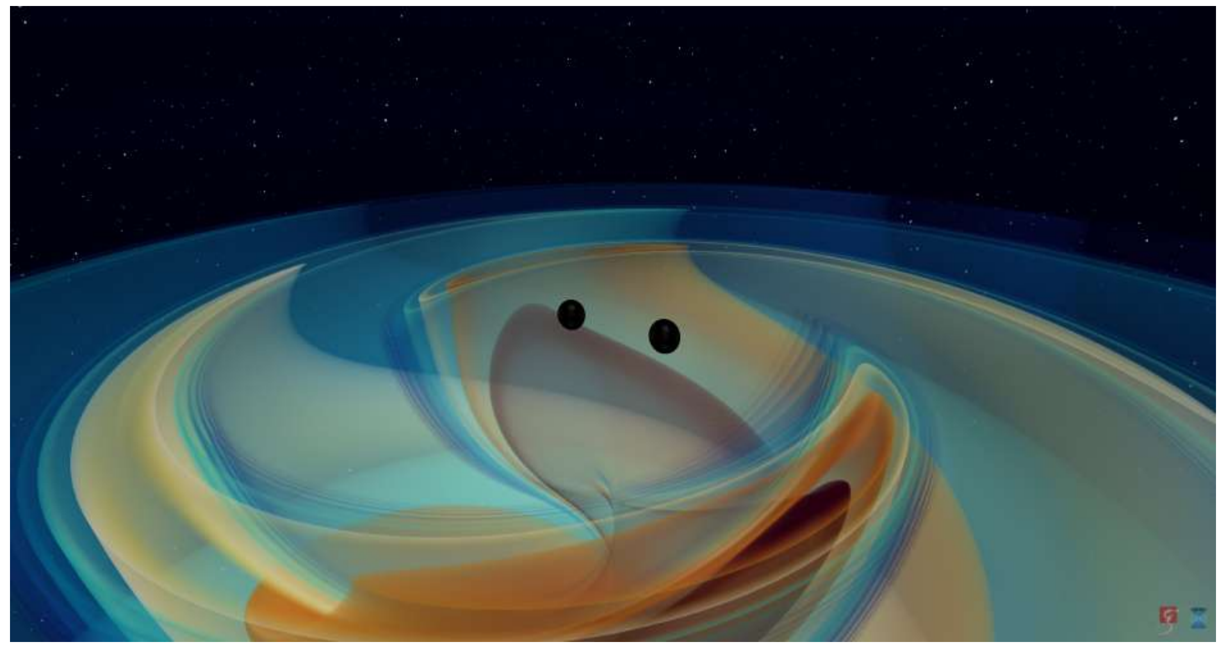
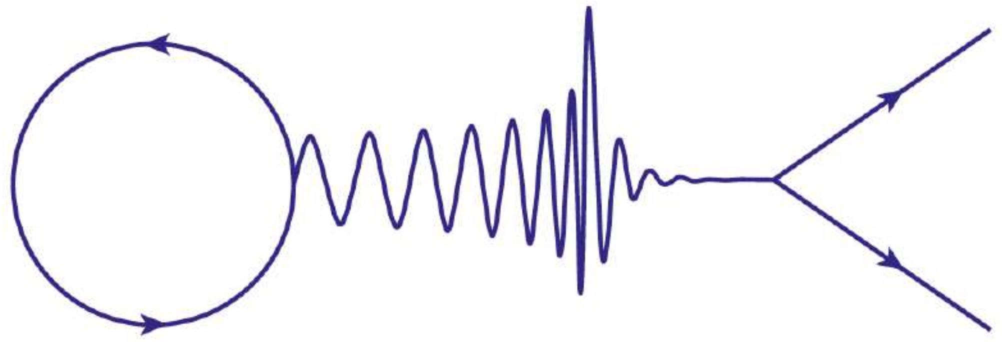
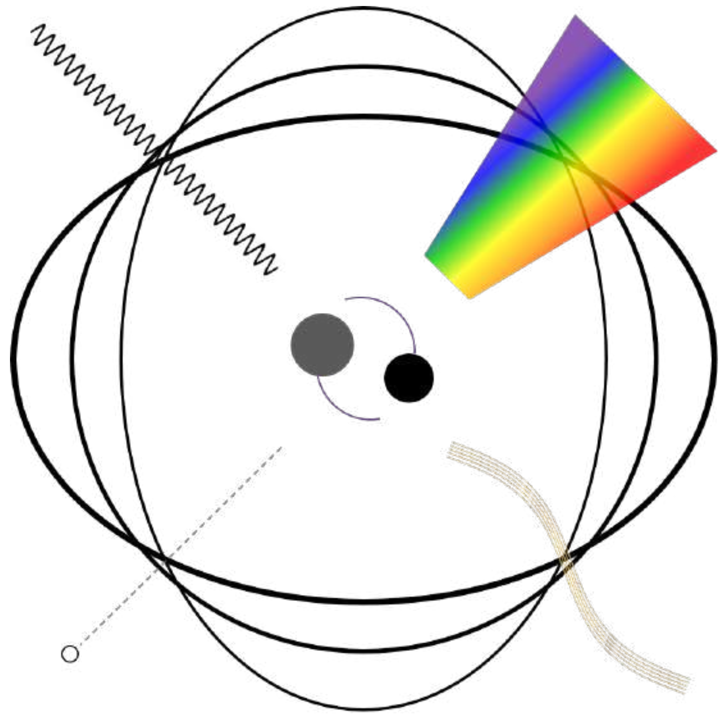
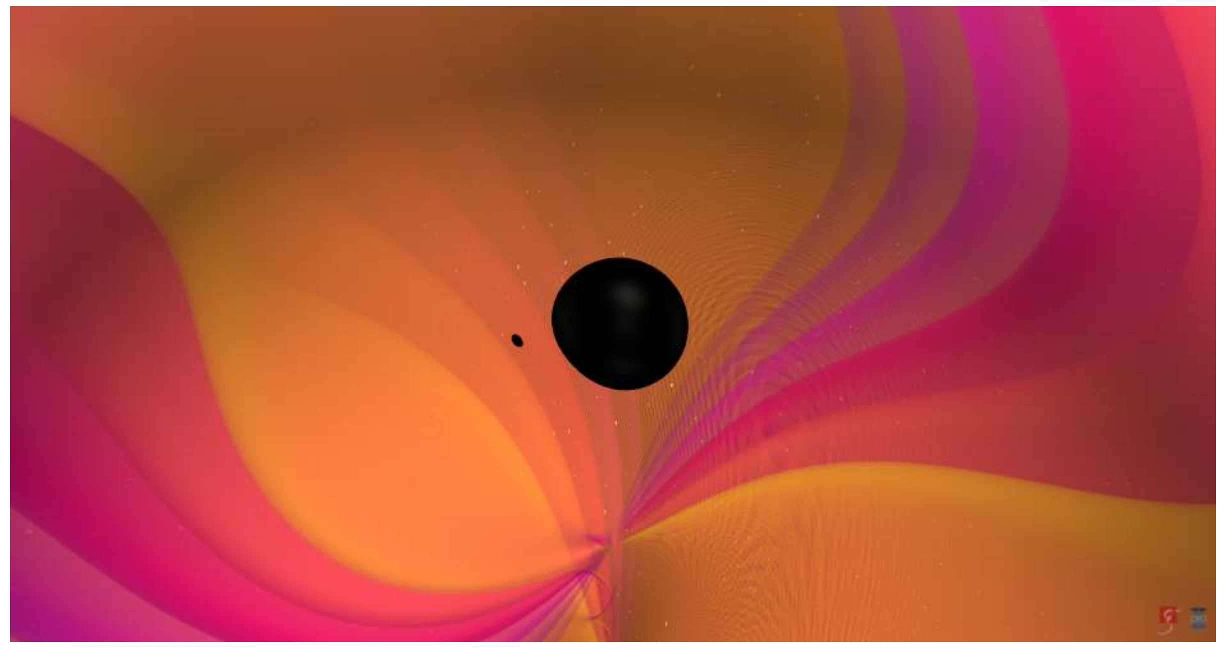
GW190521







GW190814



Status & challenges of theoretical & experimental gravitational-wave physics

Alessandra Buonanno

Max Planck Institute for Gravitational Physics

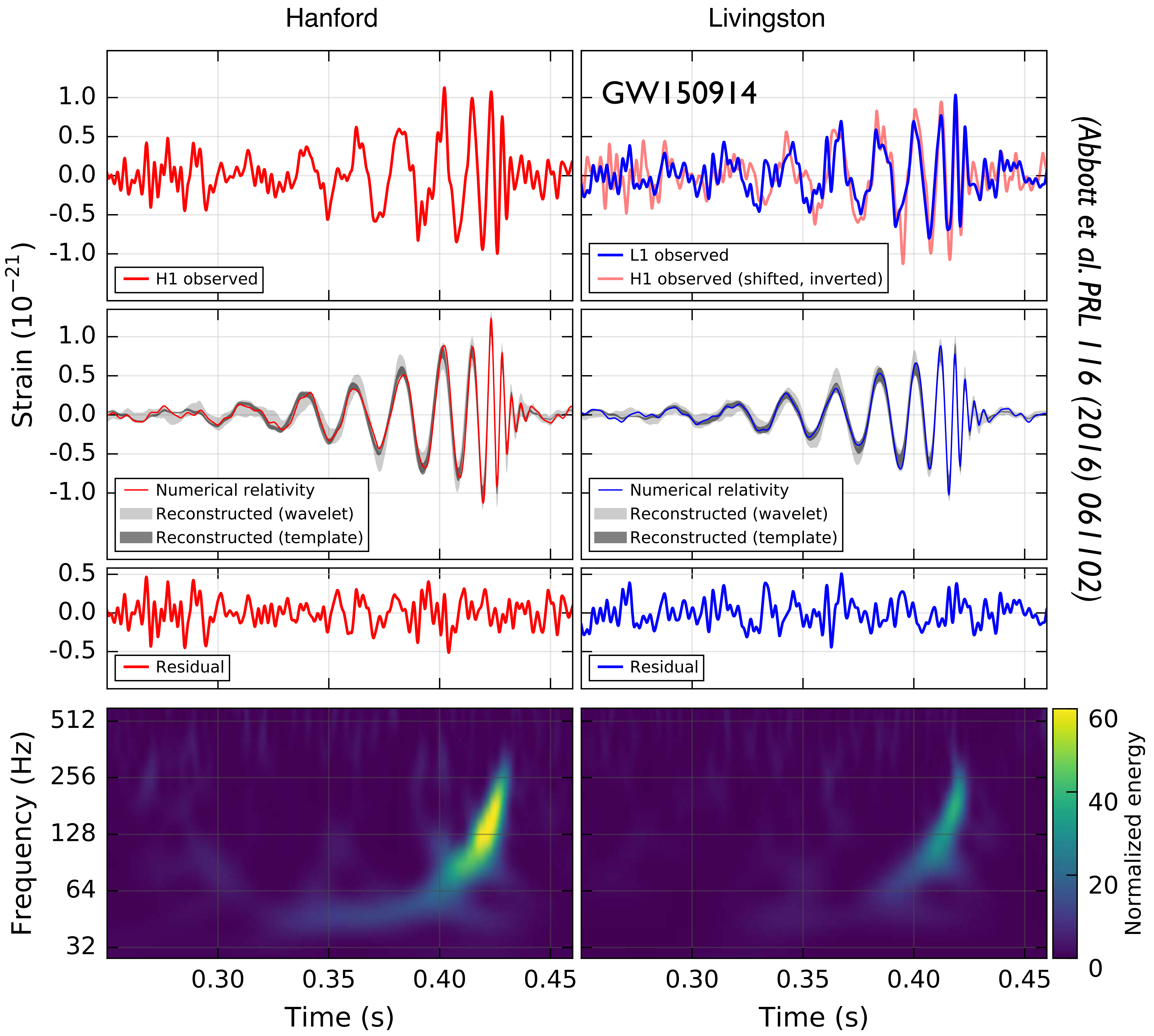
(Albert Einstein Institute)

Department of Physics, University of Maryland

“Gravitational Scattering, Inspiral and Radiation”, (virtual) Conference, GGI, Florence   April 26, 2021

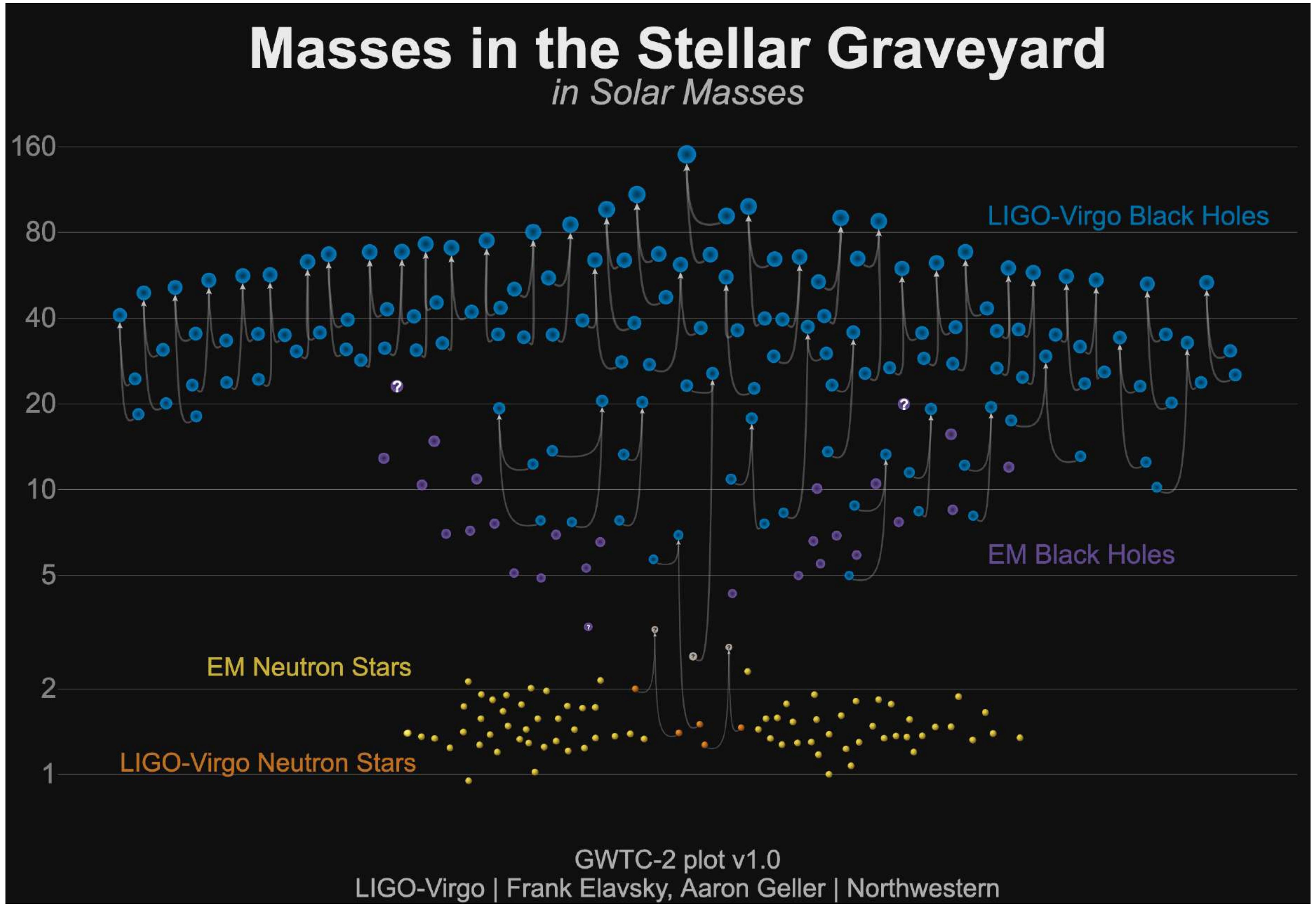
Gravitational Waves Ushered in New Era of Astrophysics

- **Discovery** of **GW** from **a binary black-hole merger** by LIGO

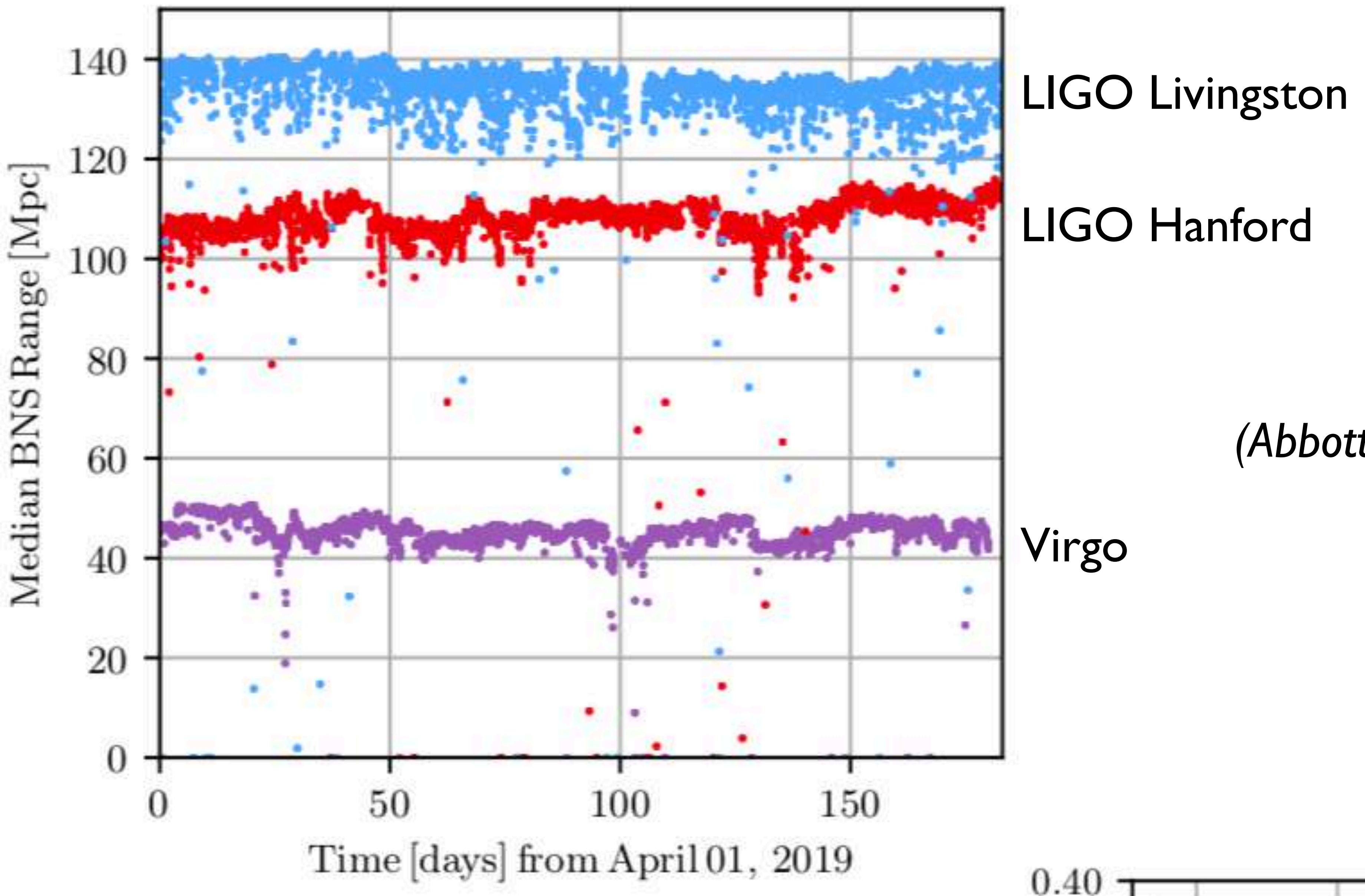


(Abbott et al. PRL 116 (2016) 061102)

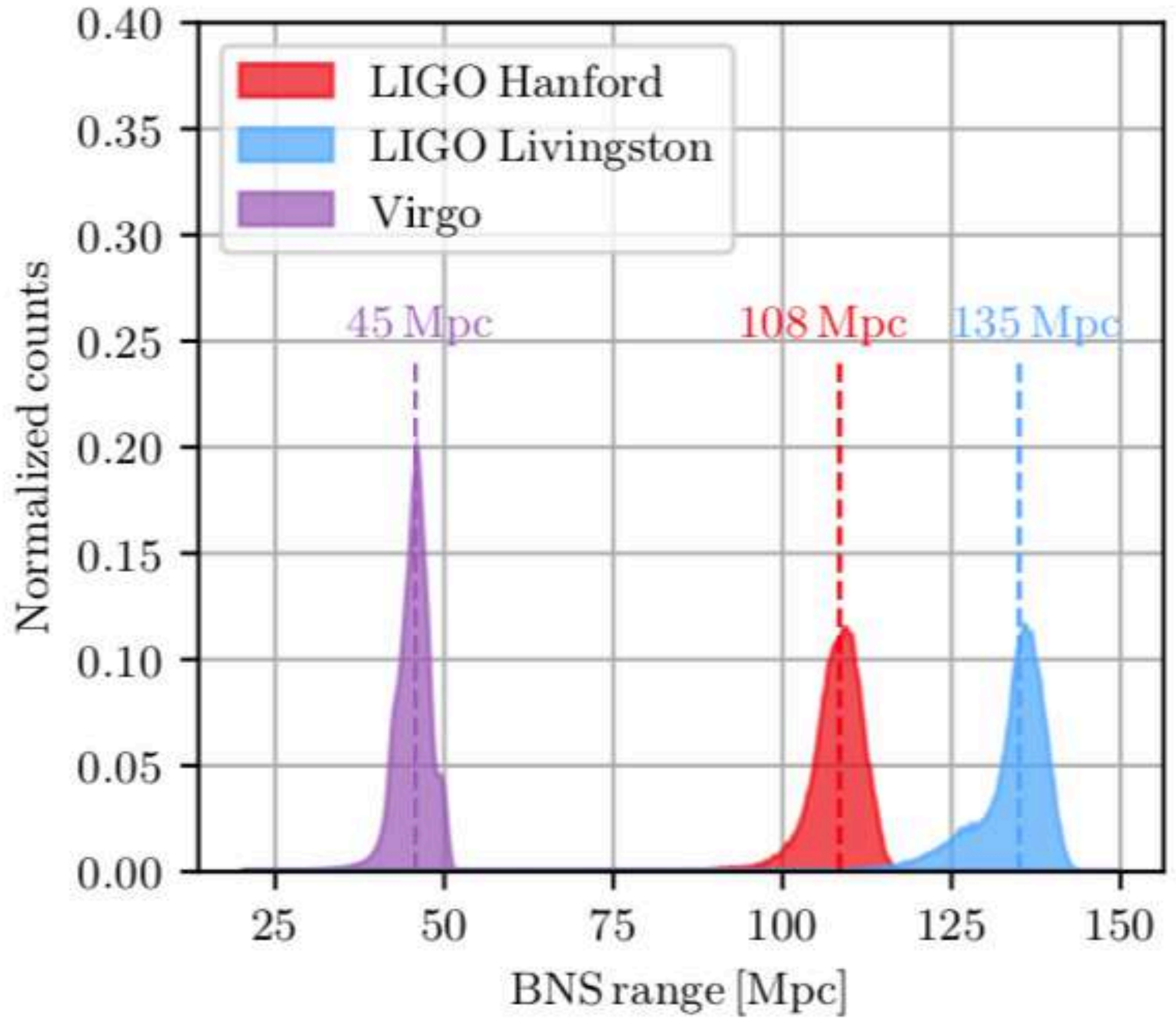
- **Since GW150914** was observed, **47 more binary black holes (BBH)** and **two binary neutron stars (BNS) discovered** by LIGO/Virgo.

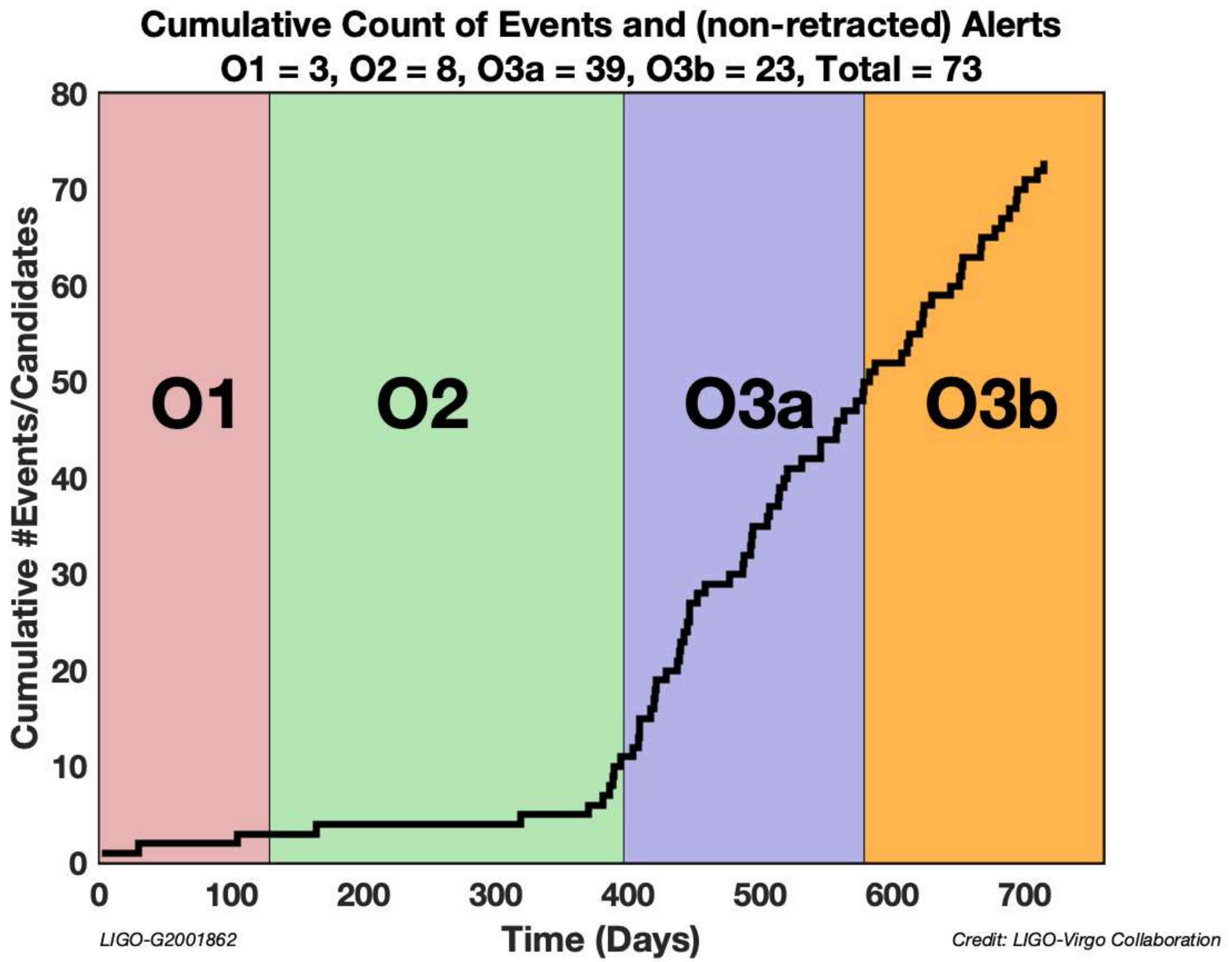


On the Third-Observing (O3) run of LIGOs and Virgo



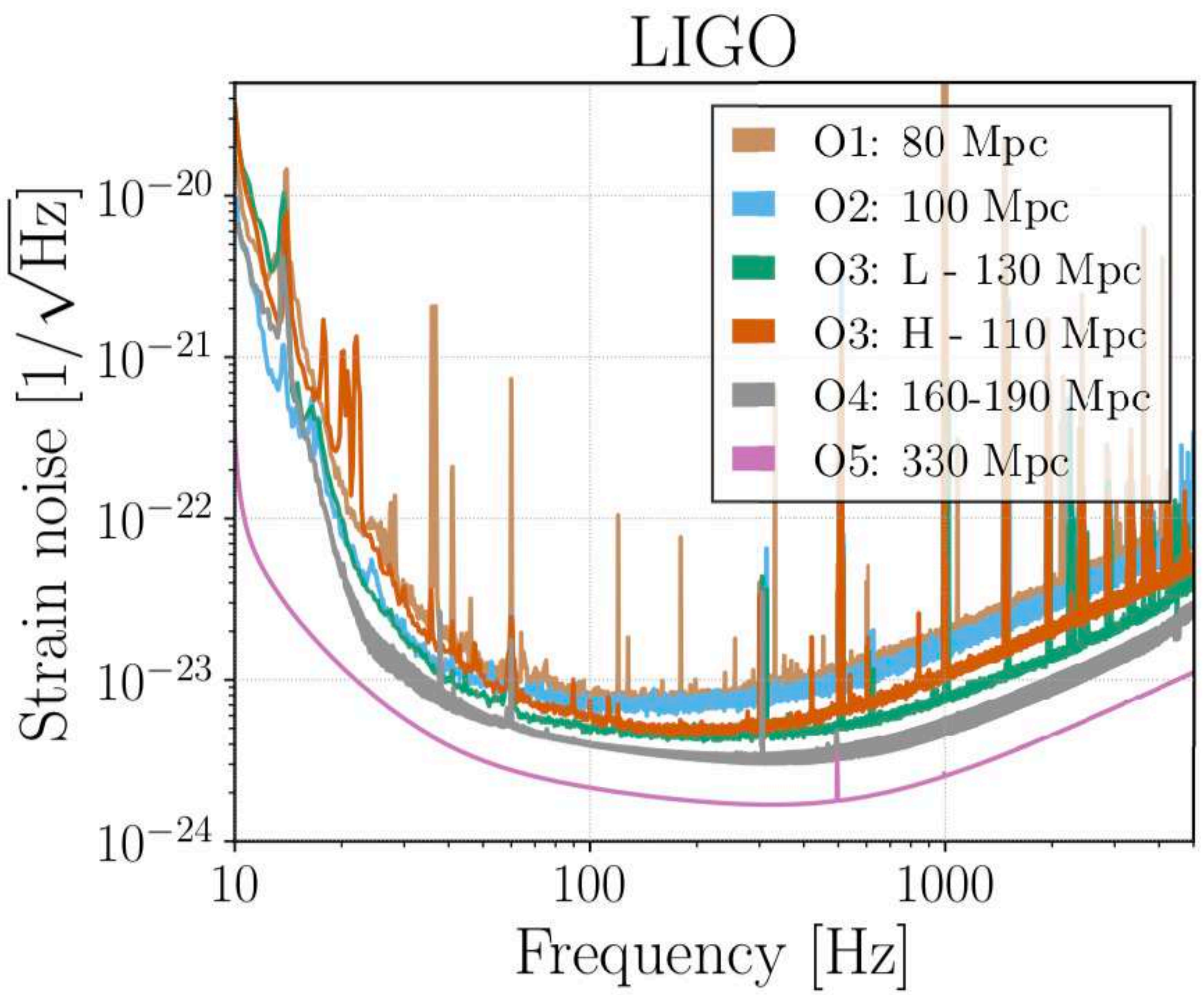
(Abbott et al. arXiv:2010.14527)



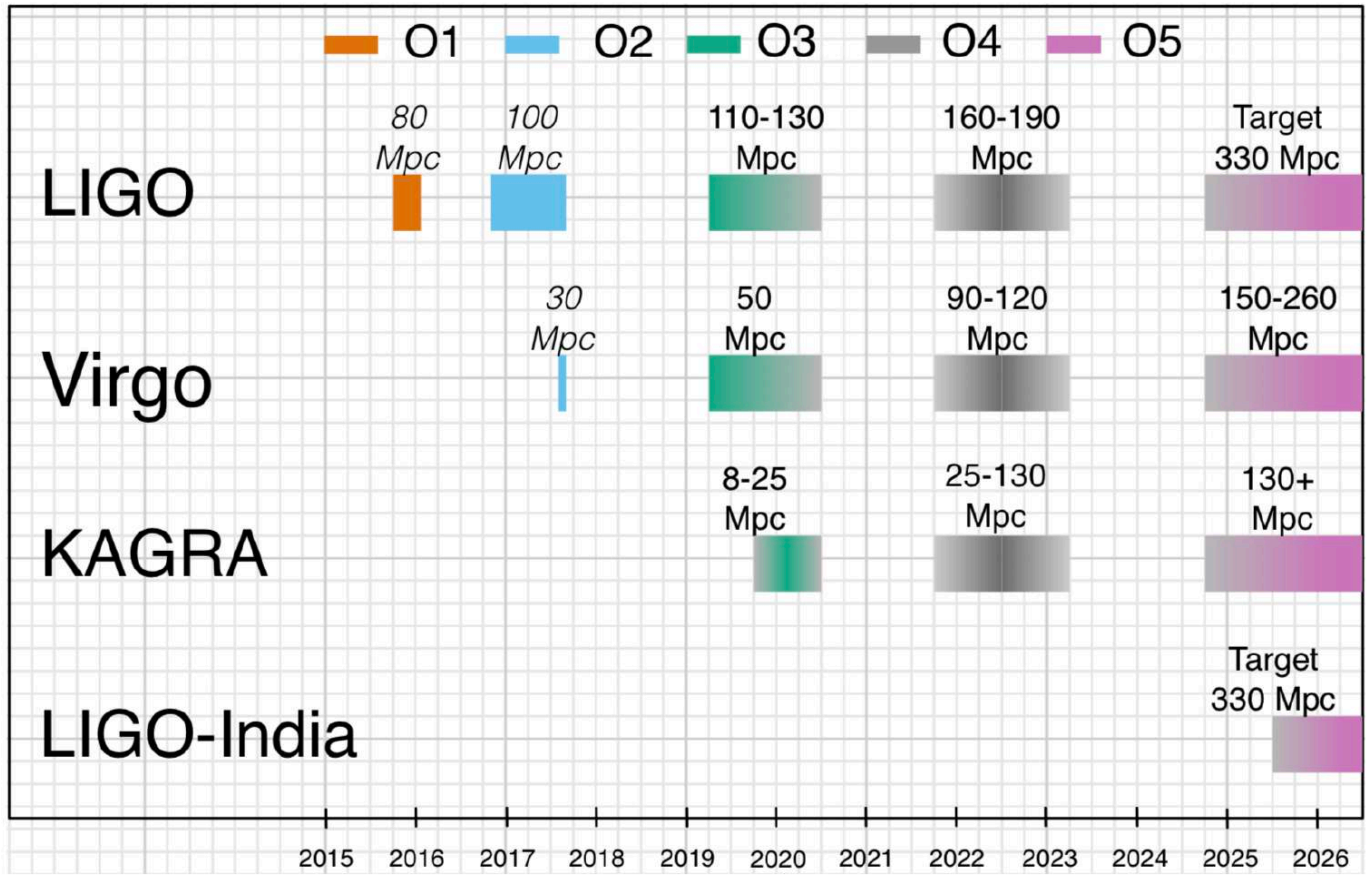


- With respect to O2, **median BNS range increased** by a factor **1.64 and 1.53** for **LIGOs**, and **1.73 for Virgo.**

Gravitational-Wave Landscape until ~2030



(Aasi et al. Living Rev. Rel. 21, 2020)



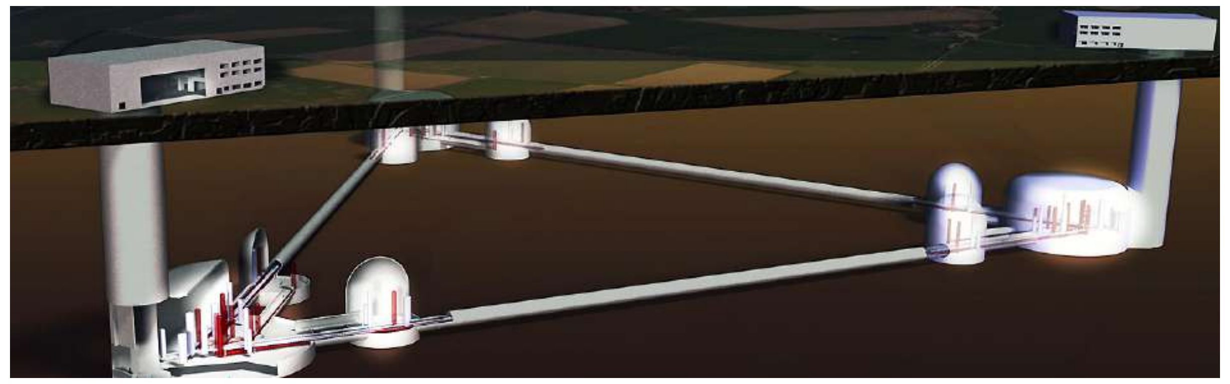
- From **several tens to hundreds** of binary detections per year.
- Inference of **astrophysical properties** of BHBs, NSBHs and BNSs **in local Universe.**

Observation run	Network	Expected BNS detections	Expected NSBH detections	Expected BBH detections
O3	HLV	1^{+12}_{-1}	0^{+19}_{-0}	17^{+22}_{-11}
O4	HLVK	10^{+52}_{-10}	1^{+91}_{-1}	79^{+89}_{-44}

(Aasi et al. Living Rev. Rel. 21, 2020)

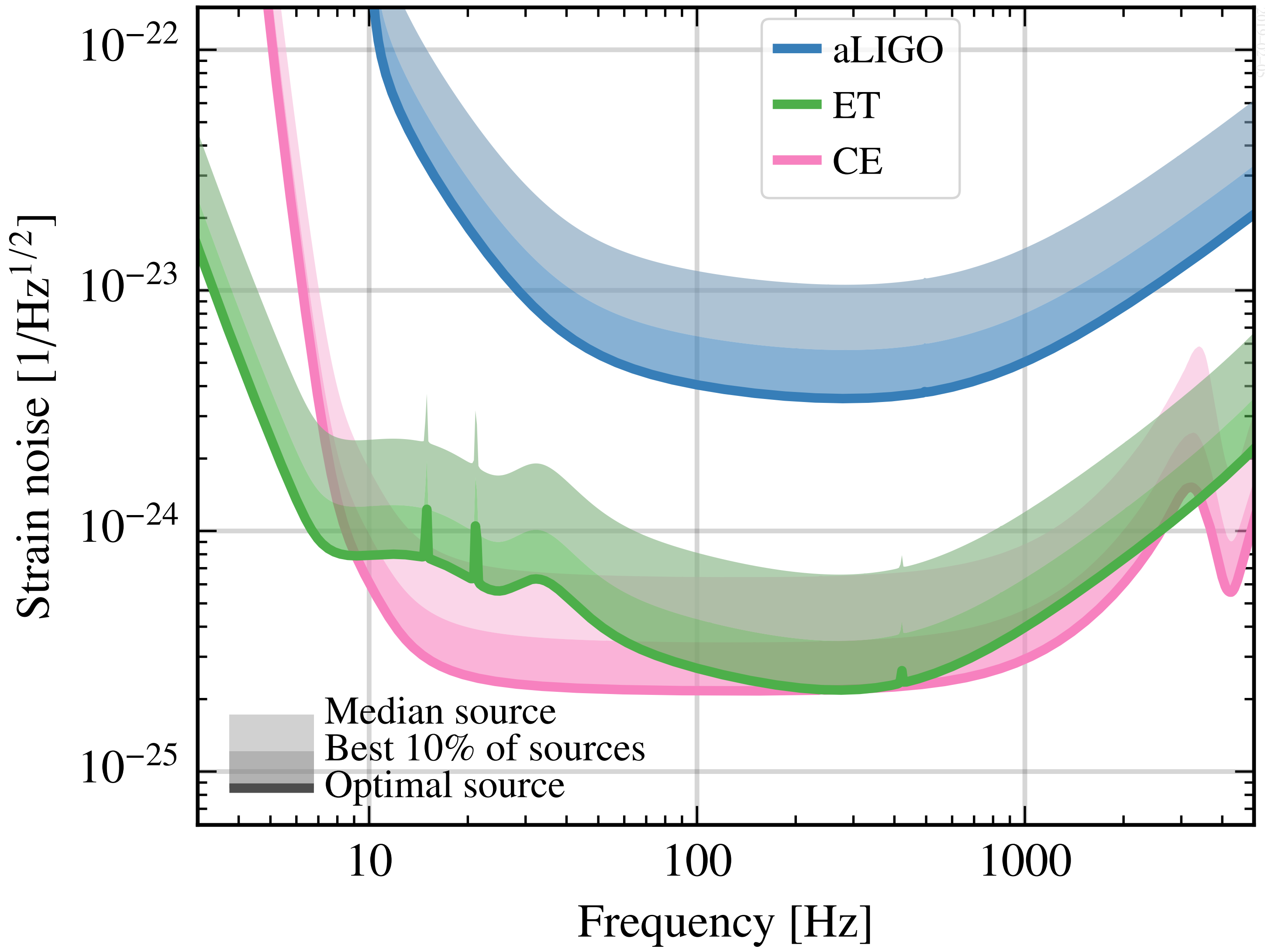
Gravitational-Wave Landscape after 2030 on the Ground and in Space

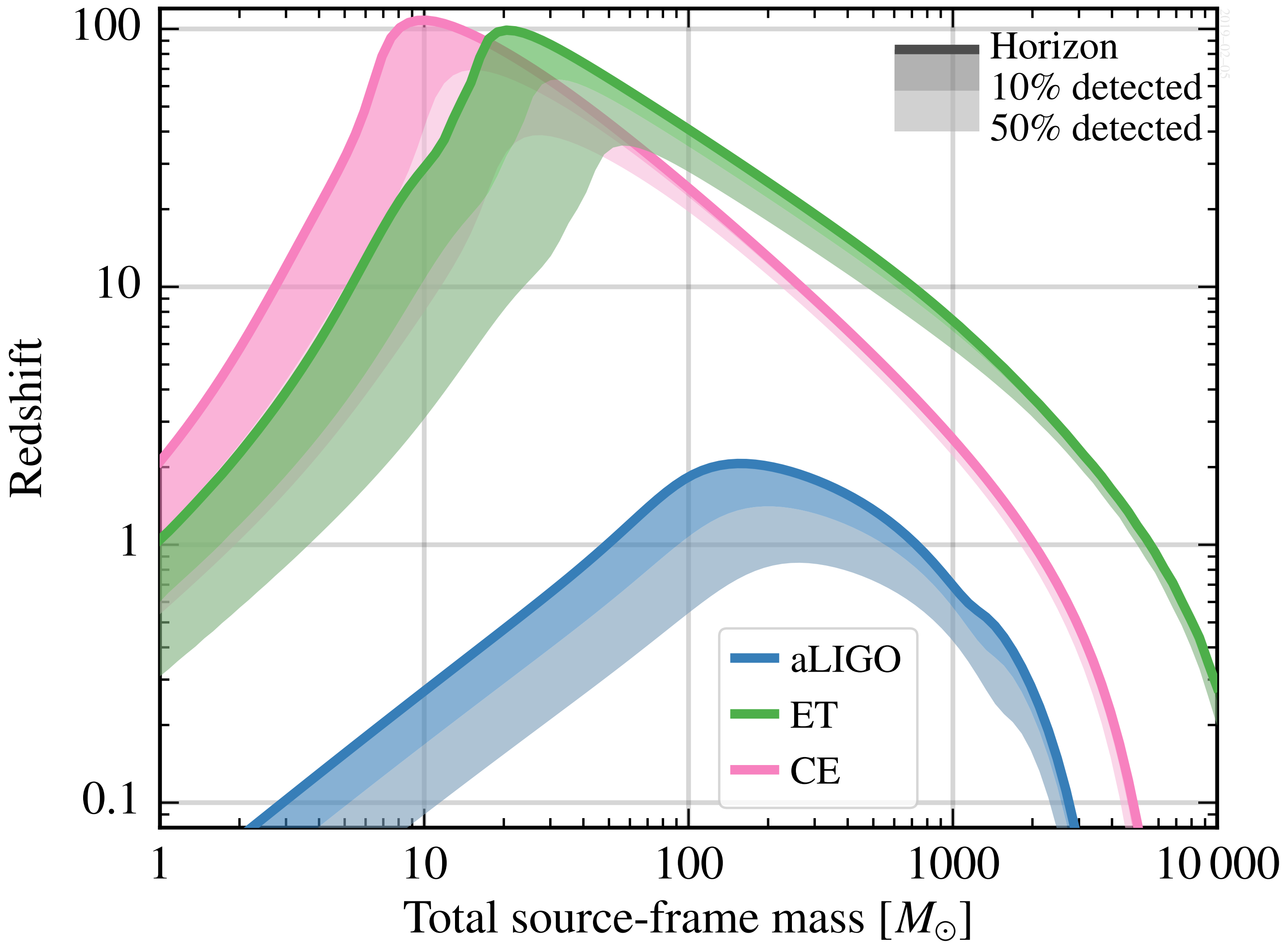
Einstein Telescope



Cosmic Explorer

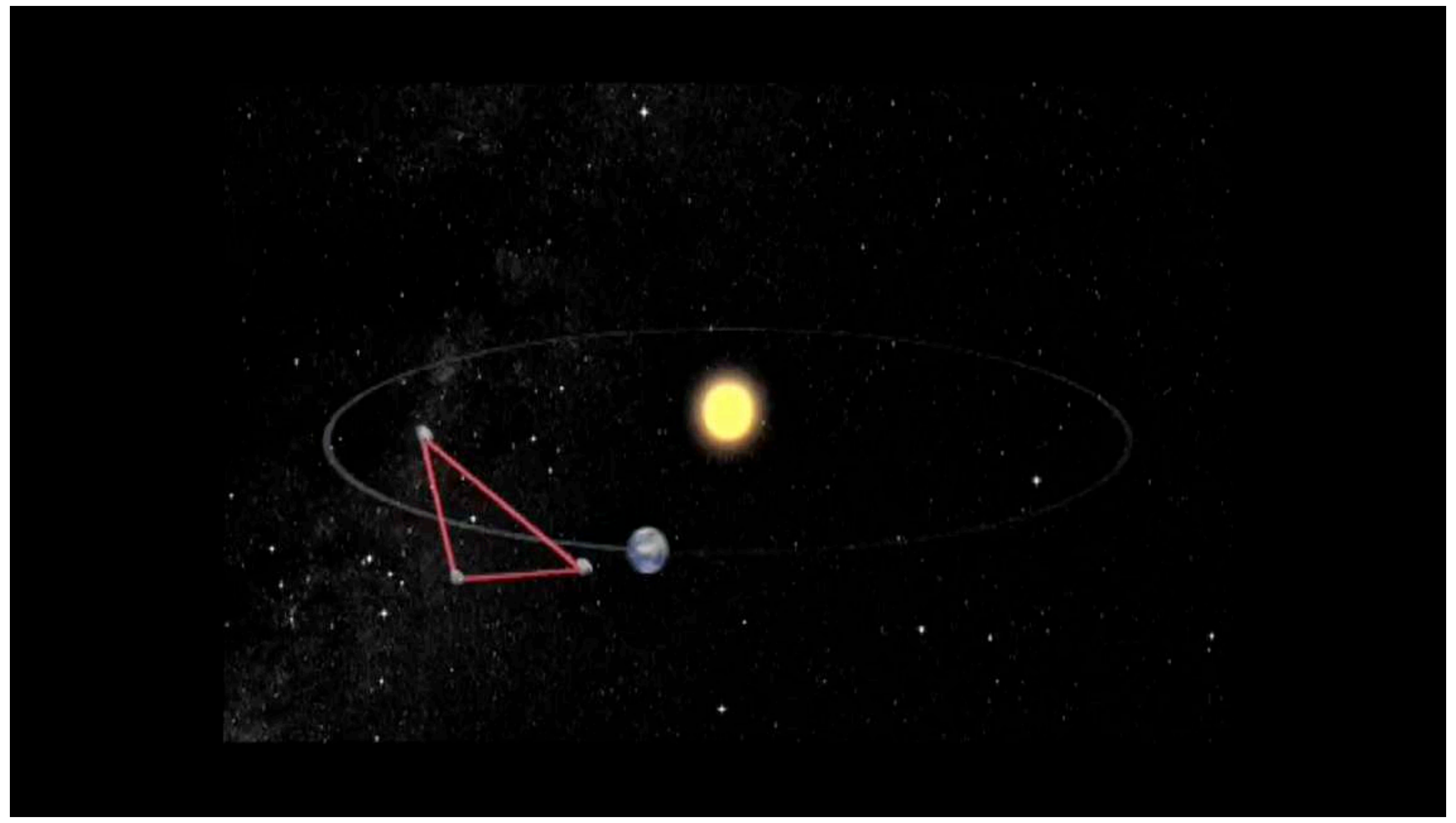
(3G Science-Case Report 21)



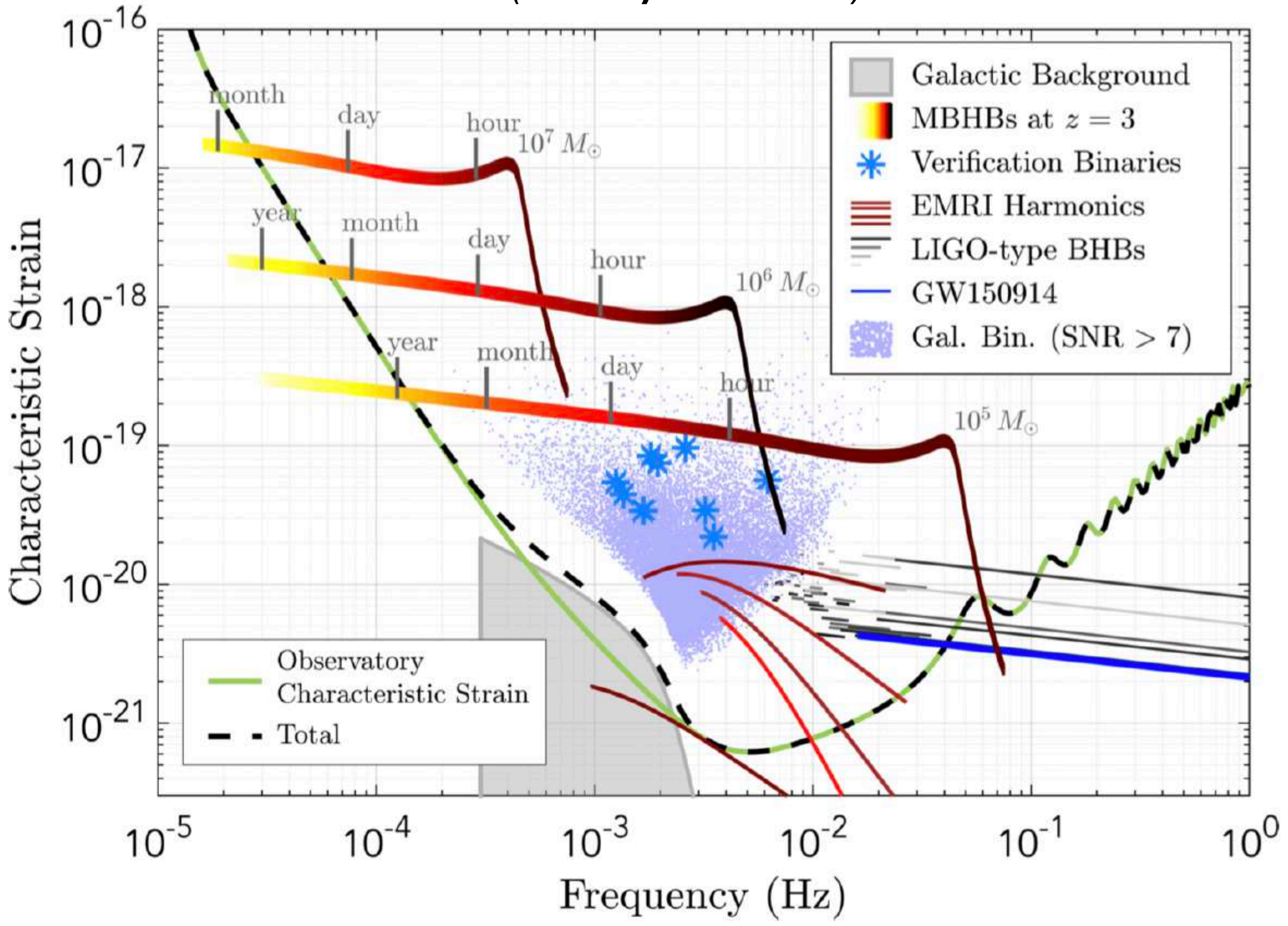


Laser Interferometer Space Antenna (LISA)

Credit: AEI/Milde Marketing



(Audley et al. 17)



Outline

- **Highlights on science** (astrophysical-source properties, tests of General Relativity) **from the latest observing run** of LIGO and Virgo.

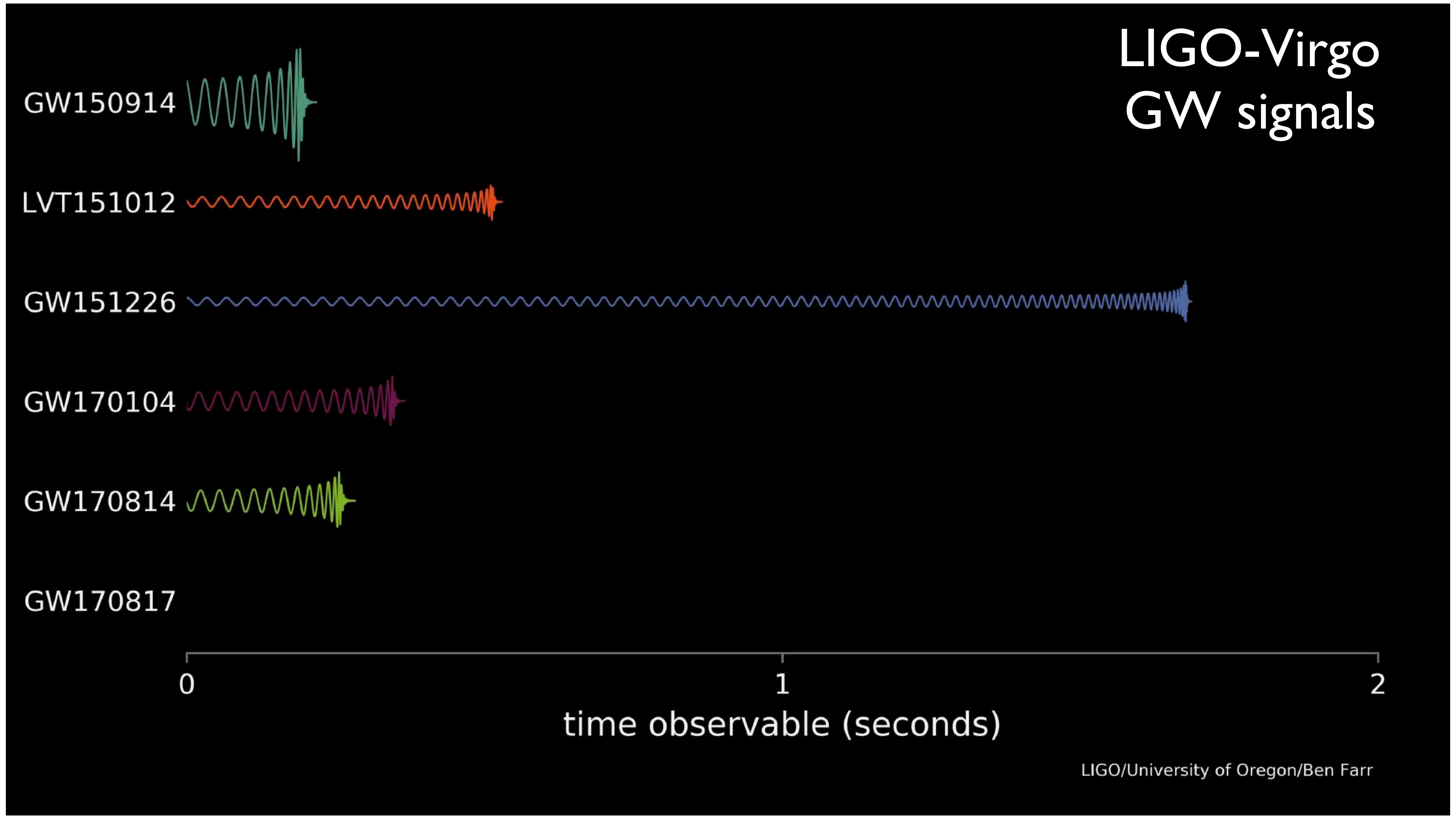
- What have we **learned from the "exceptional" GW events** of the latest observing run?

- Is a **clear picture of the population properties** (masses and spins) of compact-object binaries **emerging**?

- What has been the **role of theoretical predictions for the two-body** dynamics and gravitational radiation **in interpreting signals** and **unveiling their properties**?

- What to do (theoretically) to **keep up with the pace at which GW detectors will be improving** their sensitivity in the next years and decades?

Gravitational Waves are Fingerprints of Sources and Gravity Theory



- At fixed binary's mass, the **lower** the GW **frequency,** the **larger** the **binary's separation**, and the **earlier** the **inspiral stage**

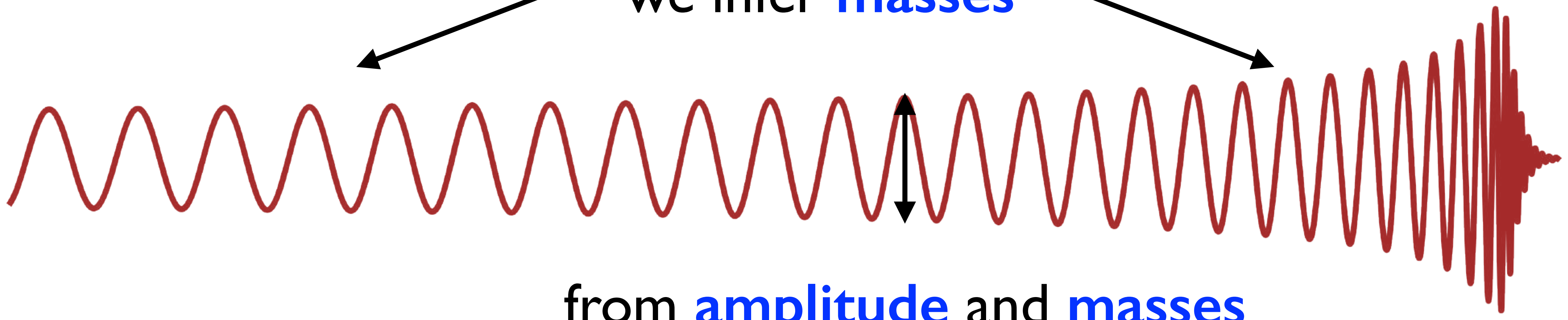
$$\omega = \sqrt{\frac{GM}{r^3}} \qquad f_{\rm GW} = \frac{\omega}{\pi}$$

orbital frequency

orbital separation

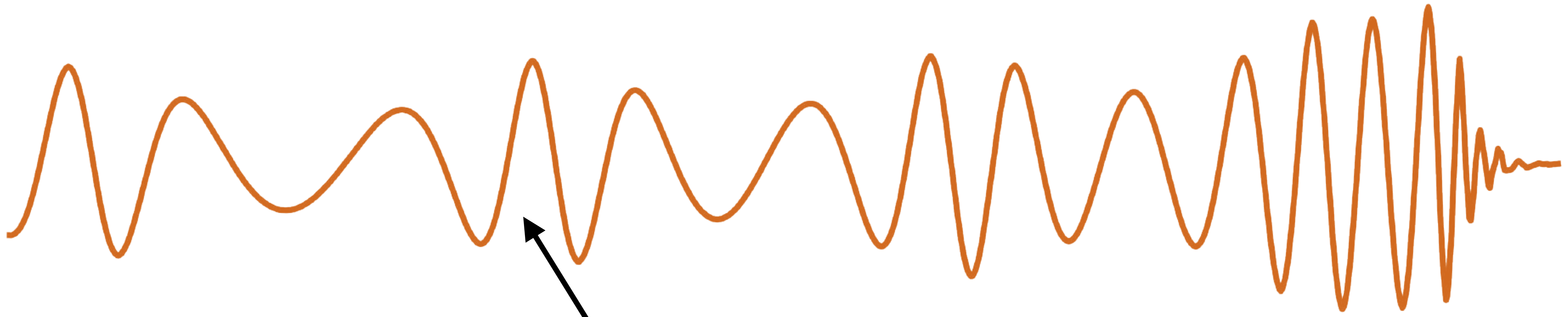
- Binary black holes **merge** at $f_{\rm GW} \sim \dfrac{4400\,{\rm Hz}}{M/M_{\rm Sun}}$

from **frequency evolution** we infer **masses**



from **amplitude** and **masses** we infer **distance**

from **time of arrival**, **amplitude and phase** at detectors we infer **sky location**



from **modulations** of **amplitude and phase** we infer **spins and eccentricity**

By **comparing to waveforms with deviations from GR,** we can **probe gravity**

Solving Two-Body Problem in General Relativity

- **GR** is **non-linear theory**. $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

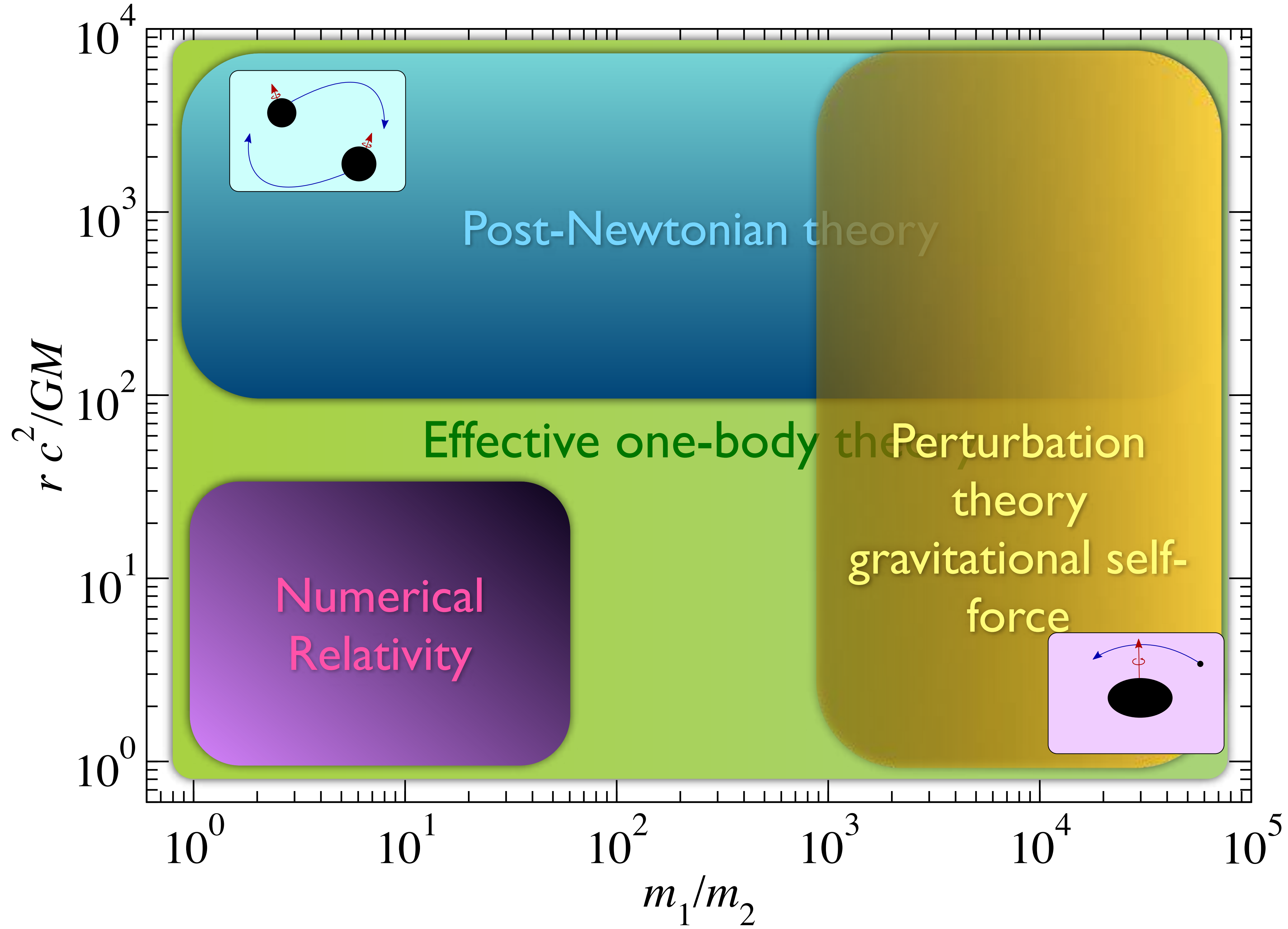
- Einstein's field equations can be solved:

-**approximately,** but **analytically** (**fast** way)

-**"exactly"**, but **numerically** on supercomputers (**slow** way)

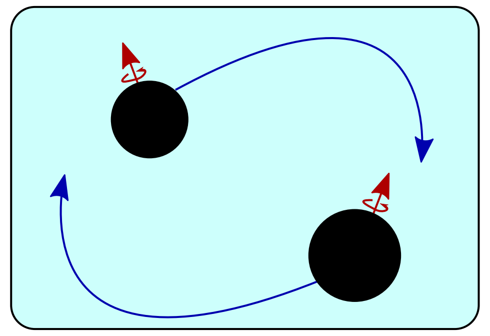
- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

bound orbits: $v^2/c^2 \sim GM/rc^2$



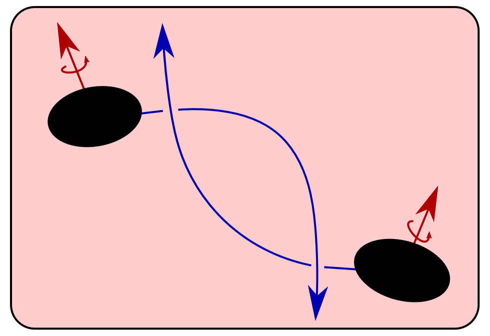
- **Post-Newtonian** (PN) (large separation, and slow motion, **bound motion**, i.e., **early inspiral**)

expansion in $v^2/c^2 \sim GM/rc^2$



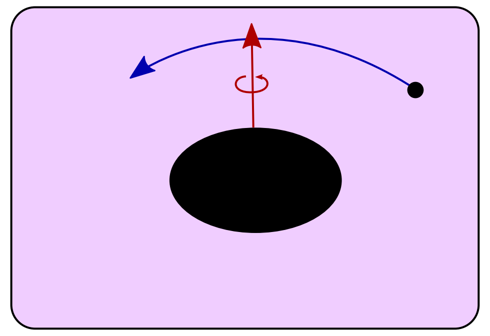
- **Post-Minkowskian** (PM) (large separation, **unbound motion**, i.e., **scattering**)

expansion in G



- **Small mass-ratio** (gravitational self-force, GSF, i.e., **early to late inspiral**)

expansion in m_2/m_1



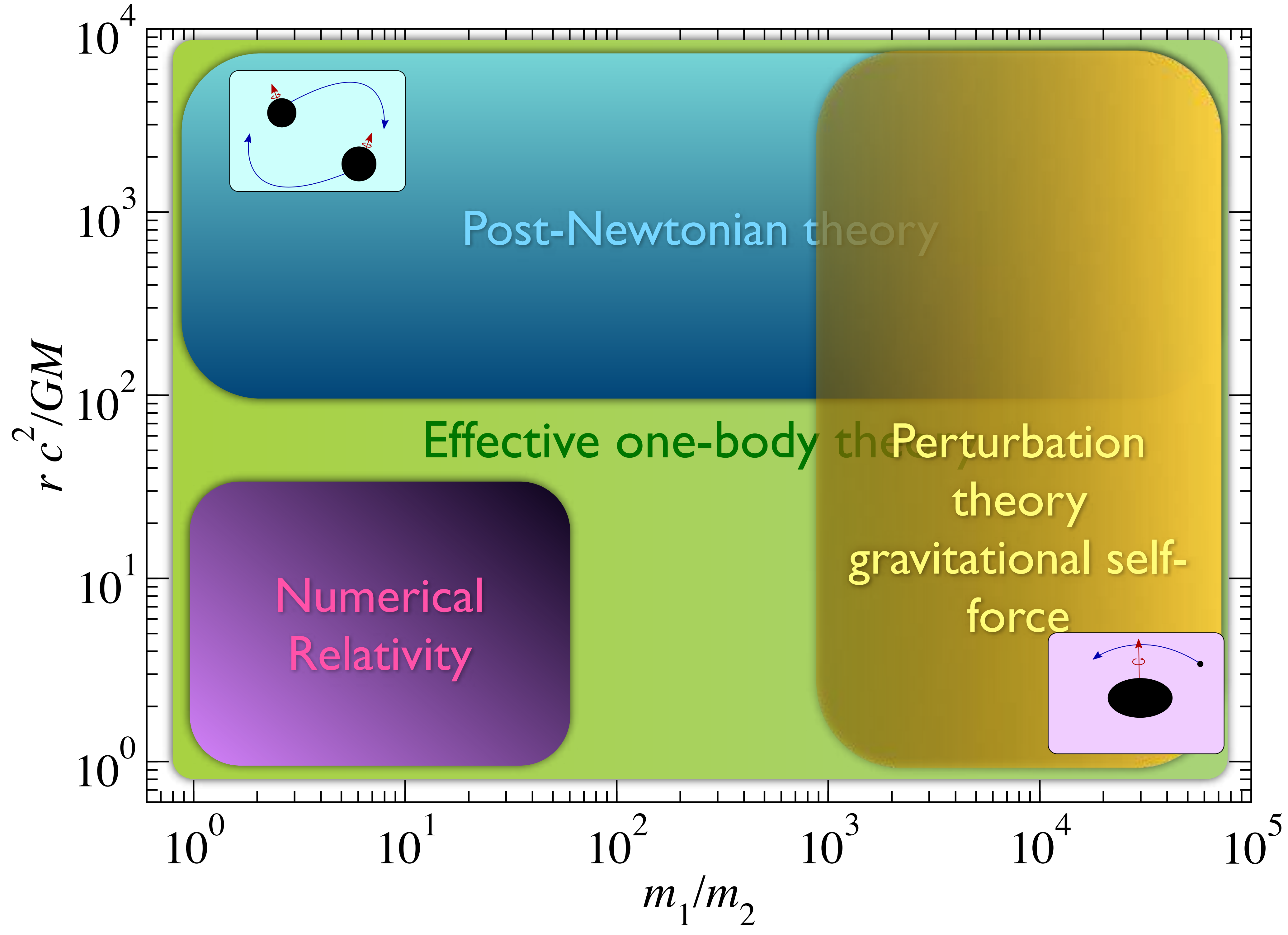
Highly Accurate Waveform Models for GW Observations

- **GR** is **non-linear theory**. $$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Einstein's field equations can be solved:
  - **approximately,** but **analytically** (**fast** way)
  - **"exactly"**, but **numerically** on supercomputers (**slow** way)

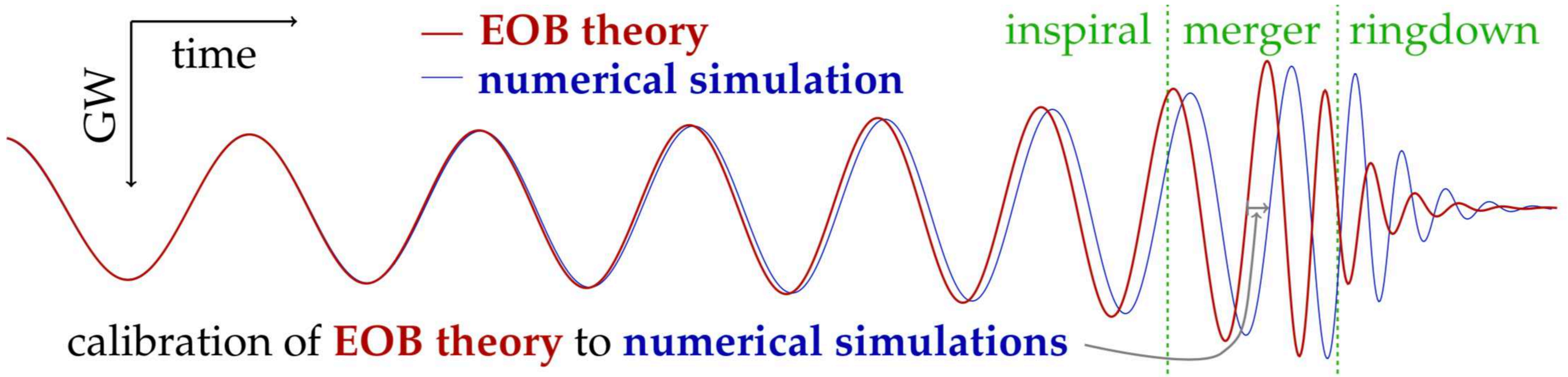
- **Synergy** between **analytical** and **numerical relativity** is **crucial** to **provide GW detectors with templates** to use for **searches** and **inference analyses**.

bound orbits: $v^2/c^2 \sim GM/rc^2$

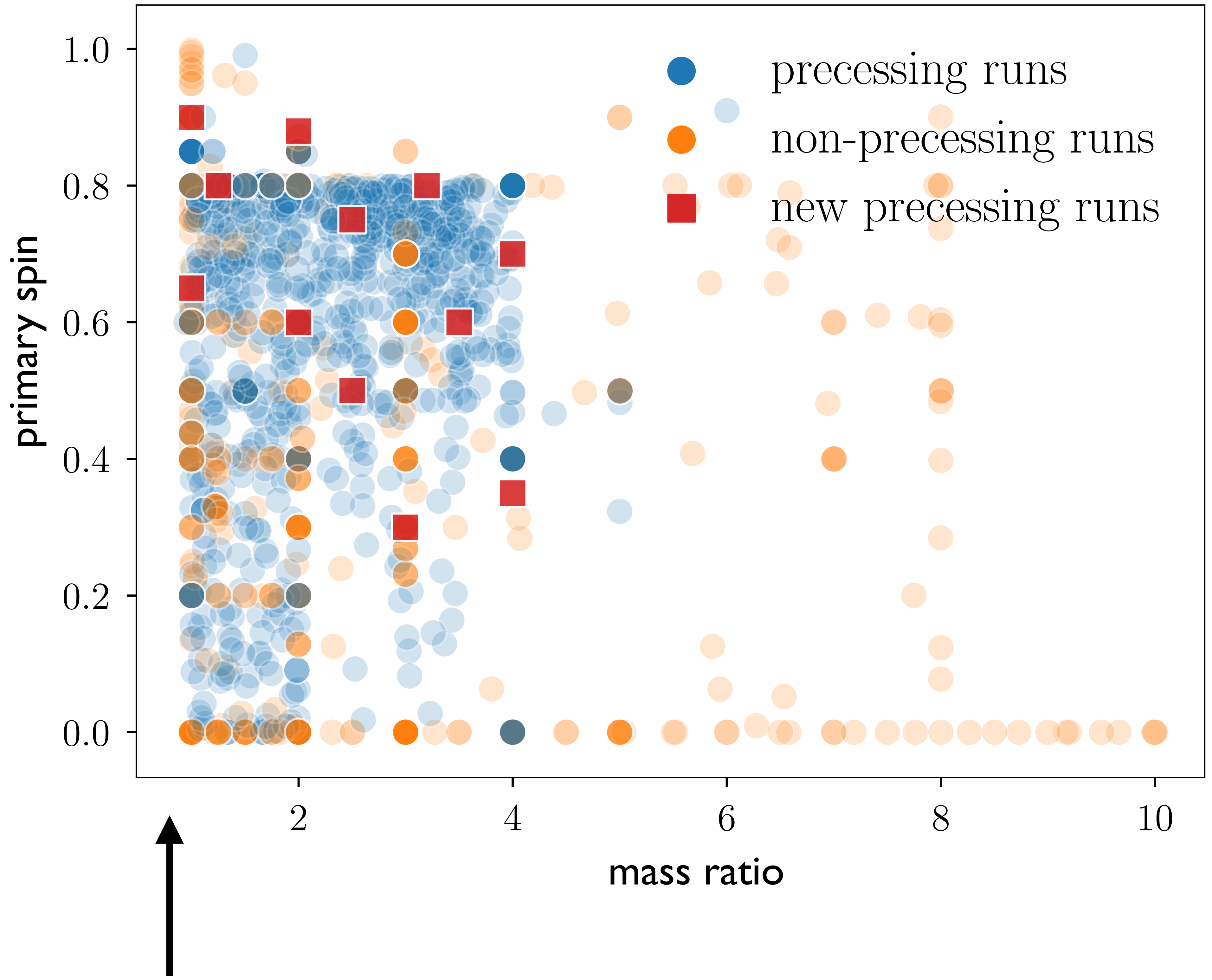


- **Effective-one-body** (EOB) (combines results from all methods, i.e., **entire coalescence**)

- **Key ideas** of EOB theory **inspired by quantum field theory.**



Numerical Relativity



- **Public Simulating eXtreme Spacetimes (SXS)** NR catalog.

(Boyle et al. 19, Ossokine et al. 20)

- **Other public NR catalogs.**

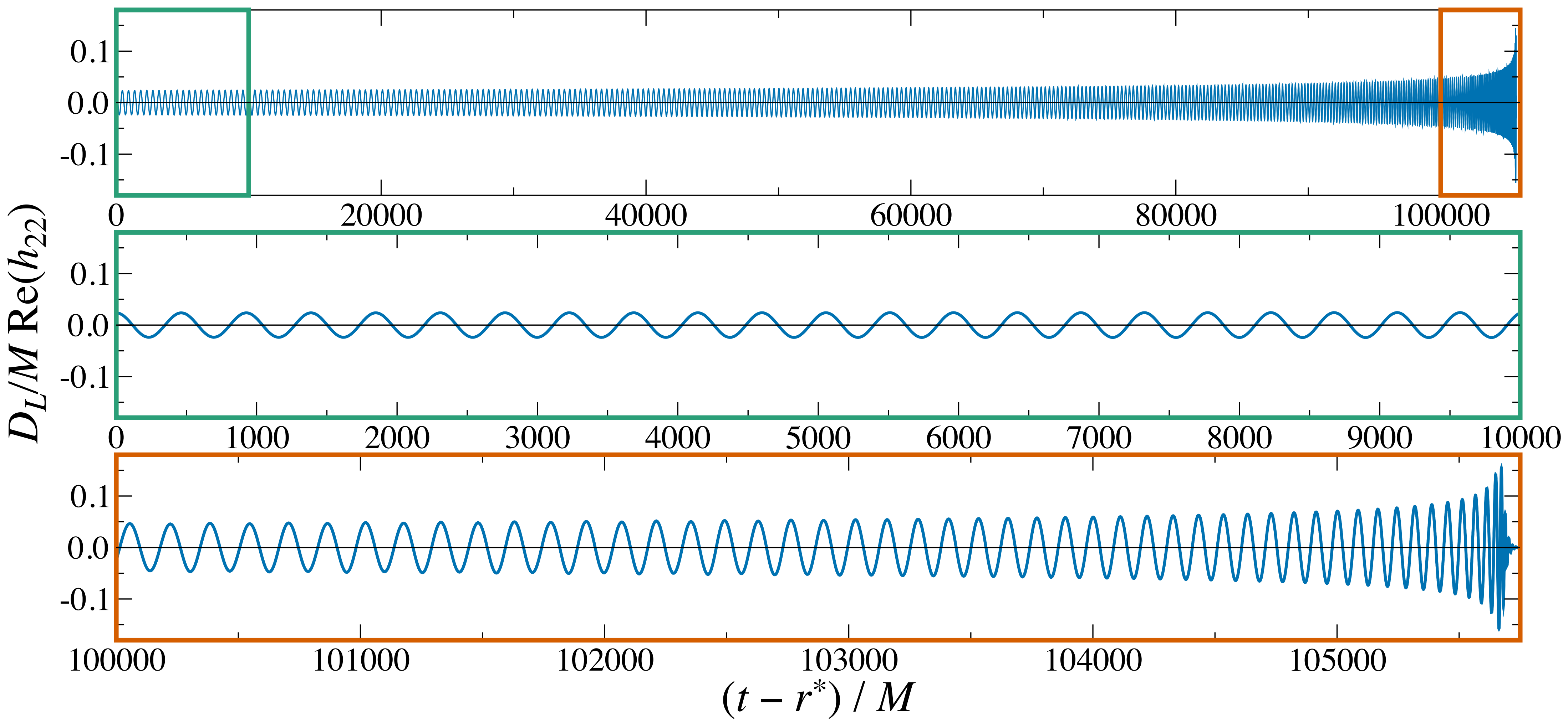
(Husa et al. 15, Jani et al. 17, Healy et al. 17, 19, 20)

- Einstein's equations solved **numerically**

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- **376 GW cycles**, zero spins & mass-ratio 7 (8 months, few millions CPU-h)

(Szilagyi, Blackman, AB, Taracchini et al. 15)



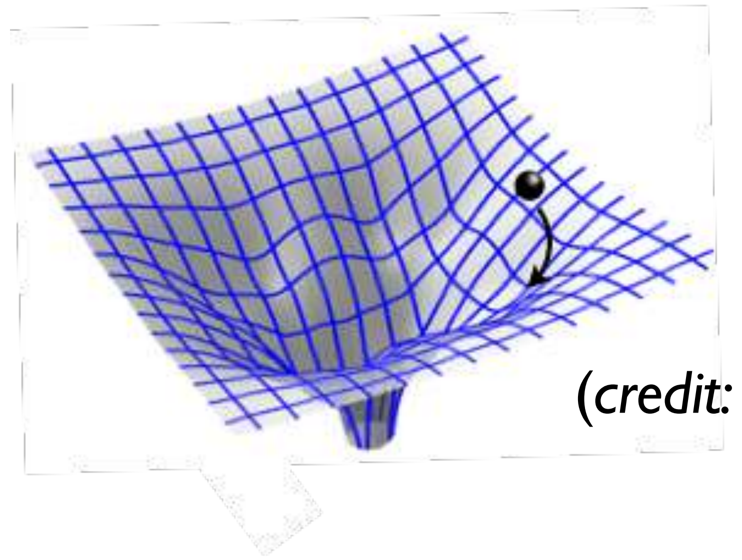
EOB Hamiltonian: Resummed Conservative Dynamics

- **Real Hamiltonian**

$$H_{\rm real}^{\rm PN} = H_{\rm Newt} + H_{\rm 1PN} + H_{\rm 2PN} + \cdots$$

- **Effective Hamiltonian**

$$H_{\rm eff}^{\nu} = \mu \sqrt{A_\nu(r)\left[1 + \frac{\mathbf{p}^2}{\mu^2} + \left(\frac{1}{B_\nu(r)} - 1\right)\frac{p_r^2}{\mu^2}\right]}$$



(credit: Hinderer)

$$ds^2_{\rm eff} = -A_\nu(r)dt^2 + B_\nu(r)dr^2 + r^2 d\Omega^2$$

- **EOB Hamiltonian**

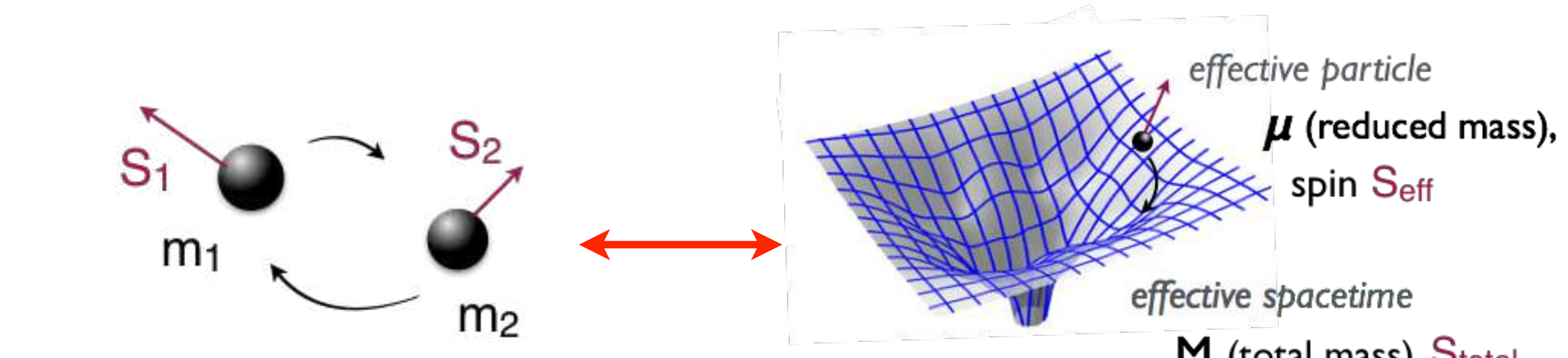
$$H_{\rm real}^{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu} - 1\right)}$$

- Dynamics condensed $A_\nu(r)$ and $B_\nu(r)$
- $A_\nu(r)$, which encodes the energetics of circular orbits, is quite simple:

$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu)\log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

5PN unknown as today

EOB Conservative Spin Dynamics & Waveforms



(credit: Hinderer)

$$H_{\rm real}^{\rm EOB} = M\sqrt{1+2\nu\left(\frac{H_{\rm eff}{}^{\nu}}{\mu}-1\right)}$$

- **EOB equations of motion** *(AB et al. 00, 05; Damour et al. 09)*:

$$\dot{\mathbf{r}} = \frac{\partial H_{\rm real}^{\rm EOB}}{\partial \mathbf{p}} \qquad F \propto \frac{dE}{dt}, \qquad \frac{dE}{dt} \propto \sum_{\ell m} |h_{\ell m}|^2$$

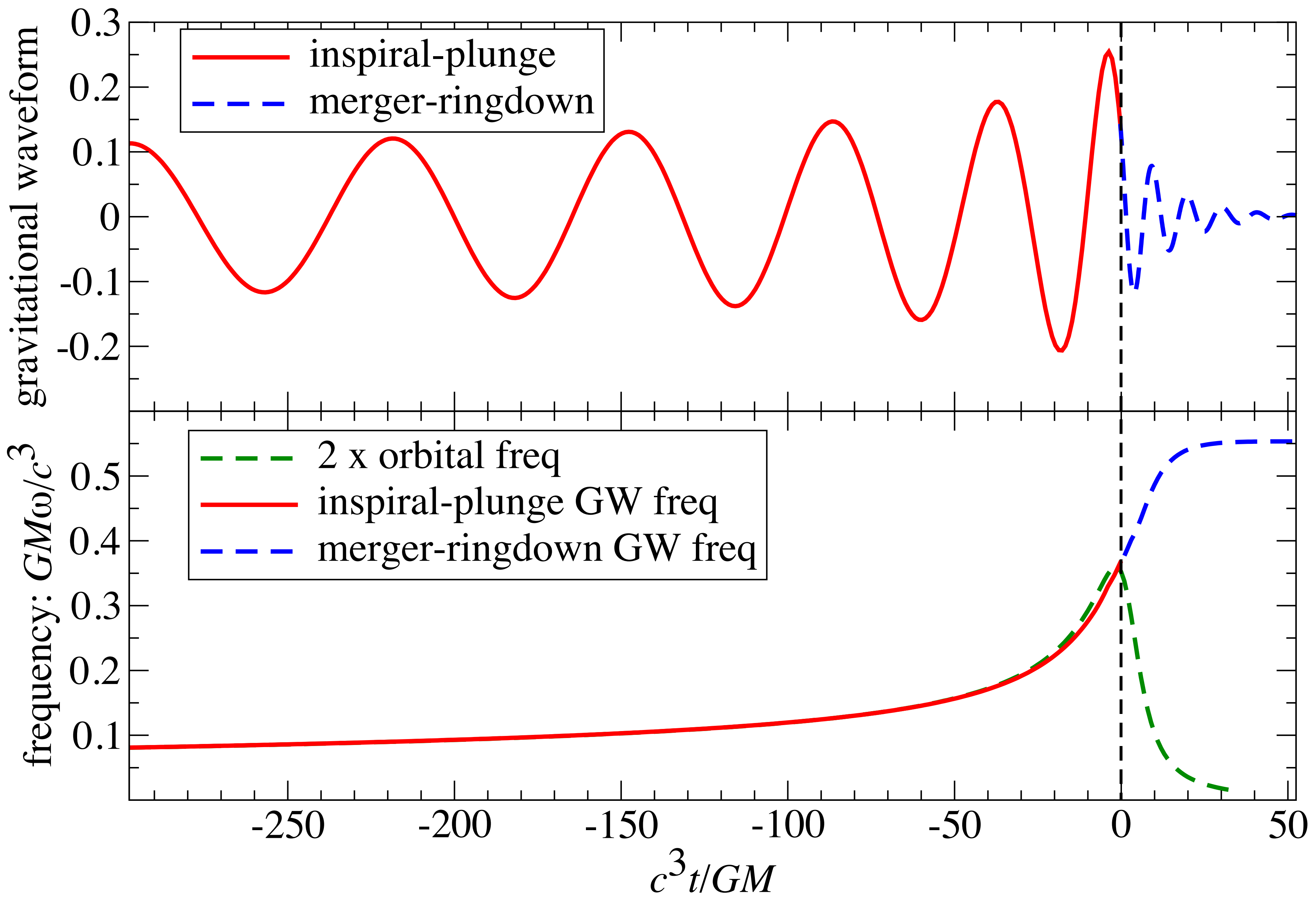
$$\dot{\mathbf{p}} = -\frac{\partial H_{\rm real}^{\rm EOB}}{\partial \mathbf{r}} + \mathbf{F} \qquad \dot{\mathbf{S}} = \{\mathbf{S}, H_{\rm real}^{\rm EOB}\}$$

- **EOB inspiral waveforms** *(AB et al. 00; Damour et al. 09, 11; Pan, AB et al. 11)*:

$$h_{\ell m}^{\rm insp-plunge} = h_{\ell m}^{\rm Newt}\, e^{-im\Phi}\, S_{\rm eff}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}\, h_{\ell m}^{\rm NQC}$$

- **EOB merger-ringdown** waveform is a **superposition of quasi-normal modes**.

(AB & Damour 00, AB et al. 07, Damour & Nagar 07, Del Pozzo & Nagar 17, Bohé et al. 17)

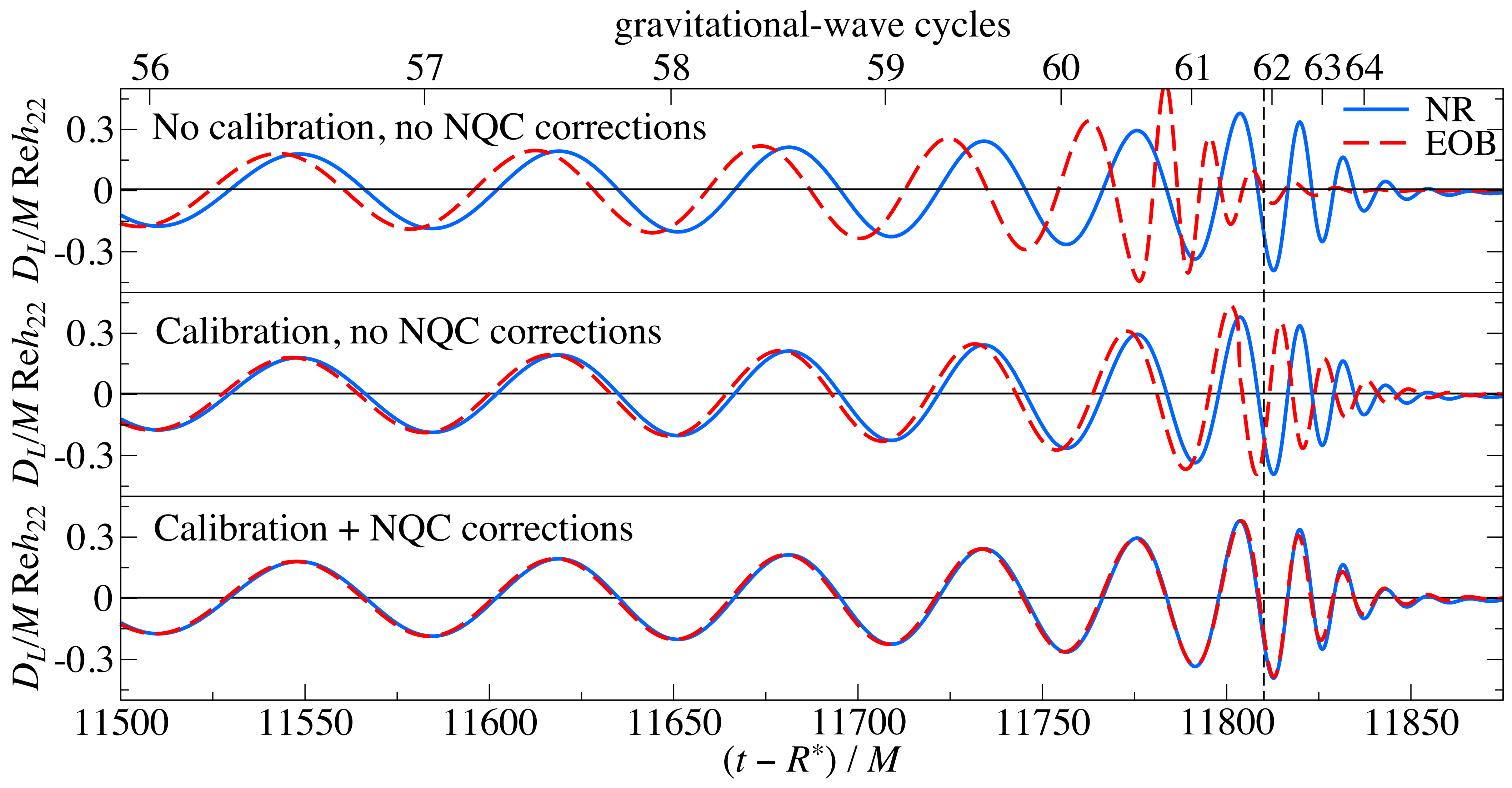


(AB & Damour 00)

Completing EOB Waveforms with NR & Perturbation Theory Information

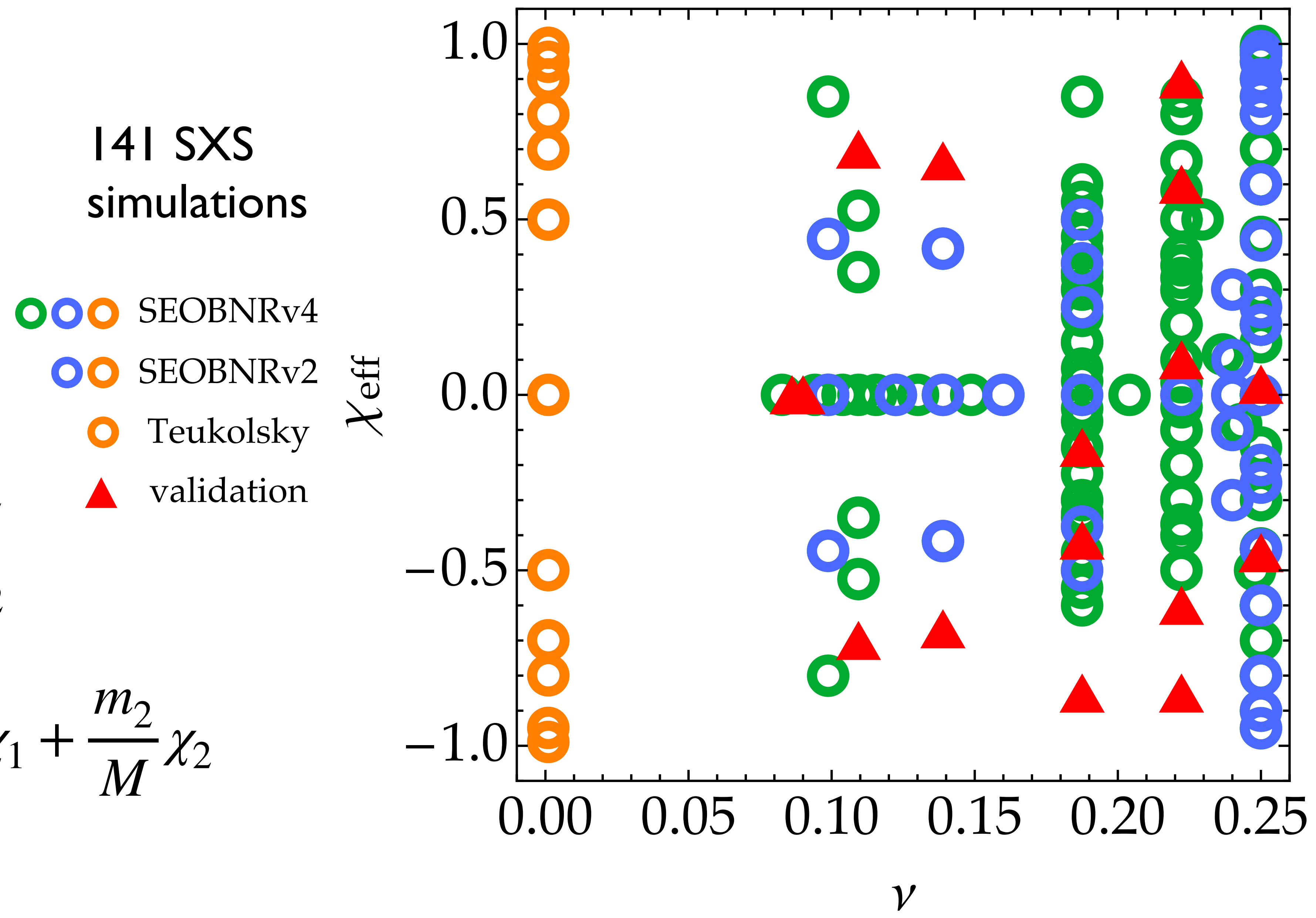
- We calibrate EOB to **inspiral-merger-ringdown NR** waveforms.

Calibration of SEOBNR for O2-O3 searches and inference studies



(credit: Taracchini)

141 SXS simulations



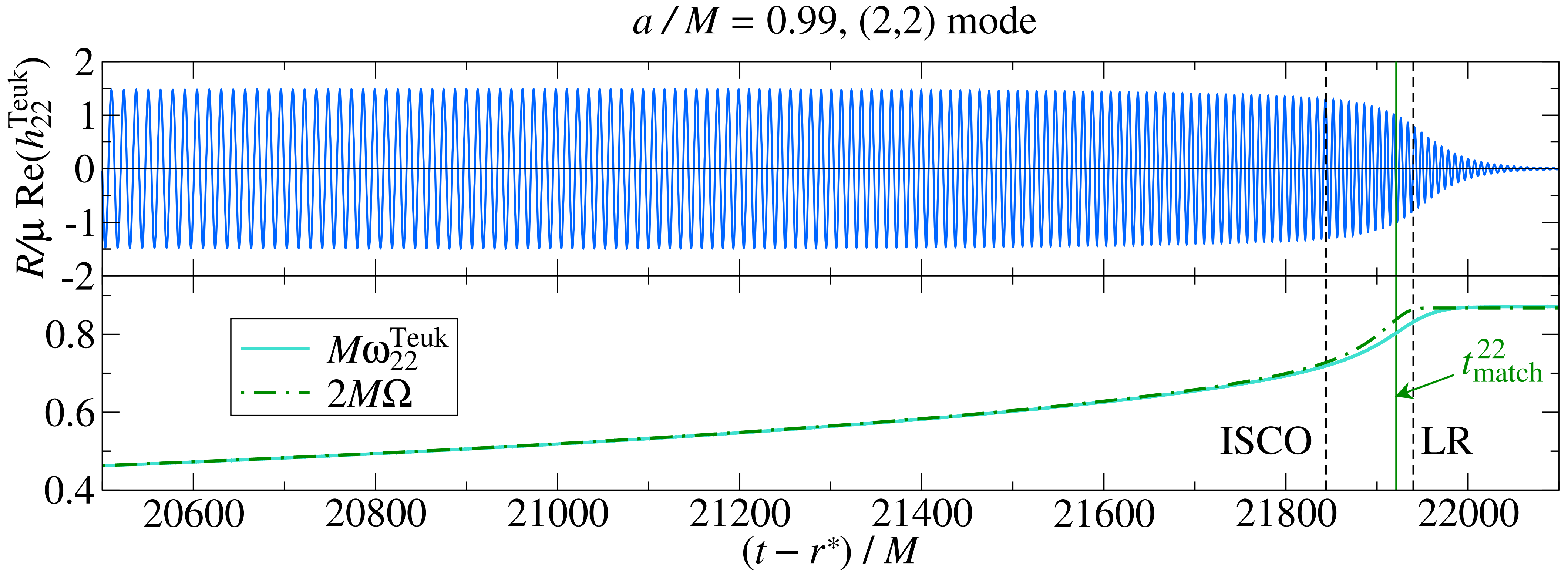
$$\chi_1 = S_1/m_1^2$$

$$\chi_2 = S_2/m_2^2$$

$$\chi_{\rm eff} = \frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2$$

(Bohé et al. 17)

- We calibrate EOB to **merger-ringdown waveforms** in **test-body limit**.



(credit: Taracchini)

(Pan, AB et al. 13, Taracchini, AB, Pan, Hinderer & SXS 14, Pürrer 15)

(Bohé, Shao, Taracchini, AB & SXS 17, Babak et al. 16; Cotesta et al. 18, 20, Ossokine et al. 20)

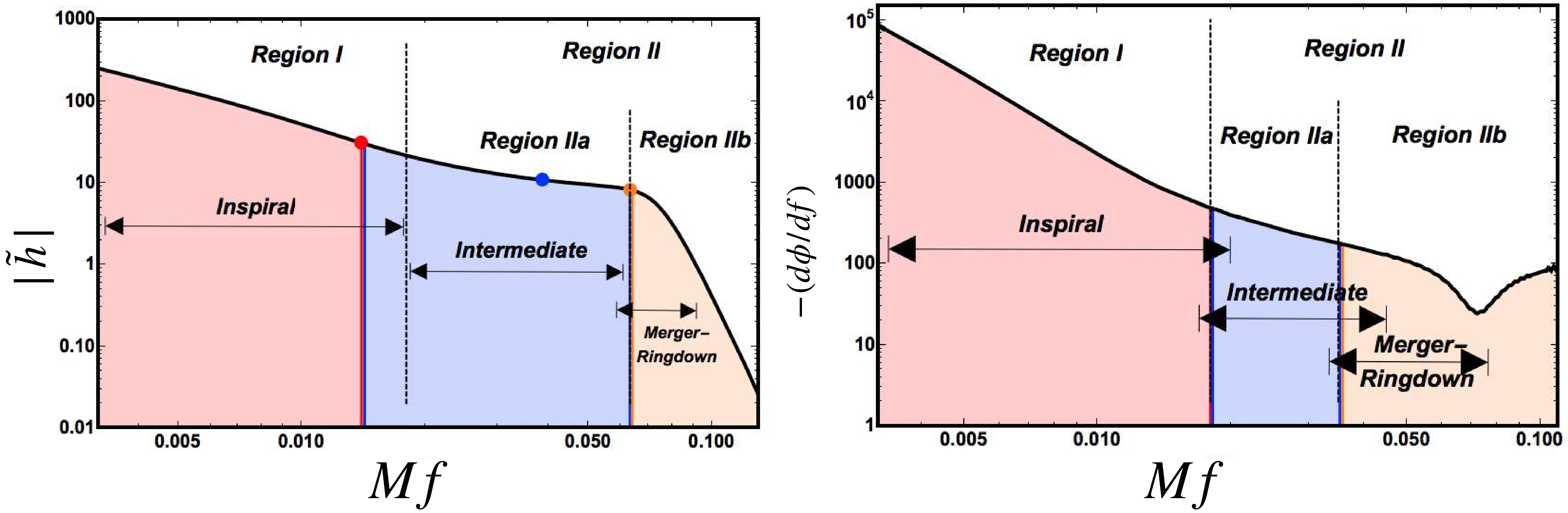
(see also Damour & Nagar 14, Nagar et al. 18, Nagar, Messina et al. 19, Nagar, Pratten et al. 20, Nagar, Riemenschneider et al. 20, Riemenschneider et al. 21)

Phenomenological & NR-Surrogate Waveforms

- **Fast, frequency-domain** waveform model hybridizing EOB & NR waveforms, and then fitting.

(Schmidt et al.12; Hannam et al. 13; Khan et al. 15; Husa et al. 15; Khan et al. 18-19; García-Quíros et al. 20, Pratten et al. 20)

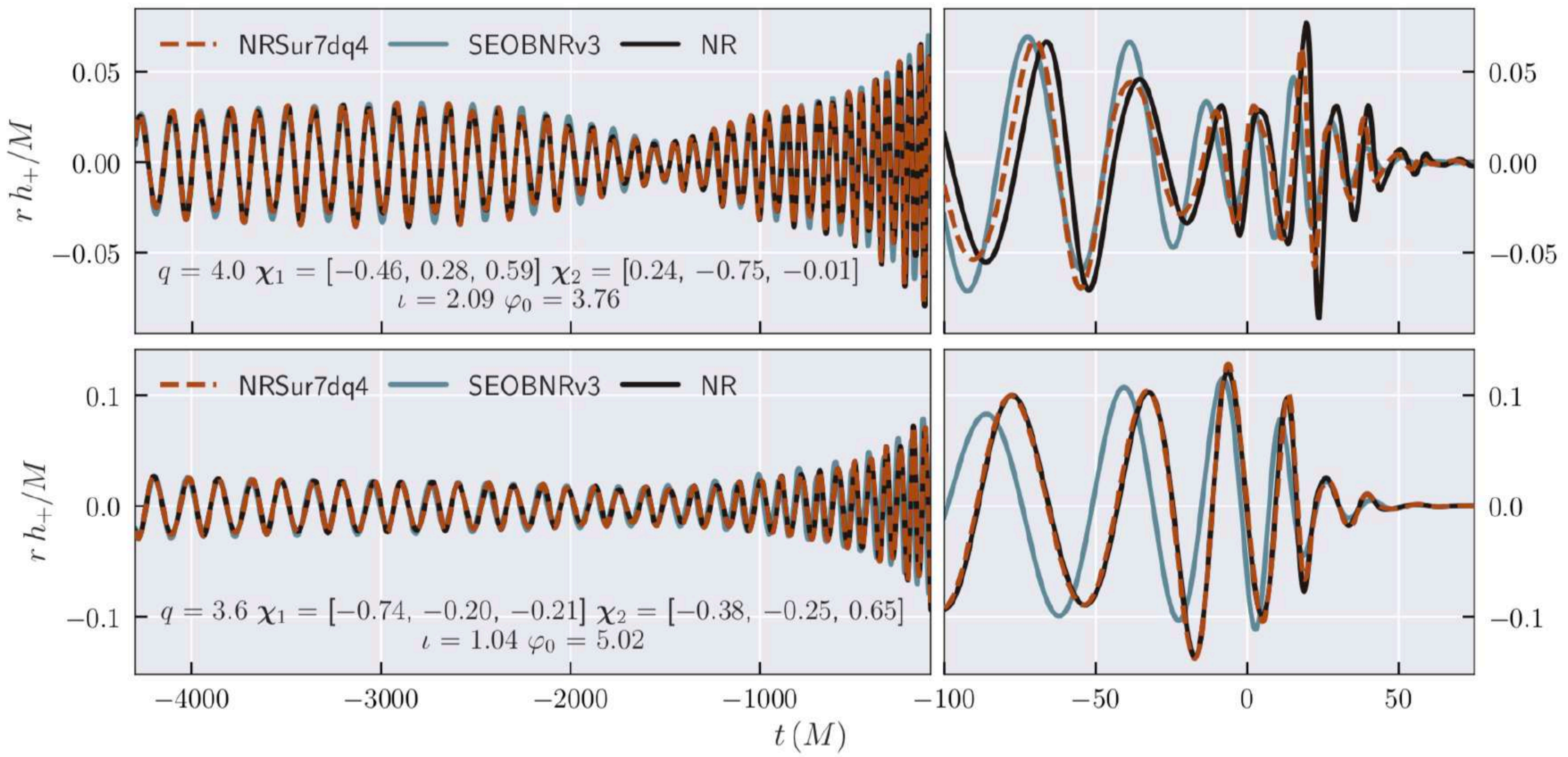
IMRPhenom $$\tilde{h}(f;\lambda_i) = \mathcal{A}(f;\lambda_i)\, e^{i\phi(f;\lambda_i)}$$



(Khan et al. 16)

- **NR surrogate models** are built **directly** by interpolating **NR simulations,** which are **selected in parameter space** using analytical waveform models.
- **Highly accurate**, but **limited in** binary's **parameter space** and **length** (~20 orbits), unless hybridized with EOBNR waveforms.

NRSur



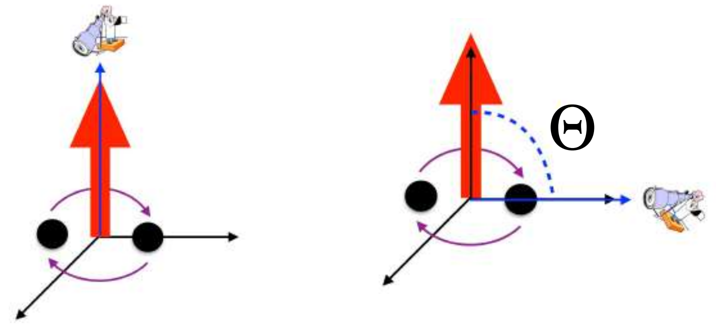
(Varma et al. 19)

Waveform Models with Spin Precession & High Harmonics

- **GWTC-2 used** for the first time **multipolar, spin precessing waveform models**.

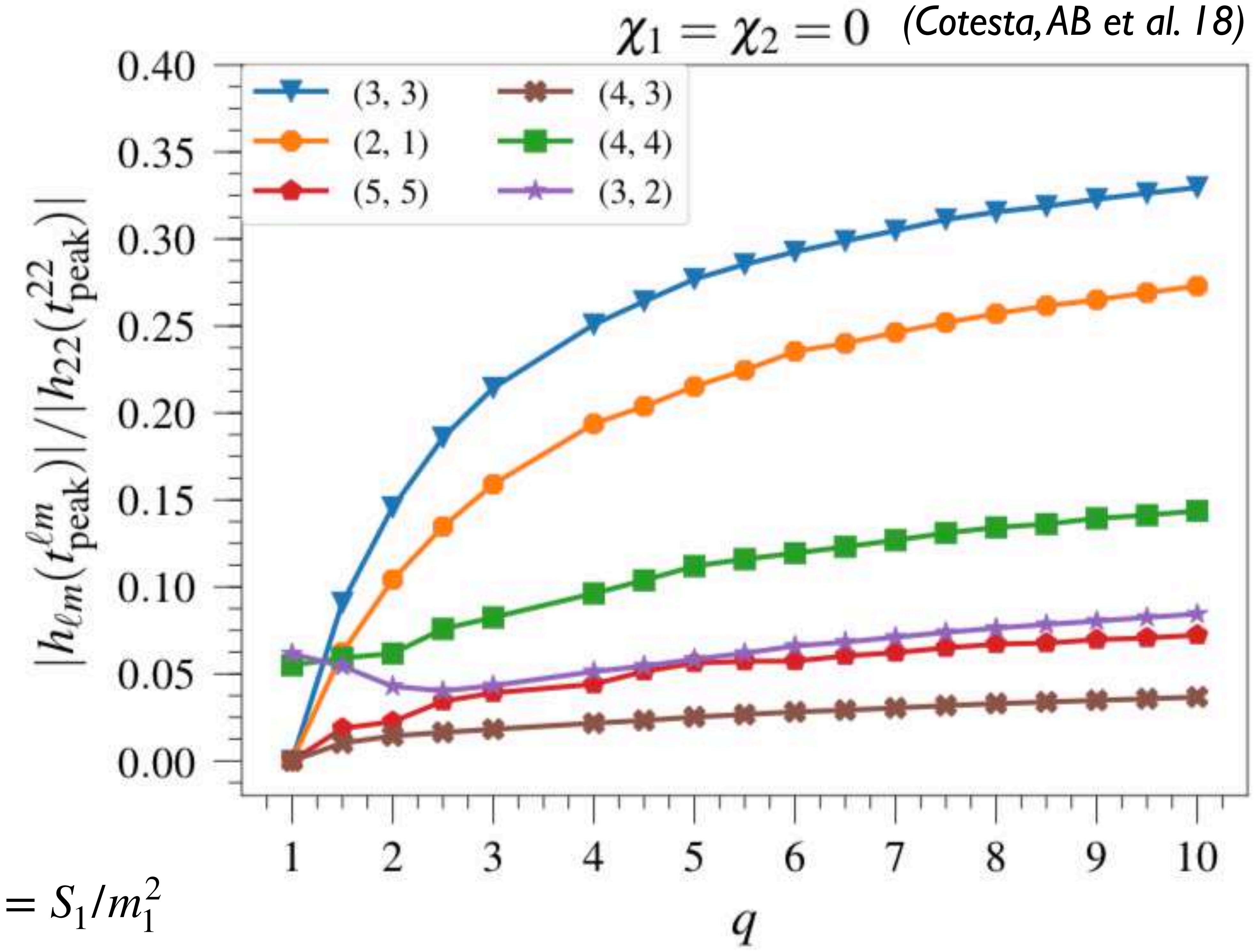
(Khan et al.19, Varma et al. 19, Ossokine et al. 20)

$$h_+(t;\Theta,\varphi) - i\,h_\times(t;\Theta,\varphi) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\Theta,\varphi)\,h_{\ell m}(t)$$



(credit: Cotesta)

(Cotesta, AB et al. 18)

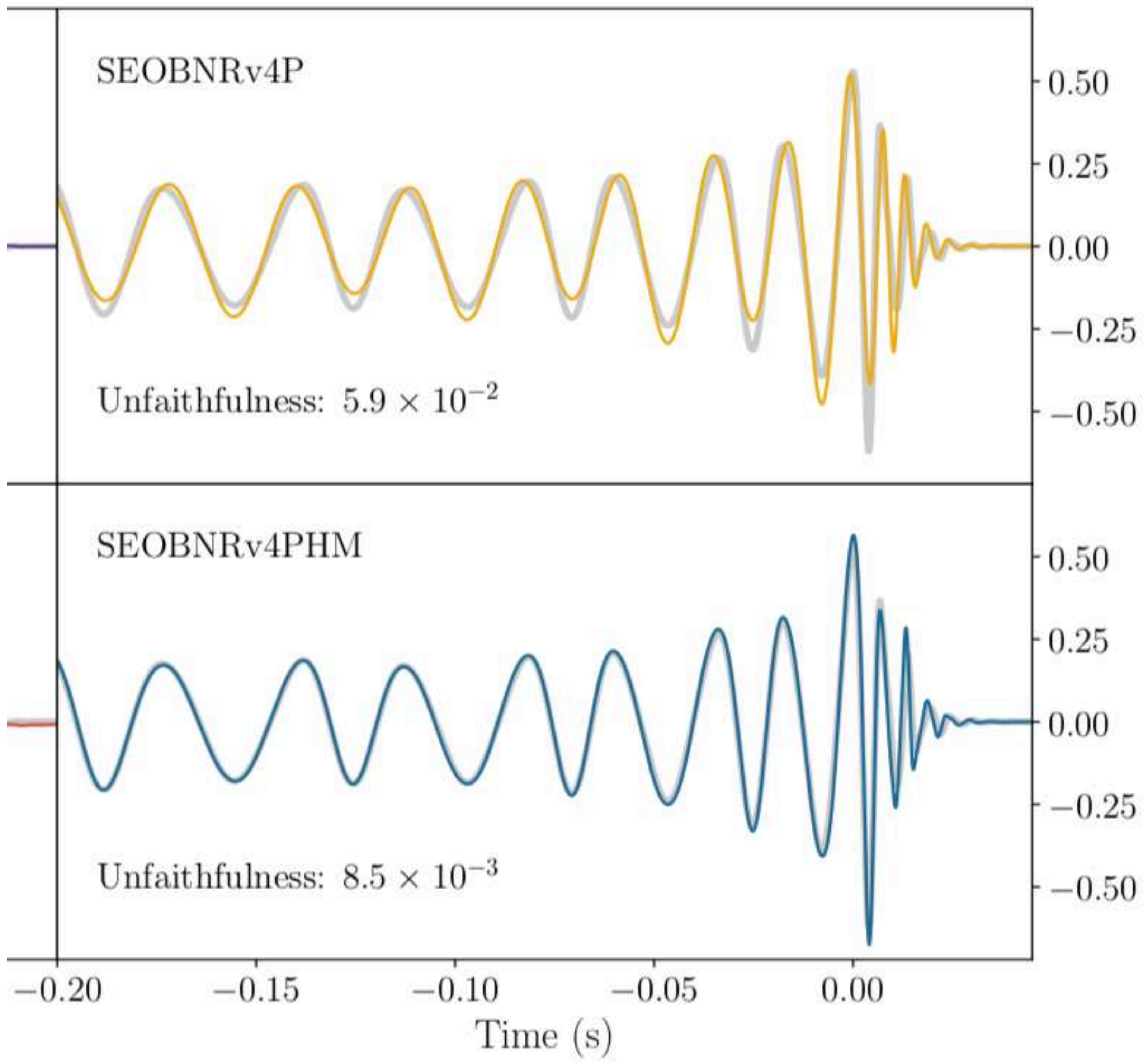


$\chi_1 = S_1/m_1^2$

$\chi_2 = S_2/m_2^2$

$q = m_1/m_2$

(Ossokine, AB, Cotesta, Marsat et al. 20)

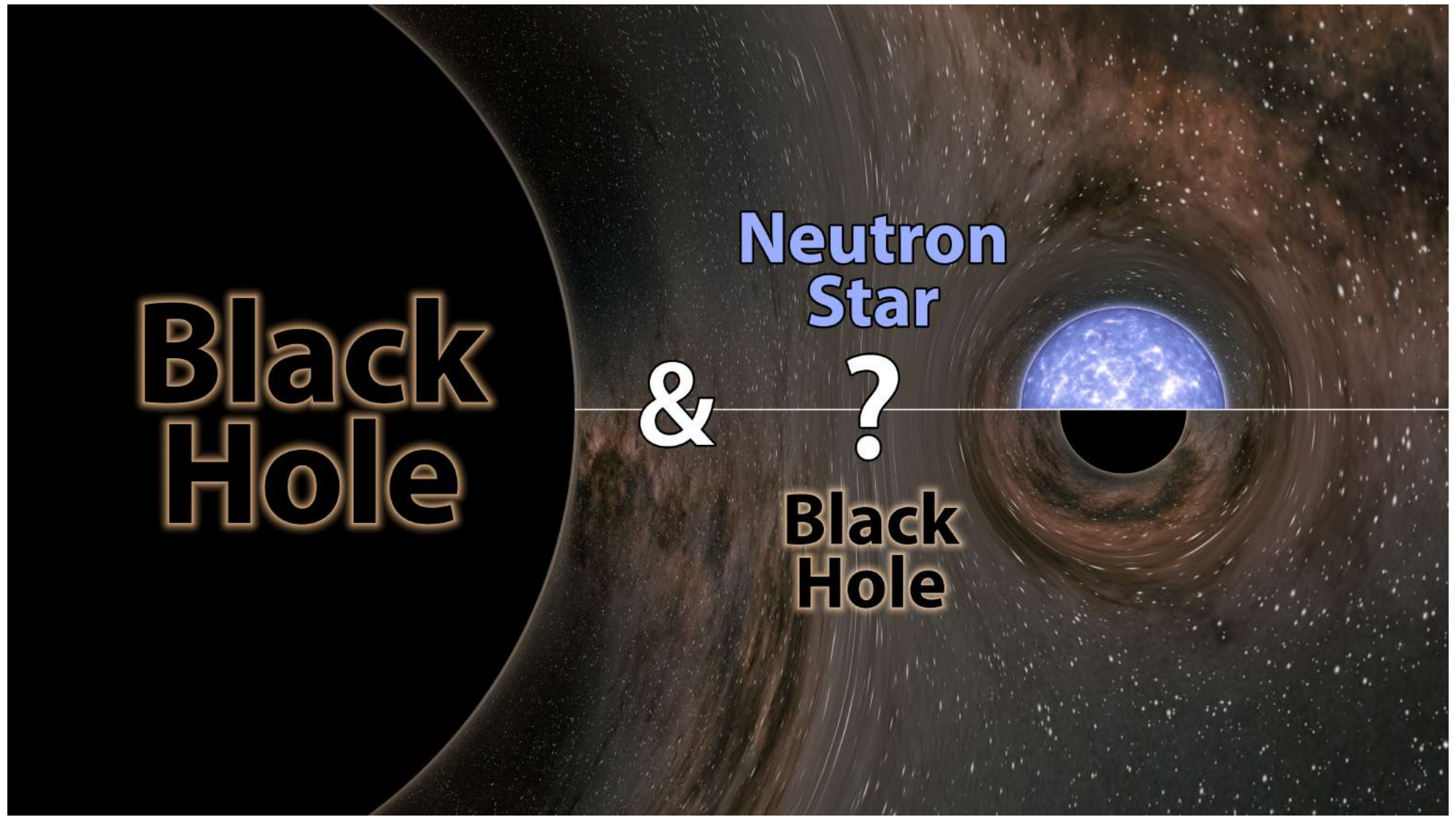


- Waveforms from **BBHs (BNSs & NSBHs)** are described by **15 (17 & 16) parameters**.

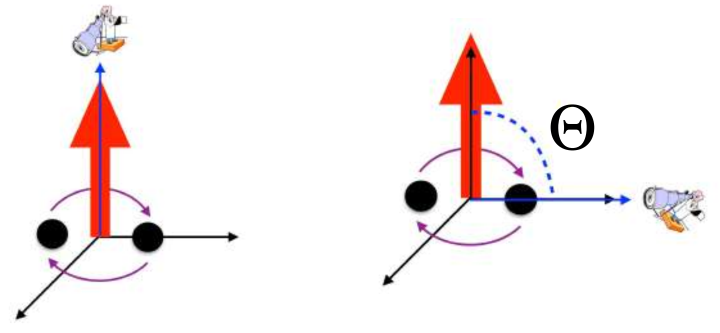
Highlights from O3a Run as we Explore the Universe

GW190814: **a binary with a puzzling companion**

$$m_1 = 23.2^{+1.1}_{-1.0}\, M_\odot \qquad m_2 = 2.59^{+0.08}_{-0.09}\, M_\odot$$

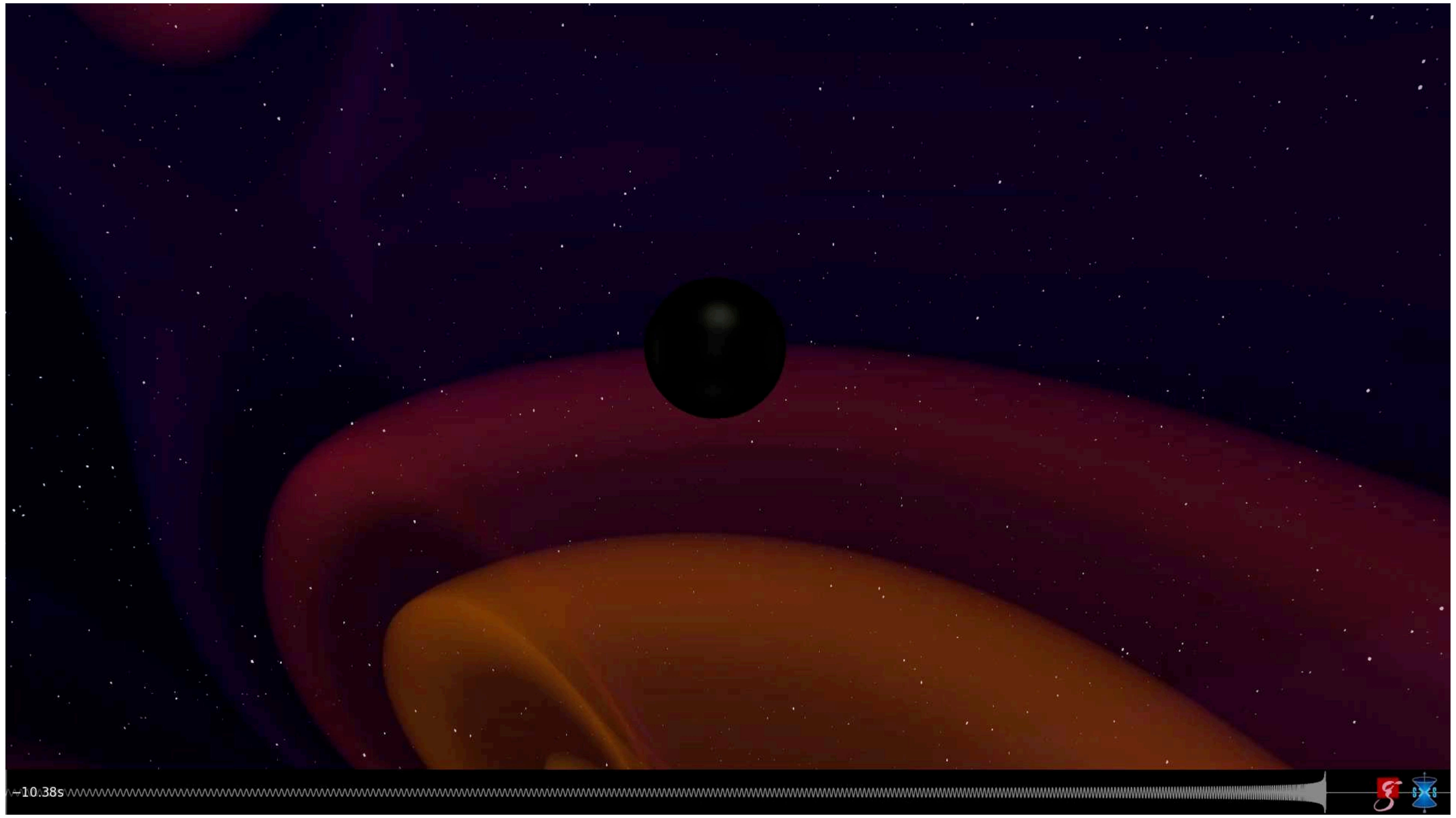


$$h_+(t;\Theta,\varphi) - i\, h_\times(t;\Theta,\varphi) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\Theta,\varphi)\, h_{\ell m}(t)$$



(credit: Cotesta)

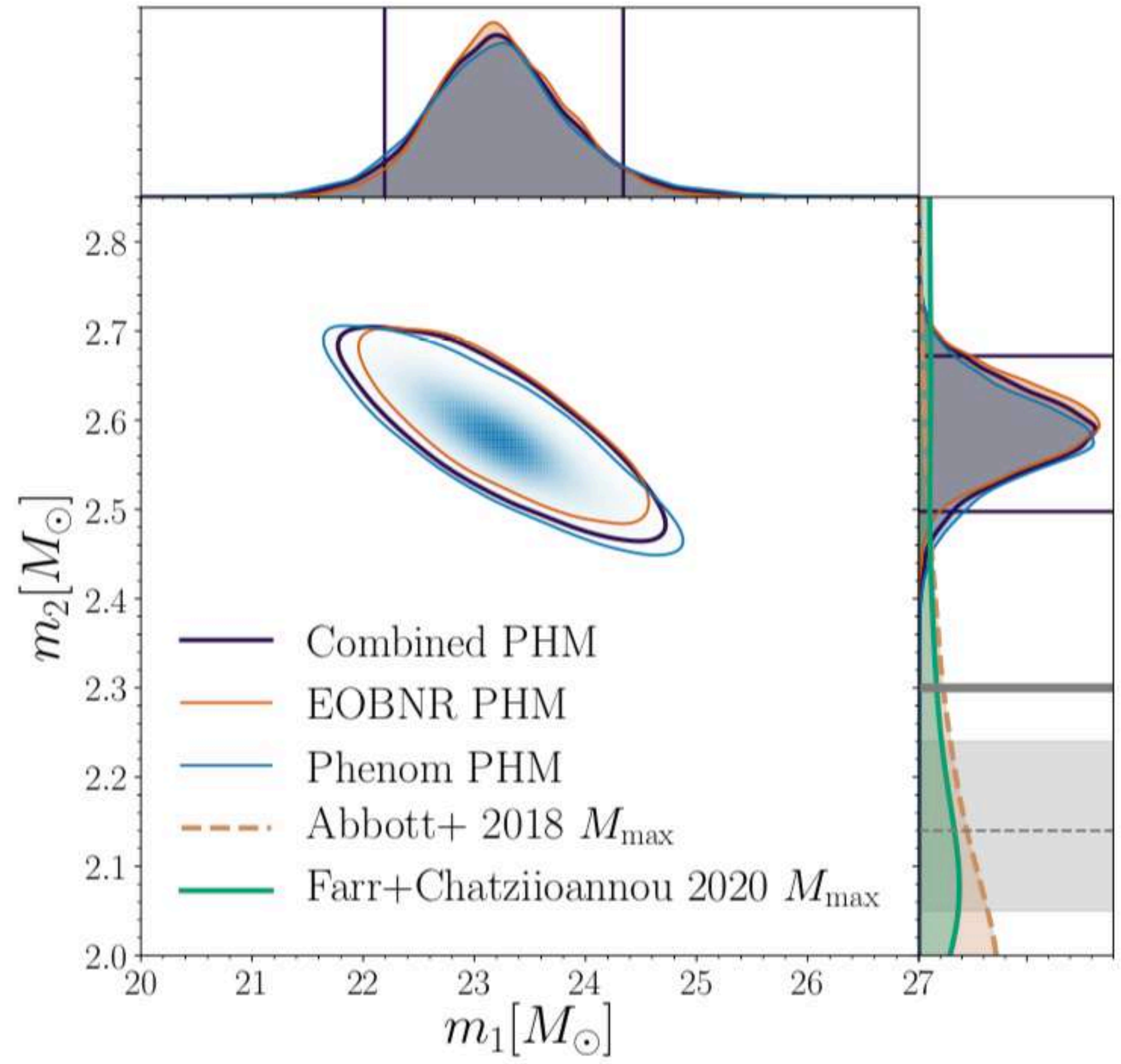
- The **more substructure and complexity** the binary has (e.g., masses or spins of BHs are different) **the richer is the spectrum of radiation** emitted.



(credit: Fischer, Pfeiffer, Ossokine & AB; SXS Collaboration)

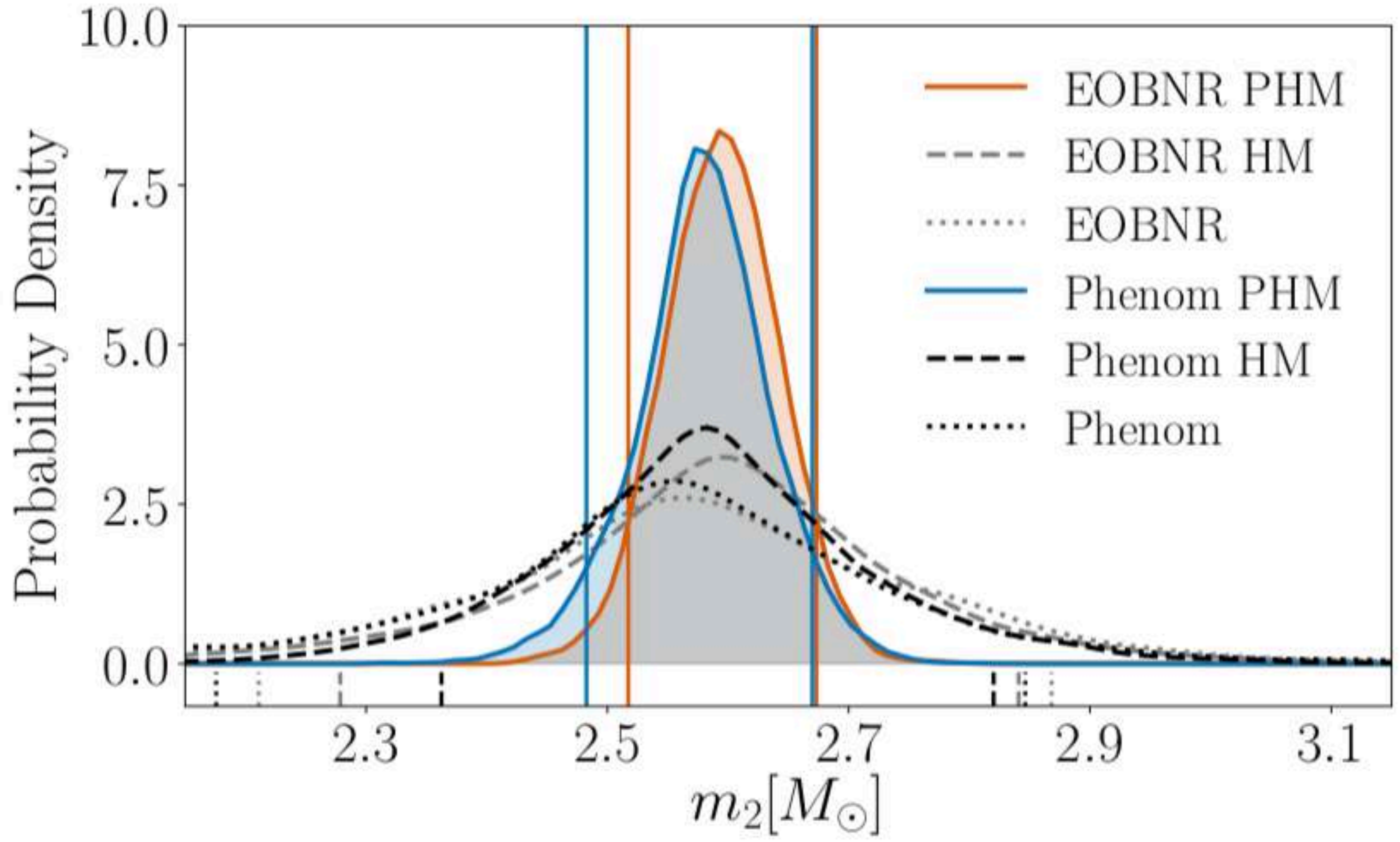
GW190814: a Binary with a Puzzling Companion

(Abbott et al. ApJ Lett 896 (2020) 2, L44)



- **More massive BH** rotated with $\chi_1 < 0.07$.
- **Systematics due to** waveform modeling **smaller than statistical** errors.

- Either the **largest neutron star** or the **smallest black hole.**



- Using waveform models with **higher-modes and spin-precession constrains more tightly** the **secondary mass.**

GW190412: a Signal Like None Before

GW190412: **a signal like none before**

$$q = 0.28^{+0.12}_{-0.06}\, M_\odot$$

- Binary black hole with **mass asymmetry**, and more massive **BH spinning at about $\chi_1 \sim 0.4$ with tilt angle $\theta_1 \sim 45^o$.**

(credit: Fischer, Pfeiffer & AB; SXS Collaboration)

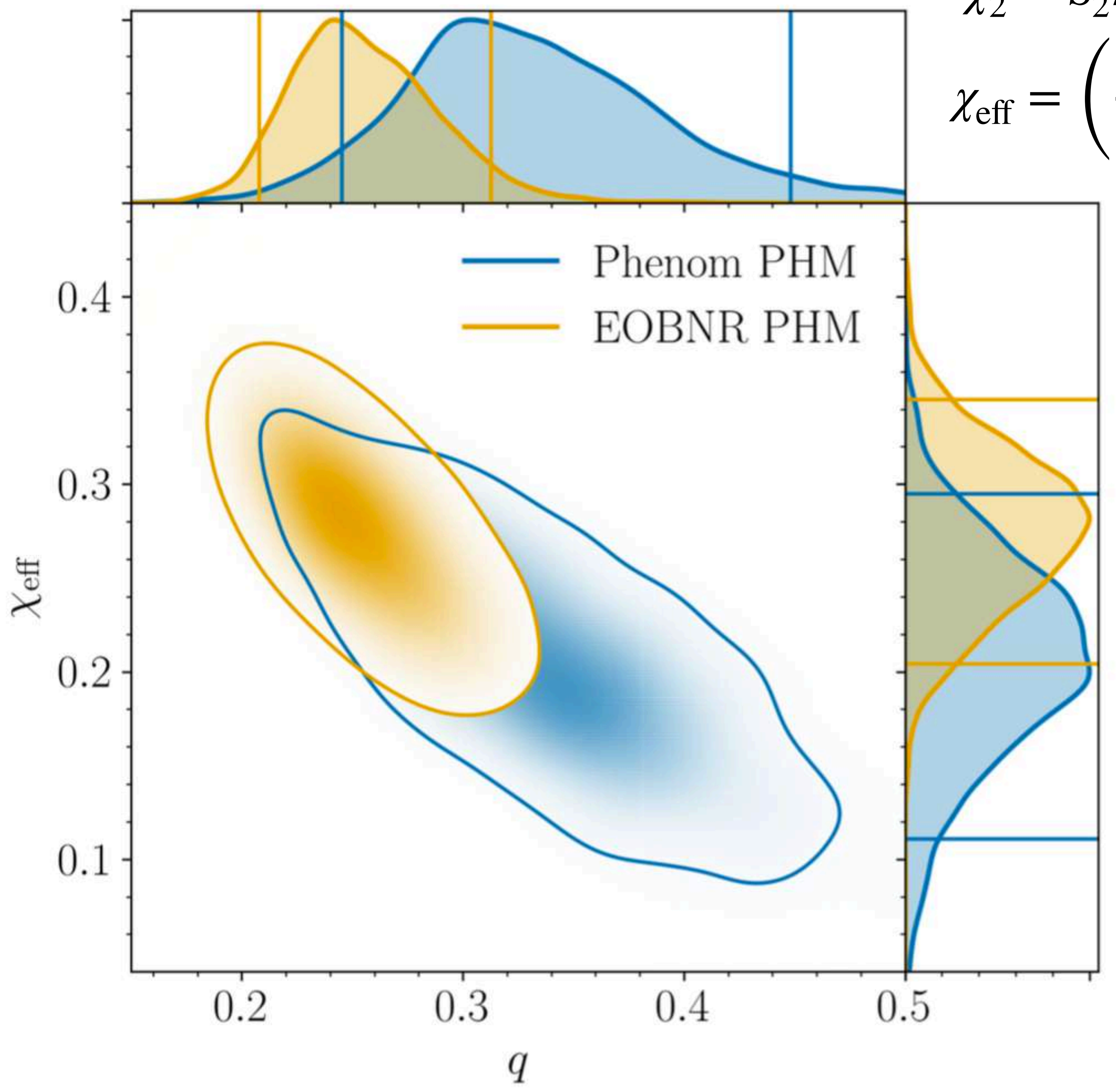
(Abbott et al. PRD 102 (2020) 4)

$$q = m_1/m_2$$

$$\chi_1 = S_1/m_1^2$$

$$\chi_2 = S_2/m_2^2$$

$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot\hat{\mathbf{L}}$$



- **More massive BH** rotated with **spin** $0.17 - 0.59$ at $90\,\%$ CI

GW190412: a Signal Like None Before

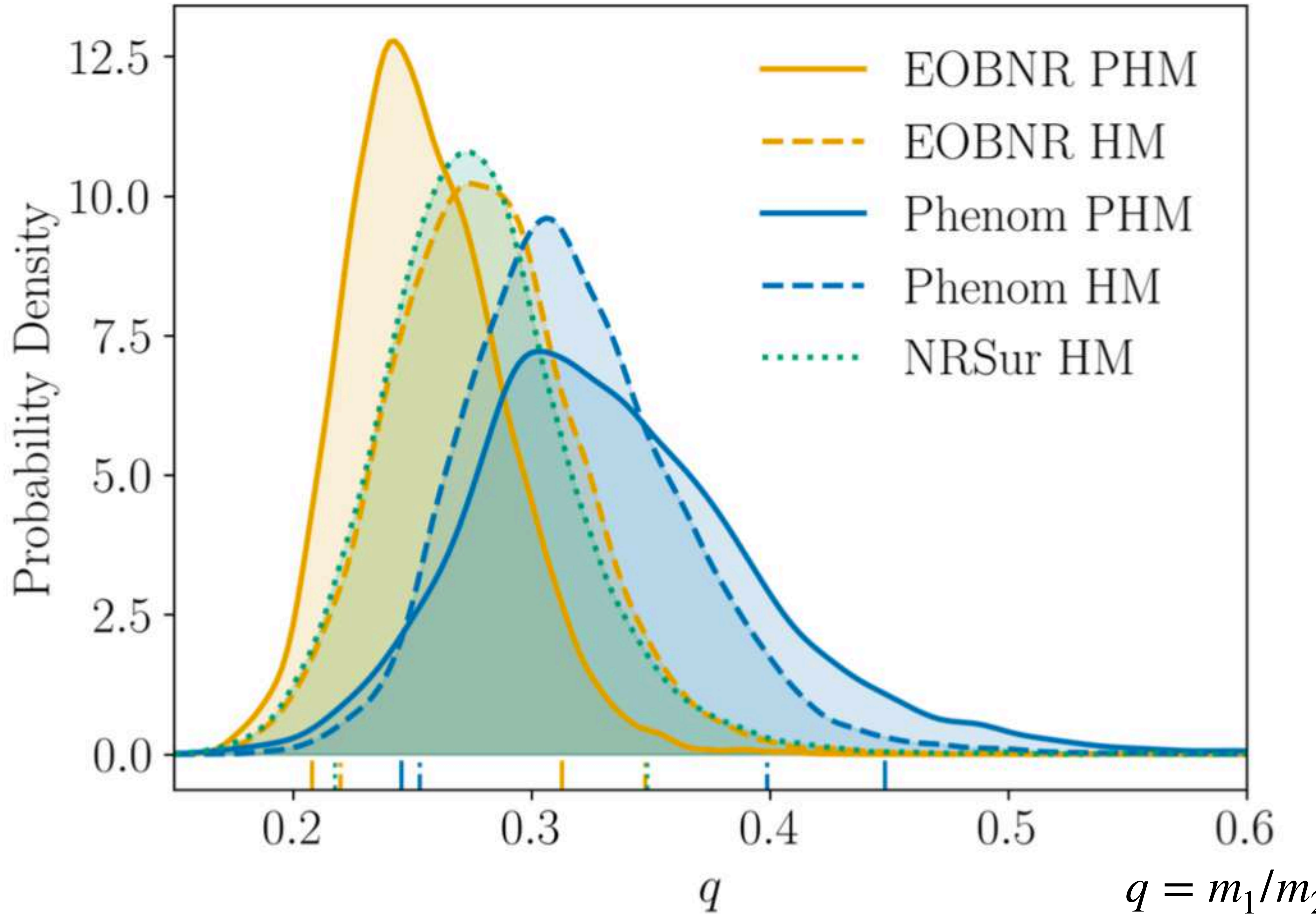
GW190412: **a signal like none before**

$$q = 0.28^{+0.12}_{-0.06} M_\odot$$

- Binary black hole with **mass asymmetry**, and more massive **BH spinning at about $\chi_1 \sim 0.4$ with tilt angle $\theta_1 \sim 45^o$.**

(credit: Fischer, Pfeiffer & AB; SXS Collaboration)

(Abbott et al. PRD 102 (2020) 4)



$q = m_1/m_2$

- **Spin-precession and non-quadrupole modes** enable **tighter inferences**

- **Systematics** due to waveform modeling **are not negligible when spins** and **higher modes are relevant.**

GW190521: a Signal Produced by the Largest BHs so far

GW190521: **a signal produced by the largest BHs so far**

(Abbott et al. PRL 125 (2020) 10, ApJ Lett 900 (2020) L13)

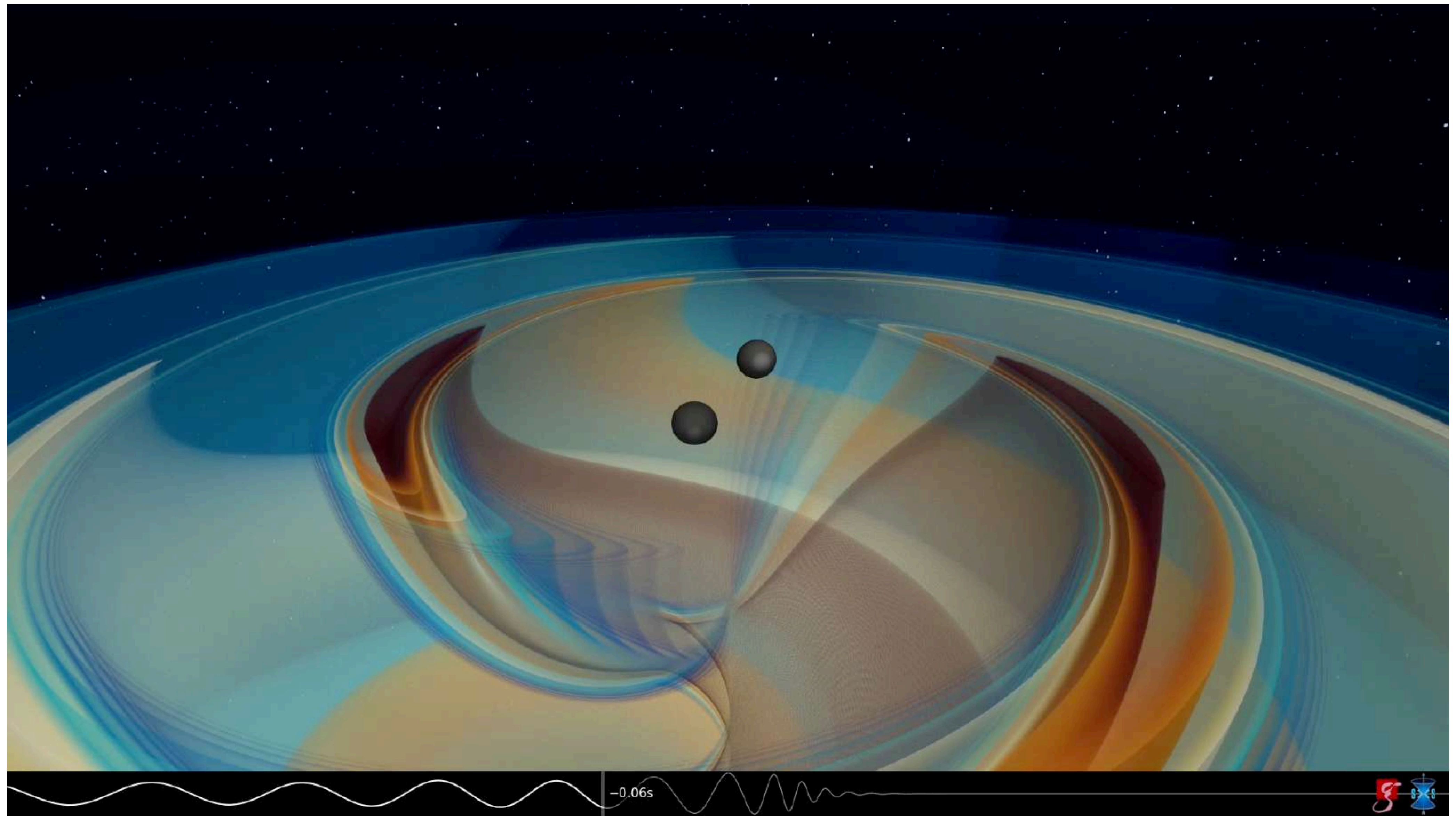
$$m_1 = 91.4^{+29.3}_{-17.5}\, M_\odot \qquad m_2 = 66.8^{+20.7}_{-20.7}\, M_\odot$$

$$q = m_1/m_2$$

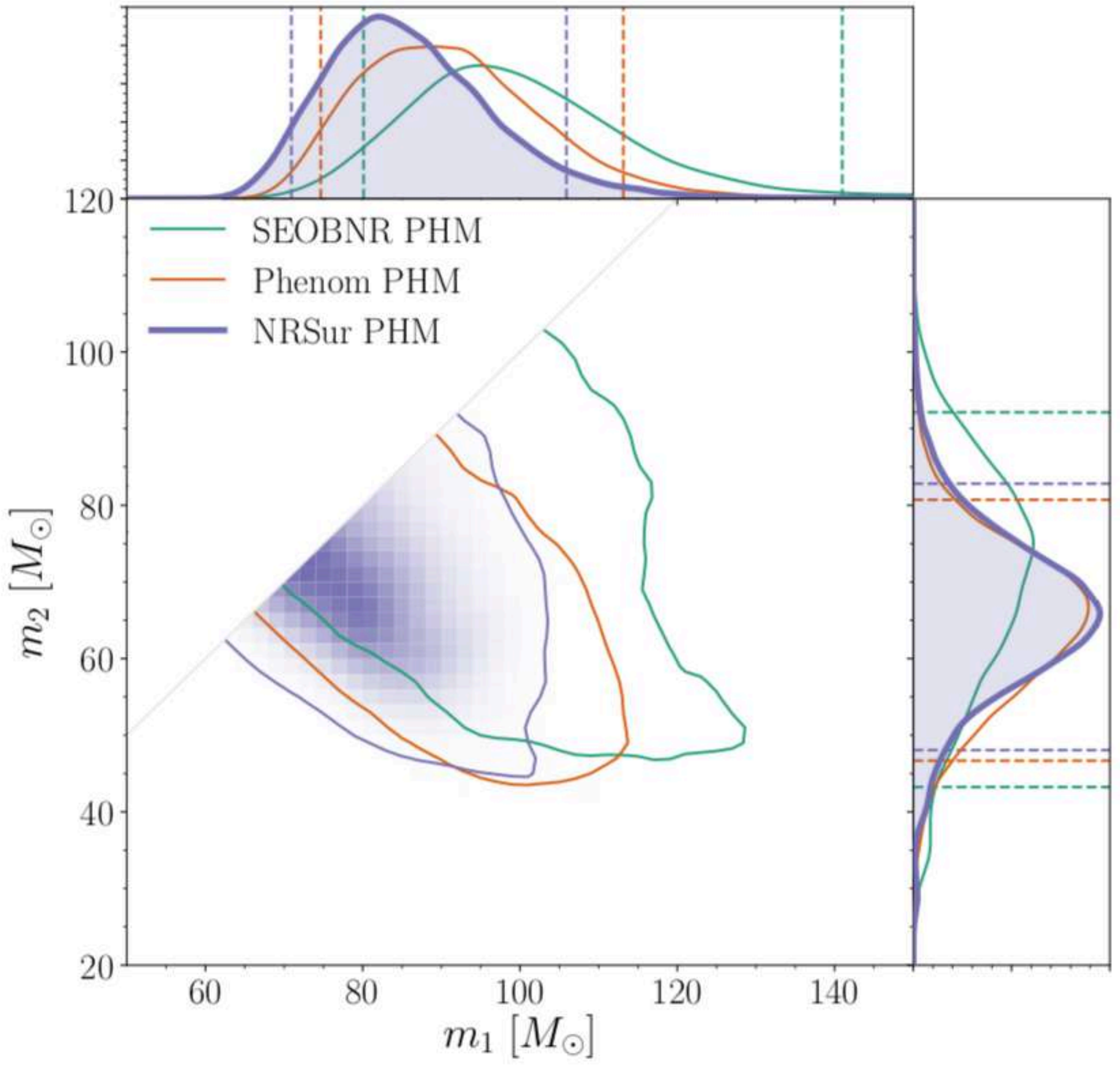
$$\chi_1 = S_1/m_1^2 \qquad \chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot \hat{\mathbf{L}}$$

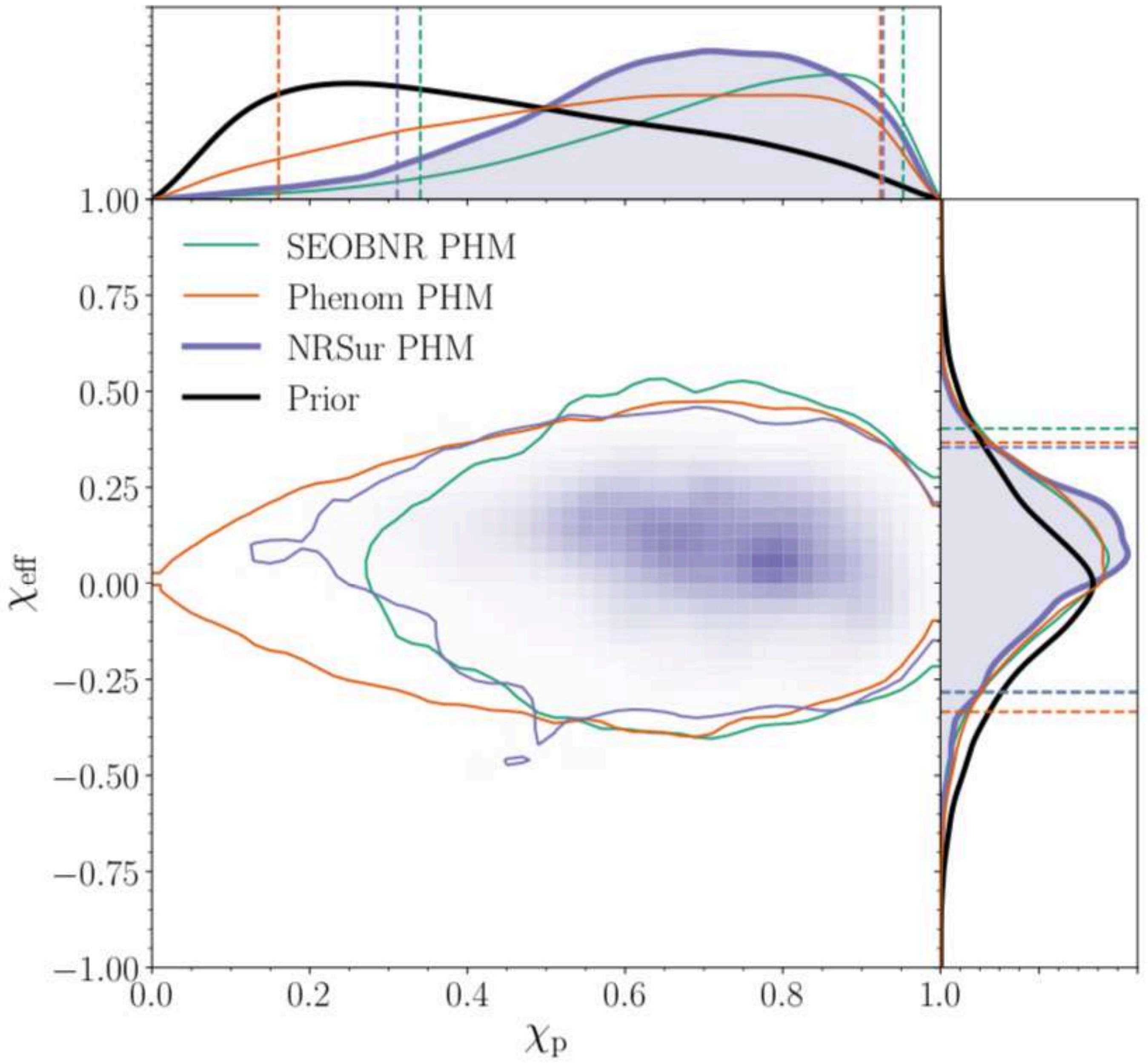
$$\chi_2 = S_2/m_2^2$$

- Likely, BHs **too massive** to have been formed **from a collapsed star, because of Pair-Instability SN.**



(credit: Fischer, Pfeiffer & AB; SXS Collaboration)

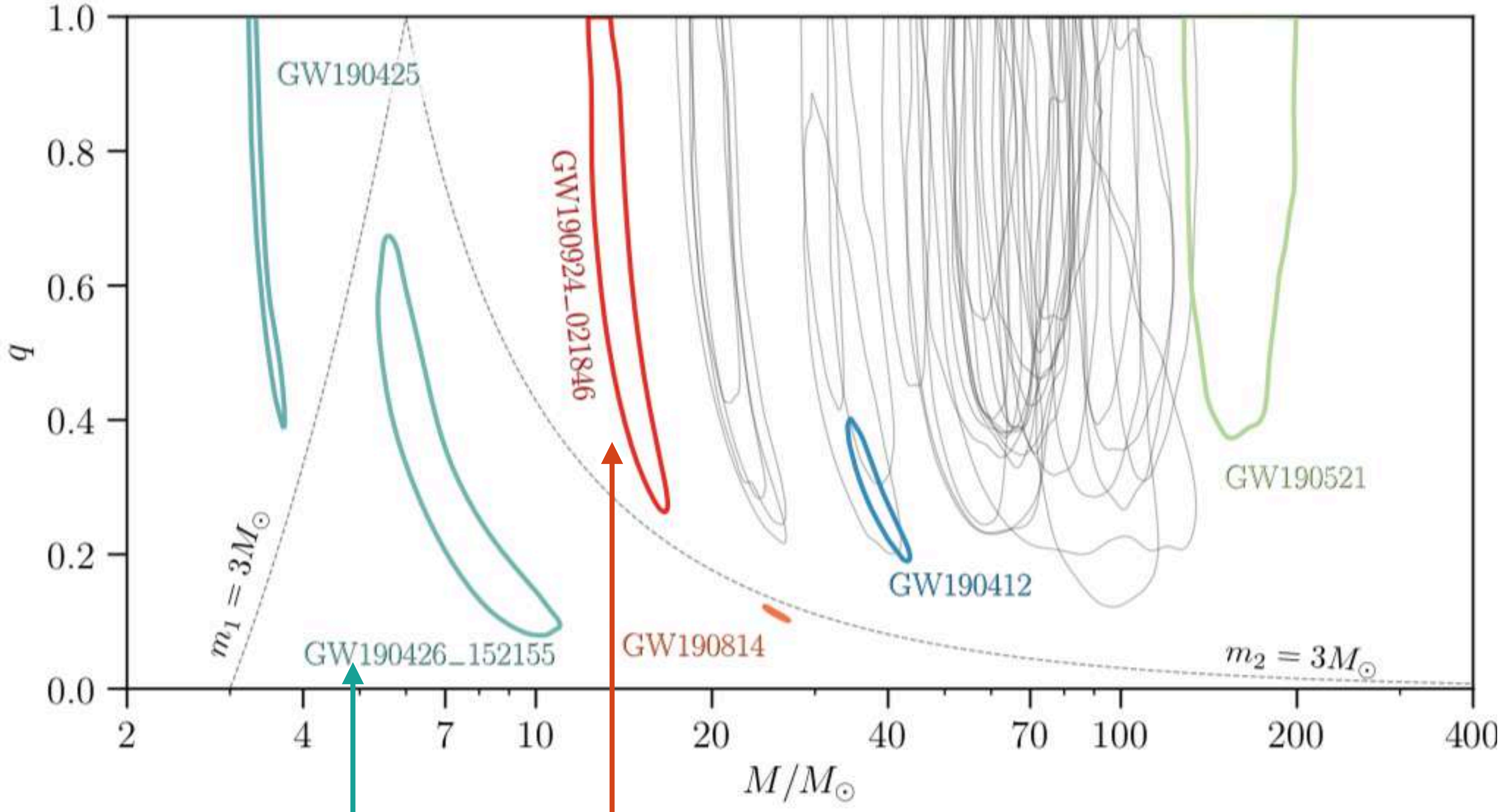




- **Systematics** due to waveform modeling **are not negligible when spin precession** and **higher modes are relevant,** but they are still **subdominant with respect to statistical uncertainty.**

Gravitational-Wave Transient Catalog 2: Source Properties

90 % credible regions



It could be a merger of a BH of about $6M_\odot$ with a NS.

Least massive system composed of two BHs

$$q = m_1/m_2 \qquad \mathscr{M} = M\,\nu^{3/5} \qquad \nu = \frac{q}{(1+q)^2}$$

$$\chi_1 = S_1/m_1^2 \qquad M = m_1 + m_2$$

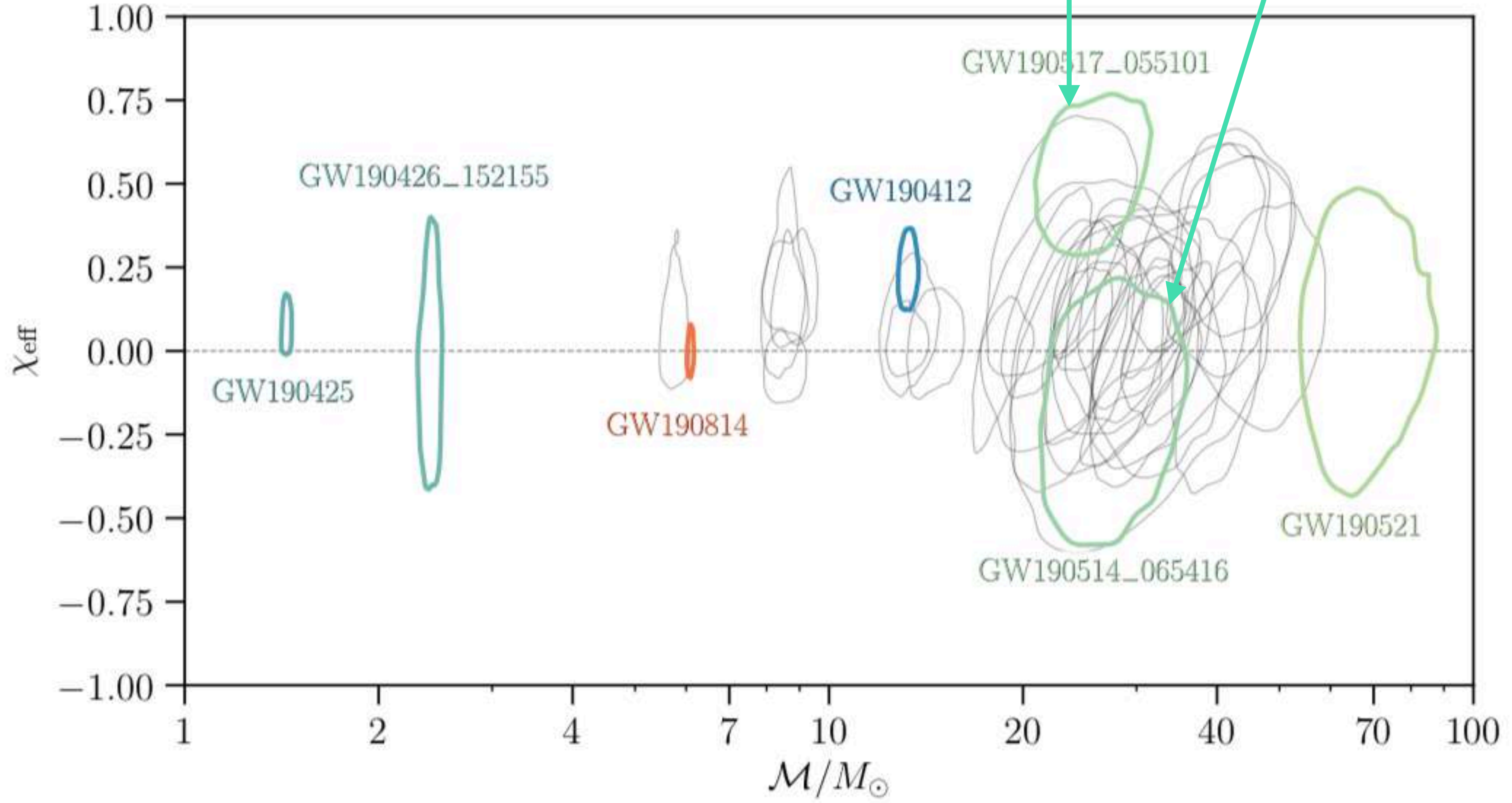
$$\chi_2 = S_2/m_2^2$$

$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot \hat{\mathbf{L}}$$

Highest probability of having the largest $\chi_{\rm eff}$

Highest probability of having the smallest $\chi_{\rm eff}$

90 % credible regions



(Abbott et al. arXiv:2010.14527)

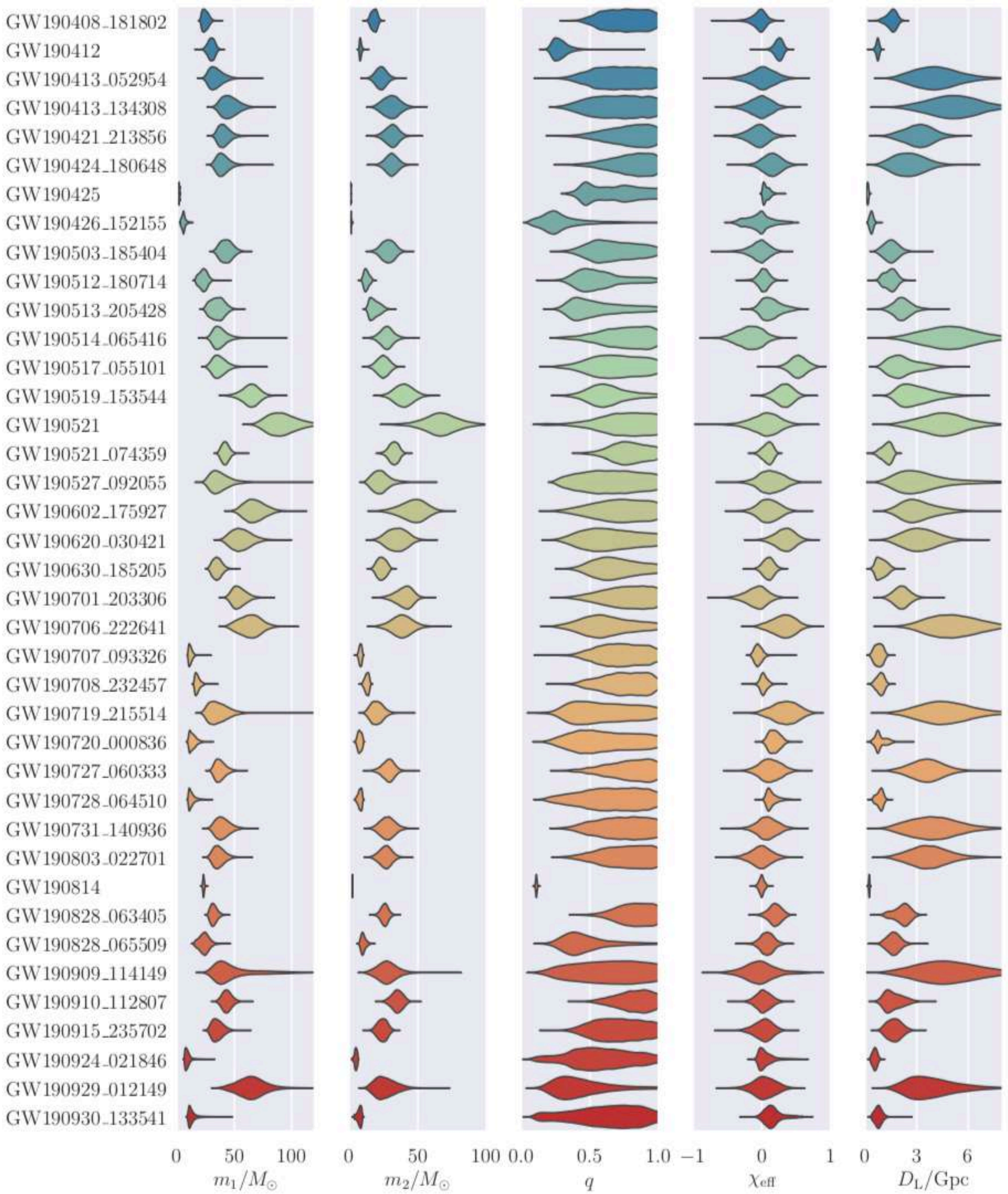
Gravitational-Wave Transient Catalog 2: Source Properties

$$q = m_1/m_2$$

$$\chi_1 = S_1/m_1^2 \qquad \chi_2 = S_2/m_2^2$$

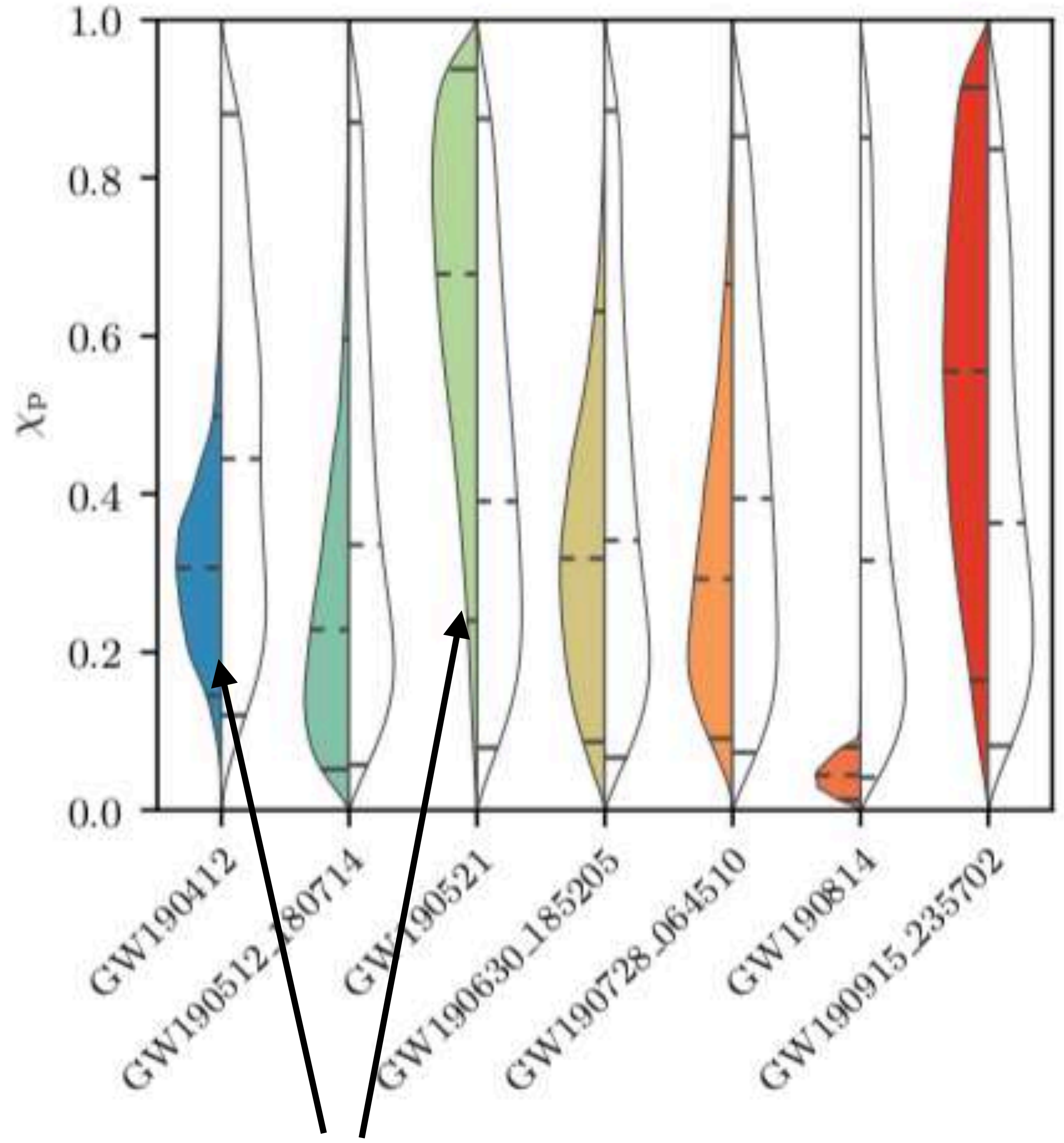
$$\chi_{\mathrm{eff}} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot\hat{\mathbf{L}}$$

- **Higher total masses** than GWTC-1
- **Three binaries** with component mass **in the lower mass gap** $\sim 2.5 - 5 M_\odot$
- **Three binaries** with component **mass larger than** $\sim 45 M_\odot$, i.e. in pair-instability SN mass gap
- At $95\,\%$ credibility, **ten sources have** $\chi_{\mathrm{eff}} > 0$



(Abbott et al. arXiv:2010.14527)

χ_p measures the spin components on the orbital plane



- Posterior distributions constrained away from zero: mild **sign of precession**.

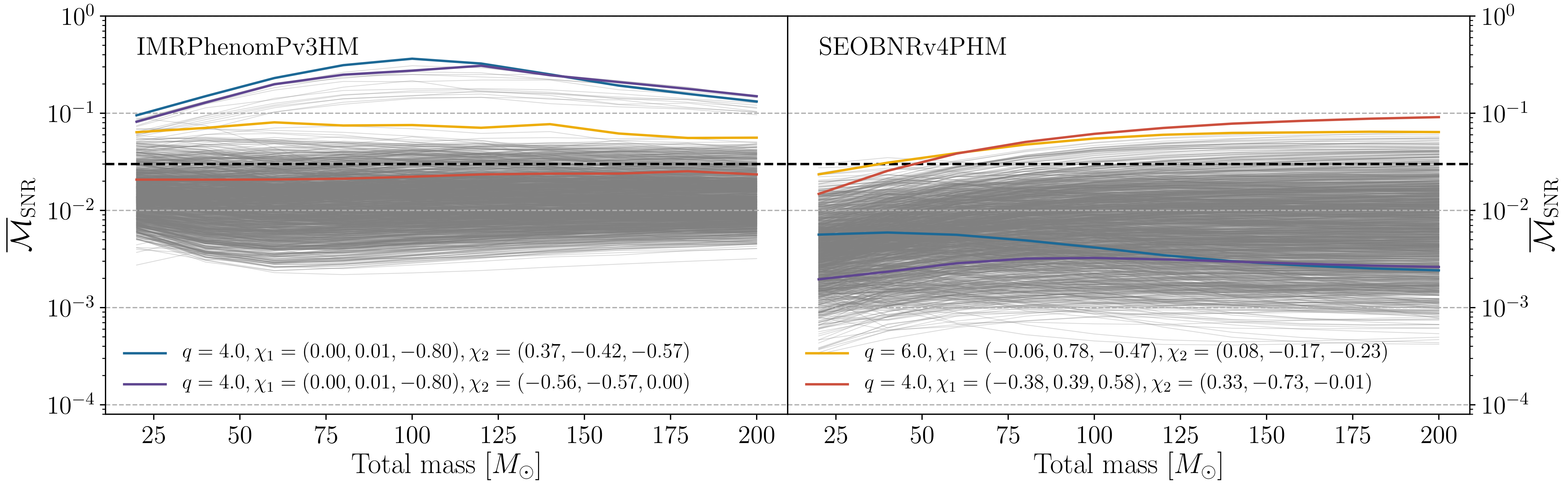
Assessing accuracy of BBH waveform models against NR

(Ossokine, AB, Marsat, Cotesta et al. 20)

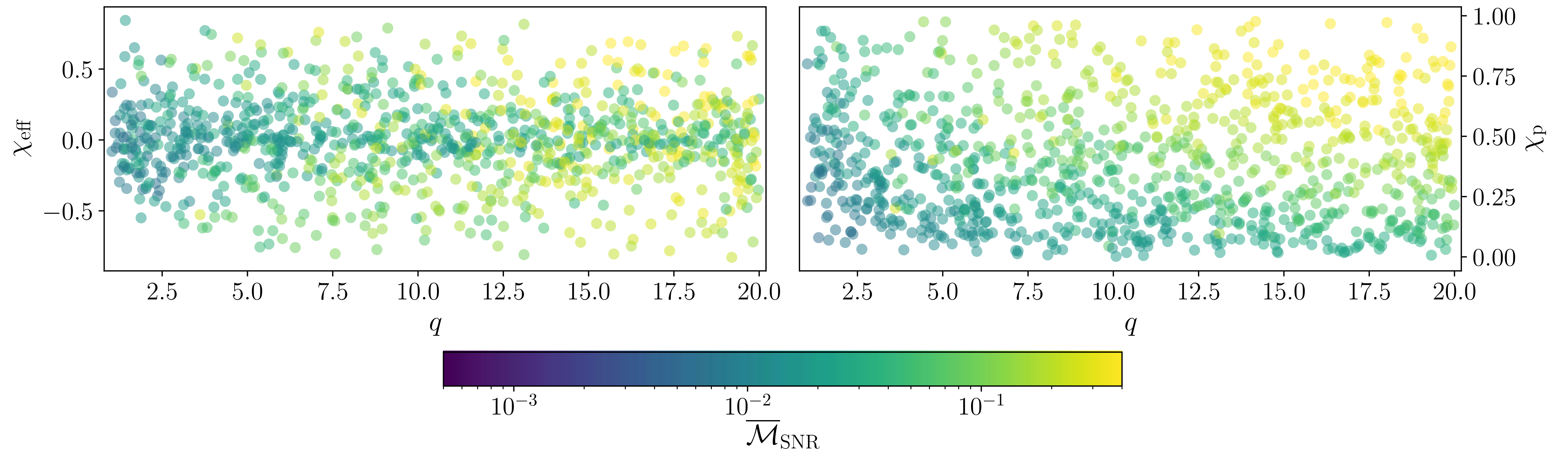
- Mismatch against **SXS NR public catalog (1344) of multipolar spin-precessing** waveforms.
- Waveform models are **calibrated** in the **non-precessing sector**.

- Mismatch **between two multipolar spin-precessing** waveforms.

unfaithfulness of SEOBNRv4PHM & IMRPhenomPv3HM against NR



unfaithfulness between SEOBNRv4PHM & IMRPhenomPv3HM



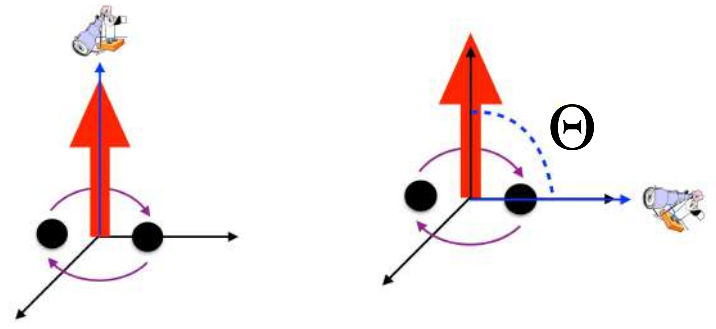
IMRPhenomPv3HM *(Khan et al. 19)*

- Waveform models differ the most for **large mass ratios** (> 10) and **large spins** (> 0.6) and **stronger precession**.

Assessing accuracy of waveform models with Bayesian analysis

(Ossokine, AB, Marsat, Cotesta et al. 20)

$$h_+(t;\Theta,\varphi) - i\,h_\times(t;\Theta,\varphi) = \sum_{\ell=2}^{\infty}\sum_{m=-\ell}^{\ell} {}_{-2}Y_{\ell m}(\Theta,\varphi)\,h_{\ell m}(t)$$



(credit: Cotesta)

NR signal is injected in O4 run:

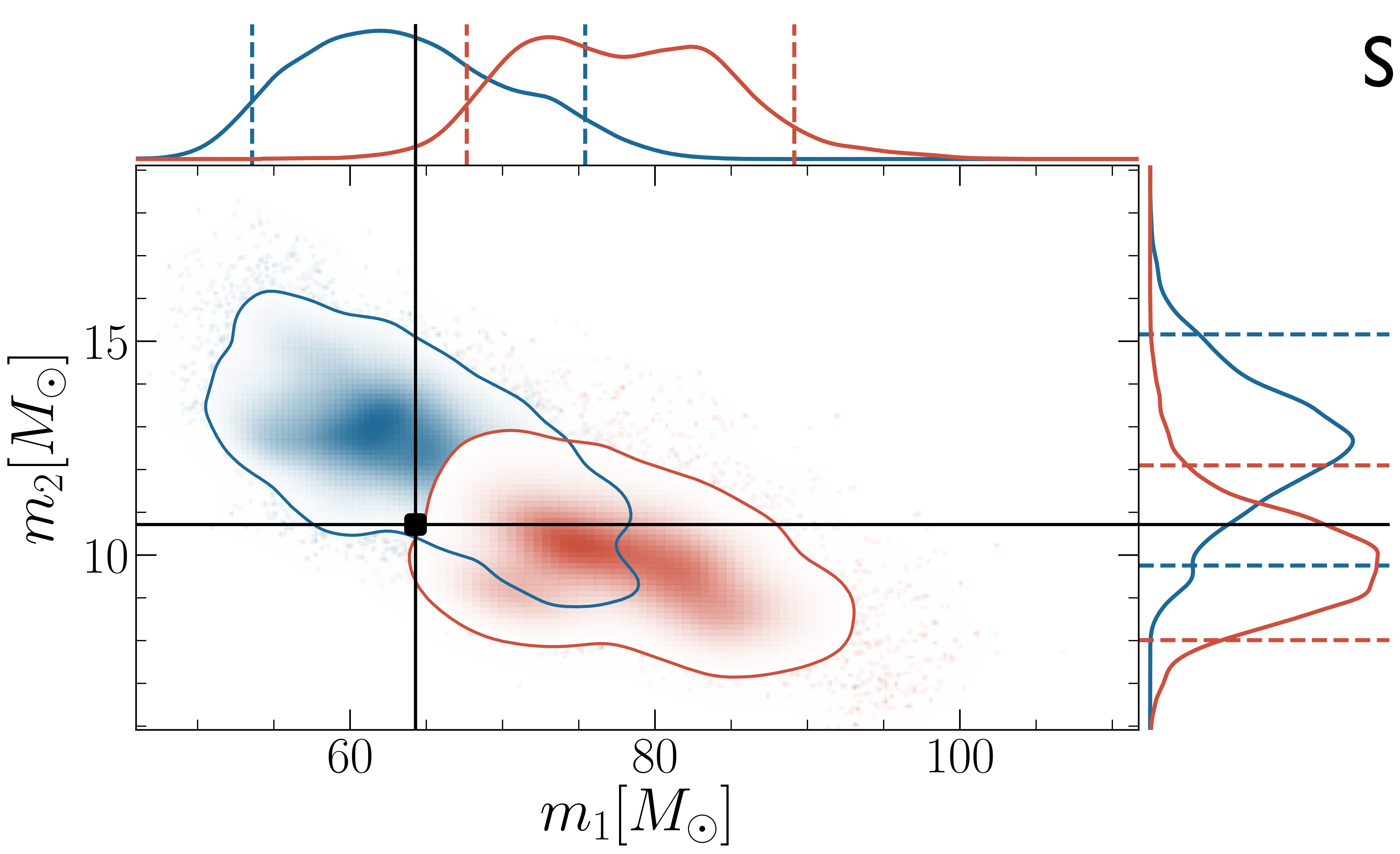
$$M = 76 M_\odot, q = 6, \chi_1 = (-0.06, 0.78, -0.47), \chi_2 = (0.08, -0.17, -0.23)$$

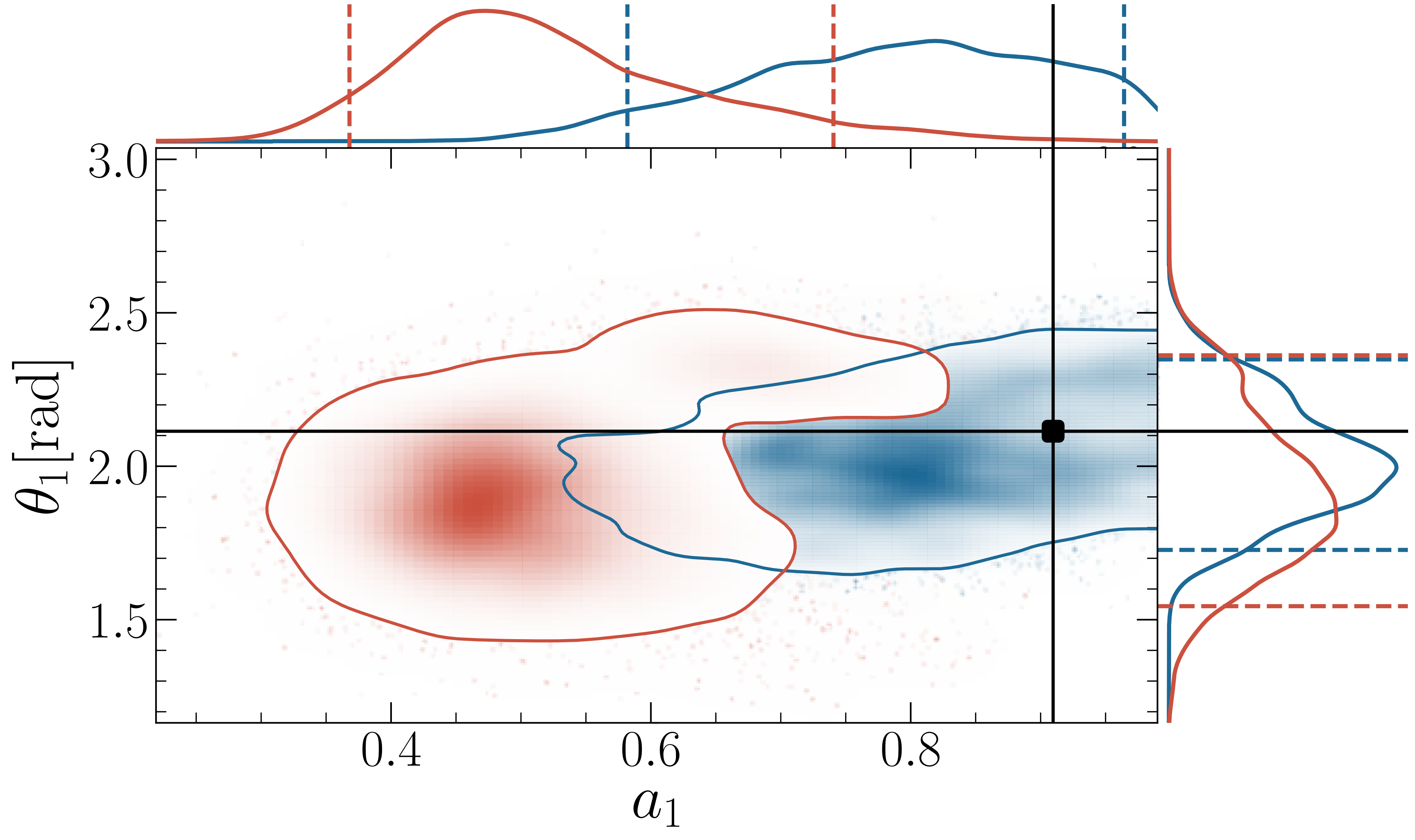
edge on: $\Theta = \pi/2$

SEOBNRv4PHM unfaithfulness = $4.4\,\%$

IMRPhenomPv3HM unfaithfulness = $8.8\,\%$

SNR = 21





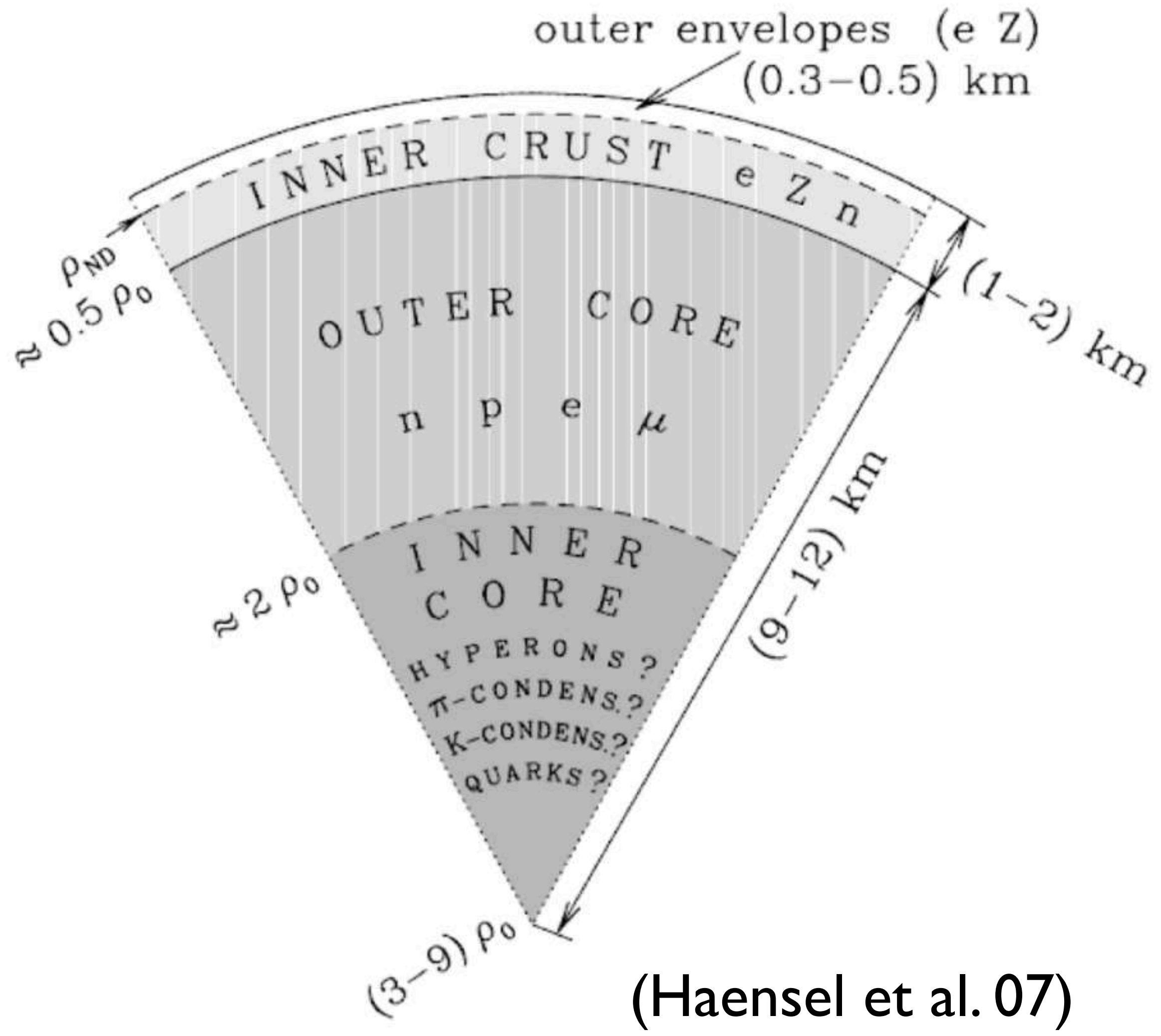
(see also Pürrer and Haster 20)

SEOBNRv4PHM IMRPhenomPv3HM

Probing Extreme-Matter with Gravitational Waves

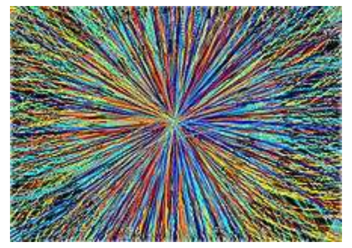
- **Neutron-star (NS)** properties:

- mass: $1 - 3\, M_{\rm Sun}$
- radius: $9 - 15\,{\rm km}$
- inner core density > $2 \times (2.8 \times 10^{14})\,{\rm g/cm^3}$ (nuclear density)
- magnetic field: $\sim 10^{15} \times$ @Earth
- surface temperature: $\sim 10^{3} \times$ @Earth
- pressure: $\sim 10^{27} \times$ @Earth

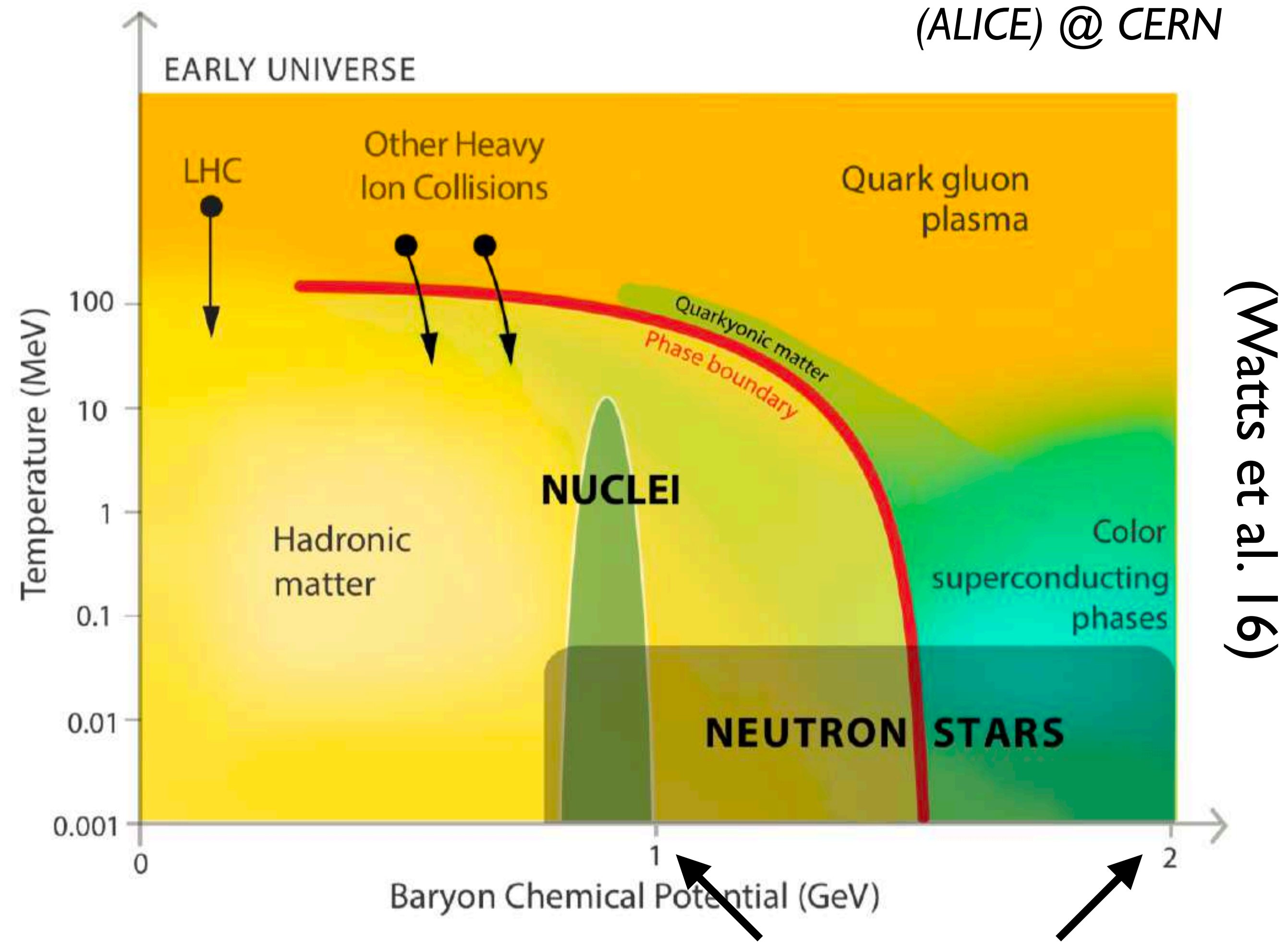


(Haensel et al. 07)

- **Conjectured** states of **matter.**

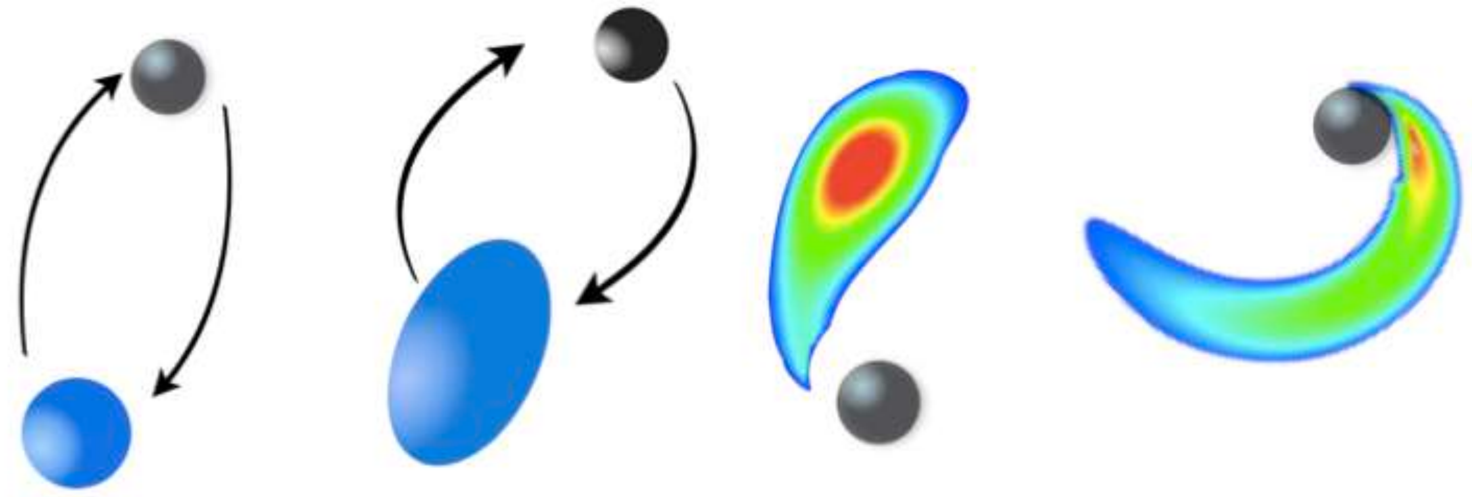


A Large Ion Collider Experiment (ALICE) @ CERN



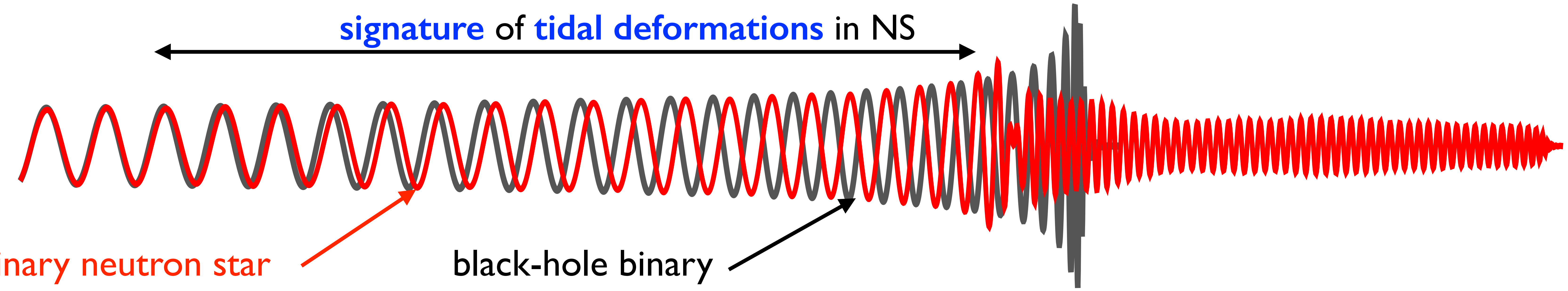
(Watts et al. 16)

- What is the **internal structure** and **composition** of **neutron stars**?



(credit: Hinderer)

- **NS equation of state** (EOS) affects gravitational **waveform** during **late inspiral, merger** and **post-merger**.



Probing Extreme-Matter with Gravitational Waves (contd.)

- **Neutron-star (NS)** properties:

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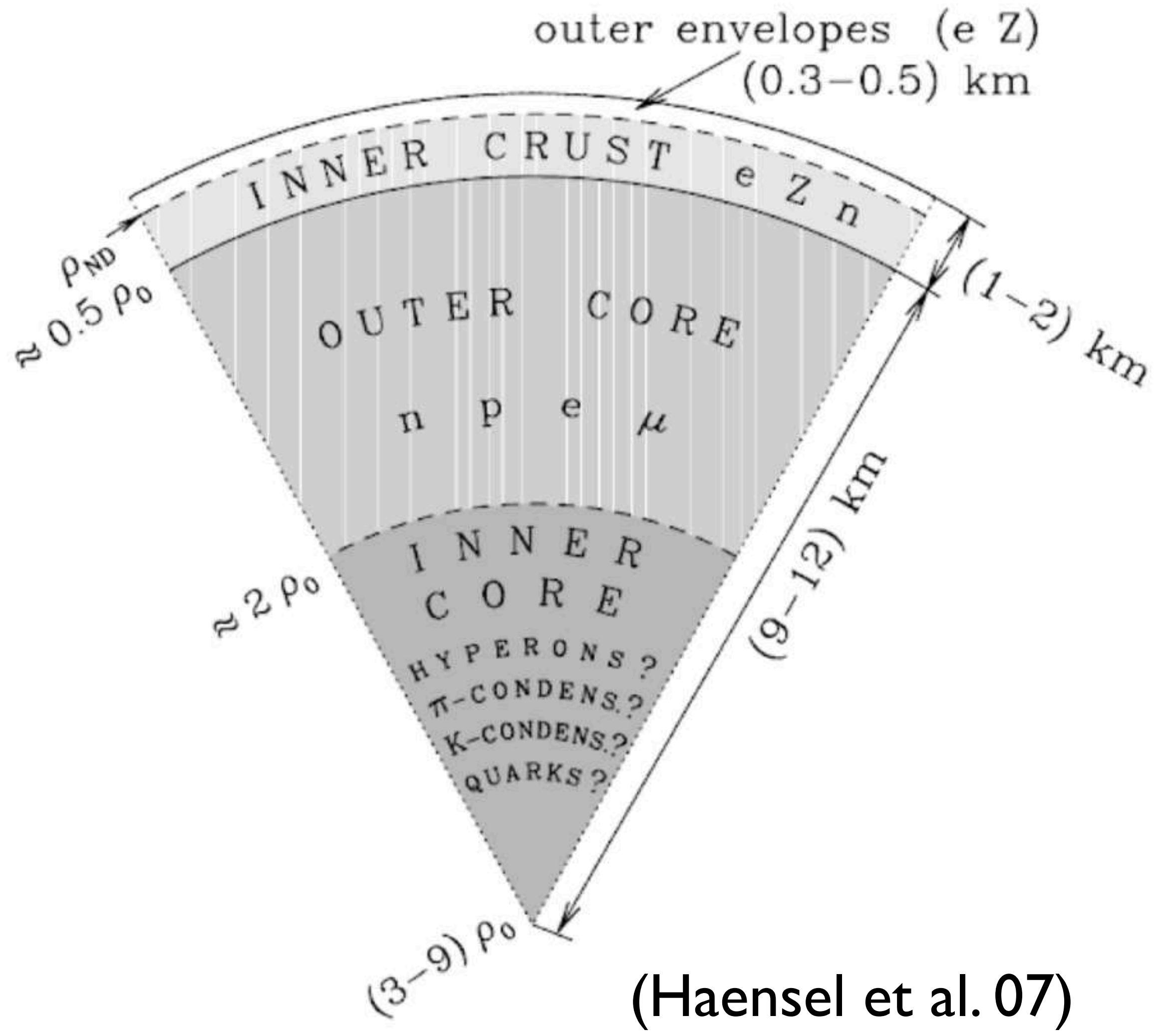
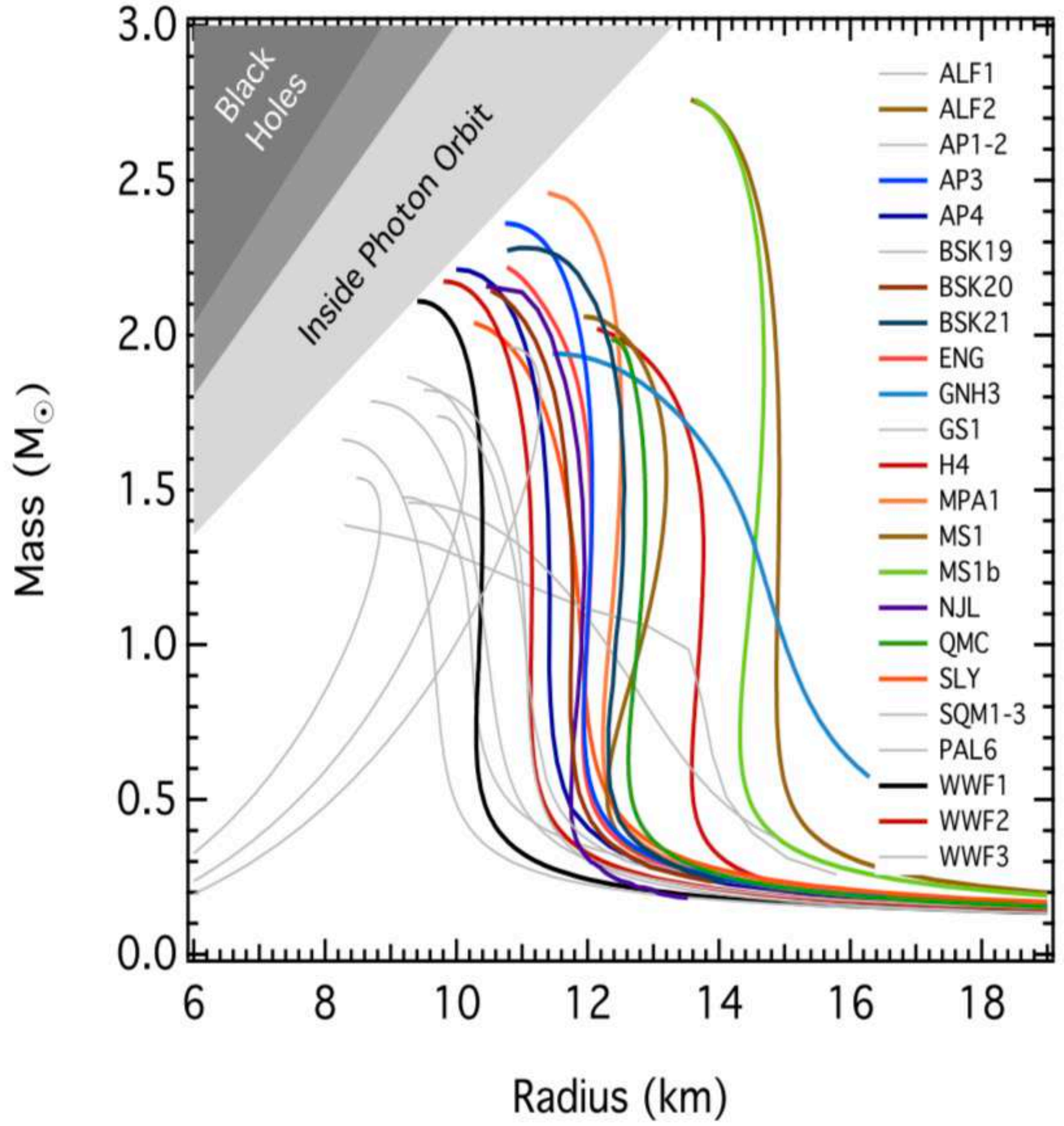
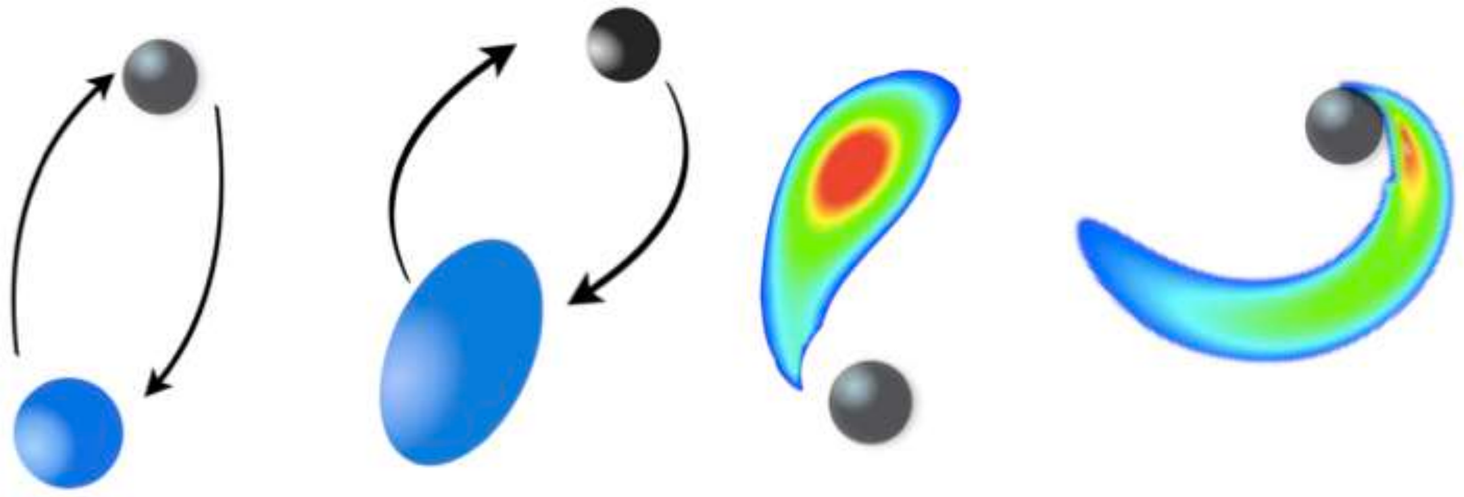


(Haensel et al. 07)



(Özel & Freire 2016)

- What is the **internal structure** and **composition** of **neutron stars**?



(credit: Hinderer)

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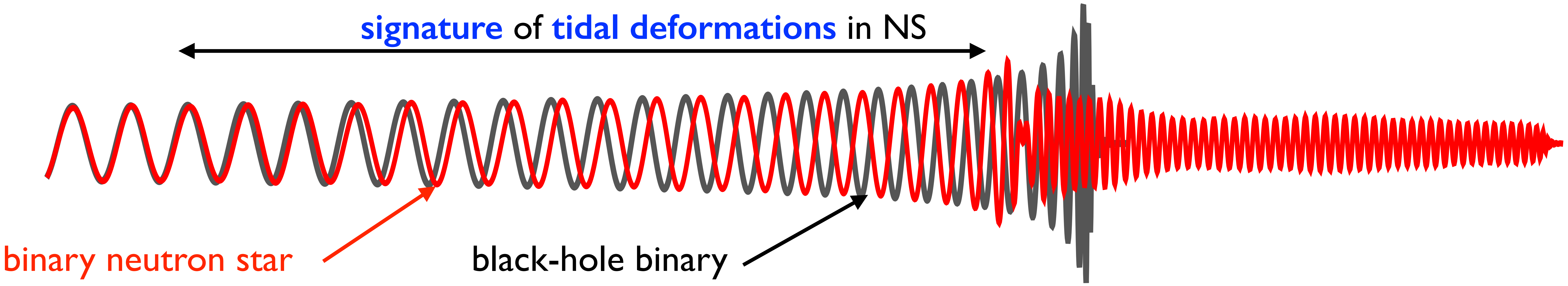
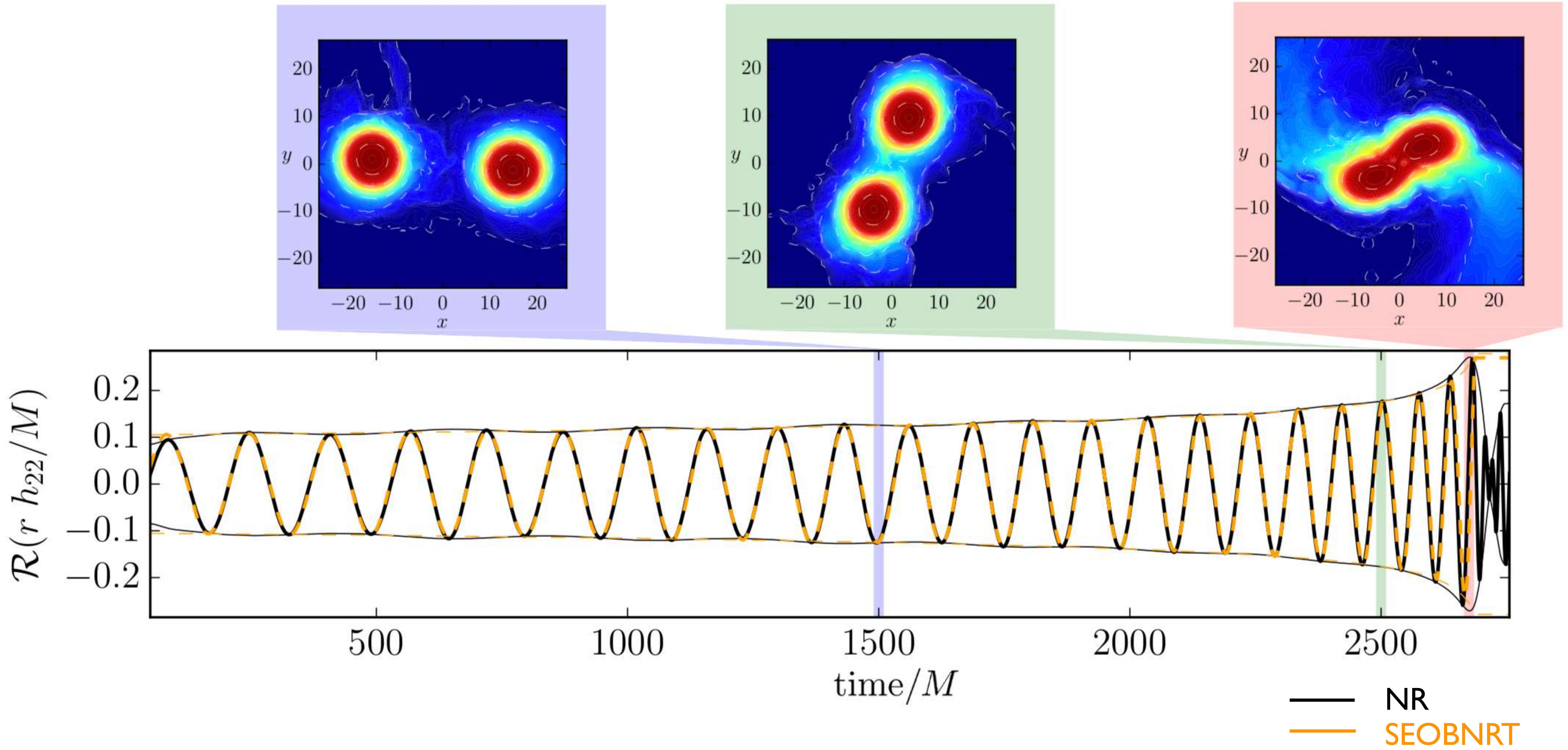


Waveforms for BNS combining Analytical & Numerical Relativity

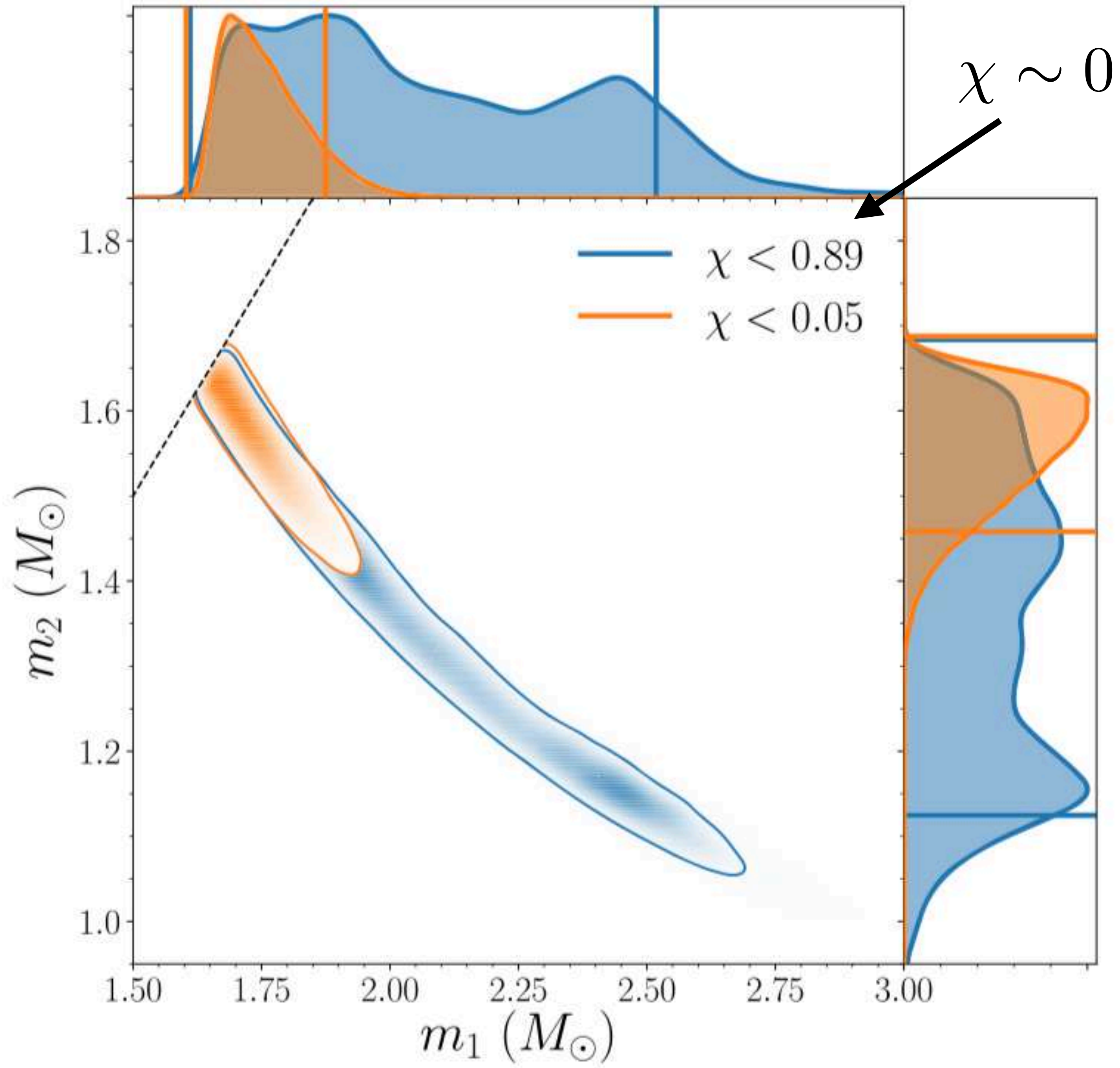
- Synergy between **analytical** and **numerical work** is **crucial.**



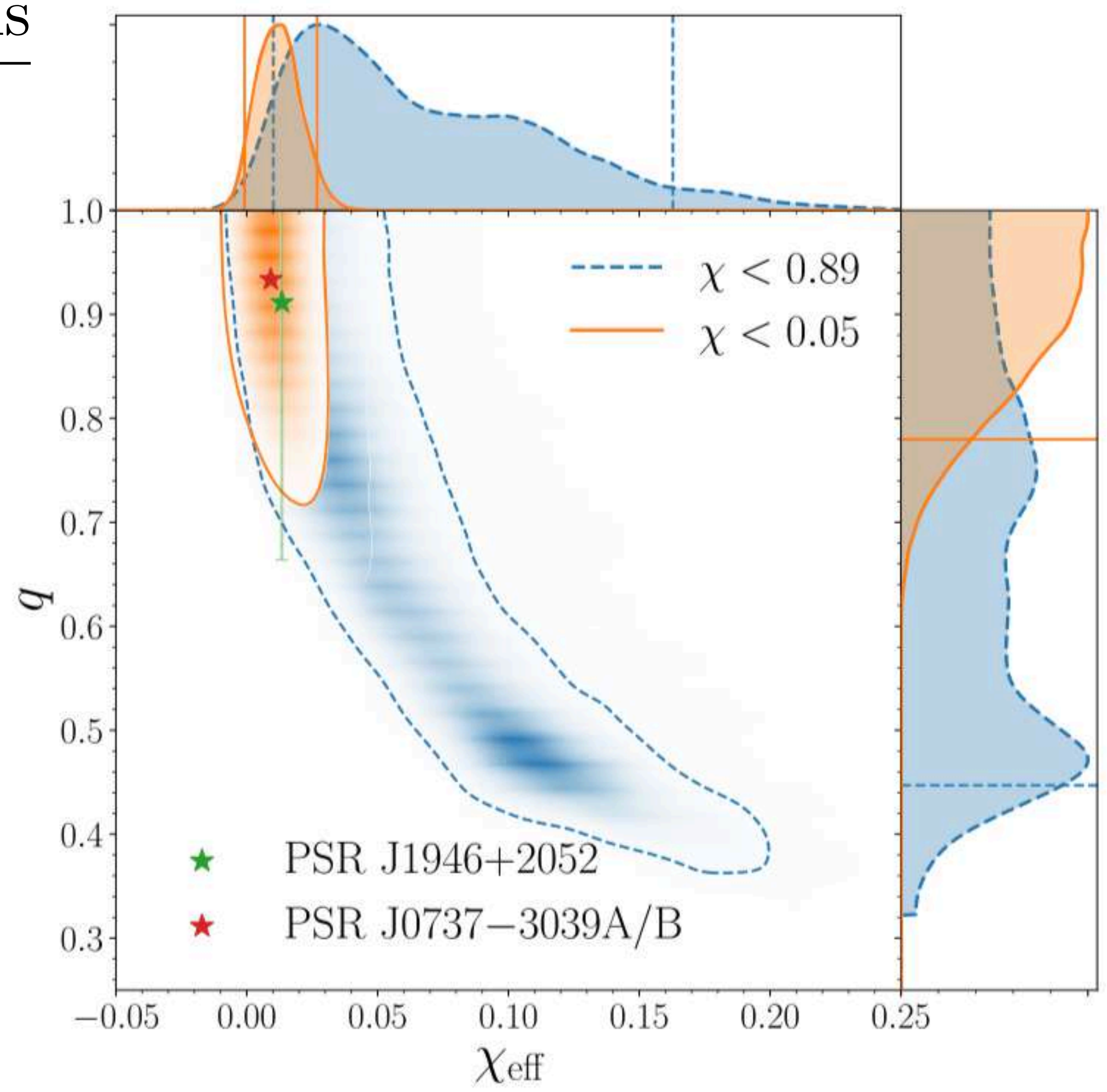
(Damour 1983, Flanagan & Hinderer 08, Binnington & Poisson 09, Vines et al. 11, Damour & Nagar 09, 12, Bernuzzi et al. 15, Hinderer, …AB … et al. 16, Steinhoff, … AB … et al. 16, Dietrich et al. 17-19, Nagar et al. 18)

GW190425: a Binary Neutron Star with Surprisingly High Mass

(*Abbott et al. ApJ Lett* 892 (2020))



$\chi \sim 0.4 \frac{1\text{ms}}{T}$



$$q = m_1/m_2$$

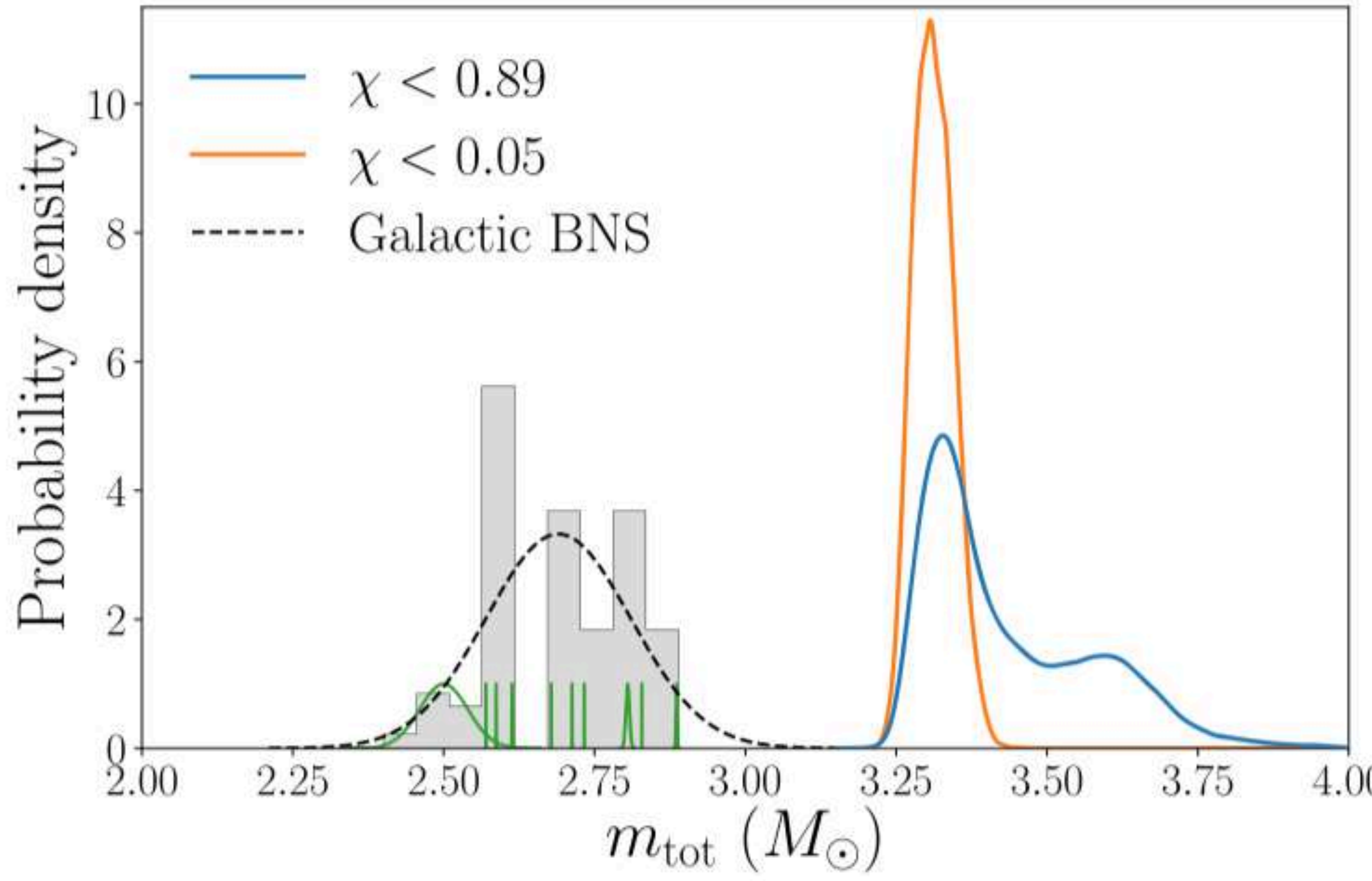
$$\chi_1 = S_1/m_1^2$$

$$\chi_2 = S_2/m_2^2$$

$$\chi_{\rm eff} = \left(\frac{m_1}{M}\chi_1 + \frac{m_2}{M}\chi_2\right)\cdot\hat{\mathbf{L}}$$

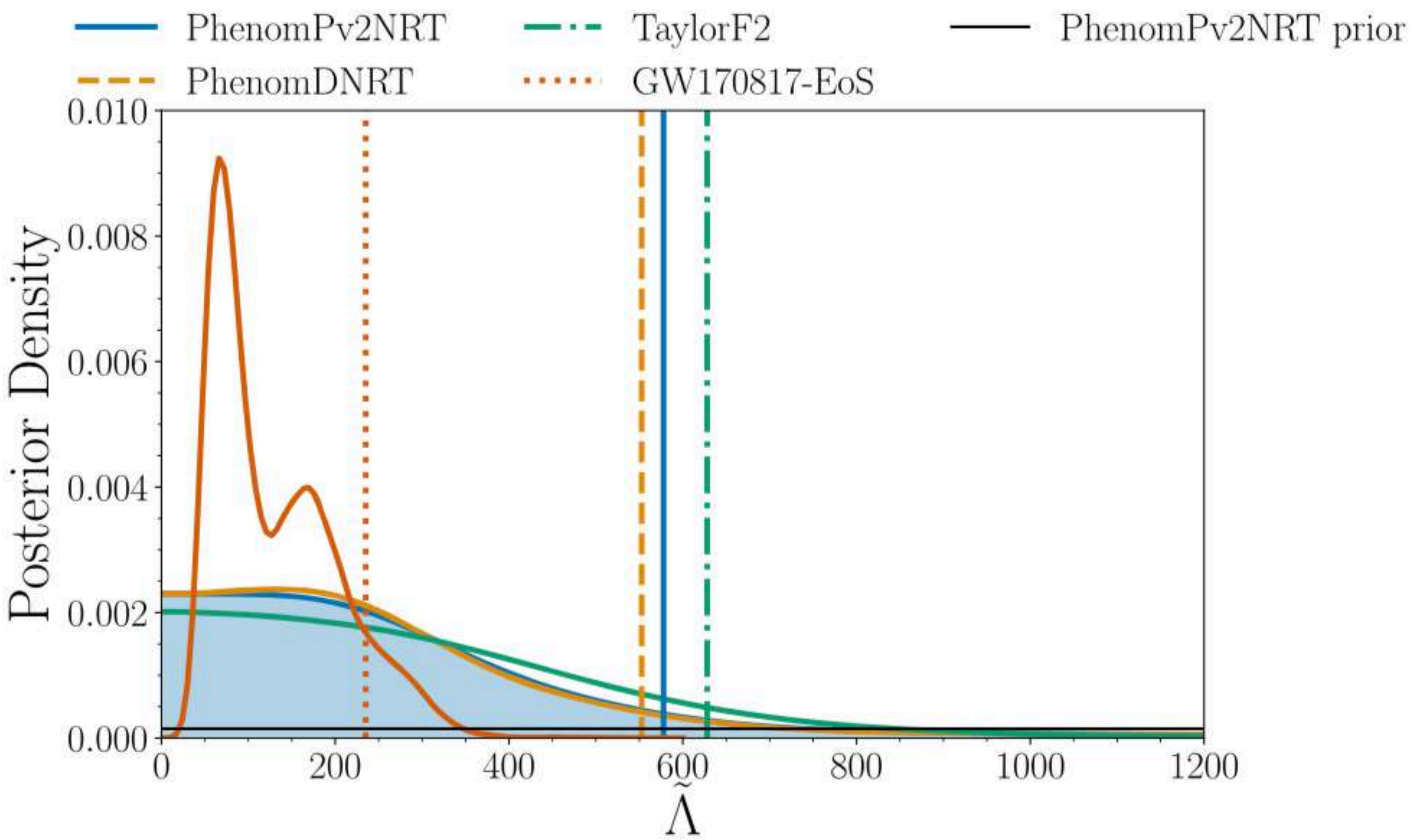
- **GW190425's masses** are **consistent with mass measurements of NSs** in binaries.

- **GW190425's total mass** $3.4^{+0.3}_{-0.1} M_\odot$ is larger than BNSs in our galaxy: **new population of BNS?**



GW190425: Inference on Tidal Deformability Parameter

(*Abbott et al. ApJ Lett* 892 (2020))

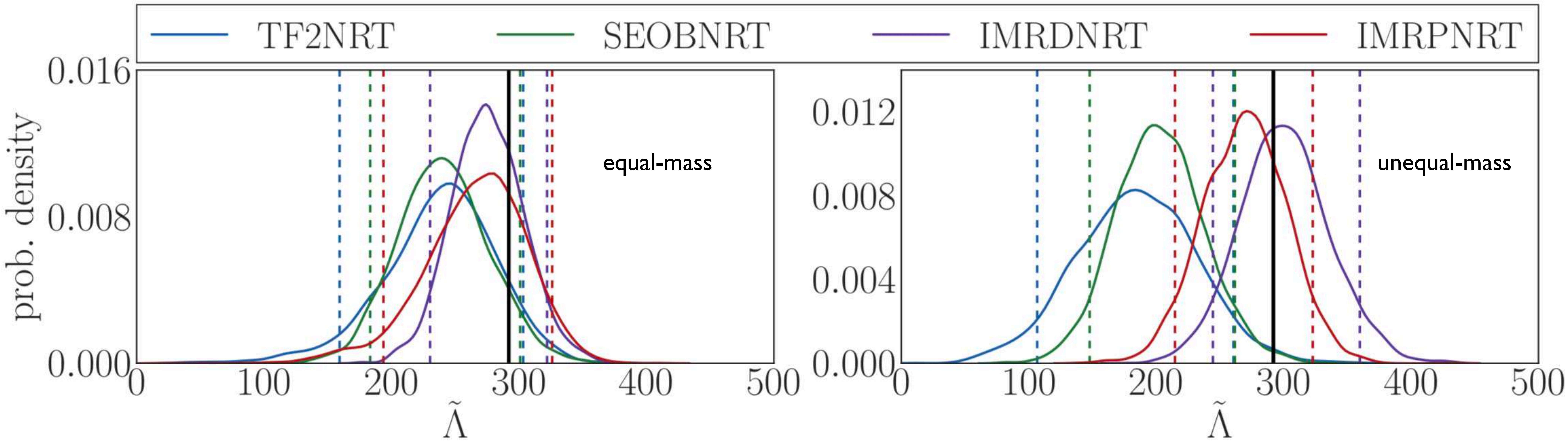


•**GW190425's SNR is lower (~ 13)** than GW170817's SNR (~ 34): **looser constraint** on tidal deformability.

Assessing accuracy of BNS waveform models against NR

- Synthetic GW signal of a **binary neutron star** at **50 Mpc** is **injected** in Gaussian noise with **O4** noise-spectral density **(SNR ~ 87)**.
- **Inference** with waveform models that have **same matter effects**, but **baseline point-mass model** is **different.**

- **IMRDNRT** is injected in O4 run



(Samajdar & Dietrich 18)

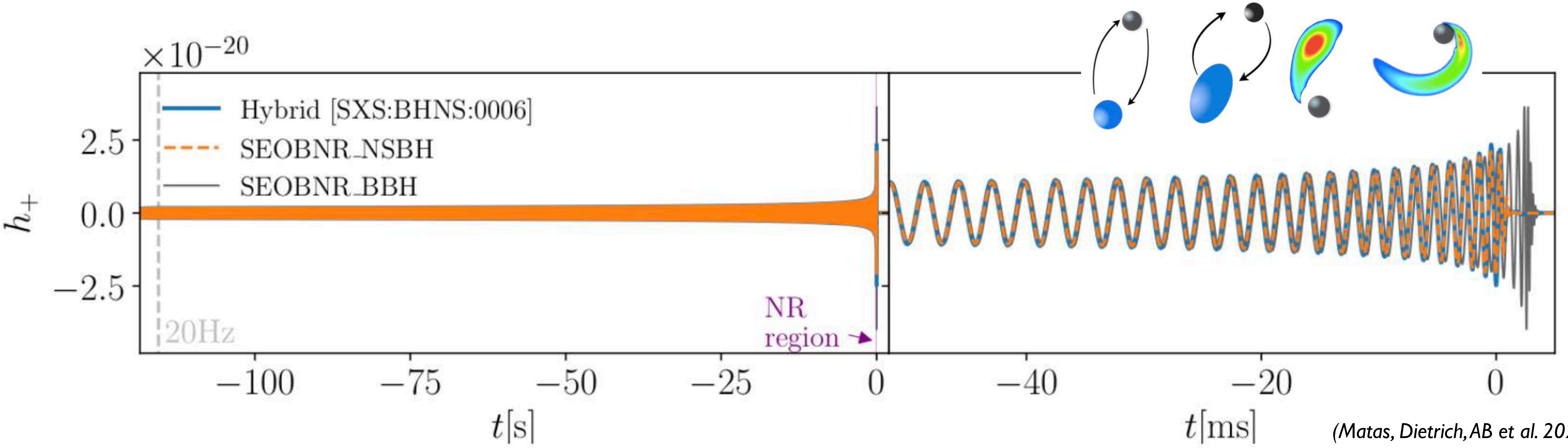
(see also Dudi et al. 18, Samajdar & Dietrich 19, Gamba et al. 20)

Systematics larger than statistical errors!

- To obtain **NS's radius with precision of 0.5-1 km**, more **accurate waveforms** are needed.

Waveforms for NSBH combining Analytical & Numerical Relativity

- Synergy between **analytical** and **numerical work** is **crucial.**



(Matas, Dietrich, AB et al. 2020)

(see also Lackey et al. 14, Pannarale et al. 15, 16, Pürrer et al. 17, Chakravarti et al. 17)

(see also Thompson et al. 2020)

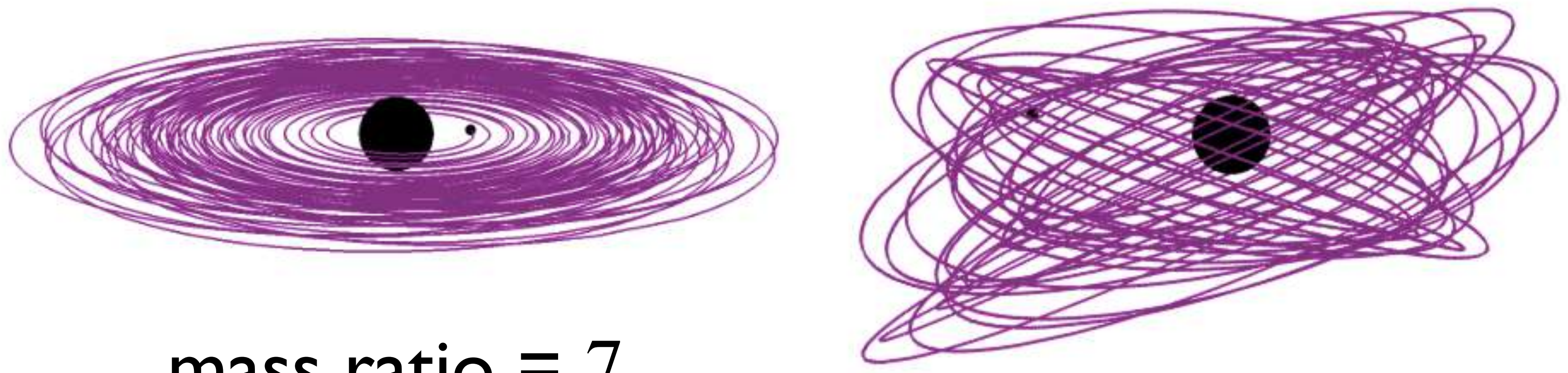
- So far, **NSBH waveforms** were used **only for** inference study of **GW190814**.
- **More accurate NSBH waveforms** are **needed for future runs** (O4, O5, CE and ET).

Waveforms for Eccentric Compact-Object Binaries

- How to **discriminate** among binary's **formation scenarios**, and **probe astrophysical environment**? **Eccentricity, spin-magnitude** and **spin-precession can disclose this information.**

- **Eccentric** compact-object **binary:**

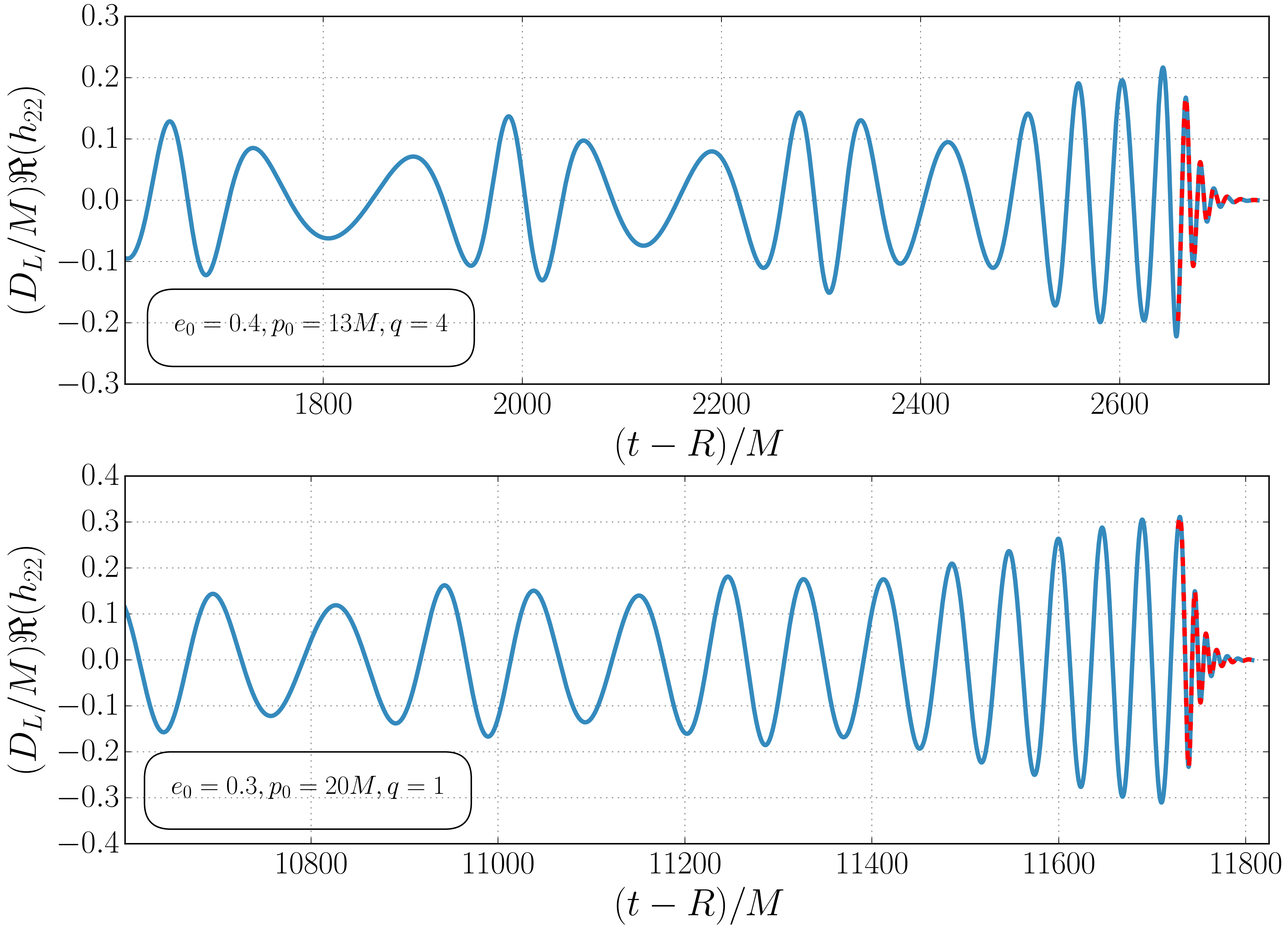
NR simulation



mass ratio = 7

(Lewis et al. 16)

(Hinderer & Babak 17)



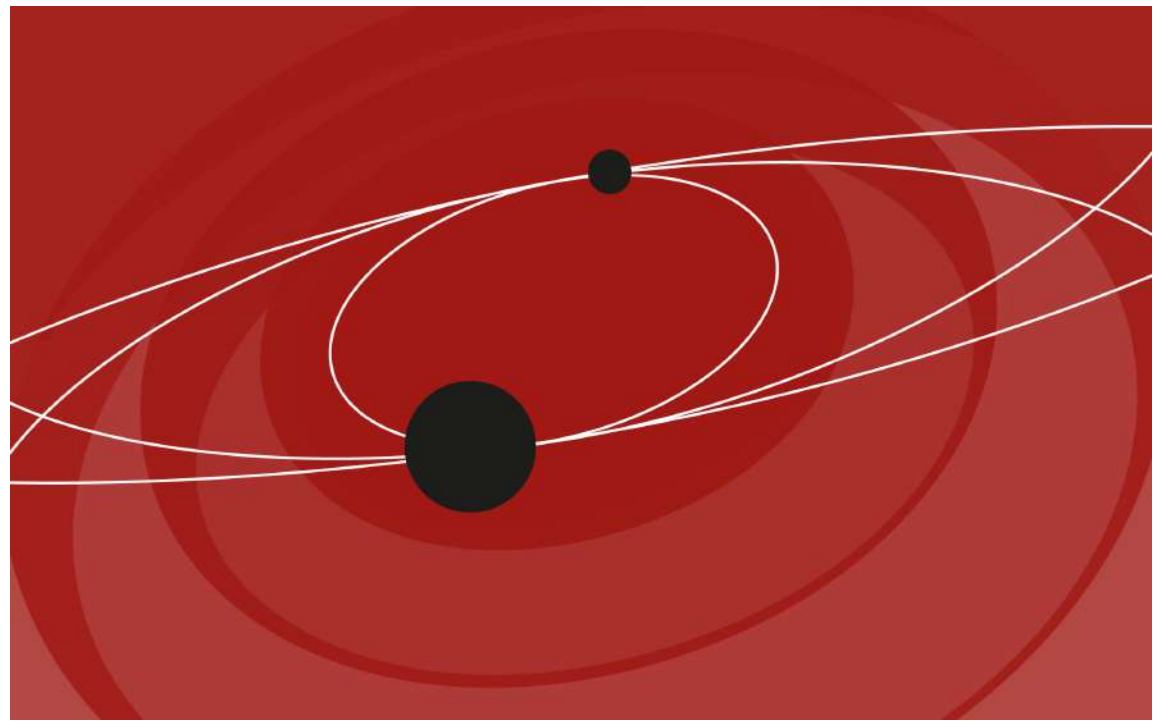
(many PN papers on eccentricity; Bini et al. 12; East et al. 13; Huerta et al. 14-19, Hinder et al. 17; Cao & Han 17; Loutrel & Yunes 16, 17; Ireland et al. 19, Moore & Yunes 19, Chiaramello & Nagar 20, Buades et al. 20, Liu et al. 21, Nagar et al. 20, 21, Islam et al. 21, Khalil et al. 21)

- **Current eccentric** models **do not cover all physical effects** (e.g., spin-precession and harmonics) **or all stages** of coalescence or **entire range** of **eccentricity, accurately** (but a lot of progress in last months!).

Scattering Amplitude: A New Way to Study 2-body Problem

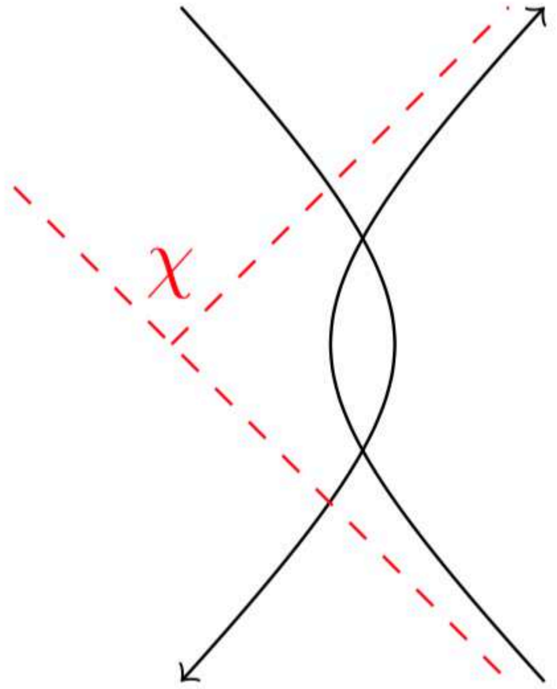
- Relativistic 2-body dynamics

(credit: Carvalho)

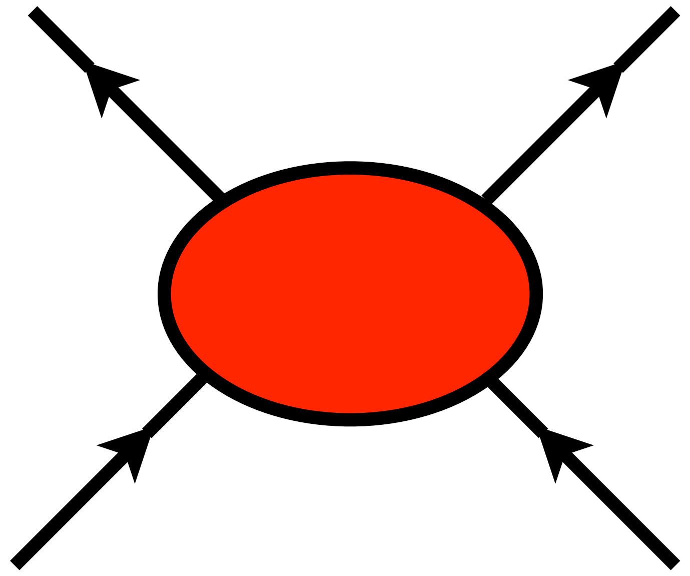


- Classical scattering: scattering angle χ

(credit: Steinhoff)



- Quantum scattering amplitude



e.g., in Born approximation: Fourier transform of potential is related to scattering amplitude

- (Local in-time) 2-body **Hamiltonian at 4PM (3 loops)** for nonspinning BHs. *(Cheung et al. 19, 20, Bern et al. 19, Blümlein et al. 20, Kälin et al. 20, Bern et al 21)*

Small parameter is **$GM/rc^2 << 1$, $v^2/c^2 \sim 1$,** large separation, natural **for unbound motion/scattering**

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r})$$

$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2) \left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$E = E_1 + E_2 \qquad \gamma = E/m$$

$$\xi = E_1 E_2 / E^2 \qquad \sigma = \frac{p_1 \cdot p_2}{m_1 m_2}$$

$$V^{(1)}(\mathbf{p},\mathbf{q}) = \int \frac{\mathbf{d^3 r}}{(2\pi)^3} \mathcal{M}^{\text{tree}}(\mathbf{p},\mathbf{q})\, \mathbf{e}^{-\mathbf{i r \cdot q}}$$

↑ amplitude

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}(1 - 2\sigma^2)$$

Results from Interplay with Scattering Amplitude Methods & EFT

(Bern et al. 19, 20)

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
1PM	$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ v^{14}$	$+\ \ldots)$	G^1
2PM		$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ v^{12}$	$+\ \ldots)$	G^2
3PM			$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ v^{10}$	$+\ \ldots)$	G^3
4PM				$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ v^8$	$+\ \ldots)$	G^4
5PM					$(1$	$+\ v^2$	$+\ v^4$	$+\ v^6$	$+\ \ldots)$	G^5
6PM						$(1$	$+\ v^2$	$+\ v^4$	$+\ \ldots)$	G^6
							$\vdots$			

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$$V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i$$

$$E = E_1 + E_2 \qquad \gamma = E/m$$

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$$V^{(1)}(\mathbf{p},\mathbf{q}) = \int \frac{d^3\mathbf{r}}{(2\pi)^3}\, \mathcal{M}^{\text{tree}}(\mathbf{p},\mathbf{q})\, e^{-i\mathbf{r}\cdot\mathbf{q}}$$

↑ amplitude

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}(1 - 2\sigma^2)$$

Comparison between PMs and NR binding energies

• 2-body non-spinning (local-in-time) **Hamiltonian at 4PM order** computed using scattering-amplitude methods.

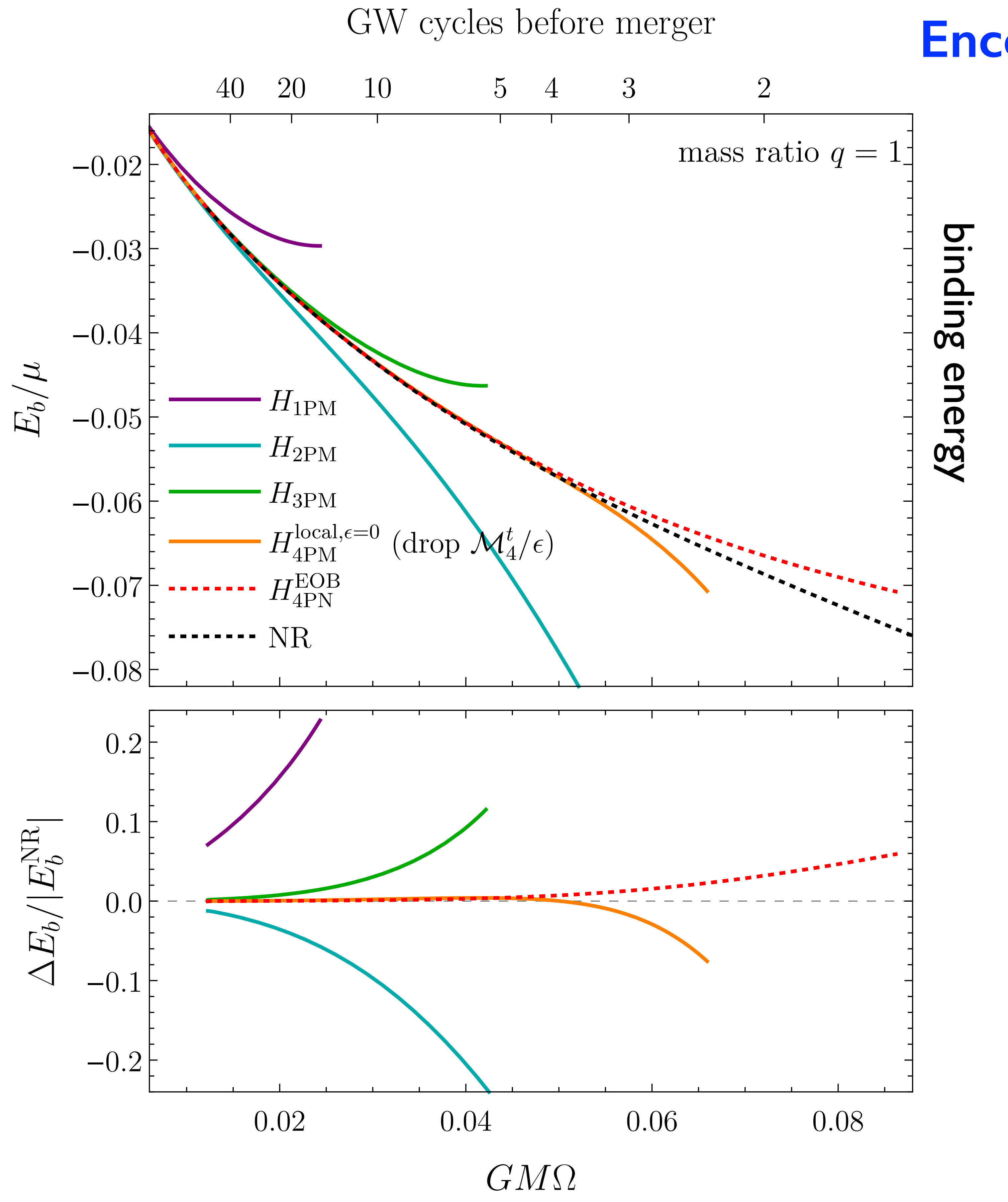
(Cheung et al. 18, Bern et al. 19, Bern et al. 21)

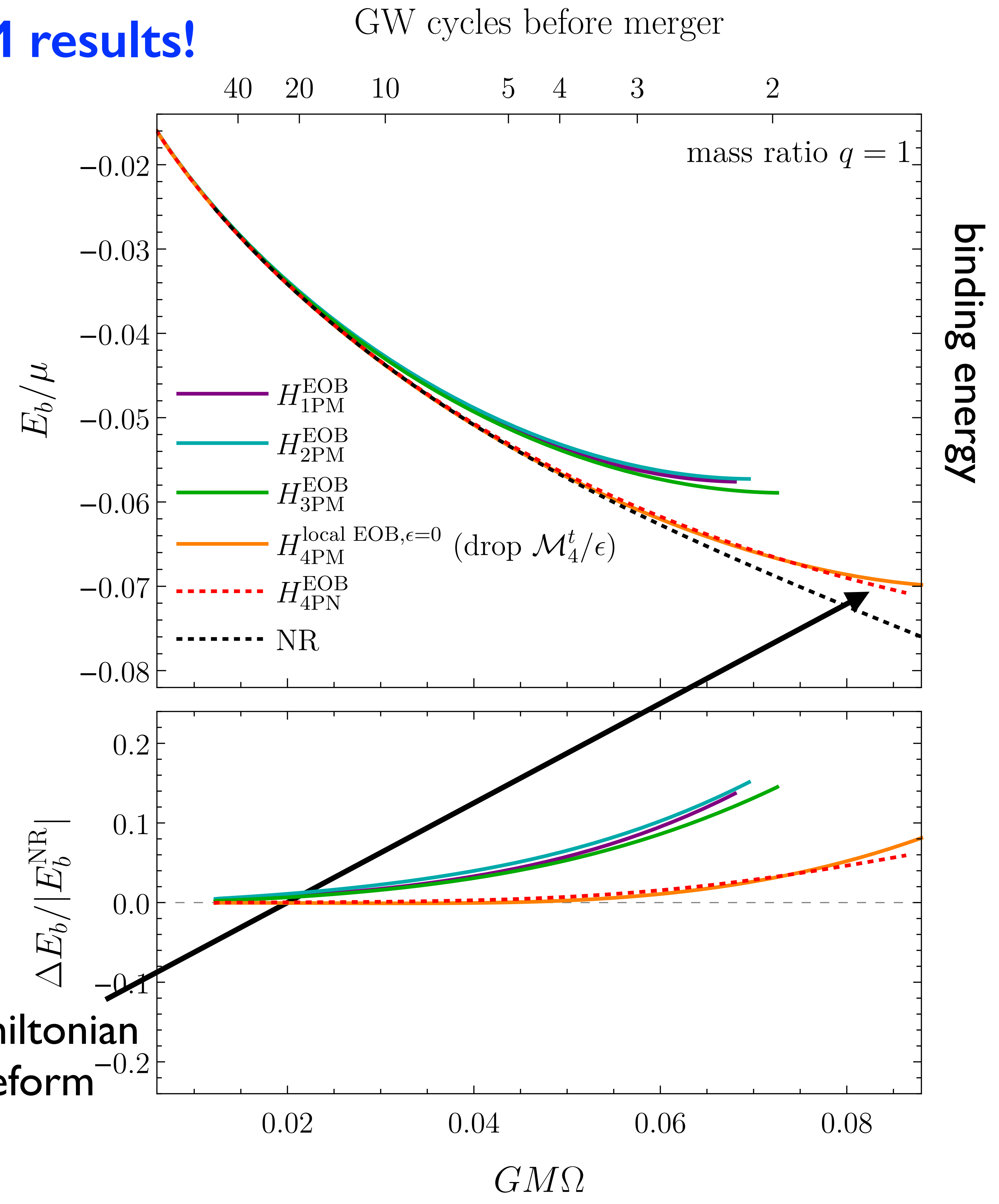
• **Crucial** to **push PM** calculations at **higher order,** and **resum them** in EOB formalism.

(Damour 19, Antonelli, AB, Steinhoff, van de Meent & Vines 19, Khalil, AB, Steinhoff & Vines in prep 21)

Encouraging (local-in-time) **4PM results!**

(Khalil, AB, Steinhoff & Vines in prep 21)



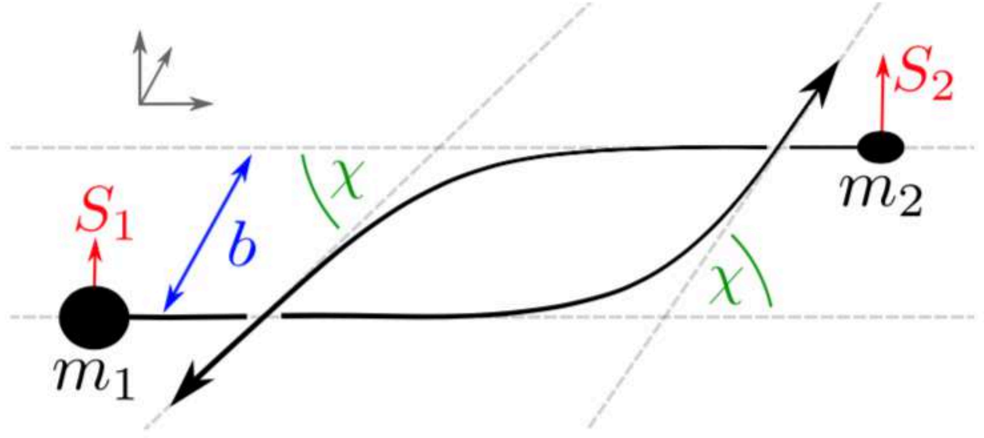


current (uncalibrated) Hamiltonian used to build EOBNR waveform models for LIGO/Virgo

Results from Interplay with Scattering Amplitude Methods & EFT/QFT

- 2-body **spin-orbit** (SO) **Hamiltonian at 4.5PN** computed using **EFT** or **interplay** between **bound and unbound orbits**, and gravitational **self-force** results.

(Levi et al. 20, Antonelli et al. 20)



- 2-body non-spinning **Hamiltonian at 5PN & 6PN** *partially* computed using **EFT** or **interplay** between **bound and unbound orbits**, and gravitational **self-force** results.

(Foffa et al. 19, Blümlein et al. 20, Damour 20, Bini, Damour & Geralico 20)

- 2-body **Hamiltonian at 2PM (1 loop)** for spinning, precessing BHs.

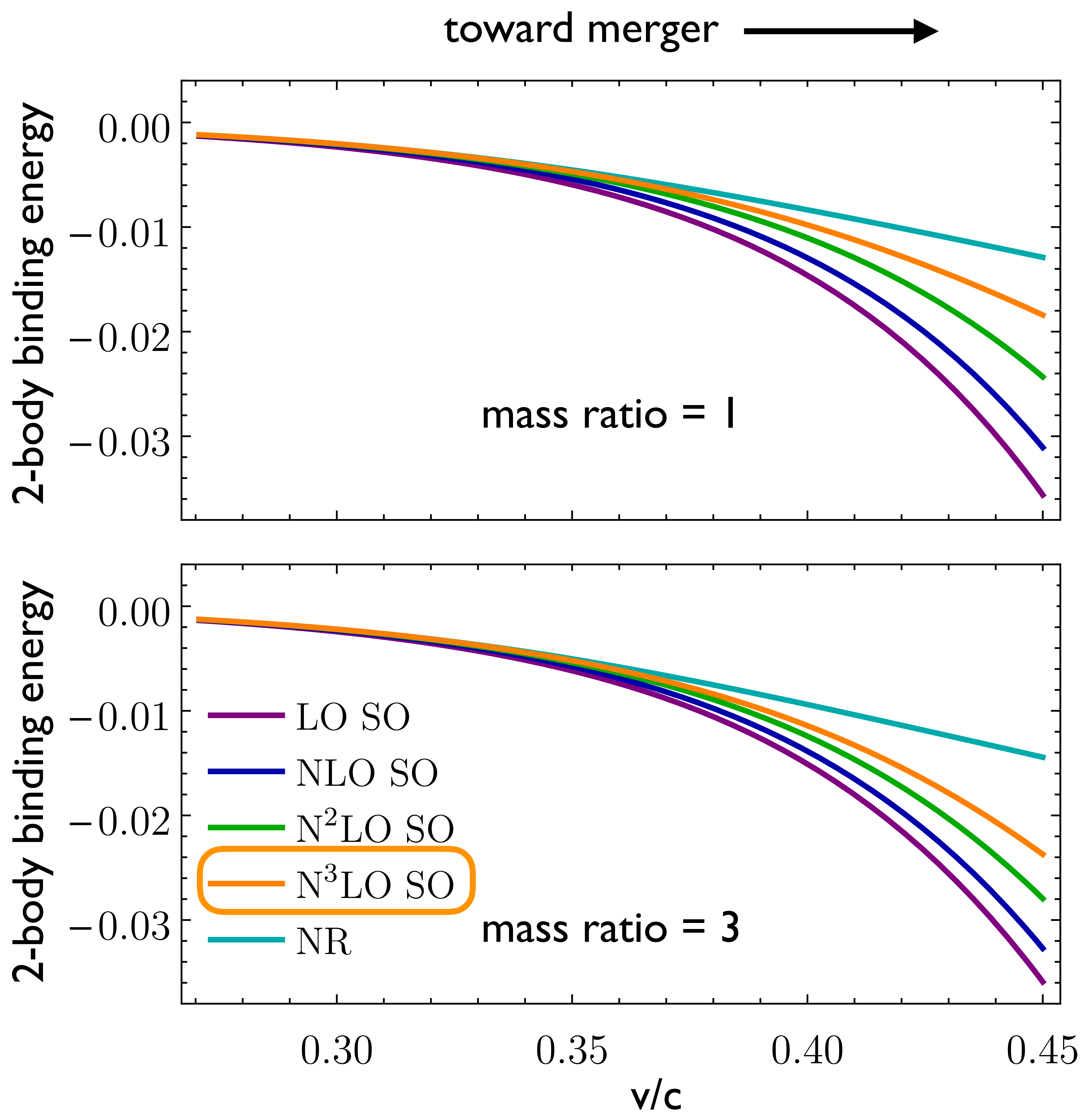
(Bini et al. 17, 18, Vines 18, Bern et al. 20, Kosmopoulos & Luna 21, Liu et al. 21)

- **Tidal effects** in 2-body dynamics **in PM expansion**

(Cheng & Solon 20, Kälin et al. 20)

- Leading-order **radiation in PM expansion**

(Damour 20, Di Vecchia et al. 20, 21, Jakobsen et al. 21, Herrmann et al. 21)



(Antonelli et al. 20)

Summary & Outlook

- Observing gravitational waves and inferring astrophysical/physical information hinges on our ability to make highly precise predictions of two-body dynamics and gravitational radiation.

- With O1 & O2 we observed the "tip of the iceberg" of the binary population, with the improved detectors' sensitivity, O3a has unveiled a richer picture and several "exceptional" sources.

- Crucial to improve waveform models for BBHs and binaries with matter for LIGO and Virgo upcoming runs and for future detectors (Cosmic Explorer, Einstein Telescope & LISA). Waveform accuracy would need to be improved by one or two orders of magnitude depending on the parameter space.

- EFT and scattering-amplitudes methods have brought new and fresh perspectives (and tools) to solve the relativistic two-body problem, unveiling new paths to intertwine the different perturbative approaches (PN, PM and GSF).

- The impact of recent PN-PM results for GW observations have not yet been fully assessed, but they will as we develop new waveform models for the upcoming O4 run.

- Unique opportunity for theoretical particle physicists to contribute!

Toward High-Precision Gravitational-Wave Astrophysics

- **Conservative dynamics** →

	0	1	1.5	2	2.5	3	3.5	4	4.5	5	5.5	6	6.5
PN order													
no spin	N	1PN		2PN		3PN		4PN		5PN		6PN	
spin-orbit			LO SO		NLO SO		N2LO SO		N3LO SO				
spin^2				LO S2		NLO S2		N2LO S2		N3LO S2			
spin^3							LO S3		NLO S3				
spin^4								LO S4		NLO S4			
spin^5											LO S5		NLO S5
spin^6												LO S6	

(credit: Vines)

need up to (→ 6PN):

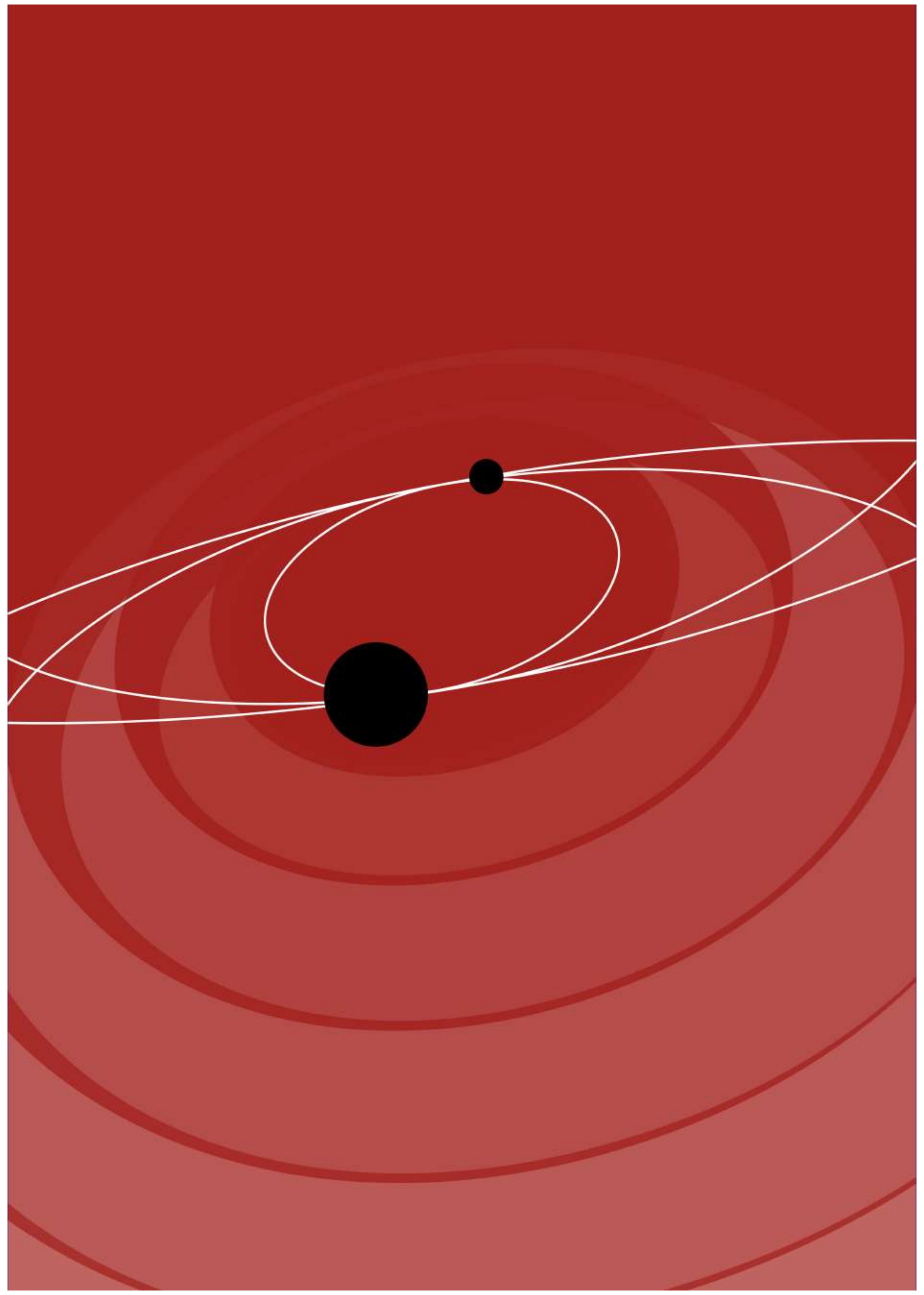
- 1PM / tree
- 2PM / 1-loop
- 3PM / 2-loop
- 4PM / 3-loop
- 5PM / 4-loop
- 6PM / 5-loop
- 7PM / 6-loop

- **Plus radiation!**

N.B. **Resummation** methods (e.g., EOB) can **accelerate accuracy**.

- Goal: **2-body dynamics and radiation (including waveforms) for spinning objects through 6PN order.**

(High precision is needed but it is not necessary that results are provided in analytical form.)



(credit: Carvalho)

Thank You!

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