Binary black hole simulations with the Spectral Einstein Code

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Apr 27, 2021

Image: Nils Fischer (AEI)
Waveform knowledge **essential for GW astronomy**

Detection by matched filtering

Detection and parameter estimation

Testing GR

Validation

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This material is based upon work supported by NSF’s LIGO Laboratory which is a major facility fully funded by the National Science Foundation.
In future, need **higher accuracy** for **more diverse** systems

**3G & LISA: expected SNRs**

needed accuracy $\sim 1/$SNR

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**LISA**

among sources: BBH $q = 1 \ldots 10^{-6}$

among science targets:

eccentricity measurement to $\delta e < 0.001$

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Methods for modeling BBH

- **Inspiral**
  - post-Newtonian theory (and PM & EOB)
  - perturbation theory in $1/q$

- **Merger**
  - BH perturbation theory

- **Ringdown**
  - perturbation theory

- Frequency

- Eccentricity, spin, …

- LIGO-Virgo Black Holes
The first 50 Years of numerical relativity for BBH

1962 ADM
3+1 formulation

1964
Hahn-Lindquist
2 wormholes

1964
Unruh
excision

1975-77
Smarr-Eppley
head-on collision

1979
York
kinematics and
dynamics of GR

1989-95
Bona-Masso
modified ADM,
(hyperbolicity)

1992,3
Choptuik;
Abrahams+Evans
critical phenomena

1994
Cook
Bowen-York
initial data

1994-95
NCSA-WashU
improved head-on collision

1994-98
BBH Grand Challenge

1999-00
AEI/PSU
grazing collisions

1997
Brandt-
Brügmann
puncture data

~2000
Choptuik;
Schnetter;Brügmann
mesh refinement

1999
BSSN
evolution system

2000-02
Alcubierre
gauge conditions

2000-04
AEI/UTB-NASA
revive crashing
codes (Lazarus)

2000
York,
Cornell, Caltech, LSU
hyperbolic formulations

2000-04
Cook, Pfeiffer ea
improved ID

2000
Ashtekar
isolated horizons

2000-04
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2003-08
Cook, Pfeiffer ea
improved ID

2004
Brügmann ea
one orbit

2005
Gundlach ea
constraint damping

2005
Pretorius
inspiral-merger-
ringdown (IMR)
w/ harmonic

2005-06
Campanelli+; Baker+
IMR w/ BSSN & moving punctures

2006,07
Baker ea;
Gonzalez ea
non-spinning BBH
kicks

2006-08
Scheel..HP+
SXS
IMR w/ spectral

2007-08
SXS
PN-NR
comparison

2007
BSSN
evolution system

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Pretorius
inspiral-merger-
ringdown (IMR)
with harmonic

2008
all of NR
NINJA

2009-11
Bishop ea;
Boyle ea
self-force studies

2009-11
Le Tiec ea
self-force studies

2010
Bernuzzi ea
C4z

2011
Lousto ea
q=100

2011
~1999

2014-15
precessing
GW models

2015
Szilagyi ea
175 orbits

2015
~2005

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Precessing parameter studies

Courtes Carlos Lousto, updated by HP

H. Pfeiffer

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2005: First working BBH inspirals

Important early result:
Simplicity of merger
Continuous transition inspiral $\rightarrow$ ringdown
Two approaches towards BBH simulations

“BSSN & Moving punctures”

LazEv, Maya, ETK, BAM, Goddard, GRchombo

Puncture initial-data
\[ \chi \lesssim 0.9 \] (but see Zlochower+ 17)

BSSN or CC4z

Moving puncture
mergers “easy”

Sommerfeld outer BC

4th to 8th order finite-difference

BHs advect through static grids

GW extrapolation
(Healy, Lousto ’20 for LazEv COM correction)

“generalized harmonic & spectral”

SpEC (SXS collaboration)

Quasi-equilibrium excision data
\[ \chi \lesssim 0.999 \]

Generalized-Harmonic Evolution System

BH excision
mergers difficult

Constraint preserving,
minimally reflective outer BC

Spectral methods

Moving grid

long, phase-accurate inspirals

GW extrapolation & COM correction

Cauchy-characteristic extraction
accurate \( m=0 \) modes, GW memory

Pretorius
Spectral Einstein Code (SpEC)

Simulating eXtreme Spacetimes collaboration

Contributors

SpEC was originally developed by Lawrence Kidder, Harald Pfeiffer, and Mark Scheel, who remain the principal maintainers of the code. Since then, many further individuals have contributed to SpEC. Most especially, Matthew Duez and Francois Foucart have developed the hydrodynamics module; Béla Szilágyi and Dan Hemberger have made numerous valuable additions throughout the code; and Lee Lindblom has contributed significantly to the algorithms used in SpEC.

The following researchers have substantially contributed to SpEC: Andy Bohn, Michael Boyle, Luisa Buchman, M. Brett Deaton, Nils Deppe, Roland Haas, Francois Hebert, Kate Henriksson, Stephen Lau, Geoffrey Lovelace, Curran Muhlbinger, Sergei Ossokine, Rob Owen, Saul Teukolsky, and Will Throwe.

Further contributions to SpEC were made by Kevin Barkett, Thomas Baumgarte, Jonathan Blackman, Wyatt Brege, Jeandrew Brink, Tony Chu, Michael Cohen, Gregory Cock, Tim Dietrich, Matt Giesler, Jason Grigsby, Casey Handmer, Frank Herrmann, Ian Hinder, Jeff Kaplan, Rez Khan, Prayush Kumar, Adam Lewis, François Limousin, Jonas Lippuner, Keith Matthews, Abdul Mroué, Lydia Nevin, Fatemeh Nouri, Maria Okounkova, David Radice, Oliver Rinne, Olivier Sarbach, Deirdre Shoemaker, Leo C. Stein, Nick Tacik, Nick Taylor, Manuel Tiglio, Vijay Varma, Trevor Vincent, John Wendell, Catherine Woodford, Anil Zenginoglu, Fan Zhang, and Aaron Zimmerman.

Finally, we thank the following undergraduate students for assisting with visualization and running simulations with SpEC: Nousha Afshari, Aliya Babul, Adam Bartnik, Deshpreet Bedi, Darius Bunandar, Iryna Butsky, Patrick Calhoun, Sourabh Chakraborty, Cameron Cogburn, Nick Demos, Patrick Fraser, Alyssa Garcia, Bryant Garcia, Yi Chen Hu, Daniel Jones, Haroon Khan, Dave Kotlis, Dongjun Li, Yor Limkumnerd, Ian MacCormack, Tamin Mansour, Robert McGhee, Dmitry Meyerson, Adam Neumann, Amin Nikbin, Hiroaki Oyaizu, Daniel Parada, Jennifer Seiler, Hsiolin Shi, Keara Soloway, Alexandre Streicher, and Allen Sussman.

http://www.black-holes.org/SpEC.html
Spectral methods

- Expand in basis-functions, solve for coefficients
  \[ u(x, t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi_k(x) \]

- Compute derivatives exactly
  \[ u'(x, t) = \sum_{k=1}^{N} \tilde{u}(t)_k \Phi'_k(x) \]

- Compute nonlinearities in physical space

- For smooth problems, **exponential convergence**
Domain-decomposition

- Many sub-domains, each with own basis-functions
  - Spheres
  - Blocks
  - Cylinders

- Advantages:
  - Excision of BH singularities
  - Adaptive Resolution
  - Parallelization

http://www.black-holes.org/SpEC.html
Einstein constraints: Formalism

\[ R + (\text{tr}K)^2 - K^2 = 0 \]
\[ \nabla \cdot (K - g \text{tr}K) = 0 \]

\[ g = \psi^4 \tilde{g} \]

Lichnerowicz 44

\[ \tilde{\nabla}^2 \psi = \ldots \]
\[ \tilde{\nabla} \cdot (\frac{1}{\sigma} \tilde{\mathcal{L}}V) = \ldots \]

coiled nonlinear elliptic PDEs in 3D

\[ K = \frac{1}{3} \text{tr}K g + A \]

conformal scaling

\[ A = \psi^{-10} \tilde{A} \]

conformal TT decompression

\[ A = A_{TT} + \frac{1}{\sigma}(\mathcal{L}V) \]
\[ A_{TT} = \psi^{-10} \tilde{A}_{TT} \]
\[ \sigma = \psi^6 \tilde{\sigma} \]

Hamiltonian picture \( \equiv \) Lagrangian picture

York(+) 72;74;99, HP,York 03

H. Pfeiffer
Applied to binary black holes

- Asymptotics/boundary conditions
  *Brandt, Brügmann 97; Cook,HP 04*

- Elliptic solver
  *HP+ 02, Ansorg 04*

- Spins > 0.9
  *Lovelace..HP+ 08*

\[ \tilde{\nabla}^2 \psi = \ldots \]
\[ \tilde{\nabla} \cdot \left( \frac{1}{N} \tilde{L} \beta \right) = \ldots \]
\[ \tilde{\nabla}^2 \tilde{N} = \ldots \]

- Control eccentricity

\[ \frac{E_{rot}}{E_{rot, \text{max}}} \]

\[ \frac{S}{M^2} \]

\[ \text{Distance} \]

\[ \text{time } t/M \]

*HP+ 05; Buonanno..HP+ 08
Chatziioannou, HP+ (in prep)*
Einstein Evolution Equations

- Einstein’s equations

\[ 0 = R_{ab}[g_{ab}] = -\frac{1}{2} \square g_{ab} + \nabla_{(a} \Gamma_{b)} + \text{lower order terms}, \quad \Gamma_a = -g_{ab} \square x^b. \]

- Generalized harmonic coordinates \( g_{ab} \square x^b \equiv H_a(x^a, g_{ab}) \)
  (Friedrich 1985, Pretorius 2005; \( H = 0 \) used since 1920’s)

\[ \square g_{ab} = \text{lower order terms}. \]

\[ \Rightarrow \text{Constraint } C_a \equiv H_a - g_{ab} \square x^b = 0 \]

- Constraint damping (Gundlach, et al., Pretorius, 2005)

\[ \square g_{ab} = \gamma \left[ t_{(a} C_{b)} - \frac{1}{2} g_{ab} t^c C_c \right] + \text{lower order terms} \]

\[ \partial_t C_a \sim -\gamma C_a. \]
BH Excision

- Excise inside BH horizons
- Domain-decomposition follows BHs continuously, conforms to shape of AH

Scheel, HP+ 08, Szilagyi+ 08, Hemberger+ 13
Outer boundary

- In SpEC:
  - Constraint preserving
  - Minimally reflective

* Lindblom, Rinne+ 06 *

- Causally connected for long simulations

* Buchman, HP, Scheel, Szilagyi, 2012 *
Accuracy of SpEC

- **Rapid convergence due to spectral methods**
- **Small errors due to moving grid**
- **Best code for long inspirals (but mergers hard)**
post-Newtonian vs. NR

PN approximants
Equally justified approaches to derive inspiral rate from energy balance

\[ \frac{dE}{dt} = -F_{GW} \]
Parameter space exploration

NINJA  Aylott .. HP+ 09

1st SXS Catalog  Mroue .. HP+ 13
Improve analytical waveform models

2nd SXS Catalog

Chu .. HP+ 15

SEOBNRv2

Taracchini .. HP+ 14

SEOBNRv4

Bohe .. HP+ 17
Improve analytical waveform models

new highly precessing runs

LIGO/Virgo analysis (GW190412)

Ossokine,..HP., 20

Abbott et al, PRD 102 043015 (2020)
2018 Waveforms
all at multiple resolutions
to assess numerical errors

\[ \frac{1}{q} \]

- \( q < \frac{1}{4} \) rare
- \( q = \frac{1}{10} \) highest public SpEC run

SXS Collaboration (Boyle, ..HP+)
CQG 2019 (1904.04831)
More parameter space exploration efforts

| Catalog          | Started | Updating? | Simulations | $m_1/m_2$ range | $|\chi_1|$ range | $|\chi_2|$ range | Precessing? | Median $N_{\text{vec}}$ | Public? |
|------------------|---------|-----------|-------------|-----------------|-----------------|-----------------|-------------|--------------------------|---------|
| NINJA [98, 115]  | 2008    | ×         | 63          | 1–10            | 0–0.95          | 0–0.95          | ×           | 15                       | ×       |
| NRAR [120]       | 2013    | ×         | 25          | 1–10            | 0–0.8           | 0–0.6           | √           | 24                       | ×       |
| Georgia Tech [122]| 2016    | √         | 452         | 1–15            | 0–0.8           | 0–0.8           | √           | 4                        | √       |
| RIT (2017) [123] | 2017    | √         | 126         | 1–6             | 0–0.85          | 0–0.85          | √           | 16                       | √       |
| RIT (2020) [124] | 2017    | √         | 777         | 1–15            | 0–0.95          | 0–0.95          | √           | 19                       | √       |
| NCSA (2019) [125]| 2019    | ×         | 89          | 1–10            | 0               | 0               | ×           | 20                       | ×       |
| SXS (2018)       | 2013    | √         | 337         | 1–10            | 0–0.995         | 0–0.995         | √           | 23                       | √       |
| SXS (2019)       | 2013    | √         | 2018        | 1–10            | 0–0.998         | 0–0.998         | √           | 39                       | √       |

SXS Collaboration (Boyle, ..HP+)
CQG 2019 (1904.04831)

And Palma group around Husa+ (data not public)
Main use for BBH simulations: waveform models

- state of the art: **Precession** and **higher modes**
- **EOB models**, **Phenom models**
- more recently: **NR surrogate models**
  - need O(1000) NR sims
  - nearly "automatic" model construction
  - model-accuracy \(\sim\) NR-accuracy
  - but: only where NR is available & requires analytic early inspiral model

\[ q = 8.00, \chi_{1z} = 0.48, \chi_{2z} = 0.75, t = 1.57, \phi_0 = 5.03 \]

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Blackman+ 15, Blackman+ 17a,b, Varma+ 18a,b, Varma+ 19a,b, Rifat+ 19, Taylor, Varma 20
Parameter space: NR records

$q=1$: $S/M^2 = 0.998$
$q<1$: $S/M^2 = 0.95$

Scheel+ 14
Lovelace..HP+15
Boyle..HP+ 19

eccentric, precessing $q=7$

Lewis, Zimmerman, HP 17

$q=1/7$, $\chi=0$

Szilagyi..HP+15

350GW cycles

$q=1/128$

Lousto, Healy 20

$q=6$ Buchman et al. ’12
Recent (1)

Improved calculation of gravitational waves
Center of mass corrections

• in NR simulations, the center of mass typically drifts slowly away from the origin (on which GW extraction spheres are centered)

• SpEC corrects this since the 2019 catalog

Woodford, Boyle, HP (PRD 2019)
Boyle..HP.. (SXS) (CQG 2019)
Weyl scalars, reduced junk radiation

- SpEC can now extract all Weyl scalars $\Psi_4, \ldots, \Psi_0$

- Impact of initial burst of radiation reduced:
  - different conformal data
  - clip burst by resetting outer boundary
  - split numerical resolutions after burst

(lozzo..HP.. (SXS) PRD 2021)

Varma, Scheel, HP PRD 2018
Ma, Giesler, Scheel, Varma PRD 2021
Cauchy Characteristic Extraction

- New spectral implementation
  
  Barkett+ PRD 2020  Moxon, Scheel, Teukolsky PRD 2020

  - faster, more accurate
  than PittNull code

  test: GW for bouncing Schwarzschild BH
  
  \[ x \rightarrow x + a \sin^4(\Omega t) \]
BMS balance laws & GW memory

- Waveforms must satisfy balance laws

\[
h = \frac{1}{2} \bar{\delta}^2 \mathcal{D}^{-1} \left[ \frac{1}{4} \int_{-\infty}^{u} |\dot{h}|^2 \, du - \left( \Psi_2 + \frac{1}{4} \dot{h}\bar{h} \right) \right]
\]

- for finite length NR waveform, define

\[
J \equiv \frac{1}{2} \bar{\delta}^2 \mathcal{D}^{-1} \left[ \frac{1}{4} \int_{u_1}^{u} |\dot{h}|^2 \, du - \left( \Psi_2 + \frac{1}{4} \dot{h}\bar{h} \right) \right] + \alpha
\]

- two possibilities:
  - validity test: \( h_{\text{NR}} - J[h_{\text{NR}}] \equiv 0 \)
  - Improve: \( h_{\text{improved}} = J[h_{\text{original}}] \)
BMS balance laws & GW memory

Mitman, Iozzo, Khera, .. HP.. PRD 2021
Iozzo, Khera .. HP.. 2104.07052

violation of BMS balance law for standard (extrapolated) SpEC $h_{2,0}$

corrected standard SpEC $h_{2,0}$

BMS Balance Law Violation (by system)

violation of BMS balance law for standard (extrapolated) SpEC GW

corrected standard SpEC GW

CCE
Recent (2)

Harmonic coordinates
Toward Harmonic Coordinates in NR

- **Prayush Kumar**, HP (in prep)

- **Evolve harmonic coordinates** as extra variables
  - easier contact with analytical calculations
  - at merger, harmonic coord singularities **outside** common AH
Recent (3)

Eccentricity
Community is beginning to explore eccentricity

\[ q \geq 1/10, \ e_0 \leq 0.18 \]
energy emission

remnant properties

Huerta+ 1901.07038

Eccentric waveform models w/ NR input:
Hinder+ 08, Huerta+ 16, Hinder+ 17,

\[ q \geq 1/4, \ \chi_{1,2} \leq 0.75 \]

hybridization & PE studies

Ramos-Buades+ 1909.11011

Injection: \( q = 1, \ \chi_{1,2} = 0, \ e_0 \neq 0 \)

Recovery with quasi-circular waveform models
SpEC: highly eccentric inspirals (q=1)

- NR phase-accurate to \( \sim 0.1 \text{rad} \)
- Eccentric PN expected to converge more slowly (Damour+ 04)

\[ e_0 = 0.79 \]

\[ e_0 = 0.53 \]
SpEC: highly eccentric inspirals (q=1)

\[(current \text{ parameter space})\]

\[m_1/m_2 = 6, \text{ no spins, } e_0 = 0.42\]
First eccentric surrogate model

- Built on SpEC runs
  - $q=1$, zero spin
  - $e<0.2$

*Islam, Varma, .. HP+ 21*
Recent (4)

BBH scattering
Recent (4): BH Scattering

- scattering angle $\alpha$
- $E_{GW}, \Delta v_\infty$
- $\Delta m_i, \Delta \vec{S}_i$
Scattering in Harmonic Coordinates

- Evolution in generalized harmonic coords
  - $x, v$

- Harmonic coords evolved along
  - $X, V$

- Post-Newtonian well converging
  - before / after encounter

$q=1$, $J=1.025$, $E_{\text{in}}=0.0226$

Rüter, HP, SXS in prep
Error analysis of scattering angle

Extrapolate (finite) NR trajectories by PN of different order

Harmonic coords reduce dependence on initial separation $D$

Cumulative uncertainty due to
• numerical resolution
• initial conditions
• coordinates
• angle extraction
less than 1 degree

PN: Memmesheimer, Gopakumar, Schäfer 04

Rüter, HP, SXS in prep
scattering angles (q=1, spin zero)

- Comparison w/ NR results of Damour+ 14
  - good agreement
  - validates both codes

E=1.0226 M

PN: Memmesheimer, Gopakumar, Schäfer 04
2PM: taken from Vines, Steinhoff, Buonanno 19, originally Westpfahl 85
From scatter -> capture

$q = \frac{2}{3}$
Recent (5)

Contact with gravitational self-force
Bridging mass-ratio gap

\( q = 1 \)

GW150914

GW190814

Intermediate mass BH

\[(10 + 1000)M_\odot\]

\[(10^3 + 10^6)M_\odot\]

S. Drasco

q = 0

EMRI

\[(10 + 10^6)M_\odot\]

NR

\[ q \gtrsim 1/20 \]

Small-mass-ratio approximation (SMR)

expansion in \( q \) or \( \nu = q/(1 + q)^2 \)

H. Pfeiffer
Challenge for NR at small \( q \)

- Scaling of number of time-steps

\[
N_{\text{steps}} \propto \frac{1}{q^2} \frac{1}{(M\Omega_i)^{8/3}}
\]

- More steps per orbit (Courant limit — numerics)
- More orbits per inspiral (physics)

\( q = 1/20, \chi_1 = 0, \chi_2 = 0 \)

\( q \leq 1/32 \): very limited convergence tests

SpEC, 4 resolutions
Ossokine, Fischer, Rüter, HP
Methods for modeling BBH

At what mass-ratios is small-mass-ratio perturbation theory accurate?
SMR orbital phasing

\[ \Phi(M\Omega) = \frac{1}{\nu} \Phi_0(M\Omega) + \Phi_1(M\Omega) + \nu \Phi_2(M\Omega) + \ldots + \frac{1}{\nu^{1/2}} \Phi_{\text{resonances}} + \frac{1}{\nu^{1/5}} \Phi_{\text{plunge}} \]

adiabatic order:
generic orbits known
Schmidt 02, Fujita+Hikida 09
Drasco+Hughes 06

1-PA: needs parts of second order GSF
circular orbits around Schwarzschild (Pound+ 1908.07419)
full 1-GSF from van de Meent

2-PA

BBH resonances
Flanagan, Hinderer 10

transition to plunge
Buonanno+Damour 00
Ori+Thorne 00
SMR orbital phasing

$$\Phi(M\Omega) = \frac{1}{\nu} \Phi_0(M\Omega) + \Phi_1(M\Omega) + \nu \Phi_2(M\Omega) + \ldots + \frac{1}{\nu^{1/2}} \Phi_{\text{resonances}} + \frac{1}{\nu^{1/5}} \Phi_{\text{plunge}}$$

- $$\Phi^{\text{NR}}(M\Omega)$$ from 55 SXS simulations with $$q = 1\ldots 1/10$$
- Fit to $$\sum_k \nu^{k-1} \Phi_k(M\Omega)$$
  - $$\Phi_0(M\Omega)$$ - agrees with 0PA
  - $$\Phi_1(M\Omega)$$ - hereby computed
  - $$\Phi_2(M\Omega)$$ - remarkably small

- $$\Phi_1$$ contributes 10’s of radians to orbital phase ⇒ significant at any $$\nu$$
- $$\Phi_2$$ small only if expanded in $$\nu$$ (not in $$q$$) and written as function of $$M\Omega$$ (not $$m_1\Omega$$)

van de Meent, HP 2020
Applicability of approximation schemes

• For non-spinning, quasi-circular, at phase-errors $\sim 1$ radian: mass-ratio gap bridged!

• Note: eccentric PN expected to converge more slowly (Damour + 04)
Summary

- NR simulations **accurate for today’s GW detectors** and at parameters of GW events so far

- **Improvements under way** for future GW detectors:
  - accuracy
  - length
  - parameter space
  - high spins

- **Biggest challenges**
  - high mass-ratio
  - near extreme spins
  - high energy encounters

- Promising results for **eccentric inspirals** and **scattering encounters**