text{EOB}} = M \sqrt{1 + 2\nu \left( \frac{H_{\text{eff}}^{\nu}}{\mu} - 1 \right)} \]

- **Dynamics condensed** \( A_\nu(r) \) and \( B_\nu(r) \)

- \( A_\nu(r) \), which encodes the energetics of circular orbits, is quite simple:

\[ A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3}{3r^3} + \left( \frac{94}{3} - \frac{41}{32}\pi^2 \right) \frac{M^4}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu) \log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots \]

5PN unknown as today (credit: Hinderer)
EOB Conservative Spin Dynamics & Waveforms

- **EOB equations of motion** (AB et al. 00, 05; Damour et al. 09):

\[
\dot{r} = \frac{\partial H^\text{EOB}_{\text{real}}}{\partial p} \quad F \propto \frac{dE}{dt}, \quad \frac{dE}{dt} \propto \sum_{\ell m} |h_{\ell m}|^2
\]

\[
\dot{p} = -\frac{\partial H^\text{EOB}_{\text{real}}}{\partial r} + F \quad \dot{S} = \{S, H^\text{EOB}_{\text{real}}\}
\]

- **EOB inspiral waveforms** (AB et al. 00; Damour et al. 09, 11; Pan, AB et al. 11):

\[
h^\text{insp-plunge}_{\ell m} = h^\text{Newt}_{\ell m} e^{-i\delta \Phi} S_{\text{eff}} T_{\ell m} e^{i\delta \ell m} (\rho_{\ell m})^\ell h^\text{NQC}_{\ell m}
\]

- **EOB merger-ringdown waveform is a superposition of quasi-normal modes.**

  (AB & Damour 00, AB et al. 07, Damour & Nagar 07, Del Pozzo & Nagar 17, Bohé et al. 17)
Completing EOB Waveforms with NR & Perturbation Theory Information

- We calibrate EOB to **inspiral-merger-ringdown NR** waveforms.

- We calibrate EOB to **merger-ringdown waveforms in test-body limit**.

Calibration of SEOBNR for O2-O3 searches and inference studies

\( \chi_1 = \frac{S_1}{m_1^2} \)

\( \chi_2 = \frac{S_2}{m_2^2} \)

\( \chi_{\text{eff}} = \frac{m_1}{M} \chi_1 + \frac{m_2}{M} \chi_2 \)

(Pan, AB et al. 13, Taracchini, AB, Pan, Hinderer & SXS 14, Pürrer 15)

(Bohé, Shao, Taracchini, AB & SXS 17, Babak et al. 16; Cotesta et al. 18, 20, Ossokine et al. 20)

(see also Damour & Nagar 14, Nagar et al. 18, Nagar, Messina et al. 19, Nagar, Pratten et al. 20, Nagar, Riemenschneider et al. 20, Riemenschneider et al. 21)
**Fast, frequency-domain** waveform model hybridizing EOB & NR waveforms, and then fitting.

(Schmidt et al. 12; Hannam et al. 13; Khan et al. 15; Husa et al. 15; Khan et al. 18-19; García-Quiros et al. 20, Pratten et al. 20)

\[ \tilde{h}(f; \lambda_i) = A(f; \lambda_i) e^{i\phi(f; \lambda_i)} \]

**NR surrogate models** are built directly by interpolating NR simulations, which are selected in parameter space using analytical waveform models.

**Highly accurate**, but limited in binary’s parameter space and length (~20 orbits), unless hybridized with EOBNR waveforms.

**IMRPhenom**

**NRSur**
Waveform Models with Spin Precession & High Harmonics

• GWTC-2 used for the first time multipolar, spin precessing waveform models.

(Khan et al. 19, Varma et al. 19, Ossokine et al. 20)

\[
\begin{align*}
    h_+(t; \Theta, \varphi) - i h_\times(t; \Theta, \varphi) &= \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} -2Y_{\ell m}(\Theta, \varphi) h_{\ell m}(t)
\end{align*}
\]

(credit: Cotesta)

\(\chi_1 = \chi_2 = 0\) (Cotesta, AB et al. 18)

\(\chi_1 = S_1/m_1^2\)
\(\chi_2 = S_2/m_2^2\)
\(q = m_1/m_2\)

• Waveforms from BBHs (BNSs & NSBHs) are described by 15 (17 & 16) parameters.

(Ossokine, AB, Cotesta, Marsat et al. 20)

SEOBNRv4P
Unfaithfulness: \(5.9 \times 10^{-2}\)

SEOBNRv4PHM
Unfaithfulness: \(8.5 \times 10^{-3}\)
Highlights from O3a Run as we Explore the Universe

GW190814: a binary with a puzzling companion

$$m_1 = 23.2^{+1.1}_{-1.0} M_\odot \quad m_2 = 2.59^{+0.08}_{-0.09} M_\odot$$

- The more substructure and complexity the binary has (e.g., masses or spins of BHs are different) the richer is the spectrum of radiation emitted.

$$h_+(t; \Theta, \varphi) - i h_\times(t; \Theta, \varphi) = \sum_{\ell=2}^{\infty} \sum_{m=-\ell}^{\ell} -2 Y_{\ell m}(\Theta, \varphi) h_{\ell m}(t)$$

(credit: Cotesta)
GW190814: a Binary with a Puzzling Companion

Systematics due to waveform modeling smaller than statistical errors.

Either the largest neutron star or the smallest black hole.

More massive BH rotated with $\chi_1 < 0.07$.

Systematics due to waveform modeling smaller than statistical errors.

Using waveform models with higher-modes and spin-precession constrains more tightly the secondary mass.
GW190412: a signal like none before

\[ q = 0.28^{+0.12}_{-0.06} \, M_\odot \]

• Binary black hole with mass asymmetry, and more massive BH spinning at about \( \chi_1 \sim 0.4 \) with tilt angle \( \theta_1 \sim 45^\circ \).

• More massive BH rotated with spin \( 0.17 - 0.59 \) at 90% CI
GW190412: a signal like none before

\[ q = 0.28^{+0.12}_{-0.06} M_\odot \]

- Binary black hole with mass asymmetry, and more massive BH spinning at about \( \chi_1 \sim 0.4 \) with tilt angle \( \theta_1 \sim 45^\circ \).

- Spin-precession and non-quadrupole modes enable tighter inferences

- Systematics due to waveform modeling are not negligible when spins and higher modes are relevant.

(credit: Fischer, Pfeiffer & AB; SXS Collaboration)
GW190521: a Signal Produced by the Largest BHs so far

GW190521: a signal produced by the largest BHs so far

\[ m_1 = 91.4^{+29.3}_{-17.5} M_\odot \quad m_2 = 66.8^{+20.7}_{-20.7} M_\odot \]

• Likely, BHs too massive to have been formed from a collapsed star, because of Pair-Instability SN.

\[ q = m_1/m_2 \]
\[ \chi_1 = S_1/m_1^2 \]
\[ \chi_2 = S_2/m_2^2 \]
\[ \chi_{\text{eff}} = \left( \frac{m_1}{M} \chi_1 + \frac{m_2}{M} \chi_2 \right) \cdot \hat{L} \]

(Systematics due to waveform modeling are not negligible when spin precession and higher modes are relevant, but they are still subdominant with respect to statistical uncertainty.

(credit: Fischer, Pfeiffer & AB; SXS Collaboration)

It could be a merger of a BH of about $6M_\odot$ with a NS.

Gravitational-Wave Transient Catalog 2: Source Properties

- Higher total masses than GWTC-1

- Three binaries with component mass in the lower mass gap \( \sim 2.5 - 5M_\odot \)

- Three binaries with component mass larger than \( \sim 45M_\odot \), i.e. in pair-instability SN mass gap

- At 95% credibility, ten sources have \( \chi_{\text{eff}} > 0 \)

\[
q = \frac{m_1}{m_2}
\]

\[
\chi_1 = \frac{S_1}{m_1^2} \quad \chi_2 = \frac{S_2}{m_2^2}
\]

\[
\chi_{\text{eff}} = \left( \frac{m_1}{M} \chi_1 + \frac{m_2}{M} \chi_2 \right) \cdot \hat{L}
\]

\( \chi_p \) measures the spin components on the orbital plane

Assessing accuracy of BBH waveform models against NR

(Ossokine, AB, Marsat, Cotesta et al. 20)

• Mismatch against SXS NR public catalog (1344) of multipolar spin-precessing waveforms.

• Waveform models are calibrated in the non-precessing sector.

• Mismatch between two multipolar spin-precessing waveforms.

• Waveform models differ the most for large mass ratios (> 10) and large spins (> 0.6) and stronger precession.
Assessing accuracy of waveform models with Bayesian analysis

(Ossokine, AB, Marsat, Cotesta et al. 20)

NR signal is injected in O4 run:

\[ M = 76M_\odot, q = 6, \chi_1 = (-0.06, 0.78, -0.47), \chi_2 = (0.08, -0.17, -0.23) \]

edge on: \( \Theta = \pi/2 \)

SEOBNRv4PHM unfaithfulness = 4.4 %

IMRPhenomPv3HM unfaithfulness = 8.8 %

SNR = 21

(see also Pürrer and Haster 20)
Probing Extreme-Matter with Gravitational Waves

• **Neutron-star (NS) properties:**
  - mass: $1 - 3 M_{\odot}$
  - radius: $9 - 15$ km
  - inner core density > $2 \times (2.8 \times 10^{14})$ g/cm$^3$
  - magnetic field: $\sim 10^{15} \times \text{@Earth}$
  - surface temperature: $\sim 10^3 \times \text{@Earth}$
  - pressure: $\sim 10^{27} \times \text{@Earth}$

• What is the **internal structure and composition** of neutron stars?

• **Conjectured states of matter.**

• **NS equation of state (EOS) affects gravitational waveform** during late inspiral, merger and post-merger.

(credit: Hinderer)
**Probing Extreme-Matter with Gravitational Waves (contd.)**

- **Neutron-star (NS)** properties:
  - mass: $1 - 3 \, M_{\text{Sun}}$
  - radius: $9 - 15 \, \text{km}$
  - inner core density: $> 2 \times (2.8 \times 10^{14}) \, \text{g/cm}^3$
  - magnetic field: $\sim 10^{15} \times @\text{Earth}$
  - surface temperature: $\sim 10^3 \times @\text{Earth}$
  - pressure: $\sim 10^{27} \times @\text{Earth}$

- What is the **internal structure** and **composition** of neutron stars?

**NS equation of state (EOS) affects gravitational waveform during late inspiral, merger and post-merger.**

- Binary neutron star
- Black-hole binary

**Signature of tidal deformations in NS**

(credit: Hinderer)
Waveforms for BNS combining Analytical & Numerical Relativity

• Synergy between analytical and numerical work is crucial.

(Damour 1983, Flanagan & Hinderer 08, Binnington & Poisson 09, Vines et al. 11, Damour & Nagar 09, 12, Bernuzzi et al. 15, Hinderer, …AB … et al. 16, Steinhoff, … AB … et al. 16, Dietrich et al. 17-19, Nagar et al. 18)
GW190425: a Binary Neutron Star with Surprisingly High Mass

$$\chi_1 = \frac{S_1}{m_1^2}$$

$$\chi_2 = \frac{S_2}{m_2^2}$$

$$\chi_{\text{eff}} = \left( \frac{m_1}{M} \chi_1 + \frac{m_2}{M} \chi_2 \right) \cdot \hat{L}$$

- GW190425’s masses are consistent with mass measurements of NSs in binaries.

- GW190425’s total mass $3.4^{+0.3}_{-0.1}M_\odot$ is larger than BNSs in our galaxy: new population of BNS?
GW190425’s SNR is lower ($\sim 13$) than GW170817’s SNR ($\sim 34$): loosen constraint on tidal deformability.
Assessing accuracy of BNS waveform models against NR

• Synthetic GW signal of a binary neutron star at 50 Mpc is injected in Gaussian noise with O4 noise-spectral density (SNR ~ 87).

• Inference with waveform models that have same matter effects, but baseline point-mass model is different.
  - IMRDNRT is injected in O4 run

![Graph showing probability density distributions for equal- and unequal-mass systems](image)

(see also Dudi et al. 18, Samajdar & Dietrich 19, Gamba et al. 20)

• Systematics larger than statistical errors!

• To obtain NS’s radius with precision of 0.5-1 km, more accurate waveforms are needed.
• Synergy between analytical and numerical work is crucial.

Matas, Dietrich, AB et al. 2020
(see also Lackey et al. 14, Pannarale et al. 15, 16, Pürrer et al. 17, Chakravarti et al. 17)

(see also Thompson et al. 2020)

• So far, NSBH waveforms were used only for inference study of GW190814.

• More accurate NSBH waveforms are needed for future runs (O4, O5, CE and ET).
Waveforms for Eccentric Compact-Object Binaries

• How to discriminate among binary’s formation scenarios, and probe astrophysical environment? Eccentricity, spin-magnitude and spin-precession can disclose this information.

• Eccentric compact-object binary:

NR simulation

mass ratio = 7

(Lewis et al. 16)

(many PN papers on eccentricity; Bini et al. 12; East et al. 13; Huerta et al. 14-19, Hinder et al. 17; Cao & Han 17; Loutrel & Yunes 16, 17; Ireland et al. 19, Moore & Yunes 19, Chiaramello & Nagar 20, Buades et al. 20, Liu et al. 21, Nagar et al. 20, 21, Islam et al. 21, Khalil et al. 21)

• Current eccentric models do not cover all physical effects (e.g., spin-precession and harmonics) or all stages of coalescence or entire range of eccentricity, accurately (but a lot of progress in last months!).
Scattering Amplitude: A New Way to Study 2-body Problem

- Relativistic 2-body dynamics
- Classical scattering: scattering angle $\chi$
- Quantum scattering amplitude

**Classical scattering: scattering angle $\chi$**

**Quantum scattering amplitude**

- Classical scattering: scattering angle $\chi$
- Quantum scattering amplitude

**Local in-time 2-body Hamiltonian at 4PM (3 loops)** for nonspinning BHs. (Cheung et al. 19, 20, Bern et al. 19, Blümlein et al. 20, Kälin et al. 20, Bern et al. 21)

**Small parameter** is $GM/rc^2 \ll 1$, $v^2/c^2 \sim 1$, large separation, natural for unbound motion/scattering

$$H(p, r) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + V(p, r)$$

$$E = E_1 + E_2 \quad \gamma = E/m$$

$$\xi = E_1 E_2/E^2 \quad \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}.$$
Results from Interplay with Scattering Amplitude Methods & EFT

(Local in-time) 2-body Hamiltonian at 4PM (3 loops) for nonspinning BHs.

Small parameter is $GM/rc^2 << 1$, $v^2/c^2 \sim 1$, large separation, natural for unbound motion/scattering

$H(p, r) = \sqrt{p^2 + m_1^2} + \sqrt{p^2 + m_2^2} + V(p, r)$

$E = E_1 + E_2 \quad \gamma = E/m$

$\xi = E_1E_2/E^2 \quad \sigma = \frac{p_1\cdot p_2}{m_1m_2}$

$V(p, r) = \sum_{i=1}^{\infty} c_i(p^2) \left( \frac{G}{|r|} \right)^i$

$V^{(1)}(p, q) = \int \frac{d^3r}{(2\pi)^3} M_{\text{tree}}(p, q) e^{-ir\cdot q}$

$c_1 = \frac{\nu^2m^2}{\gamma^2\xi} (1 - 2\sigma^2)$

(Bern et al. 19, 20)

(Cheung et al. 19, 20, Bern et al. 19, Blümlein et al. 20, Kälin et al. 20, Bern et al 21)
Comparison between PMs and NR binding energies

- 2-body non-spinning (local-in-time) Hamiltonian at 4PM order computed using scattering-amplitude methods. (Cheung et al. 18, Bern et al. 19, Bern et al. 21)

- Crucial to push PM calculations at higher order, and resum them in EOB formalism. (Damour 19, Antonelli, AB, Steinhoff, van de Meent & Vines 19, Khalil, AB, Steinhoff & Vines in prep 21)

Encouraging (local-in-time) 4PM results!
• 2-body spin-orbit (SO) Hamiltonian at 4.5PN computed using EFT or interplay between bound and unbound orbits, and gravitational self-force results. (Levi et al. 20, Antonelli et al. 20)

• 2-body non-spinning Hamiltonian at 5PN & 6PN partially computed using EFT or interplay between bound and unbound orbits, and gravitational self-force results. (Foffa et al. 19, Blümlein et al. 20, Damour 20, Bini, Damour & Geralico 20)

• 2-body Hamiltonian at 2PM (1 loop) for spinning, precessing BHs. (Bini et al. 17, 18, Vines 18, Bern et al. 20, Kosmopoulos & Luna 21, Liu et al. 21)

• Tidal effects in 2-body dynamics in PM expansion (Cheng & Solon 20, Kälin et al. 20)

• Leading-order radiation in PM expansion (Damour 20, Di Vecchia et al. 20, 21, Jakobsen et al. 21, Herrmann et al. 21)
Summary & Outlook

• Observing gravitational waves and inferring astrophysical/physical information hinges on our ability to make highly precise predictions of two-body dynamics and gravitational radiation.

• With O1 & O2 we observed the "tip of the iceberg" of the binary population, with the improved detectors’ sensitivity, O3a has unveiled a richer picture and several “exceptional" sources.

• Crucial to improve waveform models for BBHs and binaries with matter for LIGO and Virgo upcoming runs and for future detectors (Cosmic Explorer, Einstein Telescope & LISA). Waveform accuracy would need to be improved by one or two orders of magnitude depending on the parameter space.

• EFT and scattering-amplitudes methods have brought new and fresh perspectives (and tools) to solve the relativistic two-body problem, unveiling new paths to intertwine the different perturbative approaches (PN, PM and GSF).

• The impact of recent PN-PM results for GW observations have not yet been fully assessed, but they will as we develop new waveform models for the upcoming O4 run.

• Unique opportunity for theoretical particle physicists to contribute!
Toward High-Precision Gravitational-Wave Astrophysics

**Conservative dynamics**

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<th>PN order</th>
<th>1.5</th>
<th>2.5</th>
<th>3.5</th>
<th>4.5</th>
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<td>NLO SO</td>
<td>N2LO SO</td>
<td>N3LO SO</td>
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</table>

**Plus radiation!**

N.B. Resummation methods (e.g., EOB) can accelerate accuracy.

Goal: 2-body dynamics and radiation (including waveforms) for spinning objects through 6PN order.

*(High precision is needed but it is not necessary that results are provided in analytical form.)*
Thank You!

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