Gravitational Dressing of Conformal Galileons

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(based on GG, G. Tukhashvili, PLB, PRD)

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(Luty, Porrati Rattazzi
Nicolis, Rattazzi)

Galileons : a scalar field $\pi(\vec{x},t)$

$$a_0 = \int d^4x\, \pi \qquad \Pi_{\mu\nu} \equiv \partial_\mu \partial_\nu \pi$$

$$a_1 = \int d^4x\, \pi\, \mathcal{E}\mathcal{E}\Pi \simeq \int d^4x\, (\partial\pi)^2$$

$$a_2 = \int d^4x\, \pi\, \mathcal{E}\mathcal{E}\Pi\Pi \sim \int d^4x\, \underbrace{(\partial\pi)^2 \Box\pi}_{\text{in DGP}}$$

$$a_3 = \int d^4x\, \pi\, \mathcal{E}\mathcal{E}\Pi\Pi\Pi$$

$$a_4 = \int d^4x\, \pi\, \mathcal{E}\mathcal{E}\Pi\Pi\Pi\Pi$$

Gravitational dressing

π is a "gauge" mode of massive gravity (de Rham, G.G.
de Rham, G.G., Tolley)

Conformal Galileons

(Nicolis, Rattazzi)

$$A_0 = \int d^4x \; e^{4\pi}$$

$$A_1 = \int d^4x \; e^{2\pi} (\partial\pi)^2$$

$$A_2 = \int d^4x \, (\partial\pi)^2 \left(\Box\pi + \frac{1}{2}(\partial\pi)^2 \right)$$

$$A_3 = \int d^4x \; e^{-2\pi} (\partial\pi)^2 \left(\frac{1}{2}(\partial\pi)^4 \cdots\cdots \right)$$

$$A_4 = \int d^4x \; e^{-4\pi} (\partial\pi)^2 \left(-\frac{11}{4}(\partial\pi)^6 + \cdots \right)$$

Flat space $SO(3,2)/ISO(3,1)$ coset $\Rightarrow$

$\Rightarrow A_0, A_1, A_3, A_4$

(G. Goon
K. Hinterbichler
A. Joyce
M. Trodden
2012)

$WZ \Rightarrow A_2$

Gravitational dressing of $SO(n,2)/ISO(n-1,1)$

CCWZ '69
IO '75
BO '74
DEMPR '14

In particular, in 4D $\quad SO(4,2)/ISO(3,1)$

$$\Sigma \equiv \exp(\pi D)\exp(\zeta^a K_a)$$

dilatations $\quad$ special conformal

$$\Sigma^{-1}\left(d + e^a P_a - \frac{1}{2}\omega^{ab}J_{ab}\right)\Sigma =$$

$$= E^a P_a + \Omega_K^a K_a + \Omega_D D - \frac{1}{2}\Omega_J^{ab} J_{ab}$$

$$E^a = e^{\pi} e^a \qquad \Omega_D = D\pi + 2E^a\zeta_a$$

$$\Omega_K = D\zeta^a \ldots\ldots \qquad \Omega_J^{ab} = \omega^{ab} - \ldots\ldots$$

$$\mathcal{R}^{ab} = d\Omega_J^{ab} + \Omega_J^{ac}\wedge\Omega_{Jc}{}^{b} = \underbrace{R^{ab}}_{d\omega + \omega\wedge\omega} + \ldots\ldots$$

$$\mathcal{R}^a \equiv \iota_{E_a}\mathcal{R}^{ab} \qquad E_a = E_a^\mu \partial_\mu$$

• Zero derivatives

$$\int \varepsilon_{abcd}\, E^a \wedge E^b \wedge E^c \wedge E^d$$

$$\Big\downarrow \text{ Unitary gauge } \pi = 0$$

$$\boxed{\mathcal{A}_0 \equiv \int d^4x \sqrt{g}}$$

$$g \to g\, e^{2\pi}$$

$$\hookrightarrow g = g' e^{2\sigma} \qquad \pi = \pi' - \sigma$$

- Two derivatives:

$$\int \varepsilon_{abcd}\, R^{ab} \wedge E^c \wedge E^d$$

$$\Big\downarrow \text{ Unit. gauge } \pi = 0$$

$$\boxed{A_1 \equiv \int d^4x \sqrt{g}\, R}$$

$$g \to g e^{2\pi}$$

$$\hookrightarrow g = g' e^{2\sigma} \qquad \pi = \pi' - \sigma$$

- Four derivatives

$$\int \epsilon_{abcd} R^{ab} \wedge R^{cd}$$

$$\int \epsilon_{abcd} R^{a} \wedge R^{b} \wedge E^{c} \wedge E^{d}$$

$$\int \epsilon_{abcd} E^{a} \wedge E^{b} \wedge E^{c} \wedge E^{d} R^{2}$$

Unitary gauge

Tune two free parameters to get Galileons

$$\boxed{\mathcal{A}_2 = \int d^4x \sqrt{g} \left(R_{\mu\nu\rho\sigma}^2 - 4[R^2] + R^2 \right)}$$

↑ Gauss – Bonnet

- Six derivatives

$$\int \varepsilon_{abcd}\, R^{ab} \wedge R^{cd}\; R$$

$$\int \varepsilon_{abcd}\, R^{ab} \wedge R^{c} \wedge R^{d}$$

$$\int \varepsilon_{abcd}\, R^{a} \wedge R^{b} \wedge R^{c} \wedge E^{d}$$

$$\int \varepsilon_{abcd}\, R^{a} \wedge R^{b} \wedge E^{c} \wedge E^{d}\, R$$

$$\int \varepsilon_{abcd}\, E^{a} \wedge E^{b} \wedge E^{c} \wedge E^{d}\, R^{3}$$

$\pi = 0.$

Two constraints on 4 free parameters:

$$\boxed{A_3 = \int d^4x \sqrt{g} \Big[R_{\alpha\beta\mu\nu}^{2} R + 12 R_{\mu\nu}{}^{\alpha\beta} R_{\alpha}{}^{\mu} R_{\beta}{}^{\nu} + 24 [R^3] - 24 R [R^2] + 4 R^3 \Big]}$$

- Eight derivatives:

$$\int \varepsilon_{abcd} R^{ab} \wedge R^{cd} R^2$$

$$\int \varepsilon_{abcd} R^{ab} \wedge R^{c} \wedge R^{d} R$$

$$\vdots$$

$$\int \varepsilon_{abcd} E^{a} \wedge E^{b} \wedge E^{c} \wedge E^{d} R^4$$

Tree constraints on 4 parameters:

$\pi = 0$

$$\boxed{A_4 = \int d^4x \sqrt{g} \left(\frac{1}{6} R_{\mu\nu\rho\sigma}^2 R^2 - 6[R^4] + 3[R^2]^2 - 4 R_{\mu\nu}{}^{\alpha\beta} R_{\alpha}^{\mu} R_{\beta}^{\nu} R + \frac{8}{3}[R^3] R - \frac{1}{27} R^4 \right)}$$

$$\mathcal{A}_{coset} = c_0 \mathcal{A}_0 + c_1 \mathcal{A}_1 + c_2 \mathcal{A}_2 + c_3 \mathcal{A}_3 + c_4 \mathcal{A}_4$$

$$g = \bar{g} e^{2\pi} \qquad \pi = 0 \qquad \bar{g} = g$$

Weyl anomaly

$$\mathcal{A}_2^{WZ} \equiv \int_{\mathcal{M}_5} \varepsilon_{abcd} \, \Omega_D \wedge R^{ab} \wedge R^{cd}$$

$$\downarrow$$

$$\mathcal{A}_2^{WZ} \simeq \int d^4x \sqrt{g} \Big(-\pi \left(R_{\mu\nu\rho\sigma}^2 - 4[R^2] + R^2 \right)$$

$$+ 4 \partial^\mu \pi \partial^\nu \pi \left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \right) + 4 (\partial\pi)^2 \Box\pi$$

$$+ 2 (\partial\pi)^4 \Big)$$

$$\boxed{\mathcal{A}_{total} = \mathcal{A}_{coset}(g e^{2\pi}) + c_{WZ} \mathcal{A}_2^{WZ}(g, \pi)}$$

EFT 2 Cosmology

Conformal Galileon $\Leftrightarrow$ conformal mode in cosmology!

$\Rightarrow$ FLRW from Conformal Galileon.
No Ostrogradski ghosts!

How about ghosts due to tensor modes?

$g = g^B + h \qquad R_B << M_p^2$

$$\mathcal{L} = h\nabla^2 h + \frac{R_B}{M_p^4}\left(h\nabla^4 h + R_B h\nabla^2 h\right)$$

$$M_{ghost}^2 \simeq M_p^2 \left(M_p^2/R_B\right) \gg M_p^2$$

$$\mathcal{A}_{EFT} = \mathcal{A}_{total} + \int d^4x\sqrt{\tilde{g}}\sum_{k=1}^{\infty}\sum_{l=0}^{\infty}\sum_{m=1}^{\infty} b_{klm} W^k \tilde{D}^l \tilde{R}^m$$

Ghosts are outside of EFT

Example:

Cosmological ansatz: $g = g^{FLRW}$ $\quad \bar{\pi} = 0.$

$$-\frac{1}{12}\int d^4x\sqrt{g}\, R(g)\Big|_{FLRW} - \frac{1}{24\beta}\mathcal{A}_3(g)\Big|_{FLRW} - \frac{1}{48\gamma}\mathcal{A}_4(g)\Big|_{FLRW}$$

Friedmann equation:

$$\boxed{y - \frac{1}{\beta}y^3 + \frac{1}{\gamma}y^4 = 0 \qquad y \equiv H^2 + \frac{k}{a^2}}$$

New solutions: (for simplicity $\gamma \to \infty$ $k=1$)

$$\boxed{a(t) = \frac{1}{\beta^{1/4}}\cosh\left(\beta^{1/4} t\right)}$$

also spatially flat & open solutions exist.

Starobinsky-like.

Fine tuned

Conclusions:

Gravitational dressing of

- Galileons $\to$ massive gravity (dRGT)
  $R + \det_2(K) + \det_3(K) + \det(K)$.
- 3D Conformal Galileons $\to$ New Massive Gravity (BHT)
  $R + R^2$
- 4D Conformal Galileons $\to$ a special $W^2 + \kappa C + R + R^2 + R^3 + R^4$ theory

Applications to cosmology with fine tunings.