Towards gravitational-wave models informed by scattering

collaborators: Alessandra Buonanno, Andrea Antonelli, Chris Kavanagh, Jan Plefka, Justin Vines, Gustav Jakobsen, Gustav Mogull, Mohammed Khalil, Saketh Muddu

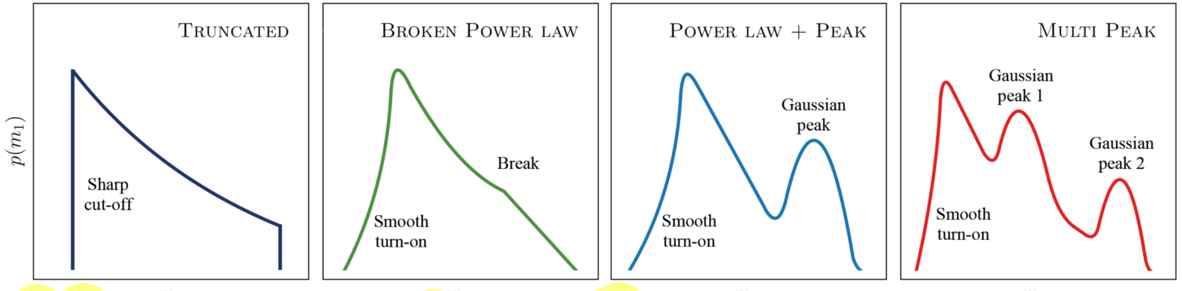
Jan Steinhoff

Albert Einstein Institute
(Potsdam)

April 29th, 2021

Gravitational scattering, inspiral, and radiation
@GGI

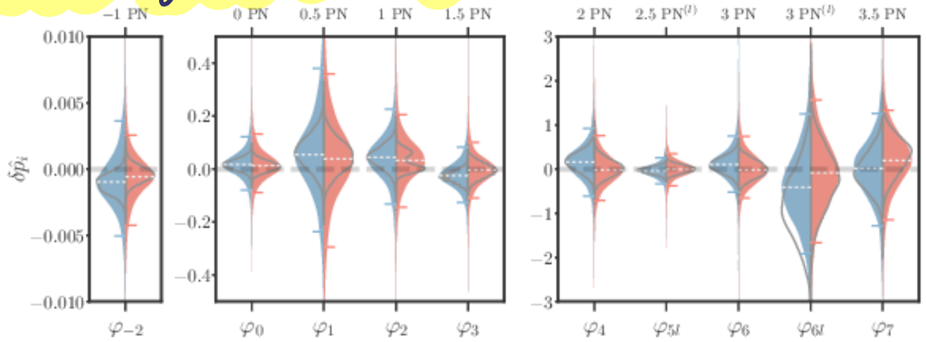
Gravitational waves (GWs)



arXiv:2010.14533

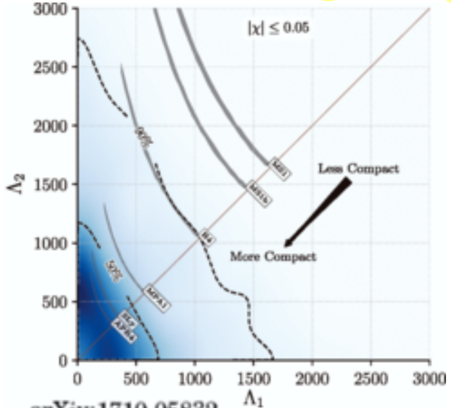
black-hole population & formation

testing gravity



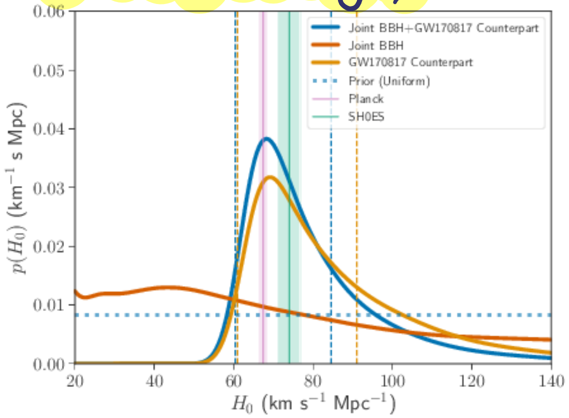
arXiv:2010.14529

neutron-star matter



arXiv:1710.05832

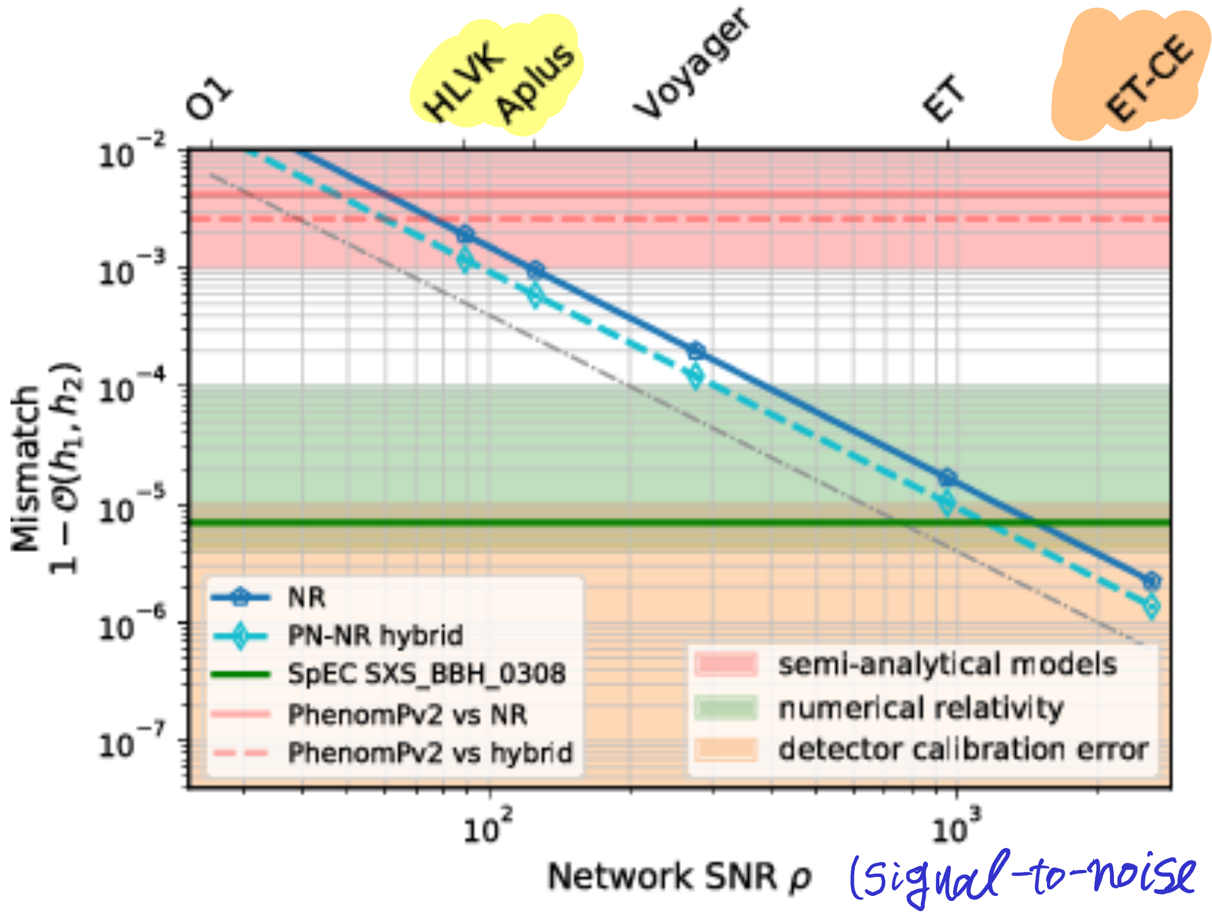
cosmology



arXiv:1908.06060

This material is based upon work supported by NSF's LIGO Laboratory which is a major facility fully funded by the National Science Foundation

Accuracy problems ahead



(signal-to-noise ratio)

Pürrer, Haster, arXiv:1912.10055

results become insufficient
when ?

{ semi analytic
2022-24

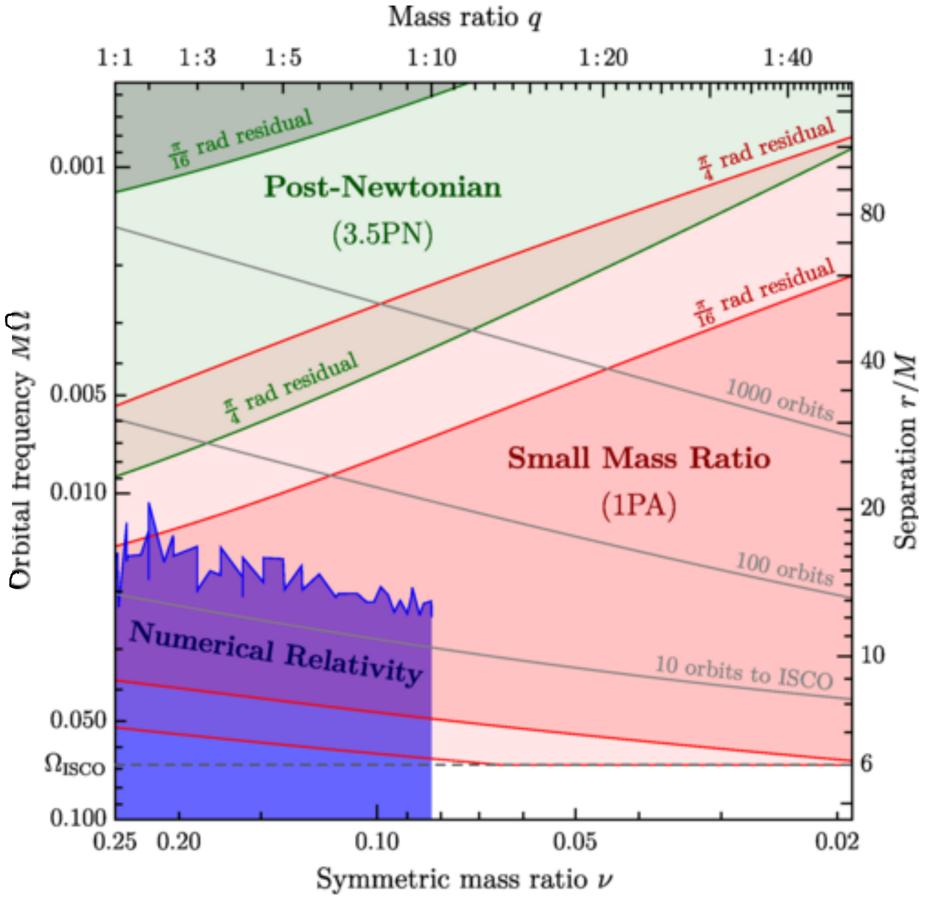
{ NR
2030's

need better predictions soon!

analytic & numeric

Binary parameter space & GW predictions

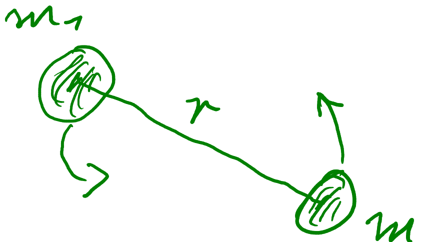
applicability of GW prediction methods:



v.d. Meent, Pfeiffer, PRL 125, 181101

post-Newtonian (PN)

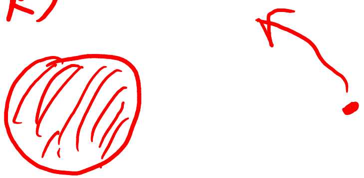
$$\frac{v^2}{c^2} \sim \frac{GM}{c^2 r} \ll 1$$



$M = m_1 + m_2$

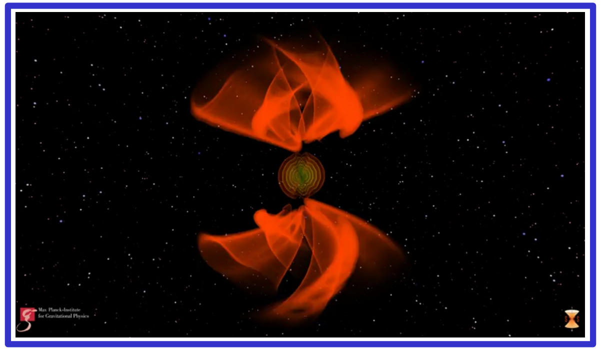
small mass ratio (SMR)

$$\nu = \frac{m_1 m_2}{M^2} \ll 1$$

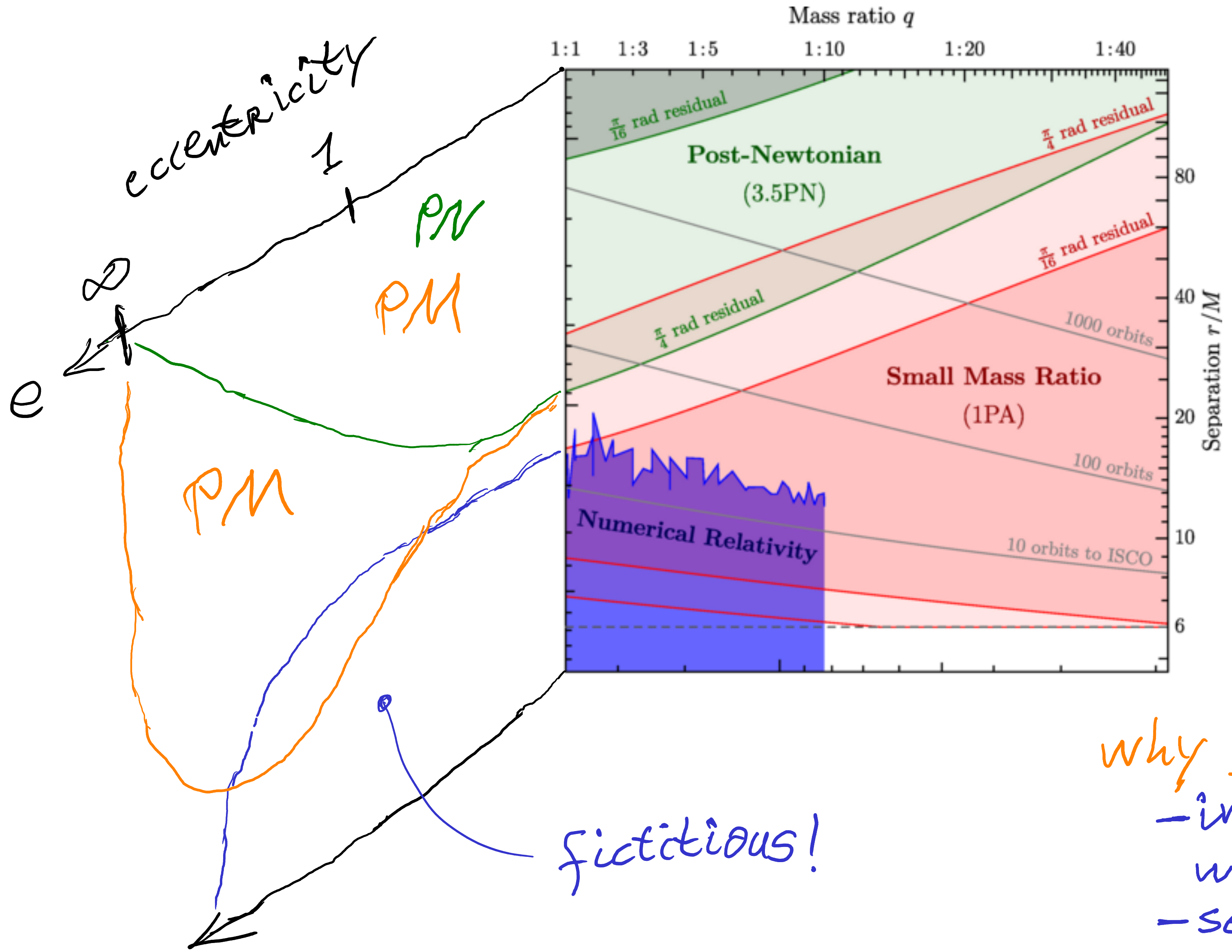


numerical Relativity (NR)

limited by computational resources



another dimension: eccentricity



PM: post-Minkowskian

$$\frac{GM}{c^2 r} \ll 1$$

Recent eccentric (EOB) waveform works by:

Albanesi, Nagar, Bernuzzi, Riemenschneider, Rettegno, Breschi, Albertini, Gamba, Bonino

Liu, Cao, Zhu

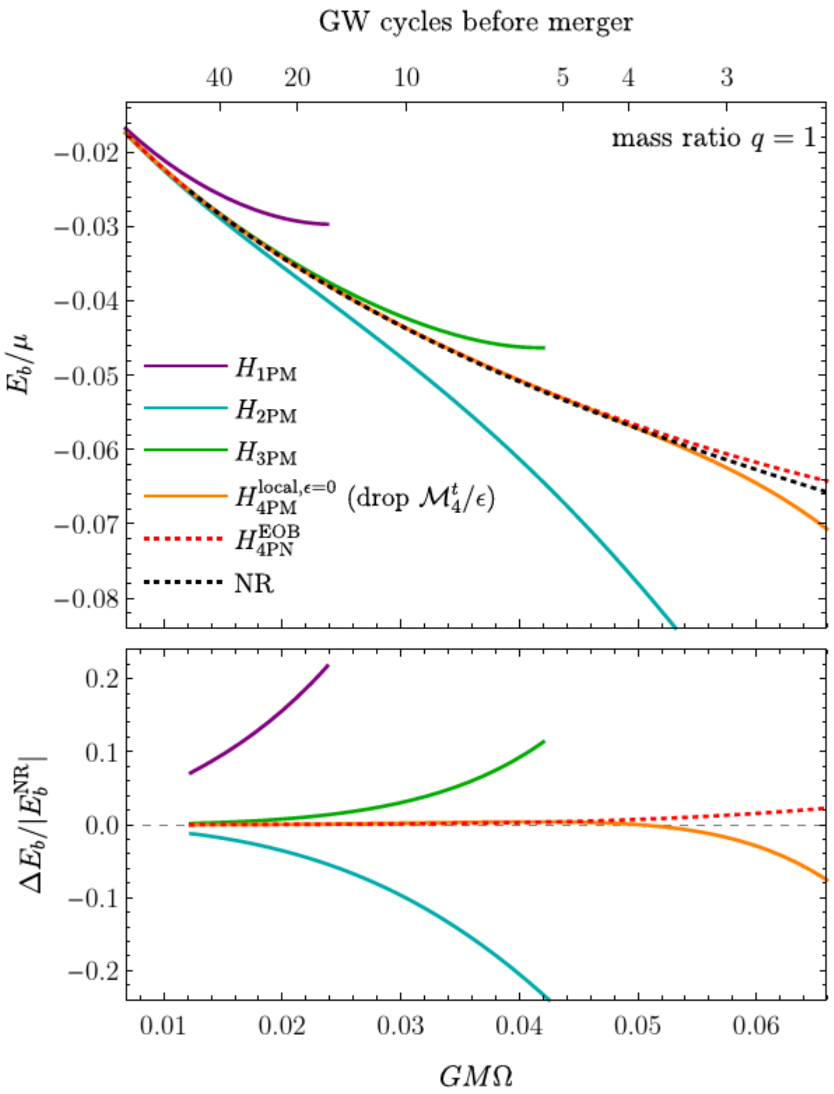
Khalil, Buonanno, JS, Vines ...

why PM:

- improve bound-orbit waveforms with eccenticity
- search for HE scattering BHs

Informing the Hamiltonian/potential with 4PM

circular-orbit binding energy →



4PM from Bern, Parra-Martinez, Roiban,
Ruf, Shen, Solon, Zeng
2101.07294

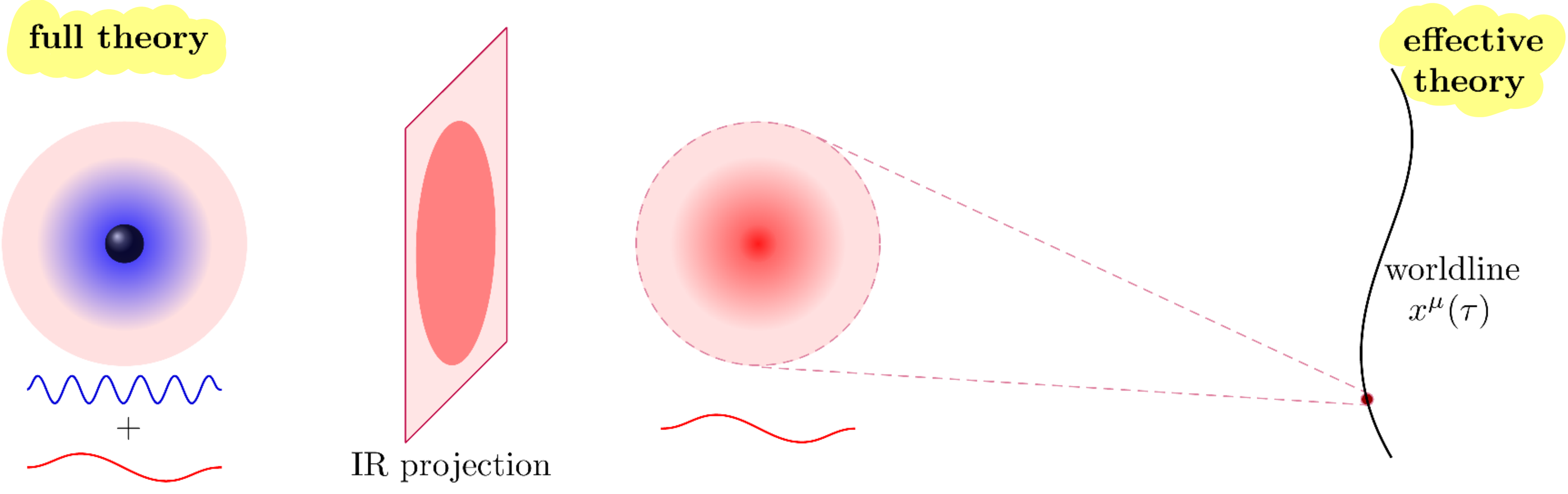
looks promising even
for circular orbits!

credit:
Mohammed
Khalil

full waveform model
requires:
- Hamiltonian/potential
- radiation-reaction force
- waveform "modes"

→ want all from PM!

Black holes as (point) particles



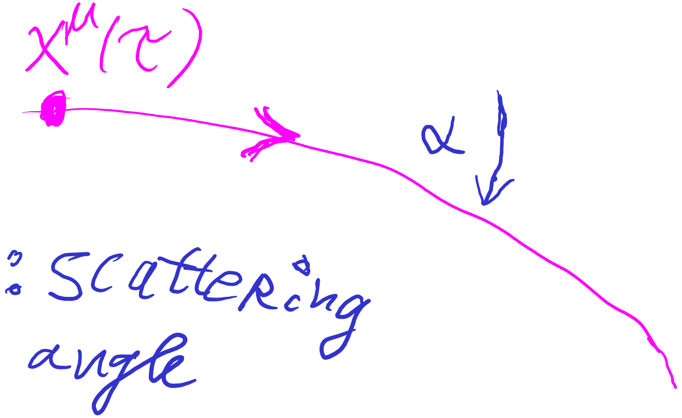
[adopted from arXiv:1906.08161]

$\Rightarrow$ effective-field-theory point-of-view
[Goldberger, Rothstein, ...]

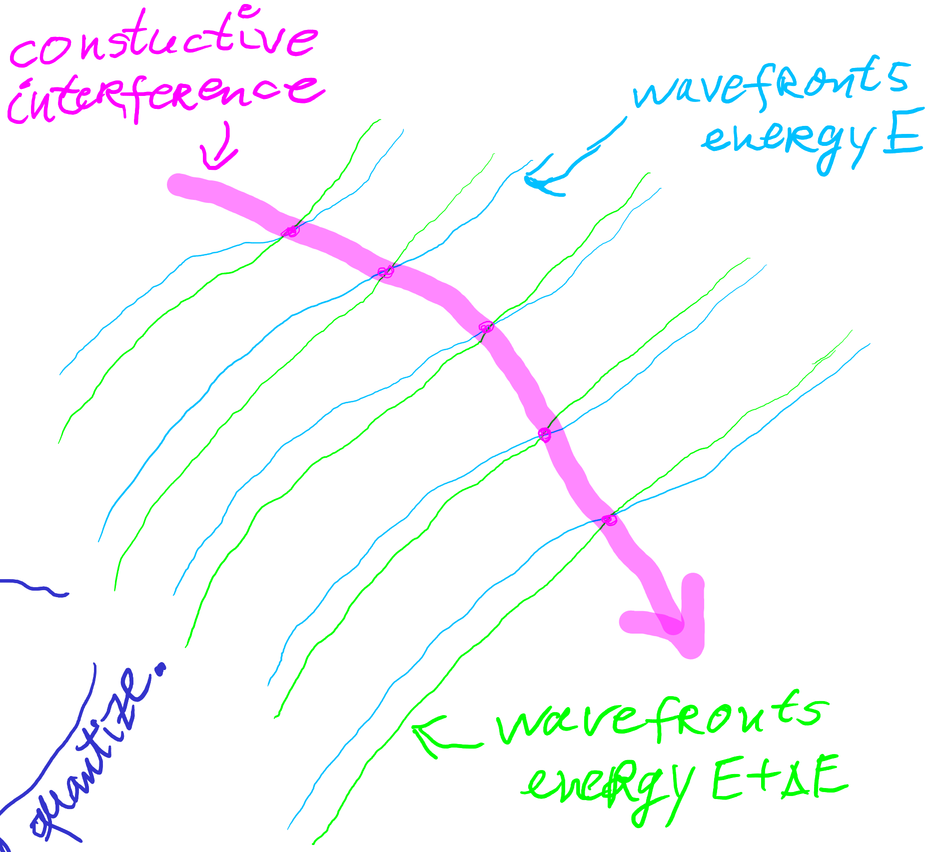
black holes $\simeq$ higher-spin massive particles

[eg. Arkani-Hamed, Huang, O'Connell (2019)]

Scattering particles vs. scattering waves



α: scattering angle

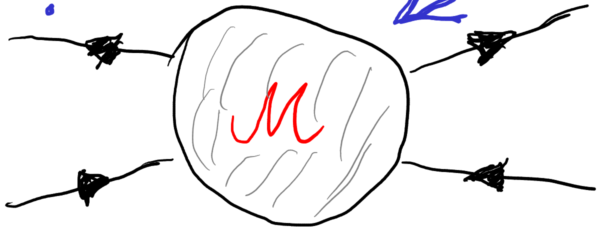


$\psi \sim \overset{\text{(slow)}}{\text{amp.}} \times e^{iS}$

eikonal approx.

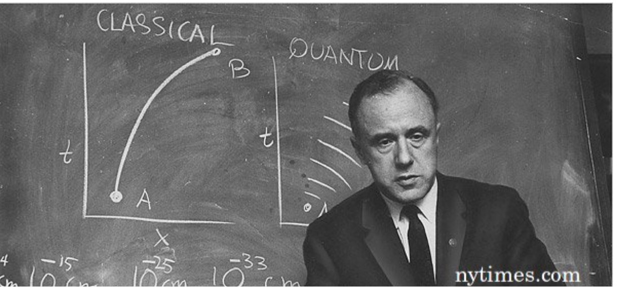
1st quantize.

2nd quantize.



$\mathcal{M}$: scattering amplitude

John A. Wheeler



nytimes.com

Connecting amplitudes and classical physics

- Matching Hamiltonians to amplitudes, eikonal phase, Lippmann-Schwinger eq., ...
  Rothstein, Neill, Cheung, Solon, Bern, Roiban, Shen, Luna, Zeng, Kosmopoulos, di Vecchia, Heissenberg, Russo, Veneziano, Parra-Martinez, Ruf, Zeng
  Bjerrum-Bohr, Cristofoli, Damgaard, Vanhove, ...

- Expectation values from amplitudes [Kosower, Maybee, O'Connell, Vines]

- Here: directly connecting amplitudes to "1st quantization" of massive particles [Mogull, Plefka, JS, 2010.1459;
  Jakobsen " , 2101.12688 PRL]

⇒ "worldline quantum field theory" (WQFT)

Related work: [Goldberger, Ridgway], [Kälin, Porto, Liu, Yang] ...

WQFT

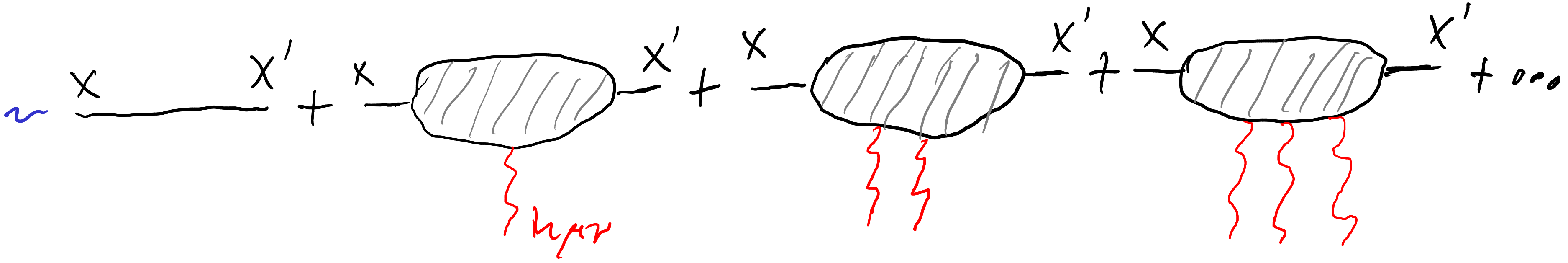
↳ Use Feynman-Schwinger rep. of dressed massive propagator $G(x,x')$

$$\boxed{\sigma = \frac{\tau}{2m}}$$

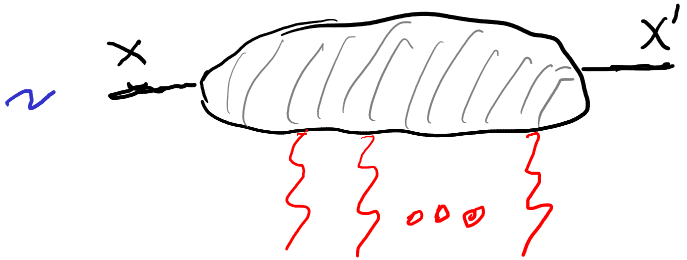
$$(\nabla_\mu \nabla^\mu + m^2 + \xi R)\, G(x,x') = \sqrt{-g}\, \delta(x-x')$$

covariant → Lee-Yang ghosts

$$G(x,x') \sim \int_0^\infty ds\, e^{-ism^2} \int_{x(0)=x}^{x(s)=x'} \mathcal{D}x \cdot \exp\left[-i\int_0^s d\sigma \left(\frac{1}{4} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + \left(\xi - \frac{1}{4}\right) R\right)\right]$$



$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$



e.g. [de Witt, Bekenstein, Parker]
[Bastianelli, van Nieuwenhuizen], ...

Classical limit in WQFT & the eikonal

[Mogull, Plefka, JS, 2010.1459]

standard approach (2nd quantized): massive propagator $\frac{1}{p^2+m^2/\hbar^2}$

$\Rightarrow$ diagrams have all powers in $\hbar$

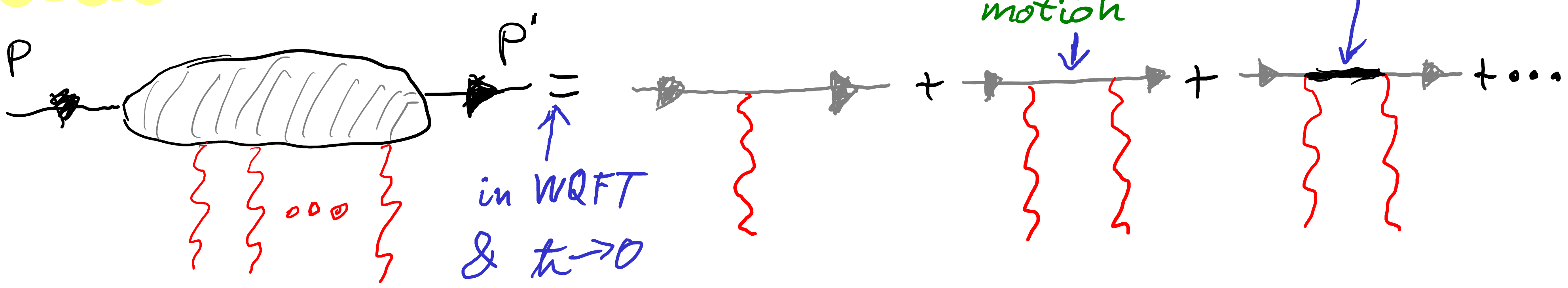
classical limit $\hbar \to 0$ subtle

WQFT: expand around straight-line motion

↪ $\hbar$-counting of diagrams, $\hbar \to 0$ limit straightforward

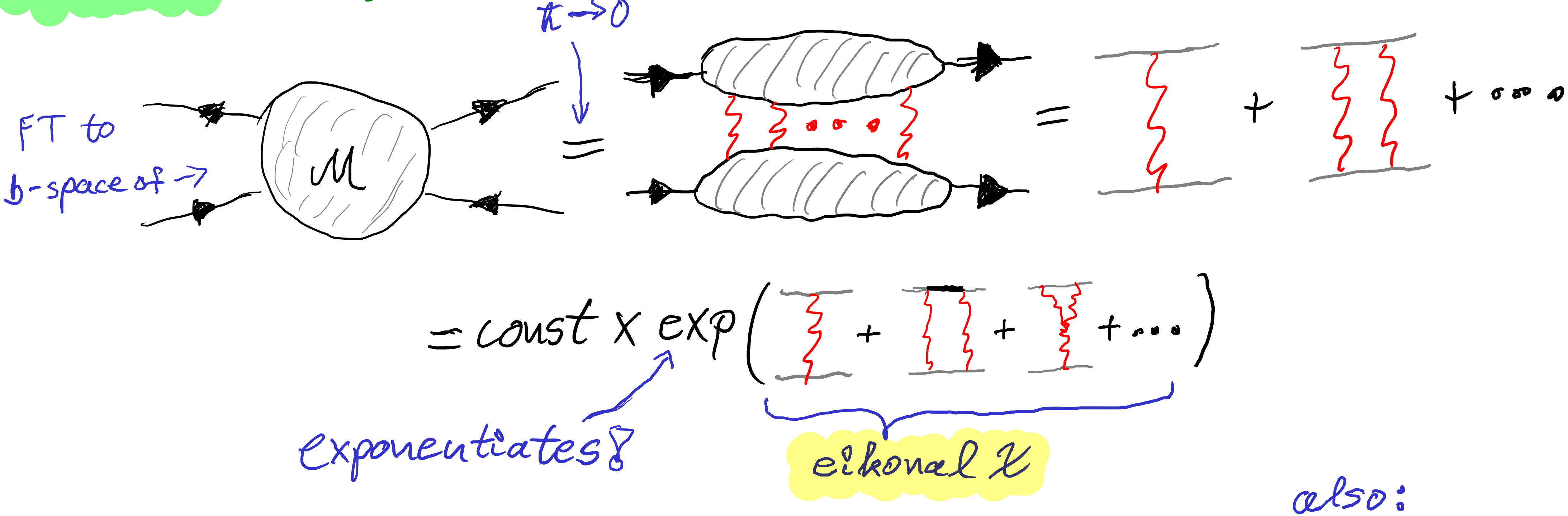
$$x^\mu(\tau) = \underbrace{b^\mu + v\tau}_{\text{straight-line motion}} + \underbrace{z^\mu(\tau)}_{\text{deflection}}$$

$\Rightarrow$ **form factor:**



Eikonal

[Mogull, Plefka, JS, 2010.1459]



$= \text{const} \times \exp\Big(\underbrace{\;\; + \;\; + \;\; + \dots}_{\text{eikonal } \chi} \Big)$

exponentiates!

$\leadsto$ $\boxed{\text{WQFT free energy} = e^{i\chi}}$ for $\hbar \to 0$

property: $\Delta p_\mu = - \frac{\partial \chi}{\partial b^\mu}$

(deflection)

$\leadsto$ $\boxed{\text{observables encoded in } \chi!}$

also:

$\boxed{\text{relation to bound orbits possible!}}$

[Kälin, Porto (2020)]

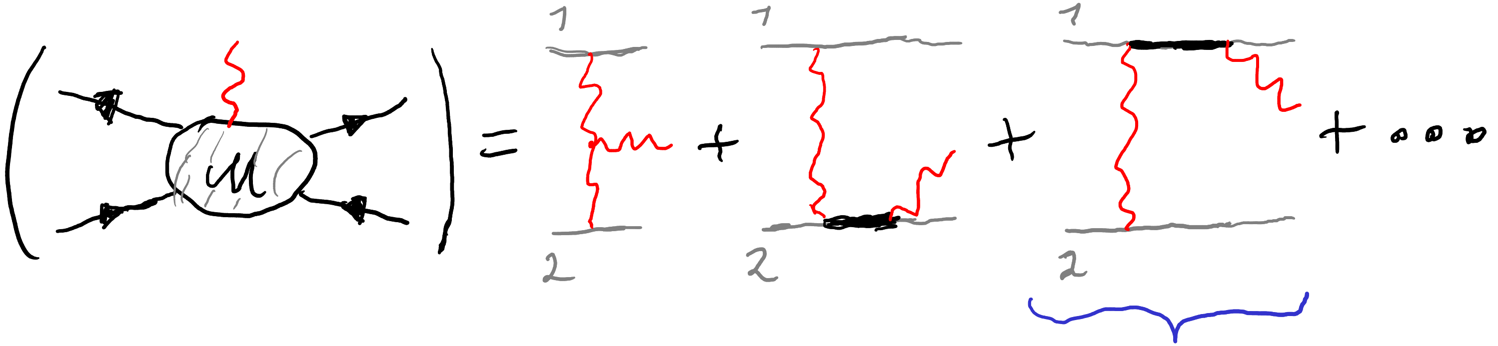
Radiation in the WQFT

[Jakobsen, Mogull, Plefka, JS, 2101.12688 PRL]
also: [Mougiakakos, Riva, Vernizzi 2102.08339]

waveform:

$\langle h^{+/\times} \rangle =$ inv. Fourier trafo

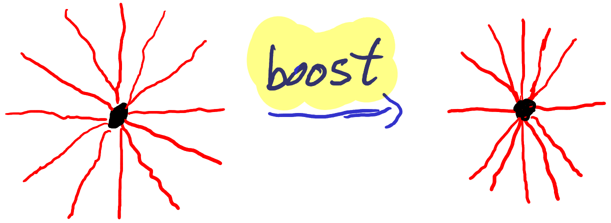
leading order



0 in rest-fame of 1

Feynman diagrams → Feyman integrals

→ integrals involve "anisotropic" propagators



$$\sim \frac{1}{p \cdot M \cdot p}$$

matrix

→ corresponds to boosted Coulomb field → still solvable

↳ reproduce [Kovacs, Thorne (1977)] "efficiently"

beyond small-angle scattering?

Radiation-Reaction

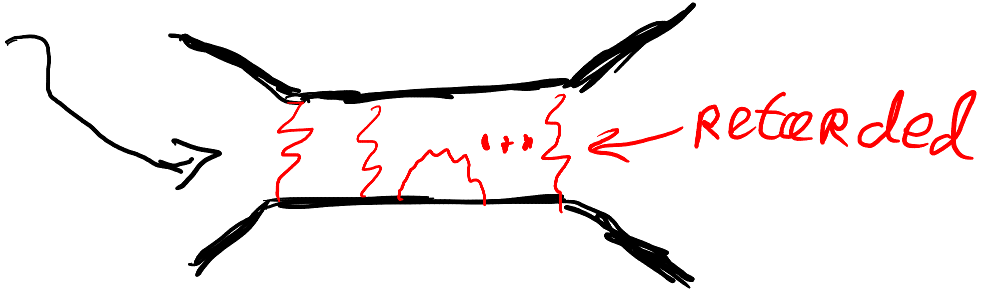
WIP with Saketh Muddu, Justin Vines, Alessandra Buonanno

↳ reconsider "the classical" approach (no amplitudes, no WQFT)

↳ linear theory for starters / QED

↳ split off self-interactions from the start

↳ iteratively compute deflected worldline



"classical" method (off-shell recursion?) seems very efficient

→ amplitude analog?

(Radiation reaction can be computed in WQFT ↳ see papers)

↳ from waveform!

Spin & WQFT

WIP with Jakobsen, Mogull, Plefka

EFT with spin: frame $\Lambda_A^{\ \mu}$ and spin $S^{\mu\nu}$ on worldline

Michèle's talk

↳ would need to integrate out $\Lambda_A^{\ \mu}$ in WQFT

↳ better method?

$$\boxed{\psi^\mu(\tau) \text{ on worldline} \rightsquigarrow S^{\mu\nu} = -2i\bar{\psi}^{[\mu}\psi^{\nu]}}$$

e.g. [Gibbons, Rietdijk, van Holten]

[Howe, Penati, Pernici, Townsend]

[Bastianelli, Benincasa, Giombi]

[Bonezzi, Meyer, Sachs]

⇒ SUSY!?

More spin: spin^2, eikonal, Radiation...

WIP with Jakobsen, Mogull, Plefka

quadrupole coupling: $\frac{1}{8m} R_{\mu\nu\alpha\beta} S^{\mu\nu} S^{\alpha\beta} + \frac{C-1}{2m} E_{\mu\nu} S^{\mu} S^{\nu}$

black hole — non-BH

SUSY! — breaks SUSY!

higher orders? SUSY $\longrightarrow$ unitarity / quantum consistency?

other symmetries?

eikonal $\longrightarrow$ generates observables $\Delta p, \Delta\psi, \Delta S$

e.g. [Maybee, O'Connell, Vines]

[Bern, Luna, Roiban, Shen, Zeng] [Kosmopoulos, Luna]

leading order radiation $\longrightarrow$ no new integrals

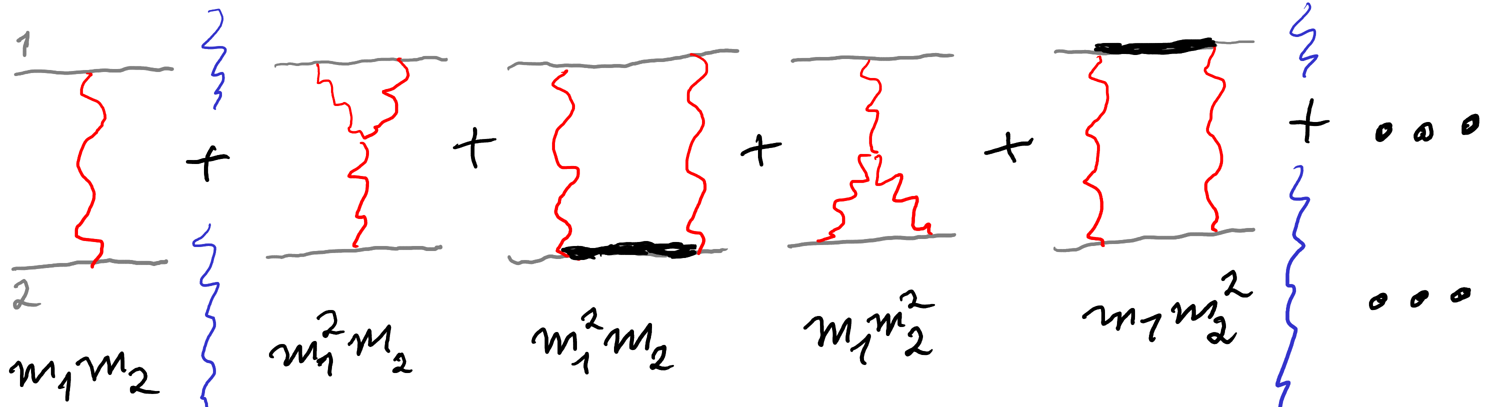
except more complicated numerators

...

Looking for structure: mass-dependence of classical scattering

[Bini, Damour, Geralico (2019)
Vines, JS, Buonanno (2019)]

eikonal =



leading order (LO)

next-to-leading order (NLO)

...

↳ LO & NLO follows from test-bodies (& symmetrizing in masses)

↳ NNLO & N^3LO 〃 1st correction in mass-ratio
(self-force)

⋮

Tutti Frutti!

N^3LO PN spin-orbit (SO) from self-force

$q = \frac{m_1}{m_2}$

[Antonelli, Khalil, ..., 2003.11391]

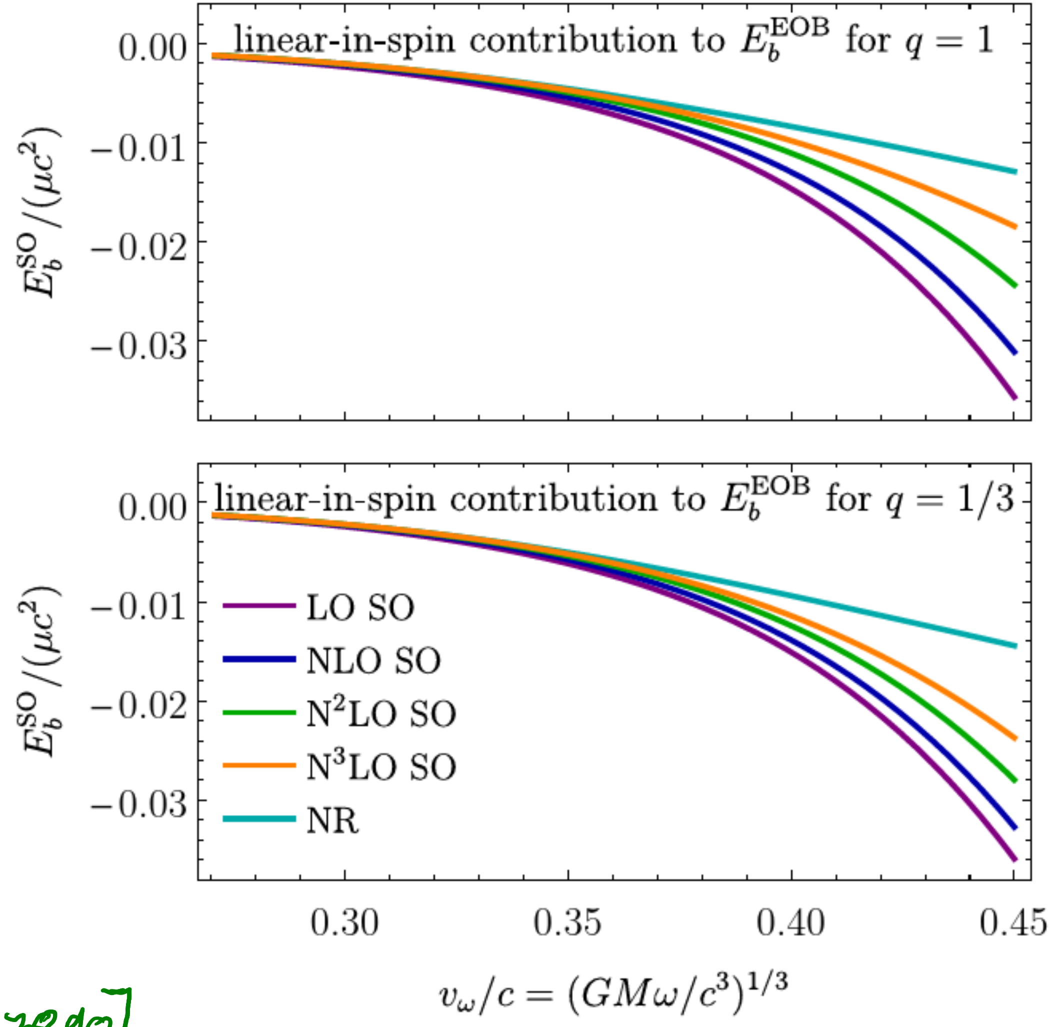
↳ naively a 3-loop calculation

↳ follows from known results for self-force without solving any integrals!

↳ extension to aligned-spin $S_1 S_2$-case possible

[Antonelli, Khalil, ..., 2010.02018]

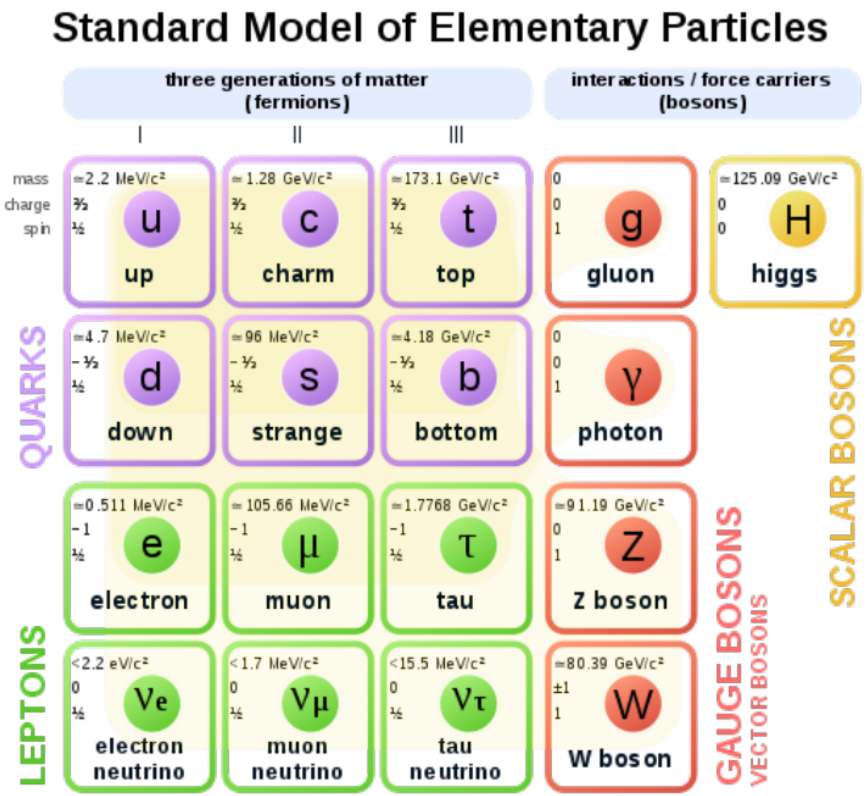
PN binding energy E_b vs NR



see also [Levi, McLeod, von Hippel, 2003.02827+07890]

Scattering amplitudes involving black holes ?

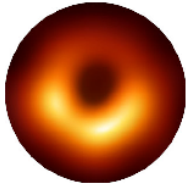
↳ of fundamental interest !



Standard Model of Elementary Particles

		three generations of matter (fermions) I	three generations of matter (fermions) II	three generations of matter (fermions) III	interactions / force carriers (bosons)	interactions / force carriers (bosons)
QUARKS	mass charge spin	≃2.2 MeV/c² ⅔ ½ u up	≃1.28 GeV/c² ⅔ ½ c charm	≃173.1 GeV/c² ⅔ ½ t top	0 0 1 g gluon	≃125.09 GeV/c² 0 0 H higgs
		≃4.7 MeV/c² −⅓ ½ d down	≃96 MeV/c² −⅓ ½ s strange	≃4.18 GeV/c² −⅓ ½ b bottom	0 0 1 γ photon	SCALAR BOSONS
LEPTONS		≃0.511 MeV/c² −1 ½ e electron	≃105.66 MeV/c² −1 ½ μ muon	≃1.7768 GeV/c² −1 ½ τ tau	≃91.19 GeV/c² 0 1 Z Z boson	GAUGE BOSONS VECTOR BOSONS
		<2.2 eV/c² 0 ½ ν_e electron neutrino	<1.7 MeV/c² 0 ½ ν_μ muon neutrino	<15.5 MeV/c² 0 ½ ν_τ tau neutrino	≃80.39 GeV/c² ±1 1 W W boson	

$+$

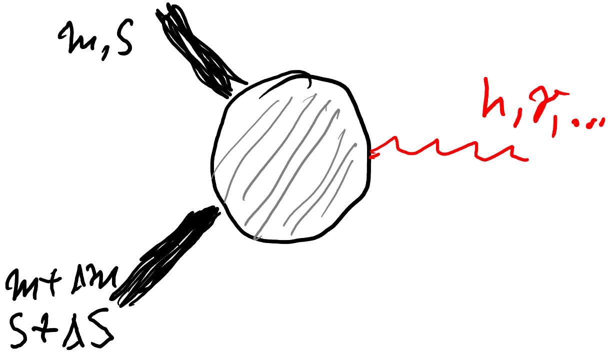


black holes
(∼ higher-spin
massive particles)

graviton
(grav. waves)

what is the "best" method for black-hole interactions ?

- classical
- Feynman rules / EFT
- amplitudes



- a synergy ?

Conclusions

↳ gravitational wave models informed by scattering?

building blocks:

potential → eikonal (radial action)

Radiation Reaction → eikonal

waveform (source multipoles) → ?

lots of progress!

connect classical & quantum physics

↳ exchange of ideas

↳ consolidation of methods & abstractions