Post-Newtonian Gravity from QFT Strategies

Michèle Levi

Niels Bohr Institute
U. of Copenhagen

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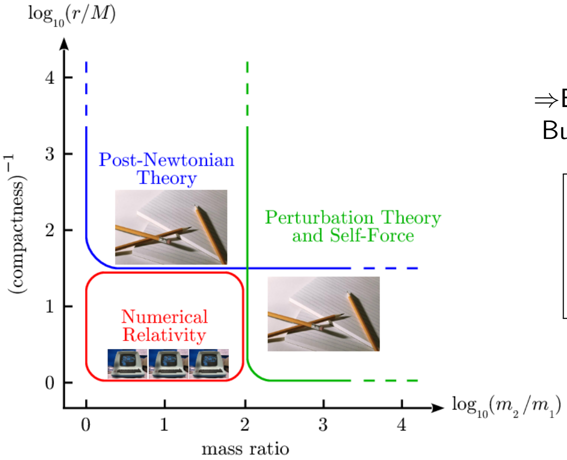
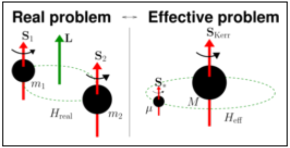
Theory of GW templates



⇒Effective One-Body (EOB) Buonanno & Damour 1999



⇒Numerical Relativity breakthrough, Pretorius 2005

- Increasing influx of real-world GW data
  ⇒ PN gravity is key for theoretical GW data → EFTs of PN Gravity
- Underlying Science: Informs on strong gravity, QFT ↔ Gravity

State of the Art

State of the Art for Generic Compact Binary Dynamics

l \ n	$(N^0)LO$	$N^{(1)}LO$	N^2LO	N^3LO	N^4LO	N^5LO
S^0	++	++	++	++	++	+
S^1	++	++	++	+		
S^2	++	++	+	+		
S^3	++	+				
S^4	++	+				

- (n, l) entry at $n + l + \text{Parity}(l)/2$ PN order
- n = highest n-loop graphs at N^nLO, l = highest multipole moment S^l
- Gray area corresponds to gravitational Compton scattering with $s \geq 3/2$ since classical $S^l \leftrightarrow$ quantum $s = l/2$
  $\Rightarrow$ Expect weird things to happen at classical level?

State of the Art

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S^1	++	++	++	+		
S^2	++	++	+	+		
S^3	++	+				
S^4	++	+				

- ++ = fully done/verified; + = partial/not verified
- Even l easier than odd l; Also in particular at $l = 0 \to n$ odd easier
- As of 2PN – UV dependence needed to complete accuracy
- At 4PN all sectors fully verified except (n,l)=(2,2) [Levi+ 2016]
- At 4.5PN & 5PN – NO sector is currently fully done/verified!

EFTs of Extended Gravitating Objects

[Goldberger & Rothstein 2006]

$$S_{\text{eff}} = S_{\text{g}}[g_{\mu\nu}] + \sum_{a=1}^{2} S_{\text{pp}}(\lambda_a); \quad S_{pp}(\lambda_a) = \sum_{i=1}^{\infty} C_i(r_s) \int d\lambda_a \mathcal{O}_i(\lambda_a)$$

$$S_{\text{g}}[g_{\mu\nu}] = -\frac{1}{16\pi G_d} \int d^{d+1}x \sqrt{g}\, R + \frac{1}{32\pi G_d} \int d^{d+1}x \sqrt{g}\, g_{\mu\nu} \Gamma^{\mu} \Gamma^{\nu},$$

$$G_d \equiv G_N \left(\sqrt{4\pi e^{\gamma}}\, R_0 \right)^{d-3},$$

To facilitate computations in PN: [Kol & Smolkin 2008]

$$ds^2 = g_{\mu\nu} dx^{\mu} dx^{\nu} \equiv e^{2\phi} \left(dt - A_i dx^i \right)^2 - e^{-\frac{2}{d-2}\phi} \gamma_{ij} dx^i dx^j,$$

$$\langle\, \phi(x_1) \;\; \phi(x_2)\, \rangle = \text{———————} = \frac{16\pi\, G_d}{c_d} \cdot \delta(t_1 - t_2) \int_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)}}{\vec{k}^2},$$

$$\langle A_i(x_1)\; A_j(x_2) \rangle = \text{- - - - - - - - -} = -16\pi\, G_d \cdot \delta(t_1 - t_2) \int_{\vec{k}} \frac{e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)}}{\vec{k}^2}\, \delta_{ij}.$$

Spin as Extra Particle DOF

Effective Action of Spinning Particle

- $u^{\mu} \equiv dy^{\mu}/d\sigma$, $\Omega^{\mu\nu} \equiv e^{\mu}_{A} \frac{De^{A\nu}}{D\sigma} \Rightarrow L_{\rm pp}\left[\bar{g}_{\mu\nu}, u_{\mu}, \Omega^{\mu\nu}\right]$
  [Hanson & Regge 1974, Bailey & Israel 1975]
- $S_{\mu\nu} \equiv -2\frac{\partial L}{\partial \Omega^{\mu\nu}}$ spin as further particle DOF – classical source
  [...Levi+ JHEP 2015]

$$\Rightarrow S_{\rm pp}(\sigma) = \int d\sigma \left[-p_{\mu}u^{\mu} - \frac{1}{2}S_{\mu\nu}\Omega^{\mu\nu} + L_{\rm NMC}\left[\bar{g}_{\mu\nu}\left(y^{\mu}\right), u^{\mu}, S_{\mu\nu}\right]\right]$$

For EFT of spin – gauge of both rotational DOFs should be fixed at level of one-particle action

- This form implicitly assumes initial "covariant gauge":
  $$e^{\mu}_{[0]} = \frac{p^{\mu}}{\sqrt{p^2}}, \quad S_{\mu\nu}p^{\nu} = 0$$
  [Tulczyjew 1959]
- Linear momentum $p_{\mu} \equiv -\frac{\partial L}{\partial u^{\mu}} = m\frac{u^{\mu}}{\sqrt{u^2}} + \mathcal{O}(RS^2)$

EFT of Spinning Particle

Effective Action of Spinning Particle

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Theory challenges tackled [...Levi+ JHEP 2015, Levi Rept. Prog. Phys. 2020]

1. Relativistic spin has a minimal finite measure S/M
   $\rightarrow$ Clashes with the EFT/point-particle viewpoint
   $\Rightarrow$ Introduce "gauge freedom" in choice of rotational variables
2. Fix non-minimal coupling part of the action, $L_{\rm NMC}$

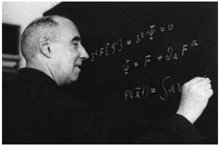
Introduce Gauge Freedom in Tetrad & Spin

[**ML** & Steinhoff, JHEP 2015]

Introduce gauge freedom into tetrad

Transform from a gauge condition

$$e_{A\mu} q^{\mu} = \eta_{[0]A} \Leftrightarrow e_{[0]\mu} = q_{\mu}$$



Ernst Stueckelberg

to

$$\hat{e}_{A\mu} w^{\mu} = \eta_{[0]A} \Leftrightarrow \hat{e}_{[0]\mu} = w_{\mu}$$

with a boost-like transformation in covariant form

$$\hat{e}^{A\mu} = L^{\mu}{}_{\nu}(w, q) e^{A\nu}$$

with q_{μ}, w_{μ} timelike unit 4-vectors

Generic gauge for the tetrad entails the generic "SSC"

$$\hat{e}_{[0]\mu} = w_{\mu} \quad \Rightarrow \quad \hat{S}^{\mu\nu} \left(p_{\nu} + \sqrt{p^2} w_{\nu} \right) = 0$$

Extra term in Minimal Coupling

[**ML** & Steinhoff, JHEP 2015]

$$\Rightarrow \hat{S}^{\mu\nu} = S^{\mu\nu} - \delta z^\mu p^\nu + \delta z^\nu p^\mu, \qquad \delta z^\mu p_\mu = 0$$

⇒ Extra term in action appears!

- From minimal coupling

$$\frac{1}{2} S_{\mu\nu} \Omega^{\mu\nu} = \frac{1}{2} \hat{S}_{\mu\nu} \hat{\Omega}^{\mu\nu} + \frac{\hat{S}^{\mu\rho} p_\rho}{p^2} \frac{D p_\mu}{D\sigma}$$

- Extra term with covariant derivative of momentum, contributes to finite size effects, yet carries no Wilson coefficient
- As of LO with spin, to all orders in spin!
- Essentially Thomas precession (later recovered as "Hilbert space matching")

- We transform between spin variables by projecting onto the hypersurface orthogonal to p_μ

$$S_{\mu\nu} = \hat{S}_{\mu\nu} - \frac{\hat{S}_{\mu\rho} p^\rho p_\nu}{p^2} + \frac{\hat{S}_{\nu\rho} p^\rho p_\mu}{p^2}$$

Why Generalized Canonical Gauge?

Here are some of the obvious reasons to use it:

1. Allows to disentangle DOFs in EFT and land on well-defined effective action
2. Standard procedure to land on Hamiltonian, similar to non-spinning sectors
3. Essential for Effective One-Body framework – needed to generate waveforms
4. Direct and simple derivation of physical EOMs for position and spin
5. Enables most stringent consistency check of Poincaré algebra of invariants
6. Natural classical treatment to be promoted/confronted with QFT

Leading Non-Minimal Couplings to All Orders in Spin

[**ML** & Steinhoff, JHEP 2014, JHEP 2015]

Key: Consider classical spin vector similar to Pauli-Lubanski vector

→ Massive spinor-helicity, Arkani-Hamed+ 2017 – resonates with this form

New Wilson coefficients of linear-in-curvature couplings → "Love numbers":

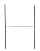
$$L_{\text{NMC}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n)!} \frac{C_{ES^{2n}}}{m^{2n-1}} D_{\mu_{2n}} \cdots D_{\mu_3} \frac{E_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}}$$
$$+ \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n+1)!} \frac{C_{BS^{2n+1}}}{m^{2n}} D_{\mu_{2n+1}} \cdots D_{\mu_3} \frac{B_{\mu_1\mu_2}}{\sqrt{u^2}} S^{\mu_1} S^{\mu_2} \cdots S^{\mu_{2n-1}} S^{\mu_{2n}} S^{\mu_{2n+1}}$$

Leading - linear in curvature - spin couplings up to 5PN order

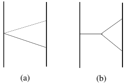
- $L_{ES^2} = -\frac{C_{ES^2}}{2m} \frac{E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu$, Quadrupole @2PN
- $L_{BS^3} = -\frac{C_{BS^3}}{6m^2} \frac{D_\lambda B_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda$, Octupole @3.5PN
- $L_{ES^4} = \frac{C_{ES^4}}{24m^3} \frac{D_\lambda D_\kappa E_{\mu\nu}}{\sqrt{u^2}} S^\mu S^\nu S^\lambda S^\kappa$, Hexadecapole @4PN

Graph Topologies up to 2-Loop

[Rept. Prog. Phys. 2020, **Levi+** + x2 2020]



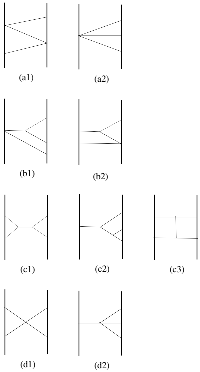
Single topology at $O(G)$:
One-graviton exchange.



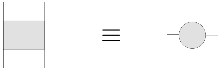
Topologies at $O(G^2)$:
(a) Two-graviton exchange;
(b) Cubic self-interaction
$\equiv$ One-loop topology.

$$\int_{\vec{p}_1} \frac{e^{i\vec{p}_1\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_1^{\,2}} \int_{\vec{p}_2} \frac{e^{i\vec{p}_2\cdot(\vec{x}_1-\vec{x}_2)}}{\vec{p}_2^{\,2}},$$

$$p_1 + p_2 \to p, \quad p_2 \to k_1, \quad \to \int_{\vec{p}} e^{i\vec{p}\cdot(\vec{x}_1-\vec{x}_2)} \int_{\vec{k}_1} \frac{1}{\vec{k}_1^{\,2}(\vec{p}-\vec{k}_1)^2}$$



Topologies at $O(G^3)$

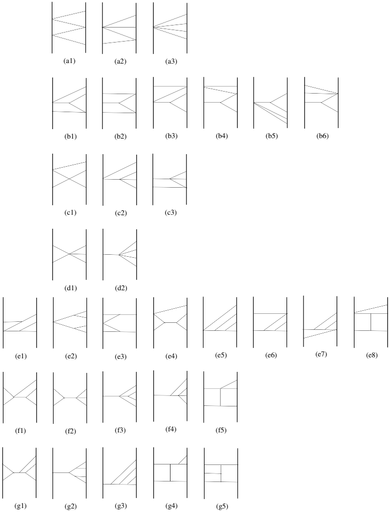


Standard QFT multi-loops:
n-loop master integrals and IBPs (Integration By Parts)
`EFTofPNG` code
[**Levi+** 2017, 2020,...]

A topology at G^{n+1} is rank r, when r basic n-loop integral types form its n-loop integral

Graph Topologies at G^4=up to 3-Loop

[**ML** Mcleod von Hippel 2020]



Topologies at $O(G^4)$

At G^n the loop order n_L

$$n_L \equiv 2n - \sum_{i=1}^{n+1} m_i$$

with m_i gravitons on insertion i

A topology at G^{n+1} is rank r, when r basic n-loop integral types form its n-loop integral

Complete N^3LO Quadratic-in-Spin

Considering linear-in-curvature couplings first

[**ML**, Mcleod, von Hippel 2020; Kim, **ML**, Yin, in prep.]

Graph distribution in N^3LO quadratic-in-spin sector in a total(?) of 1024

Order in G	1	2	3	4
No. of graphs	19	188	654	163

Do we have more contributions beyond linear in curvature? [Yes, at G^2!]

Integration and Scalability

- Building on publicly-available `EFTofPNG` code [**ML** & Steinhoff 2017] `https://github.com/miche-levi/pncbc-eftofpng`
- Higher-rank graphs reduced using IBP method, e.g. 83 at G^3, 31 at G^4
- Upgrade using projection method for integrand numerators as high as rank-8
- Upgrade from IBP "by hand" to algorithmic IBP – our variation of Laporta

Curious Findings at N^3LO/3-Loop

- Only 3-loop topologies in the worldline picture give rise to novel features, such as poles, logs, and transcendental numbers
- Only rank-3 topologies in the QFT picture give rise to transcendental numbers: Such numbers occur in quantum loop corrections as of 1-loop; In view of contact interaction terms as of N^2LO in PN gravity:
  $\rightarrow$ Not surprising that they appear in our related graphs at N^3LO
  $\rightarrow$ Next-order corrections of purely UV contributions that vanish classically
- Appearance of all special features at total 3-loop results seems to occur only in odd-in-spin sectors, e.g. does NOT occur in all known non-spinning sectors, or in quadratic-in-spin sector

Nonlinear Higher-in-Spin

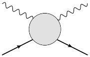
What is the nature of massive particles of $s > 2$?

- Gravitational interaction with spins $\leftrightarrow$ Scattering of graviton and massive spin

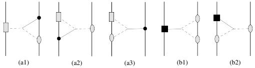
  $$\boxed{\text{Classical } S^l \leftrightarrow \text{Quantum } s = l/2}$$

- Insight on Compton scattering of graviton and massive higher-spin $s \geq 5/2$

  [Arkani-Hamed+ 2017]



NLO cubic-, quartic-in-spin [**Levi+**, Teng, JHEP 2021 x 2, + Morales in prep.]



- Graphs with "elementary" worldline-graviton couplings up to 1-loop
- Some worldline-graviton couplings become quite intricate and subtle, new "composite" multipoles in terms of "elementary" spin multipoles
- Operators quadratic-in-curvature at NLO S^4

Extending Non-Minimal Action with Spin

Extending effective action beyond linear-in-curvature

[**Levi+** 2020, JHEP 2021]

$$
\begin{aligned}
L_{\mathrm{NMC(R^2)}} = {} & C_{E^2}\frac{E_{\alpha\beta}E^{\alpha\beta}}{\sqrt{u^2}^3} + C_{B^2}\frac{B_{\alpha\beta}B^{\alpha\beta}}{\sqrt{u^2}^3} + \dots \\
& + C_{E^2S^2} S^\mu S^\nu \frac{E_{\mu\alpha}E_\nu^\alpha}{\sqrt{u^2}^3} + C_{B^2S^2} S^\mu S^\nu \frac{B_{\mu\alpha}B_\nu^\alpha}{\sqrt{u^2}^3} \\
& + C_{E^2S^4} S^\mu S^\nu S^\kappa S^\rho \frac{E_{\mu\nu}E_{\kappa\rho}}{\sqrt{u^2}^3} + C_{B^2S^4} S^\mu S^\nu S^\kappa S^\rho \frac{B_{\mu\nu}B_{\kappa\rho}}{\sqrt{u^2}^3} \\
& + C_{\nabla EBS} S^\mu \frac{D_\mu E_{\alpha\beta}B^{\alpha\beta}}{\sqrt{u^2}^3} + C_{E\nabla BS} S^\mu \frac{E_{\alpha\beta}D_\mu B^{\alpha\beta}}{\sqrt{u^2}^3} \\
& + C_{\nabla EBS^3} S^\mu S^\nu S^\kappa \frac{D_\kappa E_{\mu\alpha}B_\nu^\alpha}{\sqrt{u^2}^3} + C_{E\nabla BS^3} S^\mu S^\nu S^\kappa \frac{E_{\mu\alpha}D_\kappa B_\nu^\alpha}{\sqrt{u^2}^3} \\
& + C_{(\nabla E)^2S^2} S^\mu S^\nu \frac{D_\mu E_{\alpha\beta}D_\nu E^{\alpha\beta}}{\sqrt{u^2}^3} + C_{(\nabla B)^2S^2} S^\mu S^\nu \frac{D_\mu B_{\alpha\beta}D_\nu B^{\alpha\beta}}{\sqrt{u^2}^3} \\
& + C_{(\nabla E)^2S^4} S^\mu S^\nu S^\kappa S^\rho \frac{D_\kappa E_{\mu\alpha}D_\rho E_\nu^\alpha}{\sqrt{u^2}^3} + C_{(\nabla B)^2S^4} S^\mu S^\nu S^\kappa S^\rho \frac{D_\kappa B_{\mu\alpha}D_\rho B_\nu^\alpha}{\sqrt{u^2}^3} + \dots,
\end{aligned}
$$

- New (unstudied) Wilson coefficients
- Are there any redundant terms ("on-shell operators") here?

Curious Findings at NLO Cubic- & Quartic-in-Spin

Dependence in product of Wilson coefficients

Originating from lower-order sectors, e.g. at NLO cubic-in-spin we get $(C_{ES^2})^2$

"Composite" worldline couplings

$$p_\mu = -\frac{\partial L}{\partial u^\mu} = \tfrac{m}{u} u_\mu + \Delta p_\mu(RS^2)$$

Application of gauge at NLO as of cubic-in-spin → New type of worldine-graviton couplings to "composite" multipoles, in terms of "elementary" ones

Quadratic-in-curvature contributions

$$L_{\mathrm{S}^4(\mathrm{R}^2)} = \frac{C_{E^2S^4}}{24m^3} S^\mu S^\nu S^\kappa S^\rho \frac{E_{\mu\nu}E_{\kappa\rho}}{\sqrt{u^2}^{\,3}} + \frac{C_{B^2S^4}}{24m^3} S^\mu S^\nu S^\kappa S^\rho \frac{B_{\mu\nu}B_{\kappa\rho}}{\sqrt{u^2}^{\,3}}$$

Turns out only electric operator enters at S^4 - only single 2-graviton exchange

QFT for PN Gravity and Back

Levi Rept. Prog. Phys. 2020
Levi+ 2x 2020, 2x JHEP 2021, x in prep. + Kim, Morales, Yin

- Real-world scalability:
  - EFT of gravitating spinning objects - self-contained framework
    $\Rightarrow$ Direct derivation of useful & physical quantities
    $\Rightarrow$ Self-consistency checks
  - Precision frontier with spins being pushed to 5PN order!
  - Continuous development of public computational tools
    $\rightarrow$ `EFTofPNG` code [CQG Highlights 2017, upgrades...]

- Fundamental lessons:
  - PN gravity informs us about gravity in general
  - New features in NLO higher-spin sectors resonate with picture of composite (rather than elementary) particles at higher quantum spins
  - Possible insights for graviton Compton amplitude with higher spins?