Towards gravitational-wave models informed by scattering

collaborators: Alessandra Buonanno, Andrea Antonelli, Chris Kavanagh, Jan Plefka, Justin Vines, Gustav Jakobsen, Gustav Mogull, Mohamed Khaiel, Santh Muddu

Jan Steinhoff
Albert Einstein Institute (Potsdam)

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Gravitational scattering, inspiral, and radiation @GGI
Gravitational waves (GWs)

**Cosmology**

- **Neutrino-star matter**

- **Testing gravity**

**Black-hole population & formation**

- **Truncated**
- **Broken power law**
- **Power law + Peak**
- **Multi Peak**

- **arXiv:2010.14533**

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Accuracy problems ahead

Results become insufficient when 2.

Pürrer, Haster, arXiv:1912.10055

semi-analytic

NR

2022-24

2030's

need better predictions soon

analytic & numeric

Network SNR $\rho$ (signal-to-noise ratio)
Binary parameter space & GW predictions

Applicability of GW prediction methods:

Post-Newtonian (PN) \( \frac{v^2}{c^2} \sim \frac{GM}{c^2 r} \ll 1 \)

Small Mass Ratio (SMR) \( \nu = \frac{m_1 m_2}{M^2} \ll 1 \)

Numerical Relativity (NR)

Limited by computational resources
another dimension: eccentricity

PM: post-Minkowskian
\[ \frac{GM}{c^2 r} \ll 1 \]

Recent eccentric (EOB) waveform works by:
- Albañez, Nagar, Bernuzzi, Reisswig, Breschi, Albertini, Gamba, Bonino, Liu, Cao, Zhu, Khalil, Buonanno, Vines

PM:
- improve bound-orbit waveforms with eccentricity
- search for HE scattering BHs

fictitious!
Informing the Hamiltonian/potential with HPM

4PM from Bern, Barrau-Murcillo, Roizman, Ruf, Shen, Solou, Zeng
2101.07194

Looks promising even for circular orbits \( B \)

Full waveform model requires:
- Hamiltonian/potential
- Radiation reaction force
- Waveform "wires"

Credit: Mohammed Khalil

=> Want all from PM8
Black holes as (point) particles

Full theory

IR projection

Effective theory

Worldline \( x^\mu(\tau) \)

\[ \Rightarrow \text{effective-field theory point-of-view} \]

[Goldberger, Rothstein, ...]

Black holes \( \approx \) higher-spin massive particles

[eg. Arkani-Hamed, Huang, O'Connell (2019)]

[adopted from arXiv:1906.08161]
Scattering particles vs. scattering waves

\[ \chi(x) \]

\( \alpha \): Scattering angle

\( \Psi(x) \) is an amplitude for eikonal approximation

Conductive interference

Wavefronts energy \( E \)

Wavefronts energy \( E + \Delta E \)

John A. Wheeler

\[ M: \text{scattering amplitude} \]
Connecting amplitudes and classical physics

- Matching Hamiltonians to amplitudes, cKonal phase, Lippmann-Schwinger eq... 
  Rothstein, Nall, Cheung, Salam, Bern, Roiban, Suen, Luna, Zeng, Kosmopoulos, 
  di Vecchia, Heissenberg, Russo, Veneziano, Porrà-Martínez, Ruf, Zeng, 
  Bjerrum-Bohr, Cristoffoli, Damgaard, Vanhove, ...

- Expectation values from amplitudes [Kosower, Maybee, O'Connell, Vines]

- Here: directly connecting amplitudes to "1st quantization" of massive particles 
  [Majumdar, Sefkof, FS2010, 1499; 
  Jakobsen, 2010.12688 PRL]

⇒ "worldline quantum field theory" (WQFT)

Related work: [Goldberger, Ridgway, I, Källin, Porto, Liu, Yang]...
Use Feynman-Schwinger rep. of dressed massive propagator $G(x, x')$

$$(\nabla^\mu \nabla^\nu + m^2 + \frac{g}{2} R) G(x, x') = \sqrt{-g} S(x-x')$$

$g = \frac{c}{2m}$

$\mathcal{G}_i(x, x') \sim \int_0^s e^{ism^2} \int_{x(0)=x}^{x(s)=x'} Dx \cdot \exp \left[ -i \int_0^s ds \left( \frac{1}{4} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + (\frac{g}{2} - \frac{1}{4}) R \right) \right]$
Classical Limit in WQFT & the eikonal

standard approach: massive propagator \( \frac{1}{p^2 + m^2/\hbar^2} \)

2nd quantized

Diagrams have all powers in \( \hbar \)

\( \Rightarrow \) classical limit \( \hbar \to 0 \) subtle

WQFT: expand around straight-line motion

\( \Rightarrow \) \( \hbar \)-counting of diagrams, \( \hbar \to 0 \) limit straightforward

\( \Rightarrow \) form factor

\[ \lambda(x') = 0 + \varphi + \frac{1}{\hbar} \]

straight-line motion

deflection

in WQFT & \( \hbar \to 0 \)
Eikonal

\[ \text{FT to } \text{b-space of } \tau \]

\[ t \rightarrow 0 \]

\[ = \]

\[ = \text{const} \times \exp \left( \frac{\tau}{\tau_+ \tau_-} + \sum \ldots \right) \]

exponentiates 8

eikonal  \[ X \]

\[ \sim WQFT \text{ free energy } = e^{iX} \text{ for } t \rightarrow 0 \]

property: \[ \Delta p_{\chi} = -\frac{\partial X}{\partial b^x} \]

(deflection)

\[ \nabla \text{ observables encoded in } X \]

also:

Relation to bound orbits possible  8

[\text{Kalin, Porto (2020)}]
Radiation in the WQFT

\[ <\hat{h}^{+x}> = \text{inv. Fourier trafo} \]

\[ \text{leading order} \]

Feynman diagrams $\rightarrow$ Feynman integrals

$\rightarrow$ integrals involve "anisotropic" propagators

\[ \frac{1}{\mathbf{p} \cdot \mathbf{M} \cdot \mathbf{p}} \]

$\rightarrow$ corresponds to boosted Coulomb field

$\rightarrow$ matrix

$\rightarrow$ still solvable

$\rightarrow$ reproduce [Kovacs, Thorne (1977)] "efficiently"

beyond small-angle scattering?
WIP with Saketh Mudda, Justin Vines, Alessandra D'Urso

- reconsider "the classical" approach (no amplitudes, no WQFT)
- linear theory for starters \( \uparrow \) QED
- split off self-interactions from the start
- iteratively compute deflected worldline

"classical" method (off-shell recursion?) seems very efficient

\[ \begin{array}{c}
\text{amplitude analog?}
\end{array} \]

(Radiation reaction can be computed in WQFT \( \uparrow \) see papers)

\( \Rightarrow \text{from waveform!} \)
Spin & WQFT

WIP with Jakobsen, Mogull, Plefka

EFT with spin $\lambda^\mu_A$ and spins up on worldline

$m$ would need to integrate out $\lambda^\mu_A$ in WQFT

$m$ better method?

$\phi^\mu(\tau)$ on worldline $\Rightarrow$ $S^\mu = -2i \overline{\psi} \Gamma^\mu \psi$

$\Rightarrow$ Susy!?
quadrupole coupling: $\frac{1}{8\pi m} \mu \nu \rho \sigma \sigma \sigma^\rho \sigma^\nu + \frac{c^{-1}}{2m} E \mu \nu \sigma^\nu \sigma^\nu$

black hole SUSY!

non-BH breaks SUSY!

higher orders? SUSY $\rightarrow$ unitarity/quantum consistency? other symmetries?

eikonal $\rightarrow$ generates observables $\Delta \rho, \Delta Y, \Delta S$
	e.g. [Maybee, O’Connell, Vines]

[Bern, Luna, Roiban, Shen, Zeng] [Kosmopoulos, Luna]

leading order radiation $\rightarrow$ no new integrals except more complicated numerators
Looking for structure: mass-dependence of classical scattering

\[ \text{Eikonal} = \frac{1}{m_1 m_2} + \frac{2}{m_2^2 m_1} + \frac{2}{m_1^2 m_2} + \frac{2}{m_1 m_2^2} + \frac{2}{m_1^2 m_2^2} + \cdots \]

- Leading order (LO)
- Next-to-leading order (NLO)
- \( \cdots \)

\[ \text{LO \& NLO follows from test-bodies (\& symmetrizing in masses)} \]

\[ \Rightarrow \text{NNLO \& N^3LO} \]

1st correction in mass ratio (self-force)

\[ \text{Tutti Frutti} \]
$N^3$LO PN spin-orbit (SO) from self-force

[Antonelli, Khalil, ... 2003.11391]

\[ q = \frac{m_1}{m_2} \]

\textbf{PN binding energy $E_b$ vs $N_R$}

\begin{itemize}
  \item Naively a 3-loop calculation
  \item Follows from known results for self-force without solving any integrals
  \item Extension to aligned-spin $S_1S_2$ case possible
\end{itemize}

[Antonelli, Khalil, ... 2010.02018]

see also [Levi, Melrod, von Hippel, 2003.02827+07890]
Scattering amplitudes involving black holes? 

- of fundamental interest

What is the "best" method for black hole interactions?

- classical
- Feynman rules/EFT
- amplitudes

Graviton (grav. waves)

- a synergy?
Conclusions

- gravitational wave models informed by scattering?

Building blocks:

- potential → eikonal (Radial action)
- radiation reaction → eikonal
- waveform (source multipoles) → ?

Lots of progress!

- connect classical & quantum physics
- exchange of ideas
- consolidation of methods & abstractions