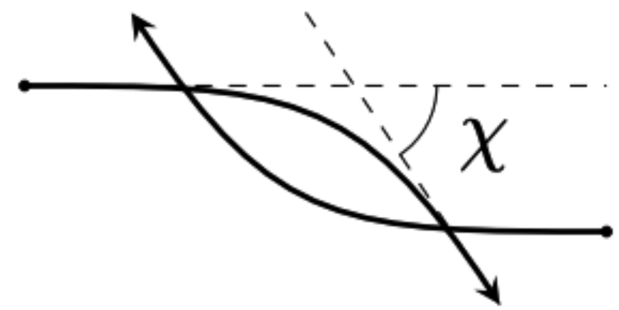
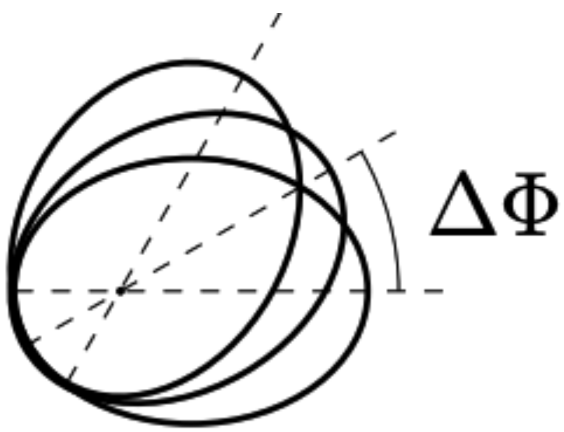
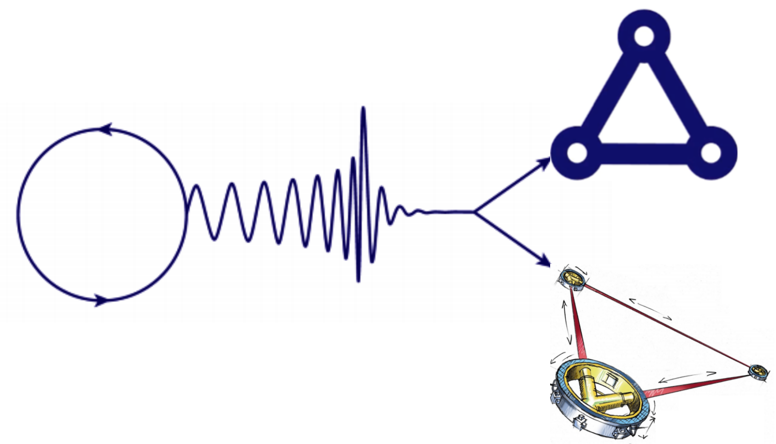
GW observables for Inspiralling Compact Objects From Post-Minkowskian scattering data







Rafael A. Porto

2006.01184
2007.04977
2008.06047
2102.10059

Based on work in collaboration with
Gihyuk Cho, Christoph Dlapa, Gregor Kälin, Zhengwen Liu and Zixin Yang

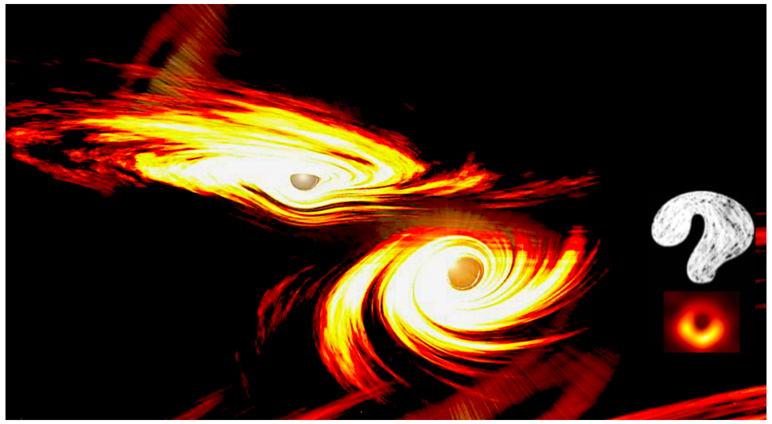
DESY.

erc European Research Council

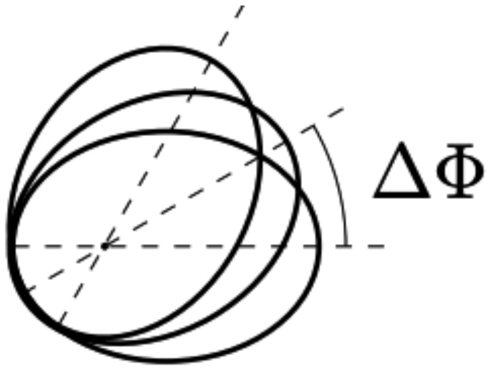
Precision Gravity: From the LHC to LISA

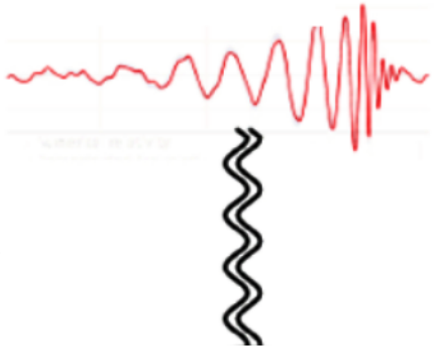
Outline

Discovery Potential =
Precise Theoretical Predictions

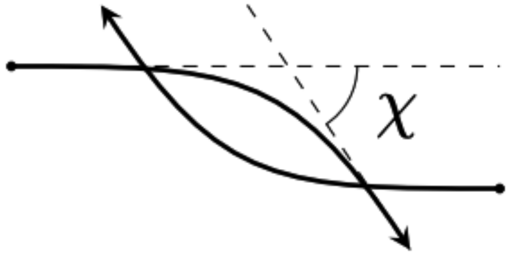


- **Part I: B2B**





- **Part II: EFT**

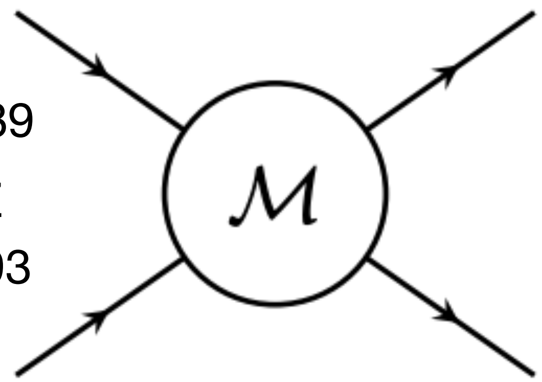


See Julio's tutorial & Zvi's talk

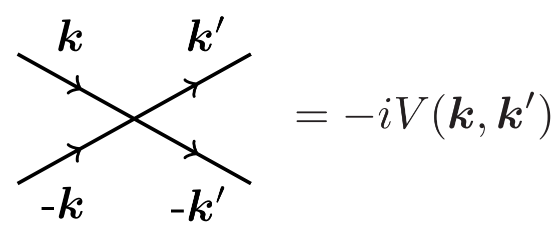
How do we compute Observables from PM data?

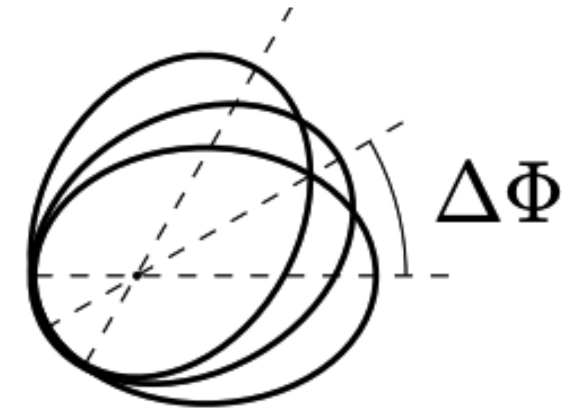
CRS 1808.02489

BCRSSZ 1908.01493



Conservative effects





The gravitational interaction is UNIVERSAL!

BUT: Do we need the Hamiltonian?

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)\,G$
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)\,G^2$
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)\,G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)\,G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)\,G^5$

Conservative 3PM Hamiltonian

ZB, Cheung, Roiban, Shen, Solon, Zeng (2019)

The O(G^3) 3PM Hamiltonian: $H(\boldsymbol{p}, \boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p}, \boldsymbol{r})$

$$V(\boldsymbol{p}, \boldsymbol{r}) = \sum_{i=1}^{3} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i,$$

Newton in here

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}\left(1 - 2\sigma^2\right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}\left(1 - 5\sigma^2\right) - \frac{4\nu\sigma\left(1 - 2\sigma^2\right)}{\gamma\xi} - \frac{\nu^2(1-\xi)\left(1 - 2\sigma^2\right)^2}{2\gamma^3\xi^2}\right],$$

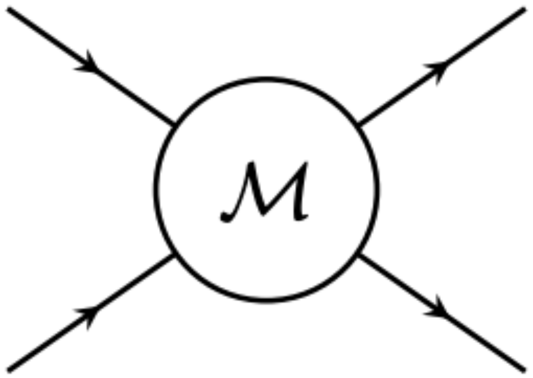
$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\left[\frac{1}{12}\left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3\right) - \frac{4\nu\left(3 + 12\sigma^2 - 4\sigma^4\right)\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}}\right.$$
$$- \frac{3\nu\gamma\left(1 - 2\sigma^2\right)\left(1 - 5\sigma^2\right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma\left(7 - 20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right)\left(1 - 2\sigma^2\right)}{4\gamma^3\xi^2}$$
$$\left. + \frac{2\nu^3(3 - 4\xi)\sigma\left(1 - 2\sigma^2\right)^2}{\gamma^4\xi^3} + \frac{\nu^4(1 - 2\xi)\left(1 - 2\sigma^2\right)^3}{2\gamma^6\xi^4}\right],$$

$$m = m_A + m_B, \qquad \mu = m_A m_B/m, \qquad \nu = \mu/m, \qquad \gamma = E/m,$$
$$\xi = E_1 E_2/E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2/m_1 m_2,$$

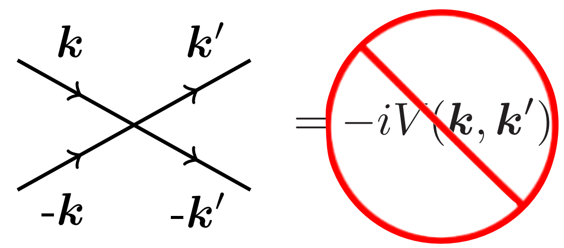
Kalin RAP
1910.03008

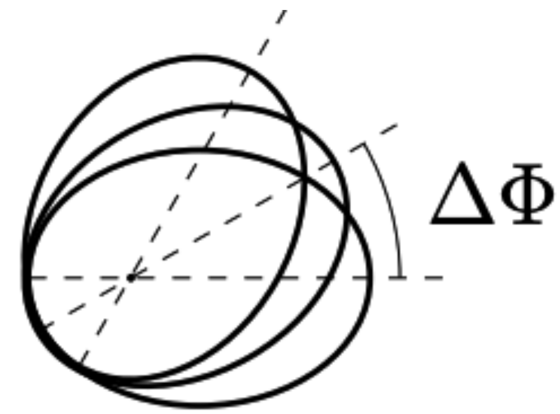
ON-SHELL SPIRIT:

gauge-invariant information!



Conservative effects





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$$- \frac{3\nu\gamma\left(1 - 2\sigma^2\right)\left(1 - 5\sigma^2\right)}{2(1 + \gamma)(1 + \sigma)} - \frac{3\nu\sigma\left(7 - 20\sigma^2\right)}{2\gamma\xi} - \frac{\nu^2\left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2\right)\left(1 - 2\sigma^2\right)}{4\gamma^3\xi^2}$$
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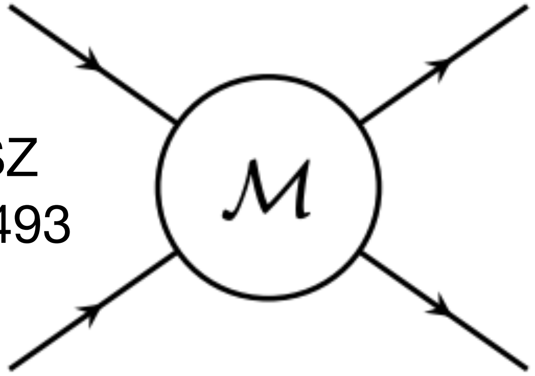
$$m = m_A + m_B, \qquad \mu = m_A m_B / m, \qquad \nu = \mu / m, \qquad \gamma = E/m,$$
$$\xi = E_1 E_2 / E^2, \qquad E = E_1 + E_2, \qquad \sigma = p_1 \cdot p_2 / m_1 m_2,$$

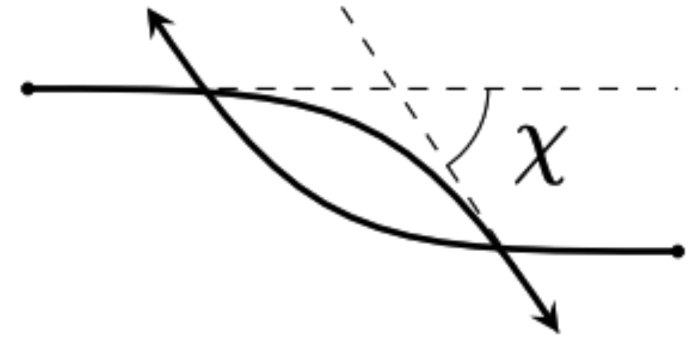
ON-SHELL SPIRIT:

gauge-invariant information!

BCRSSZ
1908.01493



Conservative effects



observed to be the same
in ‘Y-basis’ to 3PM order

$$\mathbf{p}^2(r,E) = p_\infty^2(E) + \sum_i^\infty P_i(E)\left(\frac{G}{r}\right)^i$$

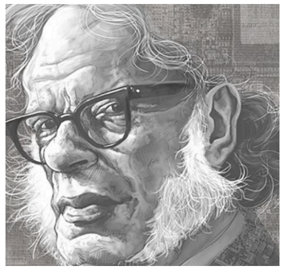
$$\widetilde{\mathcal{M}}(r,E) = \sum_{n=1}^\infty \widetilde{\mathcal{M}}_n(E)\left(\frac{G}{r}\right)^n$$

Scattering amplitude

$$\widetilde{\mathcal{M}}(r,E) \equiv \frac{1}{2E}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\,\mathcal{M}(\mathbf{q},\mathbf{p}^2 = p_\infty^2(E))e^{-i\mathbf{q}\cdot\mathbf{r}}$$

The most exciting phrase
to hear in science,
the one that heralds
new discoveries, is not
“EUREKA!”
but, “that's funny...”

—Isaac Asimov



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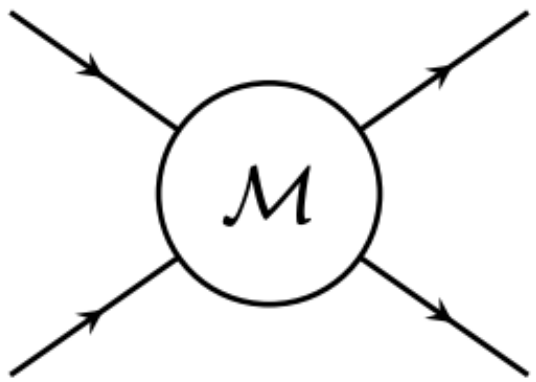
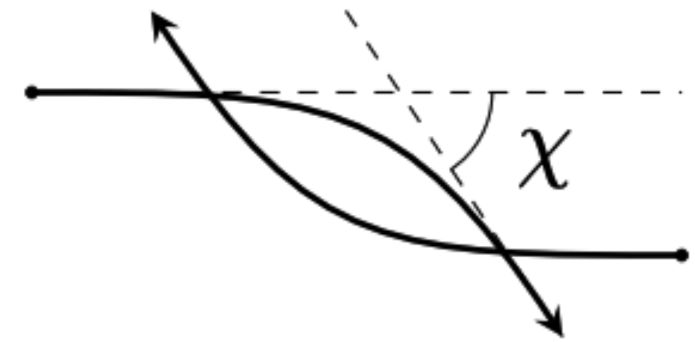
1910.03008

ON-SHELL SPIRIT:
gauge-invariant information!



Conservative effects



$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \sum_i^\infty P_i(E)\left(\frac{G}{r}\right)^i$$

$$\widetilde{\mathcal{M}}(r,E) = \sum_{n=1}^\infty \widetilde{\mathcal{M}}_n(E)\left(\frac{G}{r}\right)^n$$

'Impetus Formula'*

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$

Scattering amplitude

$$\widetilde{\mathcal{M}}(r,E) \equiv \frac{1}{2E}\int \frac{d^3\mathbf{q}}{(2\pi)^3}\,\mathcal{M}(\mathbf{q},\mathbf{p}^2 = p_\infty^2(E))e^{-i\mathbf{q}\cdot\mathbf{r}}$$

Direct algebraic relationship (Firsov Formula)

$$\bar{p}^2(r,E) = \exp\left[\frac{2}{\pi}\int_{r|\bar{p}(r,E)|}^\infty \frac{\chi_b(\tilde{b},E)\mathrm{d}\tilde{b}}{\sqrt{\tilde{b}^2 - r^2\bar{p}^2(r,E)}}\right]$$

$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell \frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$

* IR-finite part ('potentials' only)

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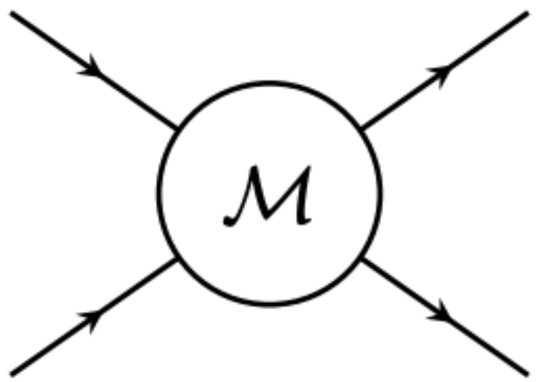
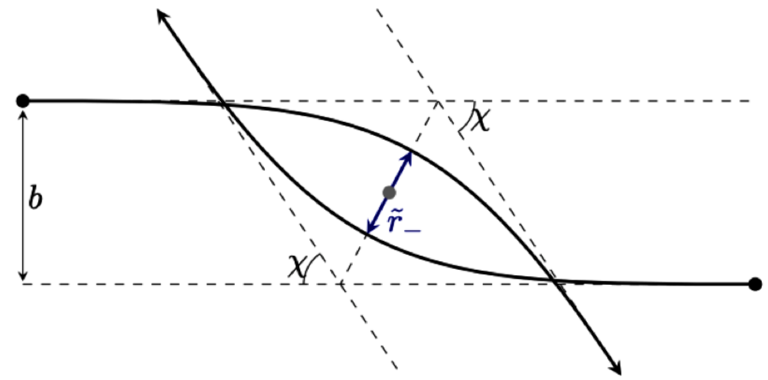
ON-SHELL SPIRIT:

gauge-invariant information!



Conservative effects



$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$

Recycle old idea from Sommerfeld

$$\mathcal{S}_r \equiv \frac{1}{2\pi}\int p_r dr$$

Radial Action

$$\mathcal{S}_r(J,\mathcal{E}) = \frac{1}{\pi}\int_{\tilde{r}_-}^{\infty} \sqrt{p_\infty^2(\mathcal{E}) + \widetilde{\mathcal{M}}(r,\mathcal{E}) - J^2/r^2}\,\mathrm{d}r$$

Scattering angle

$$\frac{1}{2} + \frac{\chi}{2\pi} = -\frac{\partial \mathcal{S}_r}{\partial J}$$

$$\mathcal{E} = \frac{E-M}{\mu} > 0$$

Un-Bound

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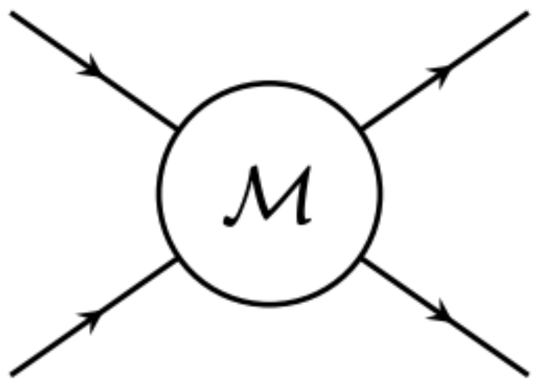
1910.03008

B2B correspondence

gauge-invariant information!



Conservative effects

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \widetilde{\mathcal{M}}(r,E)$$

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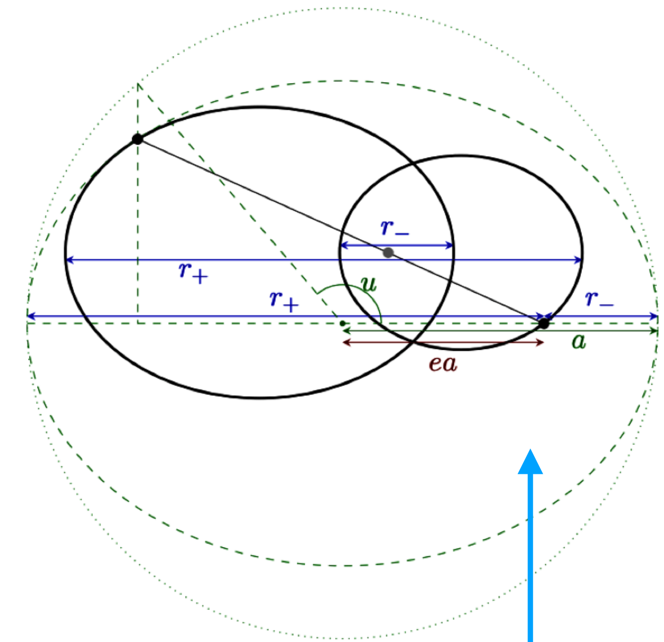

Analytic continuation!

$$\mathcal{S}_r(J,\mathcal{E}) = \frac{1}{\pi}\int_{r_-}^{r_+}\sqrt{p_\infty^2(\mathcal{E}) + \widetilde{\mathcal{M}}(r,\mathcal{E}) - J^2/r^2}\,\mathrm{d}r$$

Observables Bound Orbits

$$\delta\mathcal{S}_r(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

$$\mathcal{E} = \frac{E-M}{\mu} < 0$$

Bound

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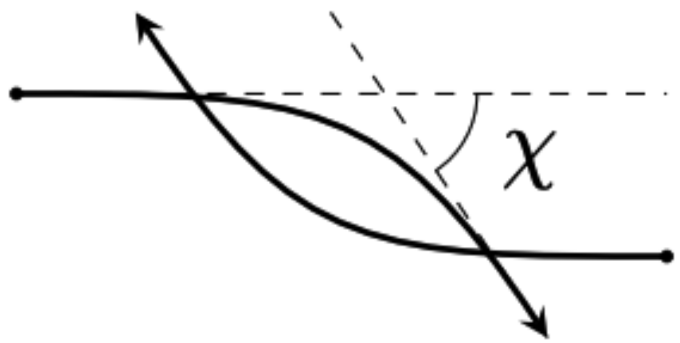
1911.09130

B2B correspondence

gauge-invariant information!



$$\bar{p}^2(r,E) = \exp\left[\frac{2}{\pi}\int_{r|\bar{p}(r,E)|}^{\infty} \frac{\chi_b(\tilde{b},E)\,d\tilde{b}}{\sqrt{\tilde{b}^2 - r^2\bar{p}^2(r,E)}}\right]$$

Conservative effects

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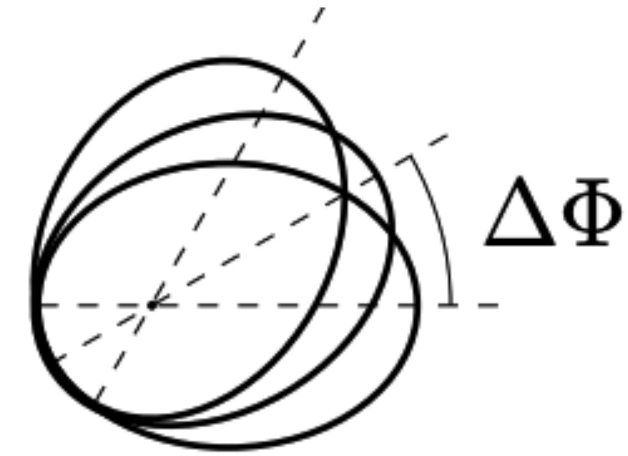

$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\,dr \qquad \frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\,dr$$

$$4\chi_j^{(4)} = \frac{3\pi\hat{p}_\infty^4}{4}\left(f_2^2 + 2f_1f_3 + 2f_4\right) = \frac{3\pi}{4M^4\mu^4}\left(\widetilde{\mathcal{M}}_2^2 + 2\widetilde{\mathcal{M}}_1\widetilde{\mathcal{M}}_3 + 2p_\infty^2\widetilde{\mathcal{M}}_4\right)$$

$$\frac{\Delta\Phi}{2\pi} = \frac{\widetilde{\mathcal{M}}_2 G^2}{2J^2} + \frac{3(\widetilde{\mathcal{M}}_2^2 + 2\widetilde{\mathcal{M}}_1\widetilde{\mathcal{M}}_3 + 2p_\infty^2\widetilde{\mathcal{M}}_4)G^4}{8J^4} + \mathcal{O}(G^6)\,,$$

$$1/j = GM\mu/J$$

The most exciting phrase to hear in science, the one that heralds new discoveries, is not "EUREKA!" but, "that's funny..."
—Isaac Asimov

Generalizes to all orders the one-loop result in
Caron-Huot Zahraee
1810.04694

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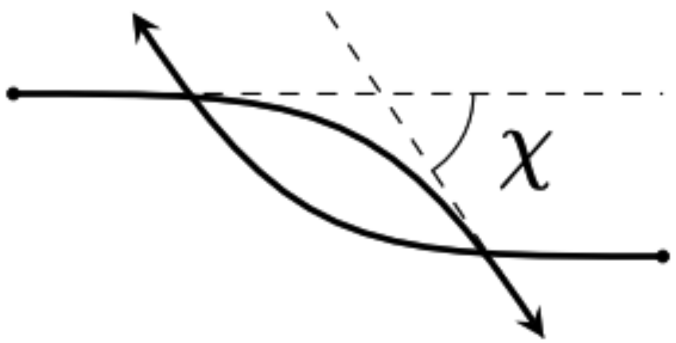
1911.09130

B2B correspondence

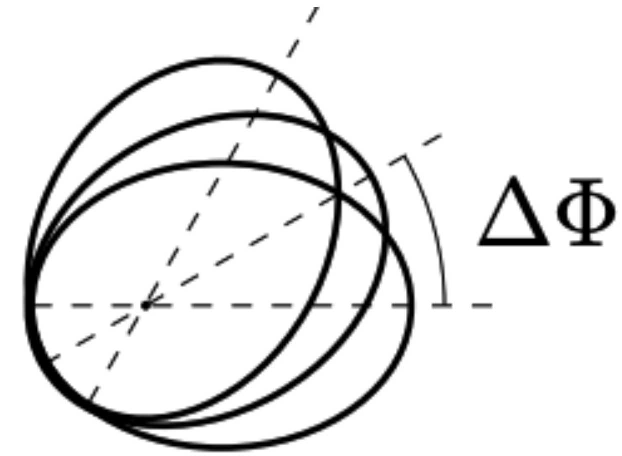
gauge-invariant information!



$$\bar{p}^2(r,E) = \exp\left[\frac{2}{\pi}\int_{r|\bar{p}(r,E)|}^{\infty} \frac{\chi_b(\tilde{b},E)\,\mathrm{d}\tilde{b}}{\sqrt{\tilde{b}^2 - r^2\bar{p}^2(r,E)}}\right]$$

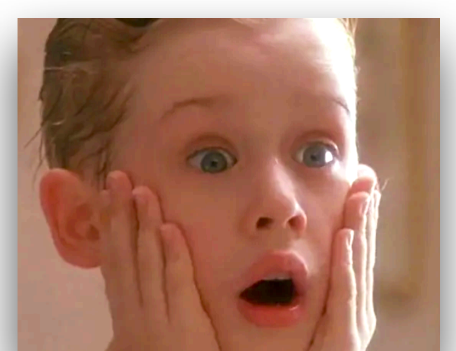
Conservative effects

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$



$$\frac{1}{\pi}\int_{\tilde{r}_-(J,\mathcal{E})}^{\infty} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r\,; \qquad \frac{1}{\pi}\int_{r_-(J,\mathcal{E})}^{r_+(J,\mathcal{E})} \frac{J}{r^2\sqrt{\boldsymbol{p}^2(\mathcal{E},r) - J^2/r^2}}\mathrm{d}r$$

Loop around infinity



Remarkably!

$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})\,, \qquad \mathcal{E}<0\,,$$

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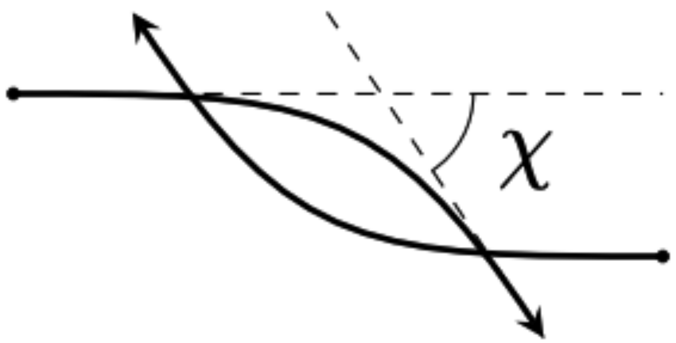
B2B correspondence

gauge-invariant information!



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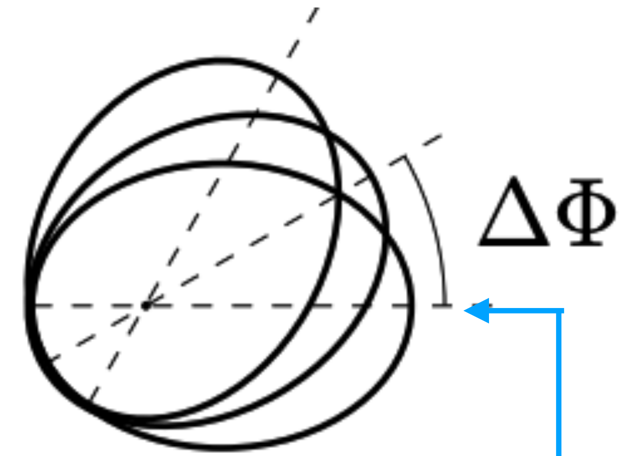


At the level of the radial action:

$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

Analytic continuation

$\mathcal{E} < 0$

Central object for the **bound** problem:

$$i_r(j,\mathcal{E}) \equiv \frac{\mathcal{S}_r}{GM\mu} = \mathrm{sg}(\hat{p}_\infty)\chi_j^{(1)}(\mathcal{E}) - j\left(1 + \frac{2}{\pi}\sum_{n=1}\frac{\chi_j^{(2n)}(\mathcal{E})}{(1-2n)j^{2n}}\right)$$

$$\delta\mathcal{S}_r(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

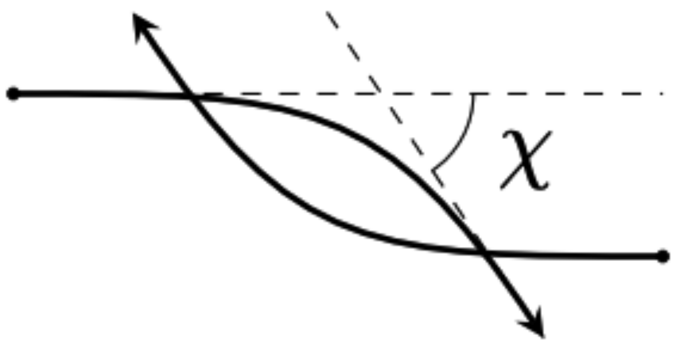
ALL conservative observables!

Kalin **RAP** 1911.09130

Liu **RAP** Yang 2102.10059

B2B correspondence

gauge-invariant information!



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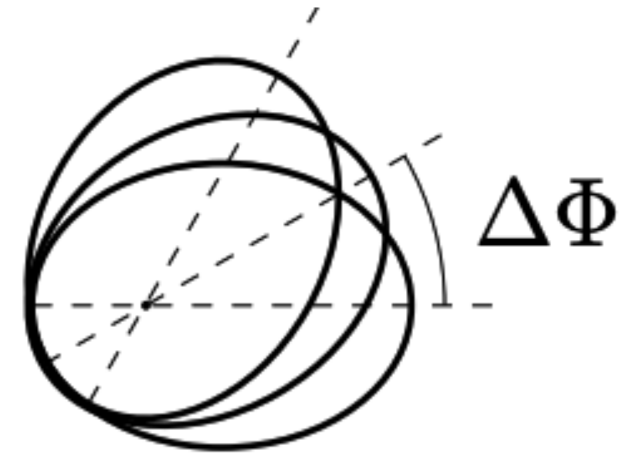


valid for spin J total (canonical) angular momentum

At the level of the radial action:

$$i_r^{(\mathrm{bound})}(\mathcal{E}<0,\ell,\tilde{a}_\pm) = i_r^{(\mathrm{unbound})}(\mathcal{E}<0,\ell,\tilde{a}_\pm) - i_r^{(\mathrm{unbound})}(\mathcal{E}<0,-\ell,-\tilde{a}_\pm)\,,$$

$$\tilde{a} = a/GM$$

Spinning bodies:

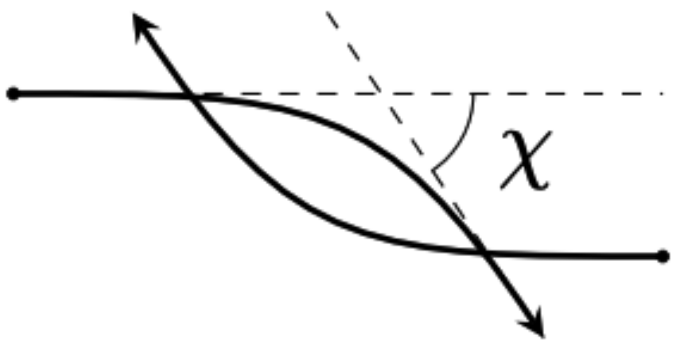
$$i_r(\mathcal{E},\ell,\tilde{a}_\pm) = \mathrm{sg}(\hat{p}_\infty)\chi_\ell^{(1)}(\mathcal{E}) + \ell\left(-1 + \frac{2}{\pi}\sum_{n=1}^{\infty}\left(\frac{\chi_{\ell,\mathrm{odd}}^{(2n+1)}(\mathcal{E},\tilde{a}_\pm)}{2n\,\ell^{2n+1}} + \frac{\chi_{\ell,\mathrm{even}}^{(2n)}(\mathcal{E},\tilde{a}_\pm)}{(2n-1)\ell^{2n}}\right)\right)$$

bound

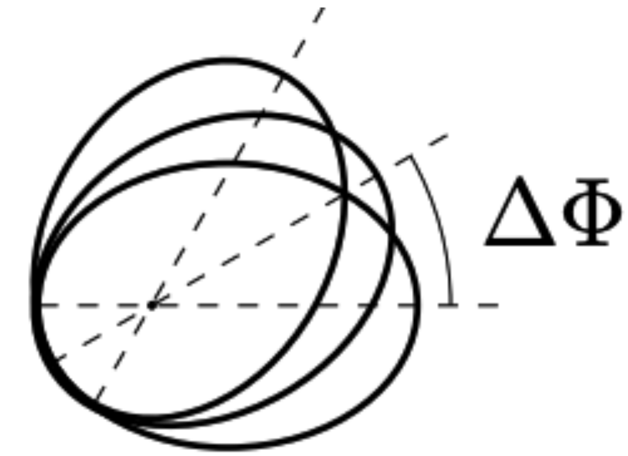
Kalin **RAP**
1911.09130

B2B correspondence

gauge-invariant information!



Conservative effects



This **"PMtoPN"** map ***formally*** connects the **G/J** coefficients of the radial action(s)

$$i_r = \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 - \frac{2}{\pi}\left(\frac{\chi_j^{(2)}}{j^2} + \frac{\chi_j^{(4)}}{3j^4}\right) + \cdots\right)$$

← even coefs.

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN		
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)$	G
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)$	G^2
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)$	G^3
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)$	G^4

Caveat:
We need various PM orders to complete PN

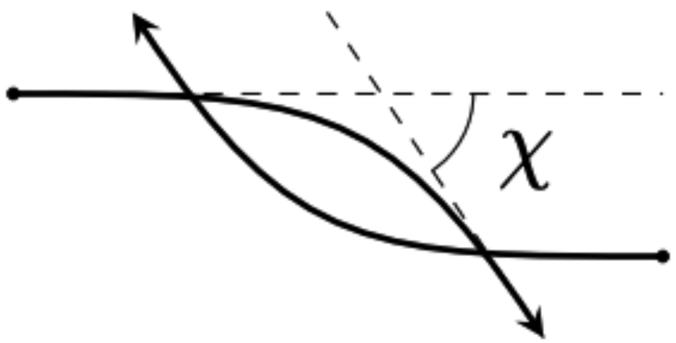
$1/J \sim |p_\infty|$

$p_\infty^2 \sim \mathcal{E}$

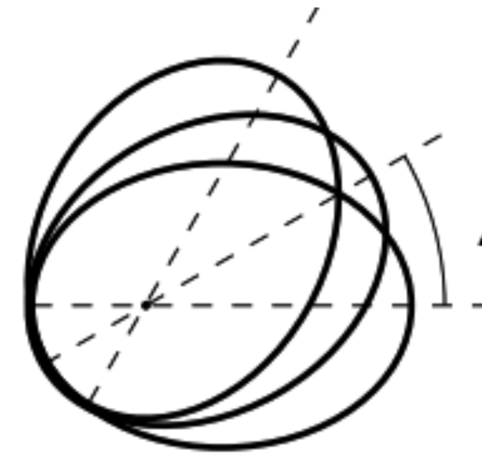
Kalin **RAP**
1911.09130

B2B correspondence

gauge-invariant information!



Conservative effects



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$$\chi_b^{(n)} = \frac{\sqrt{\pi}}{2}\Gamma\left(\frac{n+1}{2}\right)\sum_{\sigma\in\mathcal{P}(n)}\frac{1}{\Gamma\left(1+\frac{n}{2}-\Sigma^\ell\right)}\prod_\ell \frac{f_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!},$$

$$\boldsymbol{p}^2(r,E) = p_\infty^2(E) + \sum_i P_i(E)\left(\frac{G}{r}\right)^i$$

$$= p_\infty^2(E)\left(1+\sum_i f_i(E)\left(\frac{GM}{r}\right)^i\right)$$

$$\chi_j^{(4)} = \frac{3\pi}{8M^4\mu^4}\left(P_1P_3 + \frac{1}{2}P_2^2 + p_\infty^2 P_4\right),$$

$$\chi_j^{(3)} = \frac{1}{M^3\mu^3 p_\infty^3}\left(-\frac{P_1^3}{24} + p_\infty^2\frac{P_1P_2}{2} + p_\infty^4 P_3\right)$$

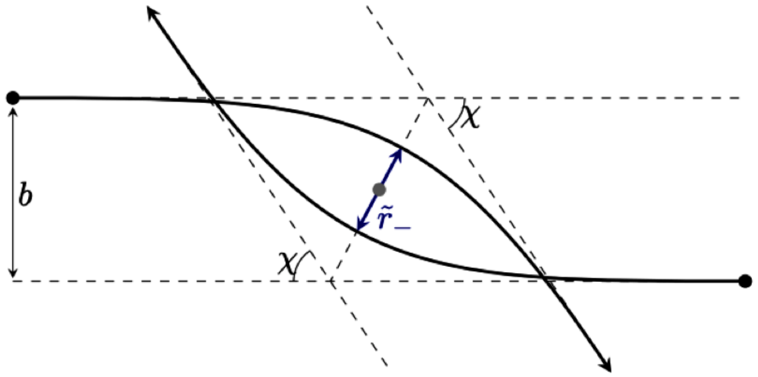
$p_\infty^2 \sim \mathcal{E}$

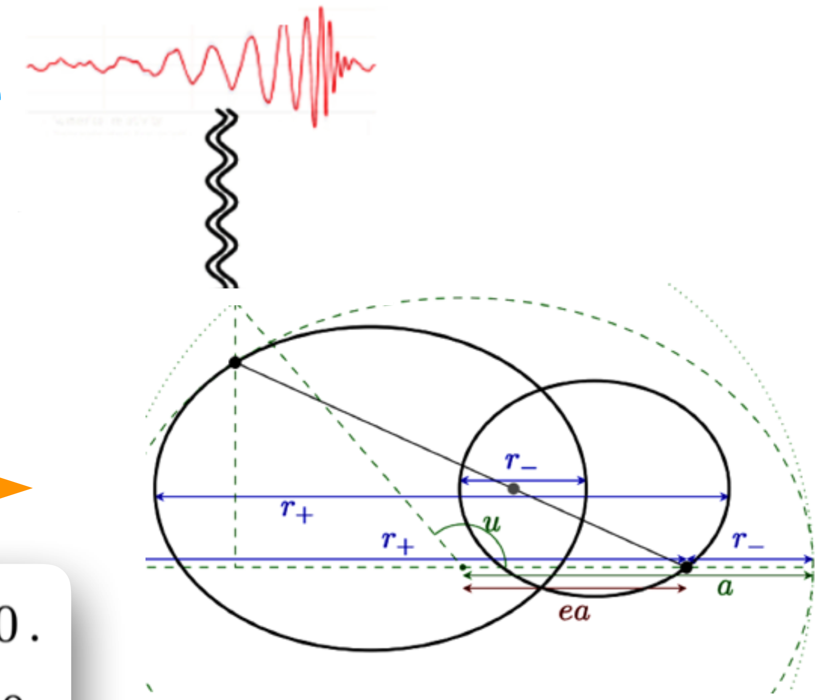
PN suppressed

To appear

B2B correspondence

gauge-invariant information!





Radiative effects?!

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\ \mathcal{E}<0\,,$$

$$\Delta E_{\rm hyp}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt \frac{dE}{dt}$$

$$\Delta E_{\rm ell}(J,\mathcal{E}) = \oint dt \frac{dE}{dt}$$

$$2\int_{\tilde{r}_-}^{+\infty} \frac{dr}{\dot{r}} \frac{dE}{dt}(r,J,\mathcal{E})$$

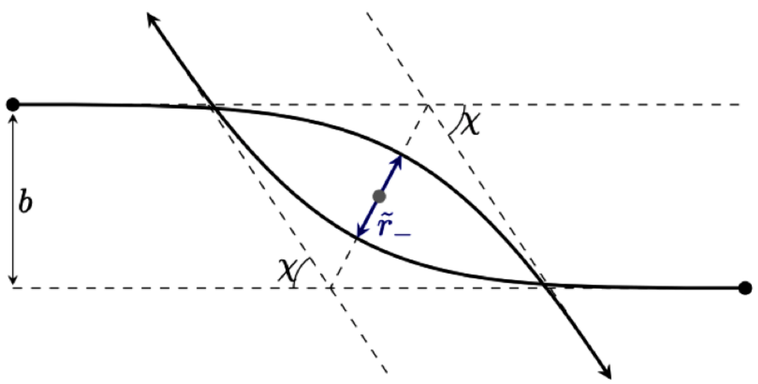
$$2\int_{r_-}^{r_+} \frac{dr}{\dot{r}} \frac{dE}{dt}(r,J,\mathcal{E})$$

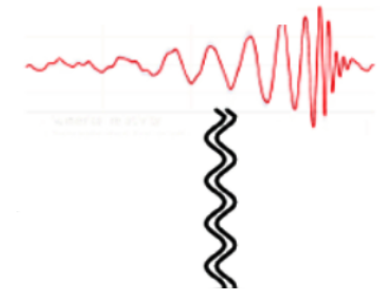
Adiabatic Approx.

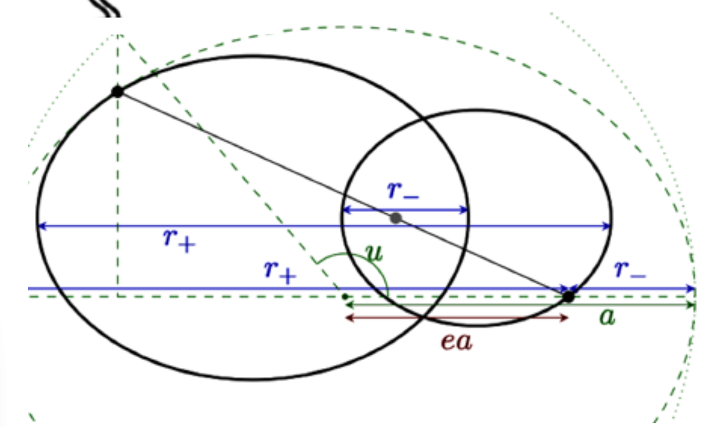
To appear

B2B correspondence

gauge-invariant information!







Radiative effects

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$$2\int_{\tilde{r}_-}^{+\infty} \frac{dr}{\dot{r}} \frac{dE}{dt}(r,J,\mathcal{E}) \qquad \frac{dE}{dt}(r,J,\mathcal{E}) = \frac{dE}{dt}(r,-J,\mathcal{E}) \qquad 2\int_{r_-}^{r_+} \frac{dr}{\dot{r}} \frac{dE}{dt}(r,J,\mathcal{E})$$

$$\dot{r} = \frac{\partial}{\partial p_r} H(p_r^2 + J^2/r^2, r)$$

Similar to radial action: **Loop-around!**

$$\boxed{\Delta E_{\rm ell}(J,\mathcal{E}) = \Delta E_{\rm hyp}(J,\mathcal{E}) - \Delta E_{\rm hyp}(-J,\mathcal{E})} \qquad \mathcal{E}<0$$

See also
Bini Damour
2007.11239

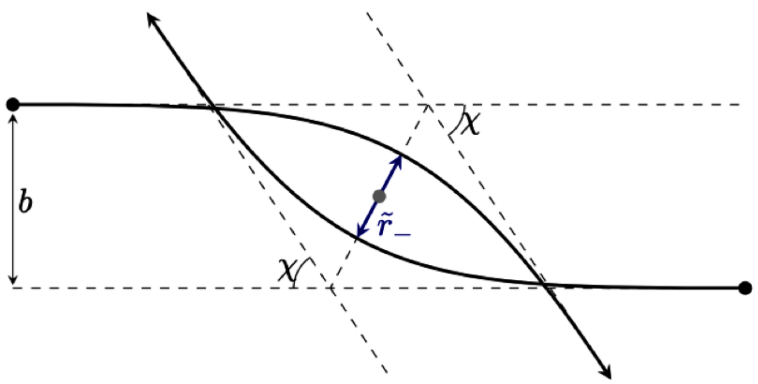
Adiabatic Approx.

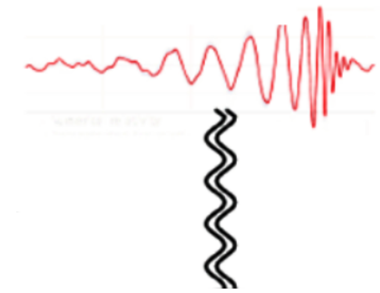
AND MORE!

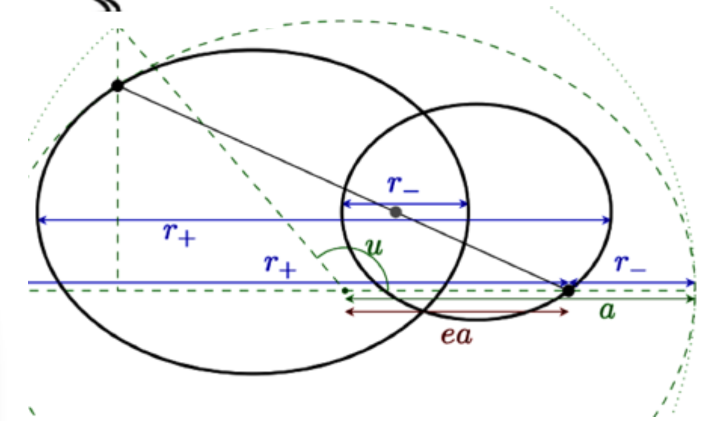
To appear

B2B correspondence

gauge-invariant information!







Radiative effects ⟷

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$$\Delta \overset{J}{\cancel{E}}_{\rm hyp}(J,\mathcal{E}) = \int_{-\infty}^{+\infty} dt\, \frac{d\overset{J}{\cancel{E}}}{dt} \quad \longleftrightarrow \quad \Delta \overset{J}{\cancel{E}}_{\rm ell}(J,\mathcal{E}) = \oint dt\, \frac{d\overset{J}{\cancel{E}}}{dt}$$

$$2\int_{\tilde{r}_-}^{+\infty} \frac{dr}{\dot{r}} \frac{d\overset{J}{\cancel{E}}}{dt}(r,J,\mathcal{E}) \qquad\qquad 2\int_{r_-}^{r_+} \frac{dr}{\dot{r}} \frac{d\overset{J}{\cancel{E}}}{dt}(r,J,\mathcal{E})$$

$$\boxed{\frac{dJ}{dt}(r,J,\mathcal{E}) = -\frac{dJ}{dt}(r,-J,\mathcal{E})}$$

$$\dot{r} = \frac{\partial}{\partial p_r} H(p_r^2 + J^2/r^2, r)$$

Similar to radial action: **Loop-around!**

$$\boxed{\Delta J_{\rm ell}(J,\mathcal{E}) = \Delta J_{\rm hyp}(J,\mathcal{E}) + \Delta J_{\rm hyp}(-J,\mathcal{E})} \qquad \mathcal{E}<0$$

Sign flip

Similar to periastron to angle

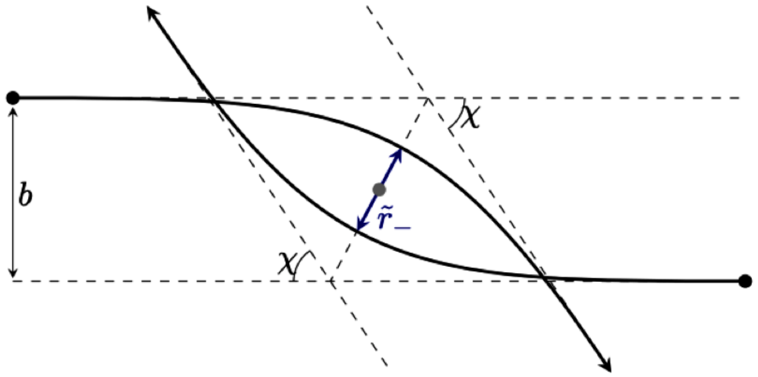
AND MORE!

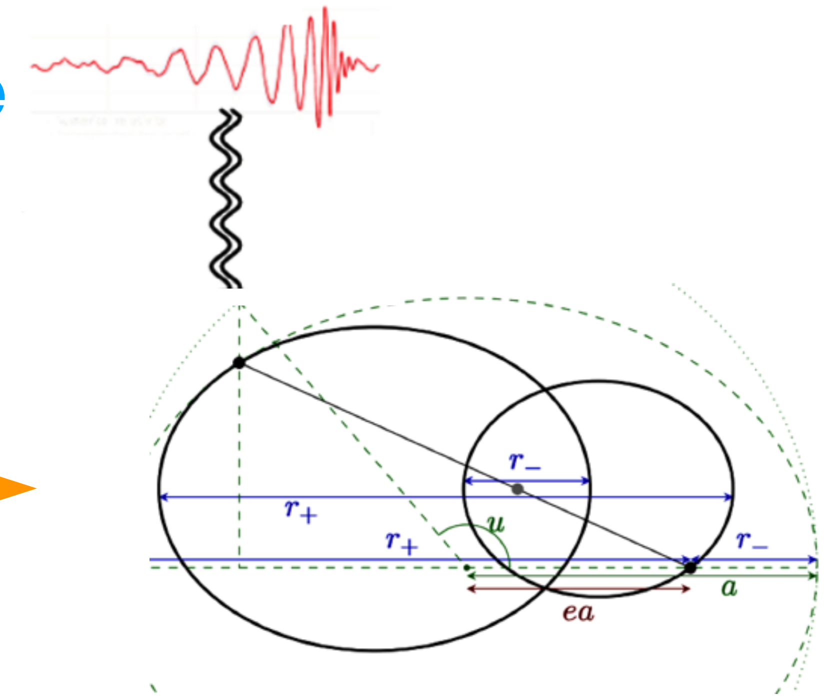
To appear

B2B correspondence

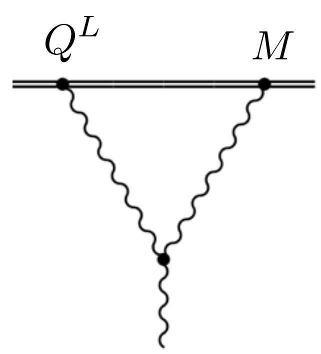
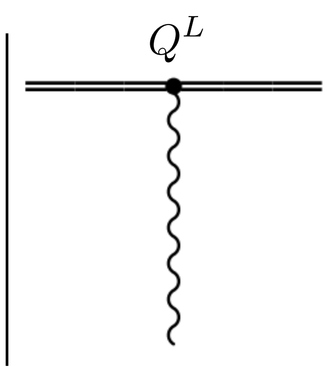
gauge-invariant information!

Conservative! Radiative effects

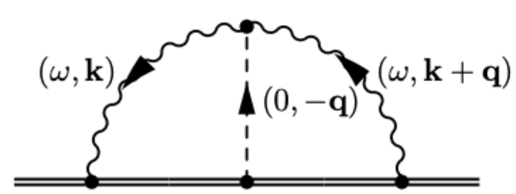




Goldberger Ross
0912.4254

$\Big|$  $\Big|^2$ = $(1 + 2\pi G M \omega)$ $\Big|$  $\Big|^2$ → $\omega \frac{d\Gamma}{d\omega}$

OPTICAL THEOREM



(Feynman b.c.)

$$= \int d\omega \left(i\pi G M \frac{dE}{d\omega} + \cdots \right)$$

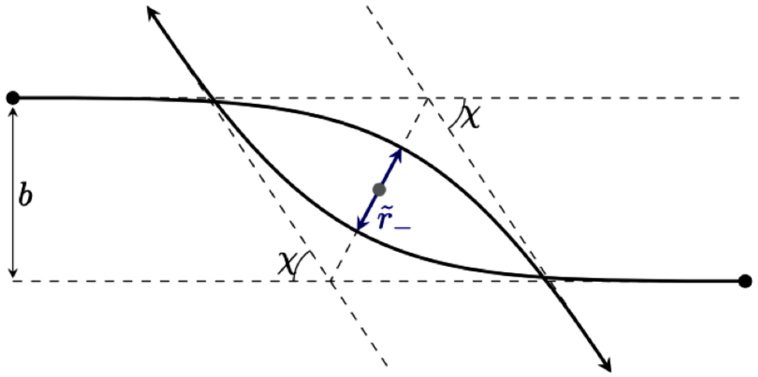
Energy spectrum!

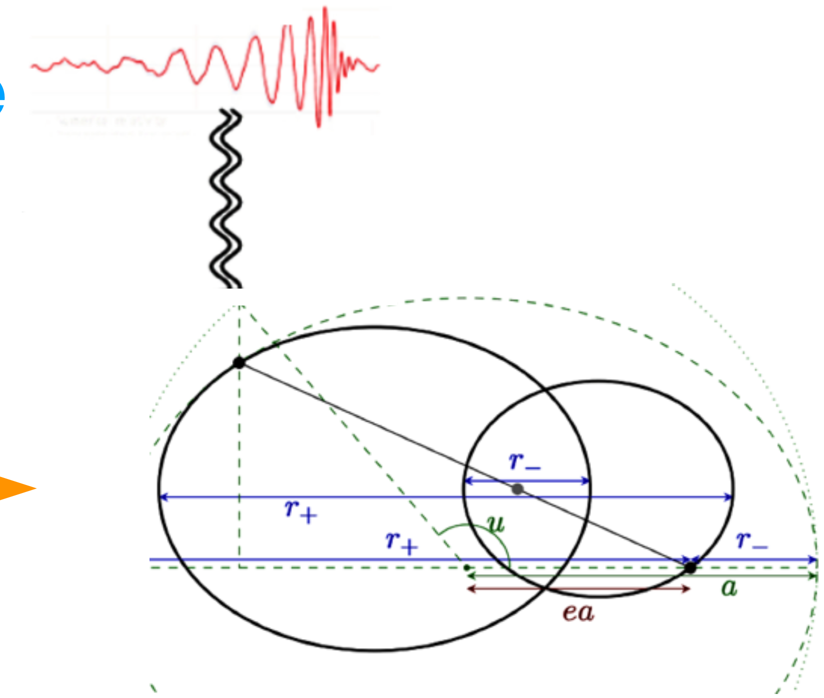
To appear

B2B correspondence

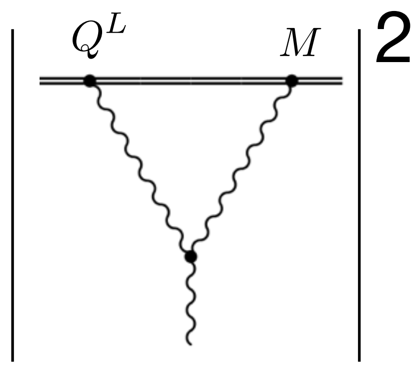
gauge-invariant information!

Conservative! Radiative effects

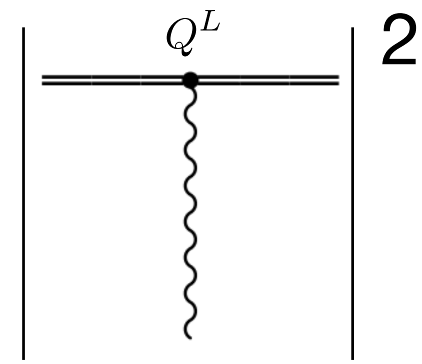




Goldberger Ross
0912.4254

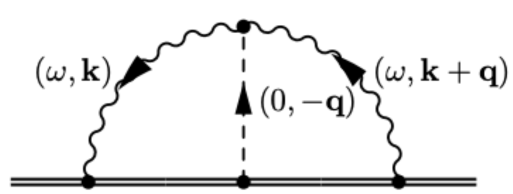


$= (1 + 2\pi G M \omega)$



See also
Foffa Sturani
2103.03190

OPTICAL THEOREM



(Feynman b.c.)

$$= \int d\omega \left(i\pi G M \frac{dE}{d\omega} + \cdots \right)$$

pole+log coincide
conservative tail

Galley Leibovich
RAP Ross
1511.07379

in-in b.c. $= GM \int d\omega \, \frac{dE}{d\omega} \left(-\frac{1}{d-4} - \log(\omega^2/\mu^2) + i\pi \mathrm{sign}(\omega) \cdots \right)$

Ret $\frac{1}{d-4}[-(\omega + i0^+)^2]^{d-4}$

Feyn $\frac{1}{d-4}[-\omega^2 - i0^+]^{d-4}$

Dissipative term

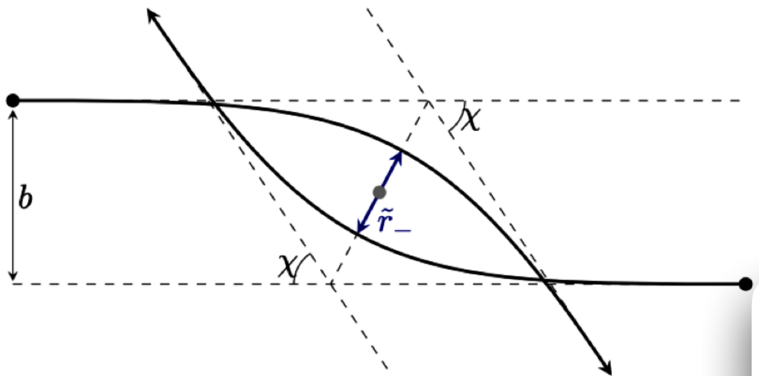
$$S_{\mathrm{eff}} = -\int H_{\mathrm{tail}} dt \longrightarrow H^{\mathrm{loc}}_{\mathrm{tail}} \propto \frac{dE}{dt}$$

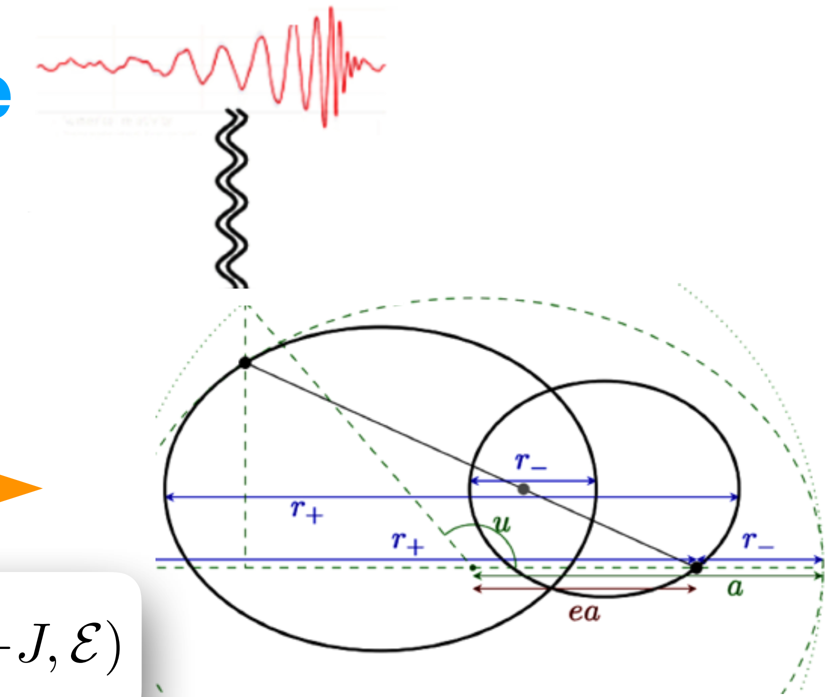
To appear

B2B correspondence

gauge-invariant information!

Local Conservative Radiative effects





$$\Delta E_{\rm ell}(J,\mathcal{E}) = \Delta E_{\rm hyp}(J,\mathcal{E}) - \Delta E_{\rm hyp}(-J,\mathcal{E})$$

See also Bini Damour 2007.11239

$$\delta\mathcal{S}_r^{bound} = -\frac{1}{2\pi}\oint H_{\rm tail}dt \qquad \delta\mathcal{S}_r^{unbound} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} H_{\rm tail}dt$$

$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$ **(local)**

The *local* conservative B2B map remains the same! (runs both ways)

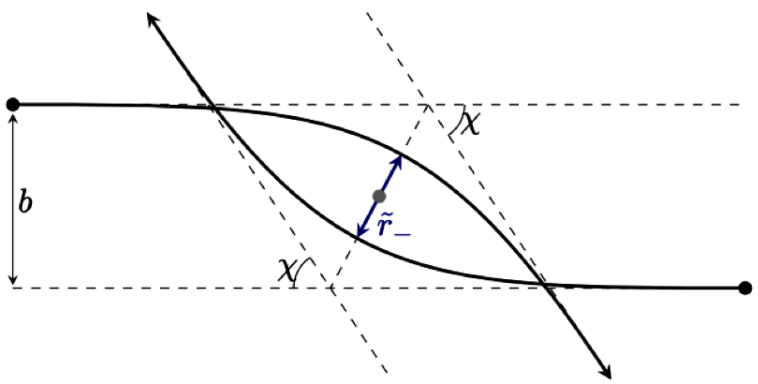
$$i_r(j,\mathcal{E}) \equiv \frac{\mathcal{S}_r}{GM\mu} = {\rm sg}(\hat{p}_\infty)\chi_j^{(1)}(\mathcal{E}) - j\left(1+\frac{2}{\pi}\sum_{n=1}\frac{\chi_j^{(2n)}(\mathcal{E})}{(1-2n)j^{2n}}\right)$$ **(local)**

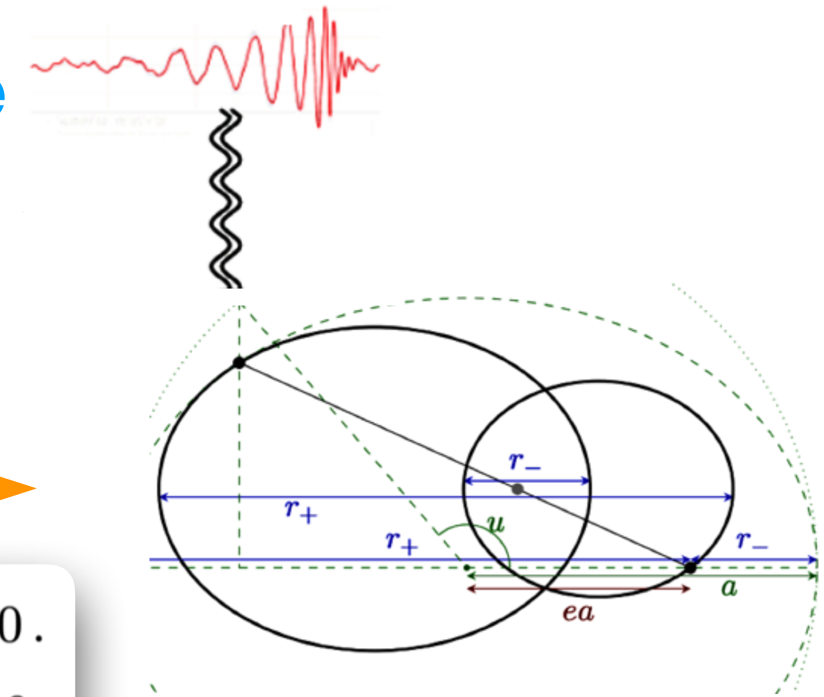
$$S_{\rm eff} = -\int H_{\rm tail}dt \longrightarrow H^{\rm loc}_{\rm tail} \propto \frac{dE}{dt} \longleftarrow$$ Same map for orbital elements and Firsov

To appear

B2B correspondence

gauge-invariant information!





Non-local **Conservative Radiative effects**

$$r_-(J,\mathcal{E}) = \tilde{r}_-(J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,.$$
$$r_+(J,\mathcal{E}) = \tilde{r}_-(-J,\mathcal{E}) \qquad J>0,\, \mathcal{E}<0\,,$$

$$\delta\mathcal{S}_r^{bound} = -\frac{1}{2\pi}\oint H_{\rm tail}dt \qquad \delta\mathcal{S}_r^{unbound} = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} H_{\rm tail}dt$$

$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

Total (L+NL) Conserv.

What about the non-local part? **Loop around again!**

$$\int_{\tilde{r}_-}^{\infty}\frac{dr}{p_r}H_{\rm tail} \qquad H_{\rm tail}(r,\mathcal{E},j) = H_{\rm tail}(r,\mathcal{E},-j) \qquad \int_{r_-}^{r_+}\frac{dr}{p_r}H_{\rm tail}$$

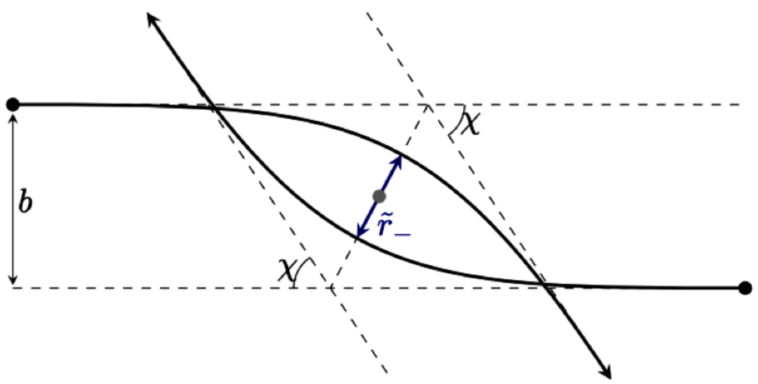
Valid in the "large-j" limit ONLY

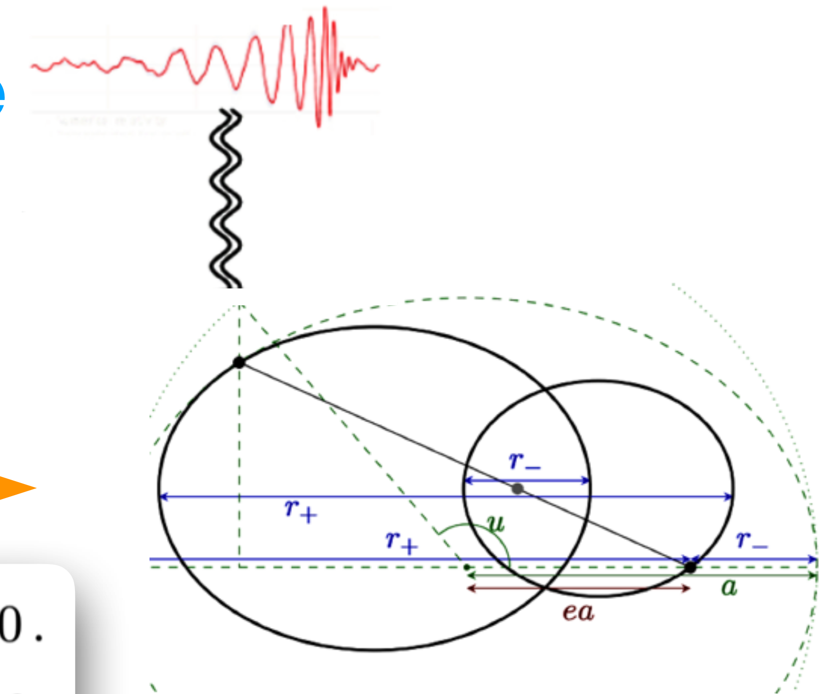
To appear

B2B correspondence

gauge-invariant information!

<u>Non-local</u> Conservative Radiative effects





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$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

Total (L+NL) Conserv.

Averaged Hamiltonian

$$\delta\bar{H}(i_r,j) = -\Omega_r\left[i_{r,\rm tail}^{unb}(j,\mathcal{E}) - i_{r,\rm tail}^{unb}(-j,\mathcal{E})\right]_{\mathcal{E}\to\mathcal{E}_0(i_r,j)}$$

<u>Valid in the "large-j" limit ONLY</u>

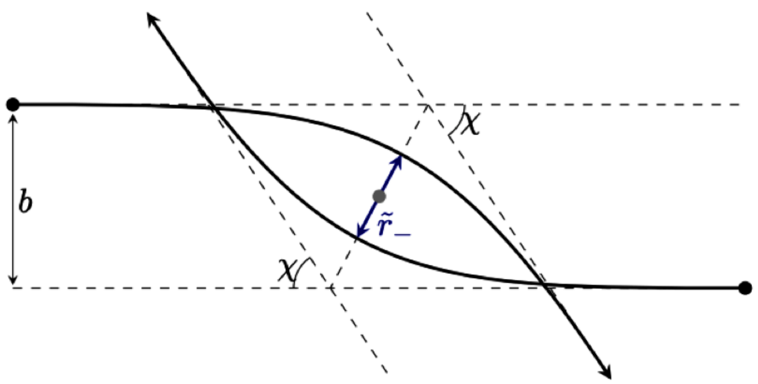
Unlike the local part this Hamiltonian does not interpolate from large to small eccentricity unscathed!

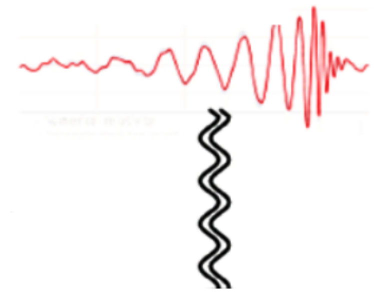
To appear

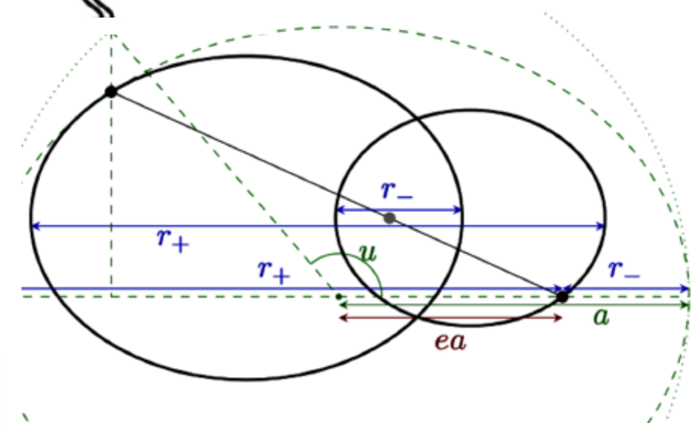
B2B correspondence

gauge-invariant information!

Non-local Conservative Radiative effects







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$$i_r^{bound}(j,\mathcal{E}) = i_r^{unbound}(j,\mathcal{E}) - i_r^{unbound}(-j,\mathcal{E})$$

Galley Leibovich
RAP Ross
1511.07379

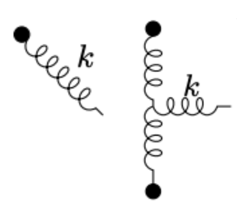
BUT we already know the ***universal*** structure of the tail term

See also
Blanchet et al.
1912.12359

Jakobsen et al.
2101.12688

Mougiakakos
Riva Vernizzi
2102.08339

Known at G^3!



dissipative

$$GM\int d\omega \frac{dE}{d\omega}\left(-\frac{1}{d-4} - \log(\omega^2/\mu^2) + i\pi\,\mathrm{sign}(\omega)\cdots\right)$$

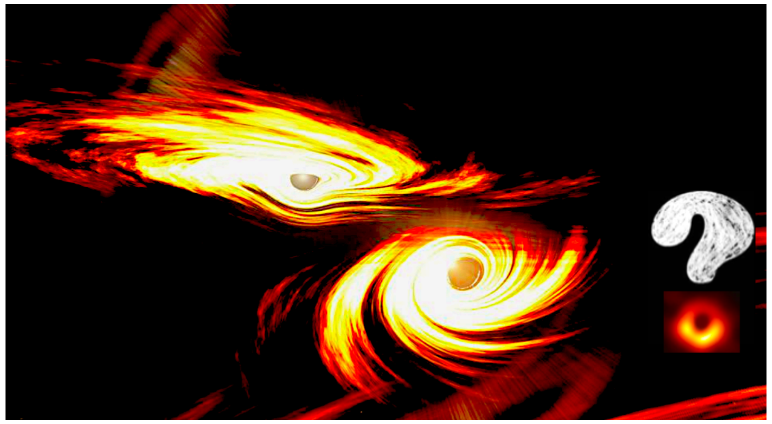
Universal UV pole cancels potential IR

Universal non-local part

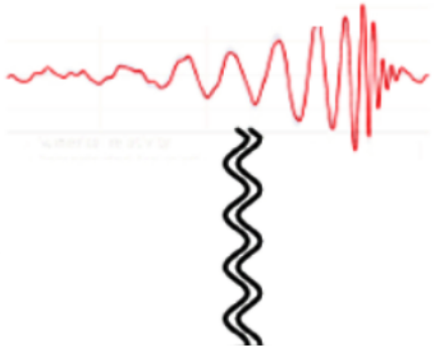
Local terms
Must compute in PM!

Outline

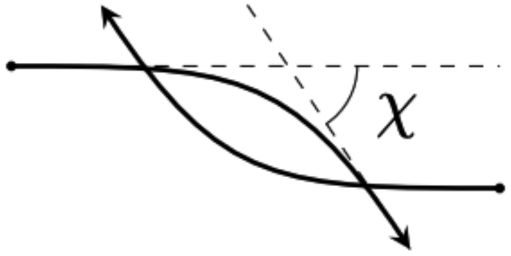
Discovery Potential =
Precise Theoretical Predictions



- Part I: B2B

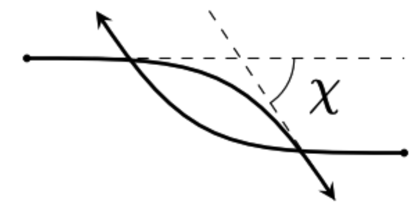


- **Part II: EFT**



Kalin **RAP** 2006.01184 Liu **RAP** Yang 2102.10059

PM EFT for scattering



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

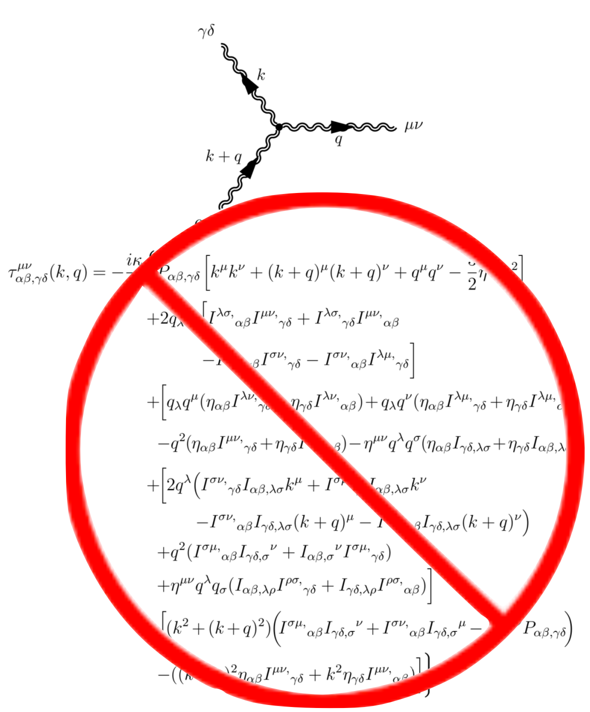
external source

$$S_{\rm pp} = -\sum_A \frac{m_A}{2}\int {\rm d}\tau_A g_{\mu\nu}\left(x_A\left(\tau_A\right)\right) v_A^\mu\left(\tau_A\right) v_A^\nu\left(\tau_A\right) - \frac{1}{2}\int {\rm d}\tau_A S_{ab}(\tau_A)\omega_\mu^{ab}(\tau_A) v_A^\mu(\tau_A).$$

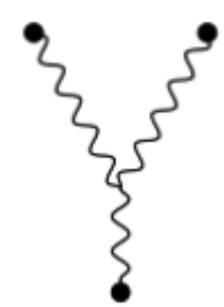
$$\frac{C_{ES^2}}{2m_A}\int \underbrace{d\tau_A E_{ab} S_A^{ac} S_{cA}^b}_{\text{spin-induced moments}} + c_{E^2}\int \underbrace{d\tau_A E_{\mu\nu}E^{\mu\nu}}_{\text{tidal effects}} + \ldots$$

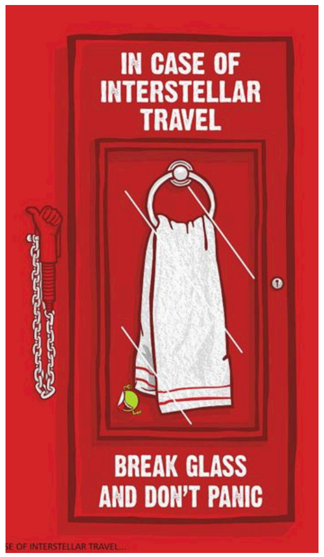
finite-size effects

Simplified Feynman rules through GF and total derivatives (but no field redef.)



$$M_{\rm Pl}\mathcal{L}_{hhh} = -\frac{1}{2}h^{\mu\nu}\partial_\mu h^{\rho\sigma}\partial_\nu h_{\rho\sigma} + \frac{1}{2}h^{\mu\nu}\partial_\rho h\partial^\rho h_{\mu\nu} - \frac{1}{8}h\partial_\rho h\partial^\rho h$$
$$+ h^{\mu\nu}\partial_\nu h_{\rho\sigma}\partial^\sigma h_\mu{}^\rho - h^{\mu\nu}\partial_\sigma h_{\nu\rho}\partial^\sigma h_\mu{}^\rho + \frac{1}{4}h\partial_\sigma h_{\nu\rho}\partial^\sigma h^{\nu\rho}\,.$$

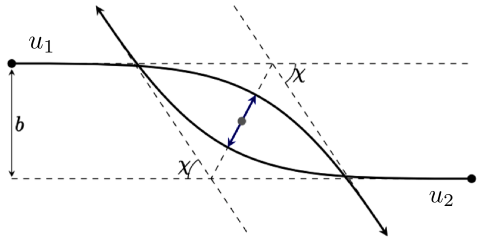




Lots of redundancy in GR — No need to panic!

Kalin **RAP** 2006.01184 Liu **RAP** Yang 2102.10059

PM EFT for scattering



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

Post-Minkowskian expanded solution (Euler-Lagrange eqs.)

Initial Data

$$v_a^\mu(\tau_1) = u_a^\mu + \sum_n \delta^{(n)} v_a^\mu(\tau_a)\,, \qquad S_A^{ab}(\tau_A) = \mathcal{S}_A^{ab} + \sum \delta^{(n)} S_A^{ab}(\tau_A)\,.$$

$$x_a^\mu(\tau_1) = b_a^\mu + u_a^\mu \tau_a + \sum_n \delta^{(n)} x_a^\mu(\tau_a)\,,$$

The true classical motion

Compute total impulse from the action…

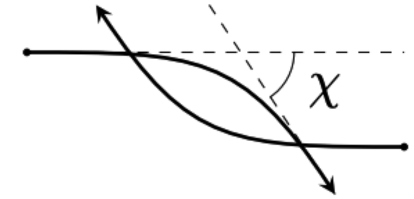
$$\Delta p_a^\mu = -\eta^{\mu\nu}\int_{-\infty}^{+\infty} {\rm d}\tau_a \frac{\partial \mathcal{L}_{\rm eff}}{\partial x_a^\nu}(x_a(\tau_a)) \qquad \Delta S_A^{ab} = \int_{-\infty}^{+\infty} d\tau \{S_A^{ab}, \mathcal{L}_{\rm eff}\}$$

… and (planar) deflection angle in the centre-of-mass

$$2\sin\left(\frac{\chi}{2}\right) = \chi - \frac{1}{24}\chi^3 + \mathcal{O}(\chi^5) = \frac{|\Delta \boldsymbol{p}_{1\rm cm}|}{p_\infty} = \frac{\sqrt{-\Delta p_1^2}}{p_\infty}\,,$$

Kalin **RAP**
2006.01184

PM EFT for scattering



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

$i\epsilon$ prescription **irrelevant** →

Potential Modes

$(k_0 \ll |\boldsymbol{k}|)$

Radiation Modes

$(k_0 \sim |\boldsymbol{k}|)$

→ *Total Flux* (optical theorem with Feynman)

$$\frac{i}{k^2 + i0} P_{\mu\nu\alpha\beta}$$

NRGR

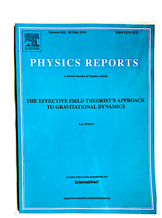
(Goldberger & Rothstein, …)

The effective field theorist's approach to gravitational dynamics

Rafael A. Porto

Physics Reports

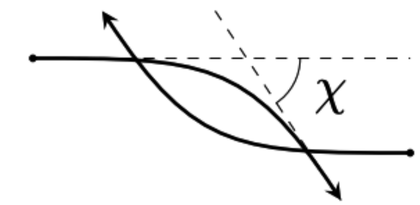
Volume 633, 20 May 2016, Pages 1-104



Dlapa Kalin Liu **RAP to appear**

PM EFT for scattering

Schwinger-Keldysh Formalism



$$e^{iS_{\text{eff}}[x_a^{1,2}]} = \int \mathcal{D}h_{\mu\nu}^{1,2}\, e^{iS_{\text{EH}}[h]+iS_{\text{GF}}[h]+iS^{C}_{\text{pp}}[x_a,h]}\,,$$

$i\epsilon$ prescription **irrelevant** → Potential Modes $(k_0 \ll |\boldsymbol{k}|)$

Radiation Modes $(k_0 \sim |\boldsymbol{k}|)$ → Radiation-Reaction (Cons + Dissip)

$$\frac{i}{(\omega + i0)^2 - \mathbf{k}^2}$$

$$\begin{pmatrix} 0 & G_{\text{adv}}(x-y) \\ G_{\text{ret}}(x-y) & 0 \end{pmatrix}$$

$$S^{\text{c}}_{\text{pp}} = S_{\text{pp}}[x^\alpha_{a(1)}, h_{\mu\nu(1)}] - S_{\text{pp}}[x^\alpha_{a(2)}, h_{\mu\nu(2)}]\,.$$

$$\Delta p^\mu_a = -\eta^{\mu\nu} \int_{-\infty}^{\infty} \left[\frac{\partial \mathcal{L}_I}{\partial x^\nu_{a(-)}(\tau_a)}\right]_{\text{PL}} d\tau_a\,,$$

$$x^\alpha_{a(-)} \to 0 \text{ and } x^\alpha_{a(+)} \to x^\alpha_a.$$

→ *

$$u^\mu_{\text{CoM}} = \frac{m_1 u^\mu_1 + m_2 u^\mu_2}{M\Gamma}\,,$$

$$2\sin(\chi/2) = \frac{|\Delta p_a|_{\text{CoM}}}{p_\infty}$$

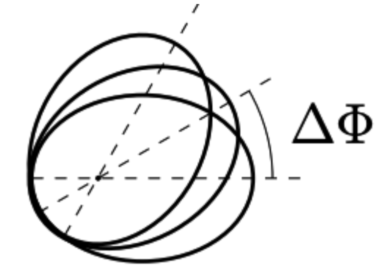
$$\Delta E_{a\text{CoM}}(J,\mathcal{E}) = \Delta p_a \cdot u_{\text{CoM}}$$

* A bit more subtle when recoil is relevant (does not affect conservative-only part)

Galley Leibovich
RAP Ross
1511.07379

PN EFT for bound states

in-in NRGR



$$e^{iS_{\rm eff}[x_a^{1,2}]} = \int \mathcal{D}h_{\mu\nu}^{1,2}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS^{C}_{\rm pp}[x_a,h]}\,,$$

Radiation-Reaction
(**Cons** + Dissip)

Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

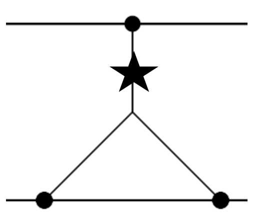
Radiation Modes
$(k_0 \sim |\boldsymbol{k}|)$

IR/UV cancelation!

RAP Rothstein
1703.06433

Foffa **RAP** Rothstein Sturani
1903.05118

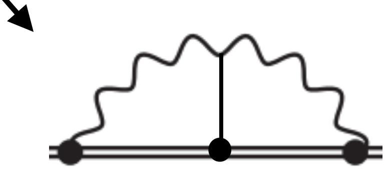
Integrals are much easier BUT spoils IR!



$$\frac{1}{p_0^2-\boldsymbol{p}^2} \simeq -\frac{1}{\boldsymbol{p}^2}\left(1+\frac{p_0^2}{\boldsymbol{p}^2}+\cdots\right).$$

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+v^{14}$	$+\cdots)\,G$
2PM		$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+v^{12}$	$+\cdots)\,G^2$
3PM			$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+v^{10}$	$+\cdots)\,G^3$
4PM				$(1$	$+v^2$	$+v^4$	$+v^6$	$+v^8$	$+\cdots)\,G^4$
5PM					$(1$	$+v^2$	$+v^4$	$+v^6$	$+\cdots)\,G^5$

Integrals are much easier BUT spoils UV!

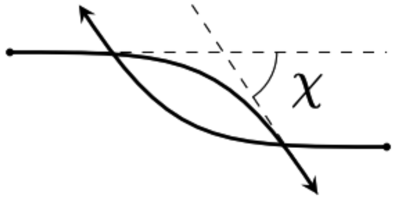


$$e^{i\mathbf{k}\cdot\mathbf{x}} = 1 + i\mathbf{k}\cdot\mathbf{x} + \cdots$$

Foffa Sturani
1907.02869
Bluemlein et al.
2010.13672

Dlapa Kalin Liu **RAP**
in progress

PM EFT for scattering



$$e^{iS_{\rm eff}[\overset{1,2}{x_a}]} = \int \mathcal{D}h_{\mu\nu}^{1,2}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+i\overset{C}{S}_{\rm pp}[x_a,h]} \,,$$

Potential Modes $(k_0 \ll |\boldsymbol{k}|)$

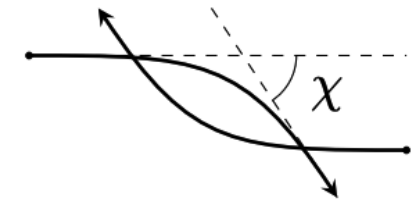
Radiation Modes $(k_0 \sim |\boldsymbol{k}|)$ → Radiation-Reaction (Cons + Dissip)

(classical) *"Soft"* Region

PM EFT for scattering

Dlapa Kalin Liu **RAP**
in progress

STAY TUNED!



$$e^{iS_{\text{eff}}[x_a^{1,2}]} = \int \mathcal{D}h_{\mu\nu}^{1,2}\, e^{iS_{\text{EH}}[h]+iS_{\text{GF}}[h]+iS_{\text{pp}}^{C}[x_a,h]},$$

Potential Modes
$(k_0 \ll |\boldsymbol{k}|)$

Radiation Modes
$(k_0 \sim |\boldsymbol{k}|)$

Radiation-Reaction
(Cons + Dissip)

(classical) "*Soft*" Region

IR/UV finite!

GOAL: Complete NNLO
Full Conservative NNNLO

Parra-Martinez
Ruff and Zeng
2005.04236

Kalin Liu **RAP**
2007.04977

Di Vecchia et al.
2104.03256

Differential Equations
b.c. from **entire region**
$(\gamma \to 1)$

$$\partial_x \vec{h}(x,\epsilon) = \epsilon\, \mathbb{M}(x)\, \vec{h}(x,\epsilon)$$

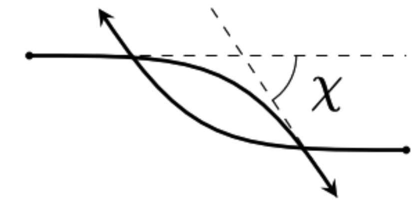
Single scale!

$$\gamma = \frac{1+x^2}{2x}, \quad \gamma \equiv u_1 \cdot u_2$$

***UV from finite-size only**

Dlapa Kalin Liu **RAP**
in progress

PM EFT for scattering



$$e^{iS_{\rm eff}[x_a^{1,2}]} = \int \mathcal{D}h^{1,2}_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS^{C}_{\rm pp}[x_a,h]}\,,$$

Not enough at NNNLO → Potential Modes $(k_0 \ll |\boldsymbol{k}|)$

Radiation Modes $(k_0 \sim |\boldsymbol{k}|)$ → Radiation-Reaction (**Cons** + Dissip)

(classical) "*Soft*" Region

IR/UV finite!

GOAL: Complete NNLO
Full Conservative NNNLO

Differential Equations
b.c. from **entire region**
$(\gamma \to 1)$

$$\partial_x \vec{h}(x,\epsilon) = \epsilon\, \mathbb{M}(x)\, \vec{h}(x,\epsilon)$$

Single scale!

$$\gamma = \frac{1+x^2}{2x}, \quad \gamma \equiv u_1 \cdot u_2$$

Full soft integrand @ 3PM &
soft-conservative @ 4PM.
Diff Eqs. "solved"

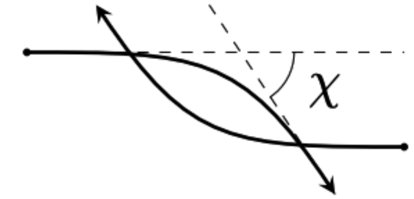
B.C.'s in progress
(only 7 in potential - solved)

NO PN SERIES EXPANSION!

PM EFT for scattering

Kalin **RAP** 2006.01184

Kalin Liu **RAP** 2007.04977



$$e^{iS_{\rm eff}[x_a]} = \int \mathcal{D}h_{\mu\nu}\, e^{iS_{\rm EH}[h]+iS_{\rm GF}[h]+iS_{\rm pp}[x_a,h]}\,,$$

Enough at NNLO →

Potential Modes $(k_0 \ll |\boldsymbol{k}|)$

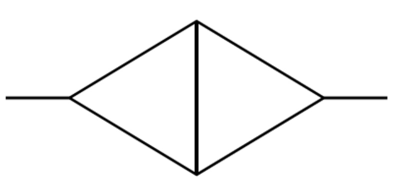
Radiation Modes $(k_0 \sim |\boldsymbol{k}|)$

(classical) "*Soft*" Region

IR/UV finite!

eight elements & canonical to NNLO

Differential Equations b.c. from potentials



$$\partial_x \vec{h}(x,\epsilon) = \epsilon\, \mathbb{M}(x)\, \vec{h}(x,\epsilon)$$

Single scale!

$$\gamma = \frac{1+x^2}{2x}, \quad \gamma \equiv u_1 \cdot u_2$$

Kalin Liu **RAP**
2007.04977

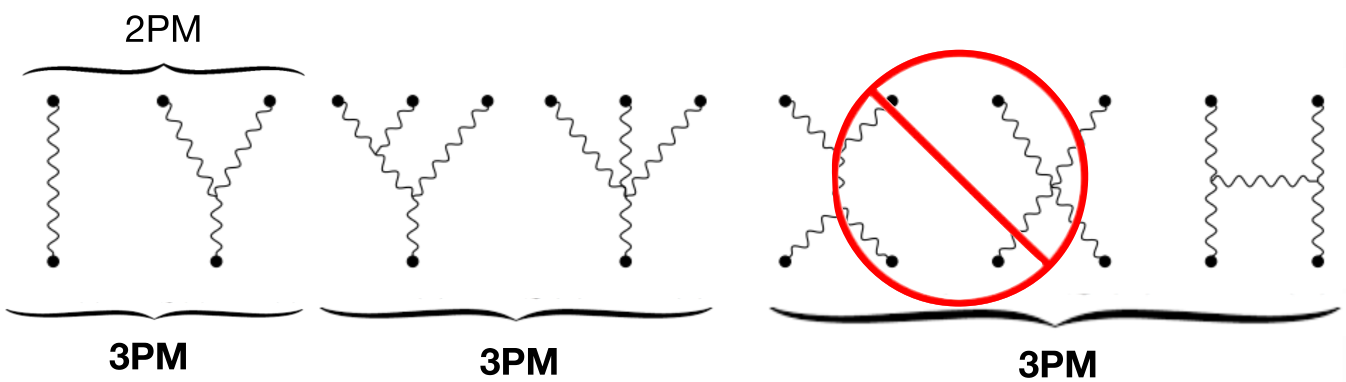
PM EFT for scattering: NNLO

Integrals (one family!):

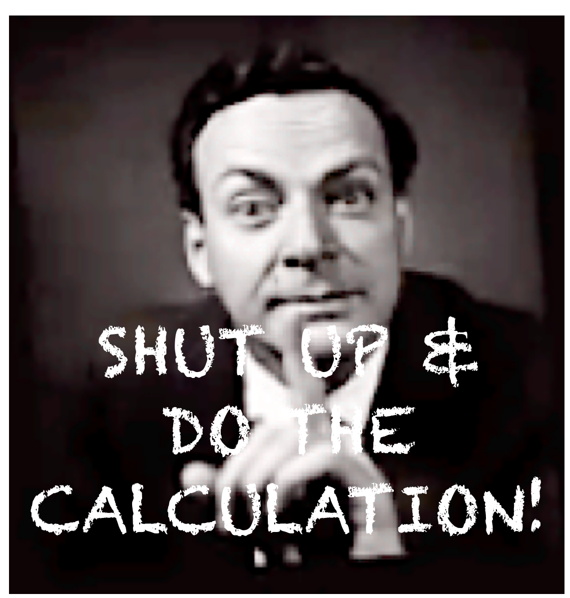
$$M^{(a,\tilde{a})}_{n_1 n_2; i_1\cdots i_5}(q,\gamma) \equiv \int_{k_1,k_2} \frac{\hat{\delta}(k_1\cdot u_a)\hat{\delta}(k_2\cdot u_{\tilde{a}})}{A^{n_1}_{1,\not{a}} A^{n_2}_{2,\not{\tilde{a}}} D_1^{i_1}\cdots D_5^{i_5}}\,,$$

$$A_{1,\not{a}} = k_1\cdot u_{\not{a}},\ A_{2,\not{\tilde{a}}} = k_2\cdot u_{\not{\tilde{a}}},\ D_1 = k_1^2,\ D_2 = k_2^2\,,$$
$$D_3 = (k_1{+}k_2{-}q)^2,\ D_4 = (k_1{-}q)^2,\ D_5 = (k_2{-}q)^2.$$

POTENTIAL REGION: DEQs with b.c. from the static limit of NRGR!



4PM: O(20) Feynman diagrams+iterations in PMEFT



Everything you wanted to know about 3PM (coincides with **IR-finite** Amplitude via **impetus** in **Y-basis**)

$$\frac{P_3}{M^3\mu^2} = \left(\frac{18\gamma^2-1}{2\Gamma} + \frac{8\nu}{\Gamma}(3{+}12\gamma^2{-}4\gamma^4)\frac{\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}}{\sqrt{\gamma^2-1}} + \frac{\nu}{6\Gamma}\left(6{-}206\gamma{-}108\gamma^2{-}4\gamma^3 + \frac{18\Gamma(1-2\gamma^2)(1-5\gamma^2)}{(1+\Gamma)(1+\gamma)}\right)\right).$$

(Objective & Subjective) Advantages

- ***We land in the classical integrand:*** Soft-expanded & cut (massless)
- ***No divergences/ambiguities:*** No "super-classical" nor region-induced in classical soft region.
- ***B2B on-shell:*** No Hamiltonian, nor extra EFT matching nor Born iterations.

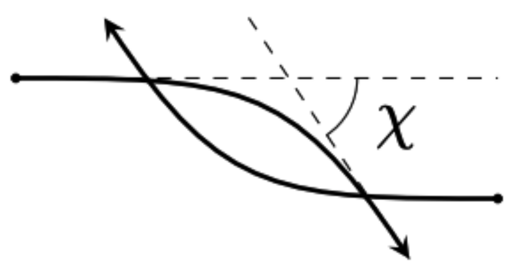
Main 'drawback':

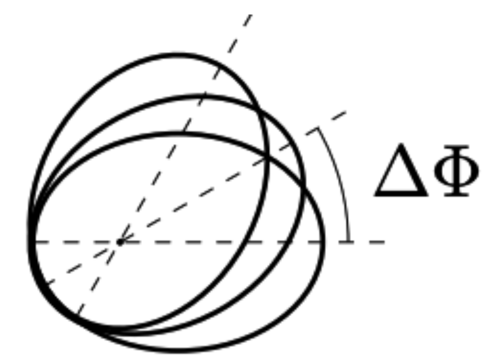
- Feynman diagrams (***significantly fewer*** than in NRGR and ***simpler***!)

Kalin Liu **RAP** 2007.04977 Kalin **RAP** 1911.09130

B2B dictionary : NNLO

Map from angle to periastron advance





<u>Full 3PM result</u> agrees with literature to 2PN

✓ 1PN ✓ 2PN ✓ 2PN ✓ 3PN ✓ 4PN

$$\left(\frac{\Delta\Phi}{2\pi}\right)_{\text{2-loop}} = \frac{3}{j^2} + \frac{3(35-10\nu)}{4j^4} + \frac{3}{4j^2}\left(10-4\nu+\frac{194-184\nu+23\nu^2}{j^2}\right)\mathcal{E}$$
$$+ \frac{3}{4j^2}\left(5-5\nu+4\nu^2+\frac{3535-6911\nu+3060\nu^2-375\nu^3}{10j^2}\right)\mathcal{E}^2$$
$$+ \frac{3}{4j^2}\left((5-4\nu)\nu^2+\frac{35910-126347\nu+125559\nu^2-59920\nu^3+7385\nu^4}{140j^2}\right)\mathcal{E}^3$$
$$+ \frac{3}{4j^2}\left((5-20\nu+16\nu^2)\frac{\nu^2}{4}\right)\mathcal{E}^4+\cdots,$$

The 5PN prediction (confirmed by tutifruti at 6PN too)

ONE-LOOP EXACT! $\frac{\Delta\Phi}{2\pi} = \frac{\widetilde{\mathcal{M}}_2 G^2}{2J^2}$

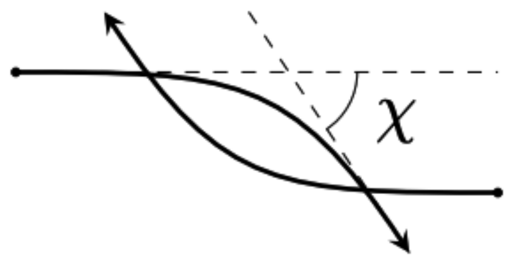
$$\chi_j^{(2)} \propto \widetilde{\mathcal{M}}_2 = \frac{3M^2\mu^2}{2}\left(\frac{5\gamma^2-1}{\Gamma}\right)$$

1/j^2 is predicted *<u>to all orders in v</u>*

(local) 4PM will complete 1/j^4

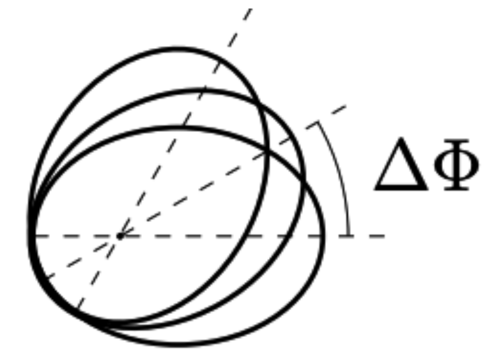
Kalin Liu **RAP** 2007.04977   Kalin **RAP** 1911.09130

B2B dictionary : NNLO



Map for _ALL_
Dynamical Invariants!

Without
HAMILTONIAN!



$$\frac{GM\Omega_r^{(L=2)}}{\epsilon^{\frac{3}{2}}} = 1 - \frac{(15-\nu)}{8}\epsilon + \frac{555+30\nu+11\nu^2}{128}\epsilon^2 + \left(\frac{3(2\nu-5)}{2j} - \frac{194-184\nu+23\nu^2}{4j^3}\right)\epsilon^{\frac{3}{2}} + \left(\frac{15(17-9\nu+2\nu^2)}{8j} + \frac{21620-28592\nu+8765\nu^2-865\nu^3}{80j^3}\right)\epsilon^{\frac{5}{2}} + \cdots$$

$$\frac{GM\Omega_\phi^{(L=2)}}{\epsilon^{\frac{3}{2}}} = 1 + \frac{3}{j^2} - \frac{15(2\nu-7)}{4j^4} + \left(\frac{1}{8}(\nu-15) + \frac{15(\nu-5)}{8j^2} - \frac{3(1301-921\nu+102\nu^2)}{32j^4}\right)\epsilon$$
$$+ \left(\frac{3(2\nu-5)}{2j} + \frac{-284+220\nu-23\nu^2}{4j^3} + \frac{3(913-728\nu+106\nu^2)}{j^5}\right)\epsilon^{\frac{3}{2}}$$
$$+ \left(\frac{1}{128}(555+30\nu+11\nu^2) + \frac{3(895-150\nu+51\nu^2)}{128j^2} - \frac{3(-270085+251236\nu-70545\nu^2+7470\nu^3)}{2560j^4}\right)\epsilon^2$$
$$+ \left(\frac{15(17-9\nu+2\nu^2)}{8j} + \frac{31520-34442\nu+10025\nu^2-865\nu^3}{80j^3}\right)\epsilon^{\frac{5}{2}}.$$

$$\delta\mathcal{S}_r(J,\mathcal{E},m_a) = -\left(1+\frac{\Delta\Phi}{2\pi}\right)\delta J + \frac{\mu}{\Omega_r}\delta\mathcal{E} - \sum_a \frac{1}{\Omega_r}\left(\langle z_a\rangle - \frac{\partial E(\mathcal{E},m_a)}{\partial m_a}\right)\delta m_a$$

$$\Omega_r(j,\mathcal{E}) \equiv \frac{2\pi}{T_p}, \quad \Omega_p(j,\mathcal{E}) \equiv \frac{\Delta\Phi}{T_p},$$

$$\Omega_\phi \equiv \Omega_r + \Omega_p = \frac{2\pi}{T_p}\left(1+\frac{\Delta\Phi}{2\pi}\right).$$

$$\epsilon = -2\mathcal{E}$$

3PN match

3PN missmatch

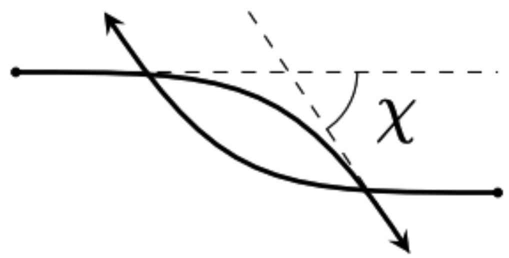
Higher orders velocity

Kalin **RAP**
1911.09130

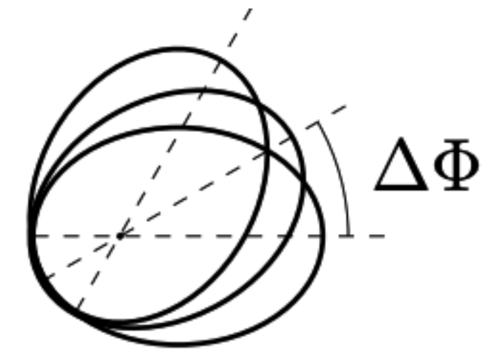
B2B dictionary/Spin

Valid for (aligned) spin!

J=canonical **total** ang. momentum



$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$



Angle from
Vines Steinhoff Buonanno
1812.00956

$$\begin{aligned}
\frac{\chi(\ell,a,\epsilon)}{2\pi} = &\left[\frac{1}{\pi}(-\epsilon)^{-\frac{1}{2}} - \frac{(\nu-15)}{8\pi}(-\epsilon)^{\frac{1}{2}} + \frac{35+30\nu+3\nu^2}{128\pi}(-\epsilon)^{\frac{3}{2}}\right]\frac{1}{\ell} \\
&+\left[3 + \frac{3(2\nu-5)}{4}\epsilon + \frac{3(5-5\nu+4\nu^2)}{16}\epsilon^2 - \frac{7\tilde{a}_+ + \Delta\tilde{a}_-}{2\pi}\epsilon^{-\frac{1}{2}} \right. \\
&\left. + \frac{5\Delta(\nu-3)\tilde{a}_- + (23\nu-25)\tilde{a}_+}{16\pi}(-\epsilon)^{\frac{3}{2}}\right]\frac{1}{2\ell^2} \\
&+\left[-\frac{7\tilde{a}_+ + \Delta\tilde{a}_-}{2} - \frac{(\nu-6)\Delta\tilde{a}_- + (7\nu-18)\tilde{a}_+}{2}\epsilon \right. \\
&- \frac{3\left((15-14\nu+2\nu^2)\Delta\tilde{a}_- + (25-38\nu+14\nu^2)\tilde{a}_+\right)}{16}\epsilon^2 \\
&\left. - \frac{2}{3\pi}(-\epsilon)^{-\frac{3}{2}} + \frac{33+\nu}{4\pi}(-\epsilon)^{-\frac{1}{2}} + \frac{3003-1090\nu-5\nu^2+128\tilde{a}_+^2}{64\pi}(-\epsilon)^{\frac{1}{2}}\right]\frac{1}{2\ell^3} \\
&+\left[\frac{3(35+2\tilde{a}_+^2-10\nu)}{4} - \frac{10080-13952\nu+123\pi^2\nu+1440\nu^2}{128}\epsilon \right. \\
&\left. - \frac{624\Delta\tilde{a}_-\tilde{a}_+ + 24(1-8\nu)\tilde{a}_-^2 - 24(12\nu-61)\tilde{a}_+^2}{128}\epsilon + \cdots\right]\frac{1}{2\ell^4} + \cdots .
\end{aligned}$$

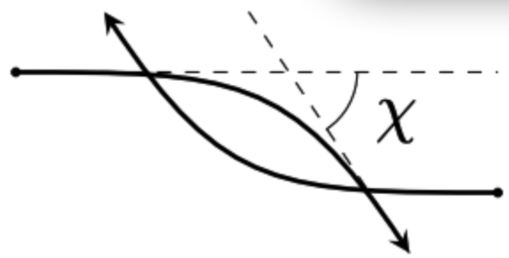
Periastron from
Tessmer Hartung Schaefer
1207.6961

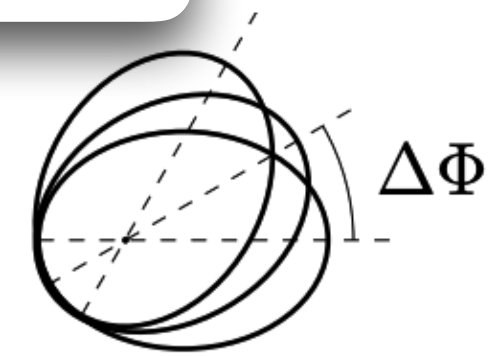
B2B dictionary/Spin

Liu **RAP** Yang
2102.10059

NEW!

$$i_r^{\rm 2PM}(\mathcal{E},\ell,\tilde a_\pm) = -\ell + \frac{2\gamma^2-1}{\sqrt{1-\gamma^2}} + \frac{3}{4\ell}\frac{5\gamma^2-1}{\Gamma} + \frac{1}{\pi}\sum_{A=\pm}\chi_A^{(3)}(\gamma)\frac{\tilde a_A}{\ell^2} + \frac{2}{3\pi}\sum_{\{A,B\}=\pm}\chi_{AB}^{(4)}(\gamma)\frac{\tilde a_A\tilde a_B}{\ell^3}\,,$$





$$\Delta\Phi(J,\mathcal{E}) = \chi(J,\mathcal{E}) + \chi(-J,\mathcal{E})$$

$$\frac{\Delta_{(a,a^2)}\chi}{\Gamma} = -\frac{GM}{|b|}\left(\frac{4\gamma}{\sqrt{\gamma^2-1}}\frac{a_+}{|b|} - \frac{2\gamma^2-1}{2(\gamma^2-1)}\frac{(\kappa_++2)\,a_+^2 + (\kappa_+-2)\,a_-^2 + 2\kappa_-\,a_-a_+}{|b|^2}\right)$$
$$-\pi\left(\frac{GM}{|b|}\right)^2\left(\frac{\gamma\,(5\gamma^2-3)}{4(\gamma^2-1)^{3/2}}\frac{7a_+ + \delta\,a_-}{|b|} - \frac{3}{256(\gamma^2-1)^2}\frac{\lambda_{++}\,a_+^2 + \lambda_{--}\,a_-^2 + 2\lambda_{+-}\,a_+a_-}{|b|^2}\right)$$

$$\lambda_{++} = 830\gamma^4 - 876\gamma^2 + 110 + (35\gamma^4 - 54\gamma^2 + 19)\,\delta\,\kappa_- + (215\gamma^4 - 222\gamma^2 + 39)\,\kappa_+,$$
$$\lambda_{--} = -450\gamma^4 + 468\gamma^2 - 82 + (35\gamma^4 - 54\gamma^2 + 19)\,\delta\,\kappa_- + (215\gamma^4 - 222\gamma^2 + 39)\,\kappa_+,$$
$$\lambda_{+-} = (215\gamma^4 - 222\gamma^2 + 39)\,\kappa_- + (\gamma^2-1)\big(70\gamma^2 + 10 + (35\gamma^2 - 19)\,\delta\,\kappa_+\big)\,.$$

$$\frac{\Delta\Phi}{2\pi} = \frac{3(5\gamma^2-1)}{4\Gamma}\frac{1}{\ell^2} + \left[\frac{6}{\Gamma+1}(5\gamma^2-1)(\gamma-1)(\Gamma\tilde a_+ - \delta\tilde a_-) - \gamma(5\gamma^2-3)(\delta\tilde a_- + 7\tilde a_+)\right]\frac{1}{4\Gamma^2\ell^3}$$
$$+\left[-\frac{64\gamma(5\gamma^2-3)(\gamma-1)}{\Gamma+1}(\delta\tilde a_- + 7\tilde a_+)(\Gamma\tilde a_+ - \delta\tilde a_-)\right.$$
$$\left.+\frac{192(5\gamma^2-1)(\gamma-1)^2}{(\Gamma+1)^2}(\Gamma\tilde a_+ - \delta\,\tilde a_-)^2 + \lambda_{--}\,\tilde a_-^2 + 2\lambda_{+-}\,\tilde a_+\tilde a_- + \lambda_{++}\,\tilde a_+^2\right]\frac{3}{256\Gamma^3\ell^4} + \cdots\,,$$

All orders in v at O(a^2/l^4)!

Confirms conjecture for Kerr in Vines et al. 1812.00956. Guevara et al. 1812.06895. ✓

Impulses agree for generic orientation Bern et al. 2005.03071 ✓

Impulses spin^2 confirmed by (concurrent) amplitude derivation

Luna Kosmopoulos 2102.10137 ✓

PN expansion agrees (after fixing it) Tessmer et al. 1207.6961 ✓

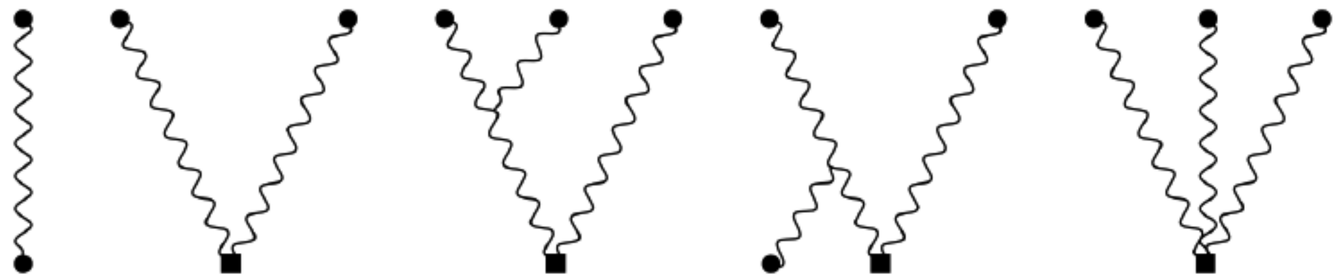
$$\frac{\Delta\Phi(\ell,a,\epsilon)}{2\pi} = \left[3 + \frac{3(2\nu-5)}{4}\epsilon + \frac{3\left(5-5\nu+4\nu^2\right)}{16}\epsilon^2\right]\frac{1}{\ell^2}$$
$$+\left[-\frac{7\tilde a_+ + \delta\tilde a_-}{2} - \frac{(\nu-6)\delta\tilde a_- + (7\nu-18)\tilde a_+}{2}\epsilon\right.$$
$$\left.-\frac{3\left(\left(15-14\nu+2\nu^2\right)\delta\tilde a_- + \left(25-38\nu+14\nu^2\right)\tilde a_+\right)}{16}\epsilon^2\right]\frac{1}{\ell^3}$$
$$+\left[\frac{3}{8}\Big(\tilde a_-^2\,(\kappa_+-2) + 2\tilde a_+\tilde a_-\kappa_- + \tilde a_+^2\,(\kappa_++2)\Big)\right.$$
$$-\frac{3}{16}\epsilon\Big(\tilde a_-^2\,(\delta\kappa_- + \kappa_+(13-3\nu) - 2\nu - 25) + 2\tilde a_+\tilde a_-\,(\kappa_-(13-3\nu) + \delta\,(\kappa_++11))$$
$$\left.+\tilde a_+^2\,(\delta\kappa_- + \kappa_+(13-3\nu) - 6\nu + 35)\Big) + \cdots\right]\frac{1}{\ell^4} + \cdots\,.$$

Kalin Liu **RAP**
2008.06047

B2B dictionary/Tidal Effects

NEW!

$$\Delta p_a^\mu = -\eta^{\mu\nu}\int_{-\infty}^{+\infty} \mathrm{d}\tau_a \frac{\partial \mathcal{L}_{\text{eff}}}{\partial x_a^\nu}(x_a(\tau_a))$$



$$S_{\text{pp}} = \sum_{a=1,2}\int \mathrm{d}\tau_a \Big(-\frac{m_a}{2} g_{\mu\nu} v_a^\mu v_a^\nu + c_{E^2}^{(a)} E_{\mu\nu}E^{\mu\nu} + c_{B^2}^{(a)} B_{\mu\nu}B^{\mu\nu} - c_{\tilde{E}^2}^{(a)} E_{\mu\nu\alpha}E^{\mu\nu\alpha} - c_{\tilde{B}^2}^{(a)} B_{\mu\nu\alpha}B^{\mu\nu\alpha} \Big)$$

$$\lambda_{E^2} \equiv \frac{1}{G^4M^5}\left(m_2\frac{c_{E^2}^{(1)}}{m_1} + m_1\frac{c_{E^2}^{(2)}}{m_2}\right),$$

$$\kappa_{E^2} \equiv \lambda_{E^2} + \frac{c_{E^2}^{(1)} + c_{E^2}^{(2)}}{G^4M^5} = \frac{1}{G^4M^4}\left(\frac{c_{E^2}^{(1)}}{m_1} + \frac{c_{E^2}^{(2)}}{m_2}\right)$$

$$\begin{aligned}
\frac{\Delta\chi_{(E,B)}}{\Gamma} = {} & \frac{45\pi}{64}\frac{(\gamma^2-1)^2}{(\Gamma j)^6}\Big[\big(35\gamma^4-30\gamma^2-5\big)\lambda_{B^2} + \big(35\gamma^4-30\gamma^2+11\big)\lambda_{E^2}\Big] \\
& + \frac{192}{35}\frac{(\gamma^2-1)^{3/2}}{(\Gamma j)^7}\Big[\big(160\gamma^6-192\gamma^4+30\gamma^2+2\big)\lambda_{B^2} + \big(160\gamma^6-192\gamma^4+72\gamma^2-5\big)\lambda_{E^2}\Big] \\
& + \frac{96\nu}{35}\frac{\sqrt{\gamma^2-1}}{(\Gamma j)^7}\kappa_{B^2}\Big[224\gamma^9-320\gamma^8-728\gamma^7+704\gamma^6+5488\gamma^5-444\gamma^4+66262\gamma^3+56\gamma^2+28084\gamma+4\Big] \\
& + \frac{96\nu}{35}\frac{\sqrt{\gamma^2-1}}{(\Gamma j)^7}\kappa_{E^2}\Big[224\gamma^9-320\gamma^8-728\gamma^7+704\gamma^6+5628\gamma^5-528\gamma^4+65982\gamma^3+154\gamma^2+28329\gamma-10\Big] \\
& - \frac{576\nu\sqrt{\gamma^2-1}}{(\Gamma j)^7}\Big[\big(440\gamma^4+474\gamma^2+32\big)\kappa_{B^2} + \big(440\gamma^4+474\gamma^2+33\big)\kappa_{E^2}\Big]\mathrm{ash}(\gamma),
\end{aligned}$$

$$\mathrm{ash}(\gamma) \equiv (\gamma^2-1)^{-1/2}\sinh^{-1}\sqrt{\frac{\gamma-1}{2}}$$

Kalin Liu **RAP**
2008.06047

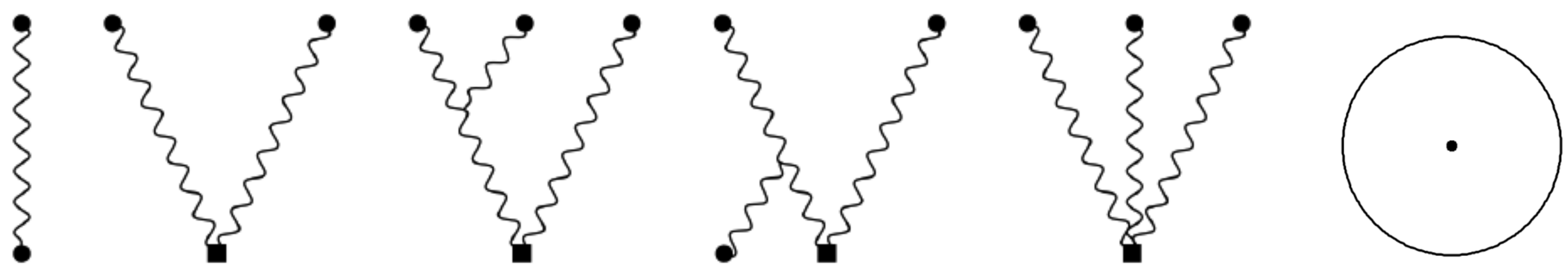
B2B dictionary/Tidal Effects

NEW!

Bound orbits
$x = (GM\omega)^{2/3} \sim v^2$

Quadrupole to NLO agrees with Cheng-Solon

2006.06665



Quadrupole/Octupole TLN in binding energy to O(G^3)

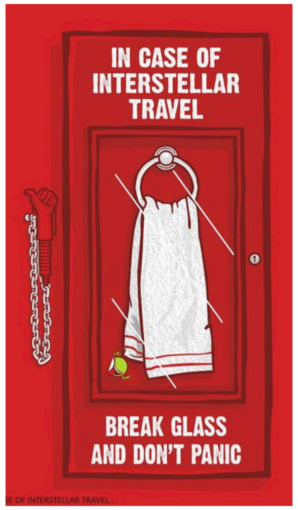
$$\Delta\mathcal{E}_{\rm T} = x\Big[18\,\lambda_{E^2}x^5 + 11\Big(3(1-\nu)\lambda_{E^2} + 6\,\lambda_{B^2} + 5\nu\,\kappa_{E^2}\Big)x^6 + \Big(390\lambda_{\tilde{E}^2} - \frac{13}{28}(161\nu^2 - 161\nu - 132)\lambda_{E^2} - \frac{1326\nu}{7}\kappa_{B^2}$$
$$+ \frac{13}{28}(616\nu + 699)\lambda_{B^2} + \frac{13\nu}{84}(490\nu - 729)\kappa_{E^2} + \frac{13}{6}\Delta\bar{P}^{(E,B)}_{8,\rm stc}\Big)x^7 + 75\big(45\nu\kappa_{\tilde{E}^2} - (13\nu+3)\lambda_{\tilde{E}^2} + 16\lambda_{\tilde{B}^2}\big)x^8$$
$$- \Big(\frac{85}{36}\left(1083\nu^2 + 1539\nu + 163\right)\lambda_{\tilde{E}^2} + \frac{27200\nu}{3}\kappa_{\tilde{B}^2} - \frac{85}{4}(270\nu + 383)\nu\kappa_{\tilde{E}^2} - \frac{680}{9}(90\nu + 173)\lambda_{\tilde{B}^2} - \frac{17}{6}\Delta\bar{P}^{(\tilde{E},\tilde{B})}_{10,\rm stc}\Big)x^9\Big]$$

NNLO terms from PM-static and probe limit

$$\Delta\bar{P}^{(E,B)}_{8,\rm stc} = \frac{1326}{7}\nu\kappa_{B^2} + \big(243 - 90\nu\big)\nu\kappa_{E^2} + \Big(45\nu^2 - \frac{885\nu}{7} + \frac{675}{14}\Big)\lambda_{E^2} - \Big(234\nu + \frac{837}{14}\Big)\lambda_{B^2}\,.$$

$$\Delta\bar{P}^{(\tilde{E},\tilde{B})}_{10,\rm stc} = \frac{1}{3}\big(2050\lambda_{\tilde{E}^2} - 13120\lambda_{\tilde{B}^2}\big) + \mathcal{O}(\nu)\,.$$

Thank you!



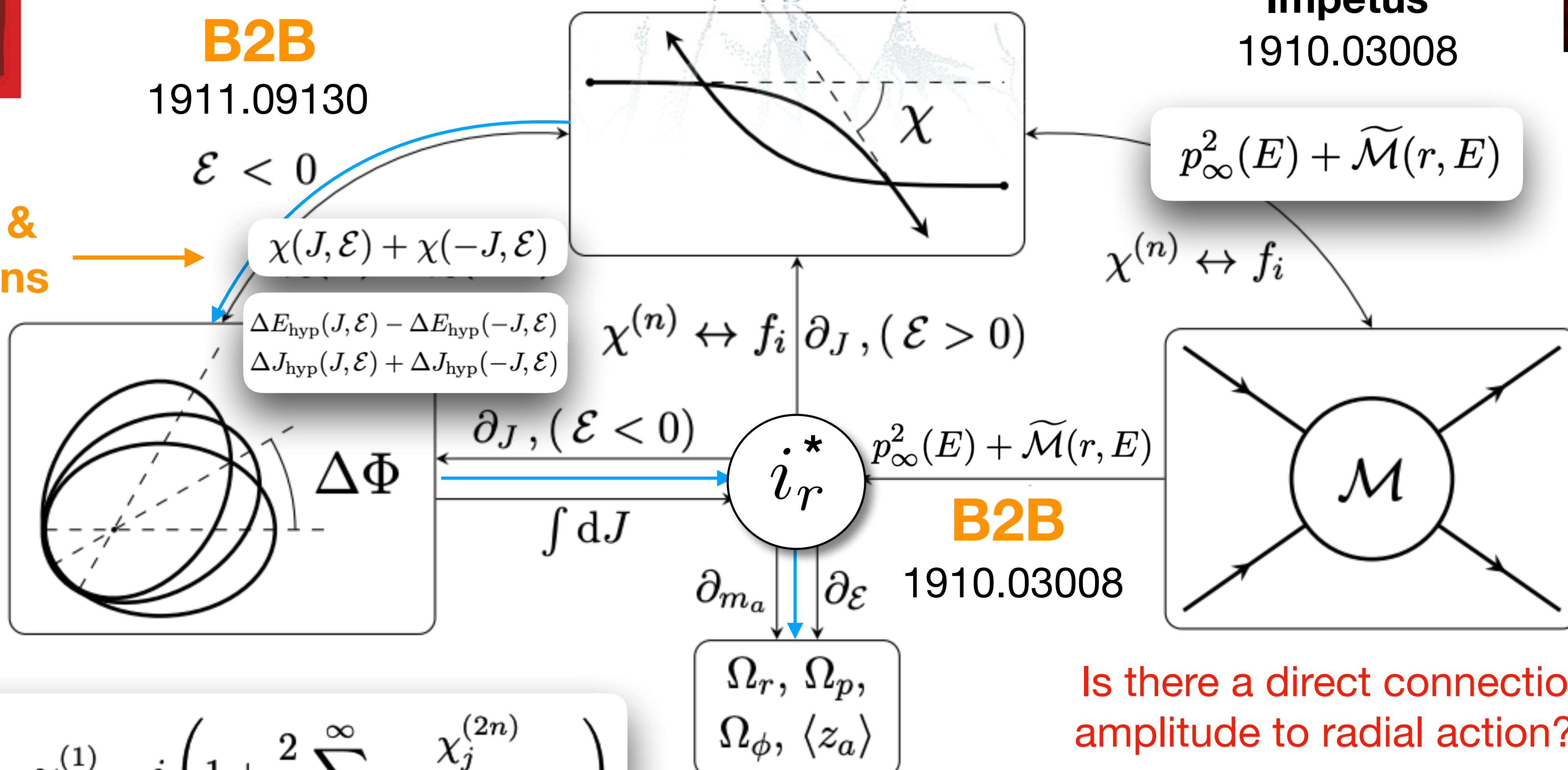
PMEFT

2006.01184
2007.04977
NEW! 2008.06047
2102.10059

B2B
1911.09130

RR (L+NL) & aligned-spins

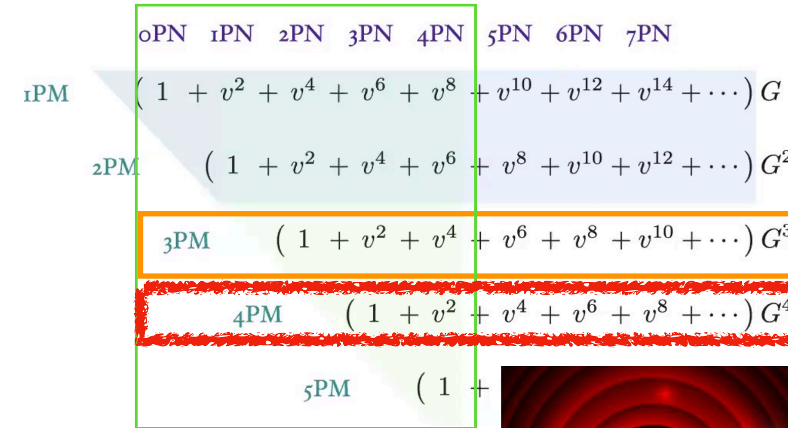
B2B



B2B
1910.03008

Impetus
1910.03008

Foffa **RAP** Rothstein Sturani
1903.05118



	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN	
1PM	(1	+ v^2	+ v^4	+ v^6	+ v^8	+ v^{10}	+ v^{12}	+ v^{14} + ⋯)	G
2PM		(1	+ v^2	+ v^4	+ v^6	+ v^8	+ v^{10}	+ v^{12} + ⋯)	G^2
3PM			(1	+ v^2	+ v^4	+ v^6	+ v^8	+ v^{10} + ⋯)	G^3
4PM				(1	+ v^2	+ v^4	+ v^6	+ v^8 + ⋯)	G^4
5PM					(1	+			

Is there a direct connection amplitude to radial action??

YES!

BPRRSSZ
2101.07254

$$* \quad i_r \equiv \frac{p_\infty}{\sqrt{-p_\infty^2}}\chi_j^{(1)} - j\left(1 + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{\chi_j^{(2n)}}{(1-2n)j^{2n}}\right)$$

Impetus/Firsov

$$\frac{1}{2\sqrt{-p_\infty^2}}\frac{\widetilde{M}_1}{M\mu} + \frac{j}{2\sqrt{\pi}}\sum_{n=0}^{\infty}\left(\frac{\Gamma\left(n-\frac{1}{2}\right)}{(\mu M j)^{2n}}\sum_{\sigma\in\mathcal{P}(2n)}\frac{p_\infty^{2(n-\Sigma^\ell)}}{\Gamma\left(1+n-\Sigma^\ell\right)}\prod_\ell\frac{\widetilde{M}_{\sigma_\ell}^{\sigma^\ell}}{\sigma^\ell!}\right)$$

ALL-ORDER