Symmetries of Gravitational Scattering

GGI Workshop
April 2021

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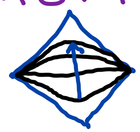
I'd like to draw attention to 3 unsolved problems in GR scattering in AF spacetimes:

1. How is $\mathcal{I}^- \to \mathcal{I}^+$ scattering defined? More subtle than 

2. What are the non-trivial symmetries/conservation laws? $E, \vec{P}, \vec{J} + \ldots \infty$ 100+ years > Einstein still unknown!

3. What are the observational signals (memory + ...) of these symmetries?

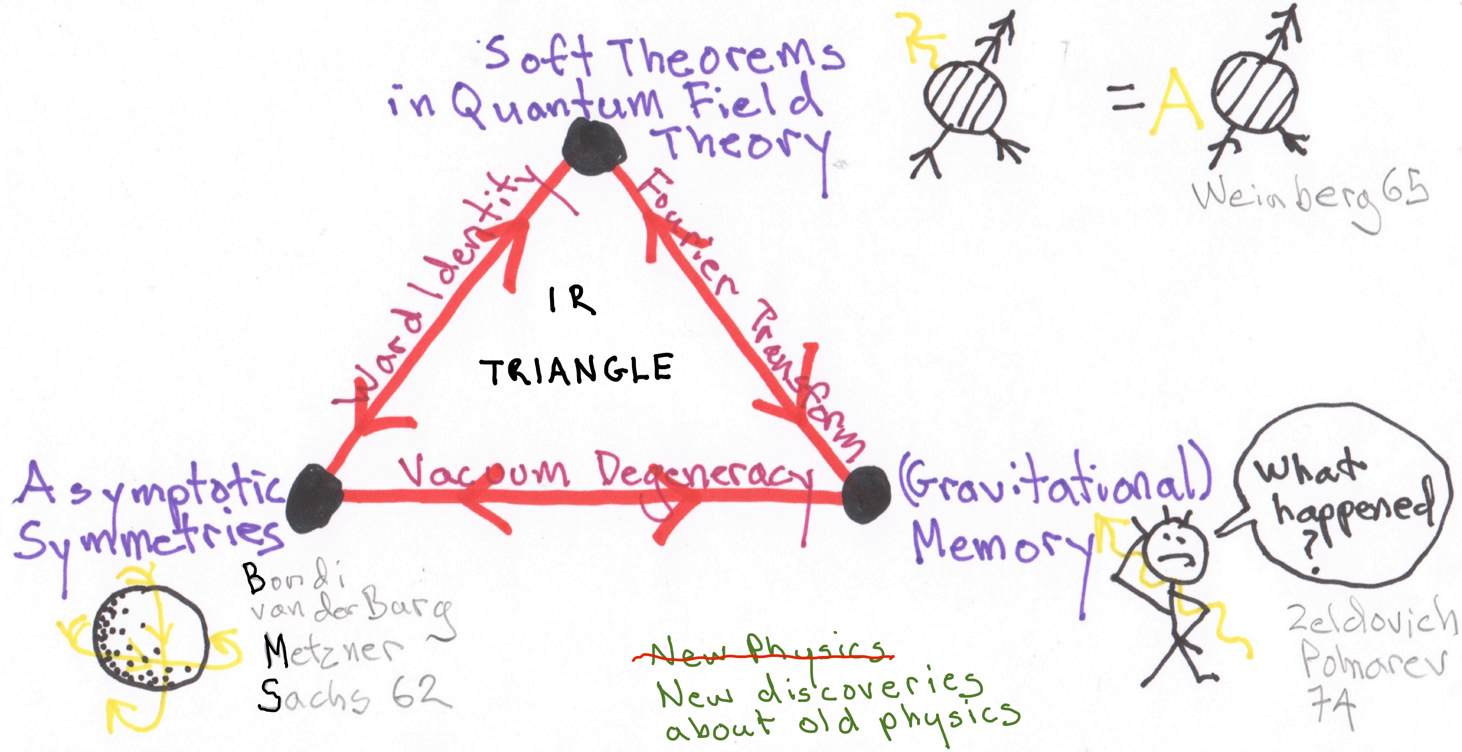
This talk: $>\frac{1}{2}$ review last 5-10 yrs progress*

$<\frac{1}{2}$ recent paper on GR symmetries with Alfredo Guevara, Mina Himwich, Monica Pate & AS 2103.03961

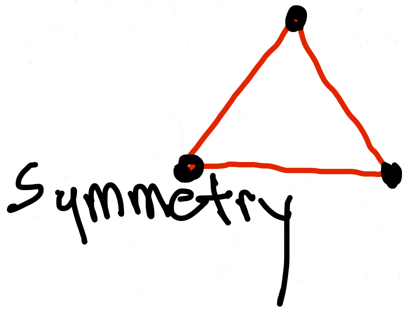
* "Lectures on the IR structure of Gravity & Gauge Theory"
Princeton Univ. Press 2018
hep-th 1703.05488

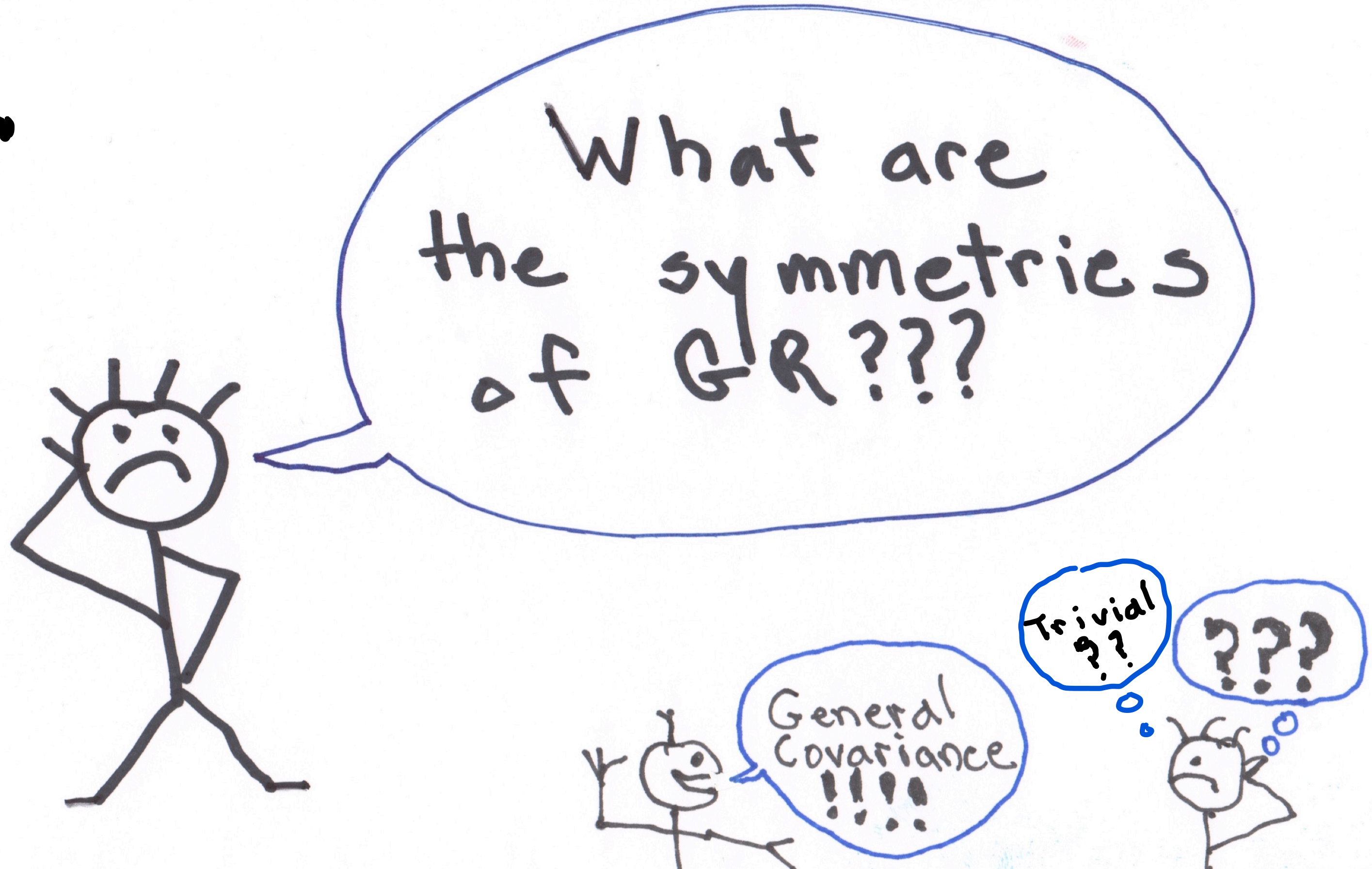
I had many wonderful collaborators refered to herein.

Over the last 7 years, an exact mathematical equivalence has been discovered of 3 previously disparate phenomena, each studied for half a century:



This has led to surprising new insights into the low-energy structure of gravitational & electromagnetic theories, implications for quantum black hole information & novel experiments at hadron accelerators, the EIC, LIGO, LISE; IR divergences & flatspace holography.





This subtle & often ill-posed question has been the source of consternation & confusion for 100 years. Our best answer has changed many times significantly, right up to the present.

In Special Relativity, Poincare symmetries lead, via Noether's theorem, to measurable conserved charges such as Energy, Momentum and Angular Momentum. For example, Energy conservation follows from time translatian symmetry

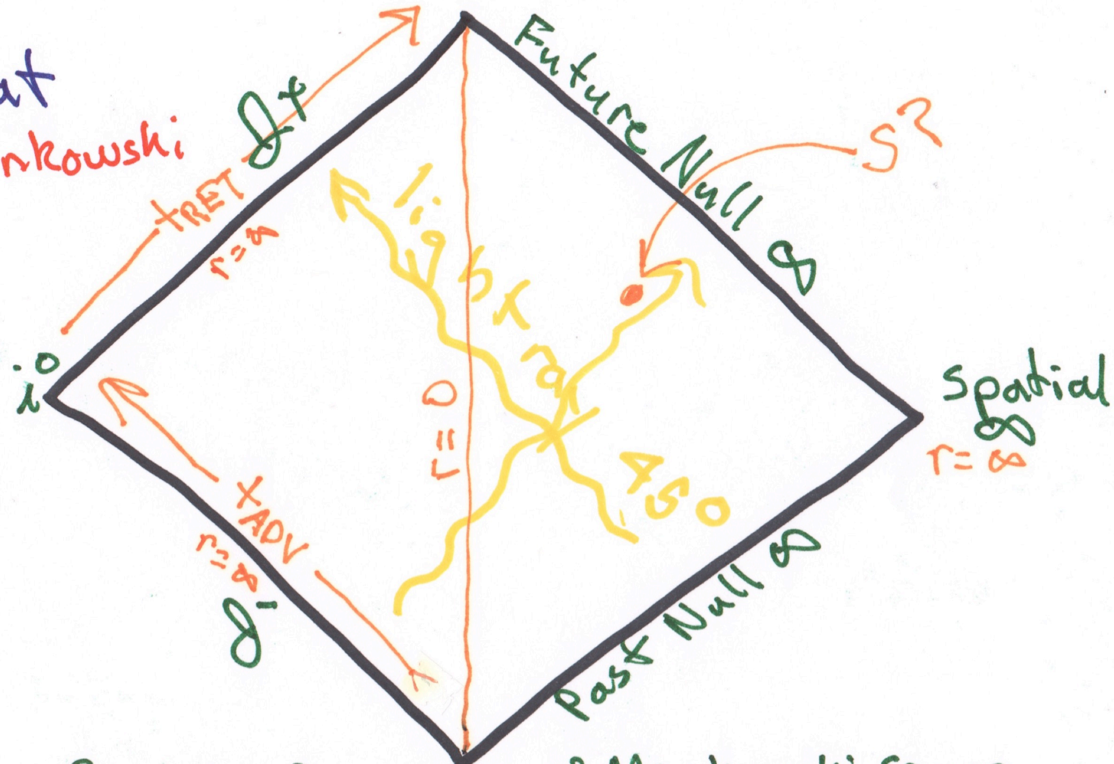
$$t \to t + \text{constant}$$

t is a time coordinate, and this is a coordinate transformation. Bondi, van der Burg, Metzner & Sachs (BMS) in 1962 asked "How do Poincare symmetries/charges appear in GR?"

The answer was a big surprise.

KEY IDEAS

1. Asymptotically Flat Spacetimes "like Minkowski near $\mathcal{I}$"
Very subtle!
Arnowitt
Deser
Misner



Penrose Diagram of Minkowski Space

2. Asymptotic Symmetries

$$= \frac{\text{Coord. Trans. preserving boundary cond.}}{\text{Trivial Coord. Trans.}}$$

$\overset{?}{=}$ Poincare **NO!!!**

ON $\mathscr{I}^+$

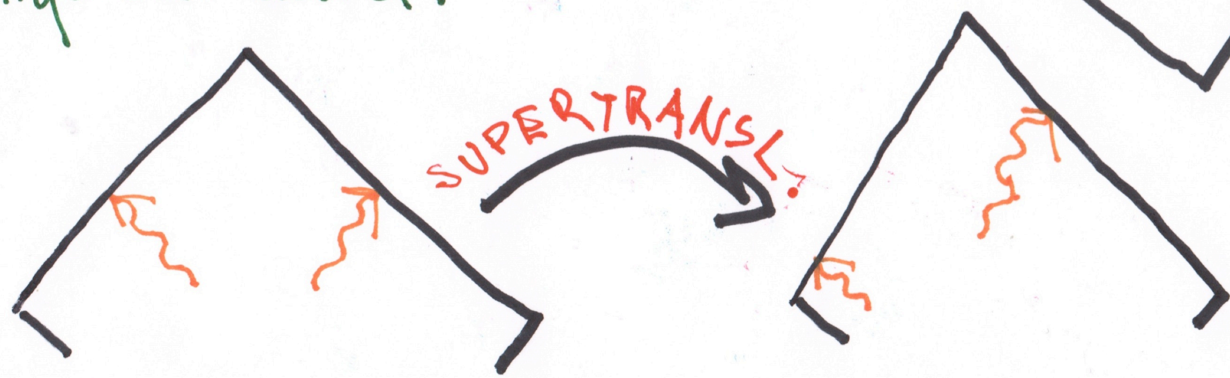
ASYMPTOTIC SYMMETRY = ∞-DIMENSIONAL BMS^+ SYMMETRY

1962

comprised of asymptotic boosts, rotations & SUPERTRANSLATIONS

$$t_{RET} \to t_{RET} + f(\theta,\phi)$$

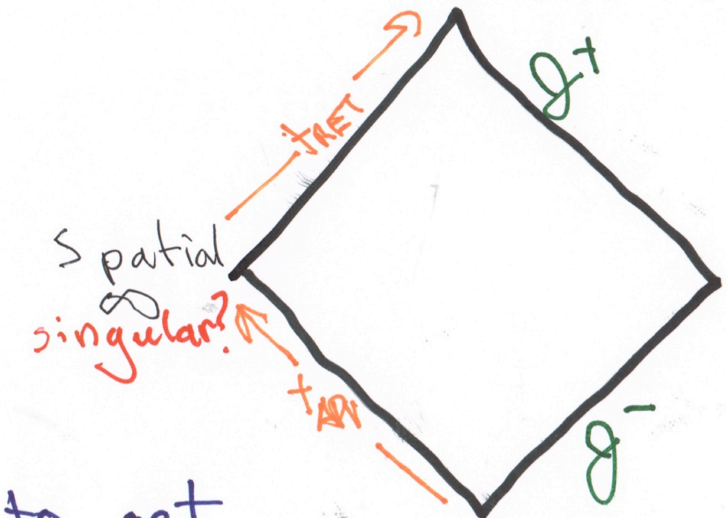
for any f. These act non-trivially on outgoing physical data:



There is also BMS^- on $\mathscr{I}^-$

$$t_{ADV} \to t_{ADV} + \tilde{f}(\theta,\phi)$$

So what do we do with this fundamental symmetry of GR?



For 50 yrs after BMS, it was widely held that BMS^+ and BMS^- acted separately on $\mathscr{I}^+$ and $\mathscr{I}^-$ and could not be tied together at spatial ∞ to get a symmetry of gravitational scattering. This was in part due to a misunderstanding about the structure of spatial ∞ which was not cleared up until the seminal 1992 work of Christodoulou & Klainerman.

Using Christodoulou & Klainerman, it was recently shown that the combined $\mathscr{I}^+$ and $\mathscr{I}^-$ supertranslations

$$t_{RET} \to t_{RET} + f(\theta,\phi)$$

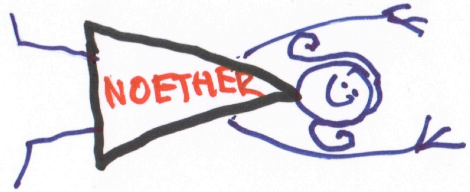
$$t_{ADV} \to t_{ADV} + f(\pi-\theta, \phi+\pi)$$ ← Antipodal

are an ∞ number of exact, nontrivial symmetries of classical gravitational scattering. AS 2013

In the language of the $\mathcal{S}$-matrix

$$B\mathcal{S} - \mathcal{S}B = 0$$

where the operator B generates infinitesmal supertranslations and acts non-trivially on physical states. These are true "Symmetries of GR"! But not all of them!

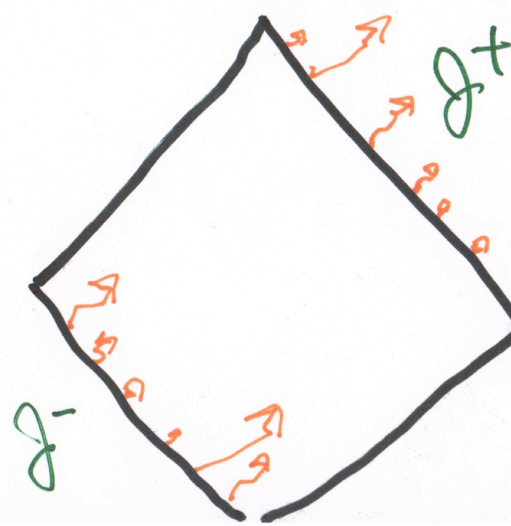
SYMMETRIES  CONSERVATION LAWS

For f = constant the conservation law is $E_{in} = E_{out}$.
More generally, we find

> Integrated total incoming energy* flux from angle θ, ϕ

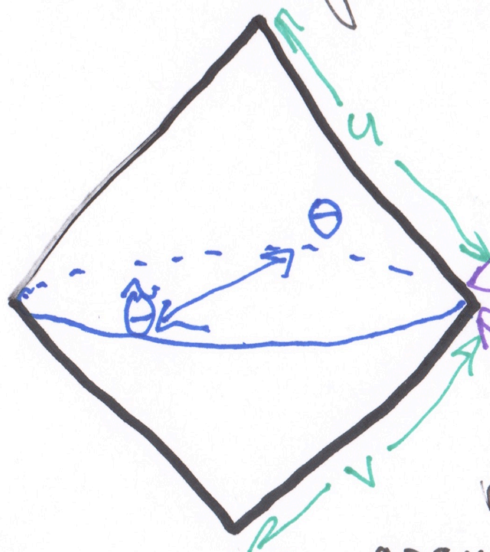
=

> Integrated total outgoing energy flux through angle $\pi - \theta, \phi + \pi$!



* Energy in GR has crucial term linear in metric fluctuation. (Bondi)

An alternate derivation of the of conservation laws, bypassing symmetries, follows from the antipodal b.c on the Bondi mass aspect



$$m_B^+(u\to-\infty,\ \Theta) = m_B^-(v\to+\infty,\ \tilde{\Theta})$$

needed to define gravitational scattering

$$B = Q_f^+ = \int d^2\Theta\, f(\Theta)\, m_B(\Theta) = Q_f^- = \int d^2\Theta\, f(\tilde{\Theta})\, m_B(\Theta)$$

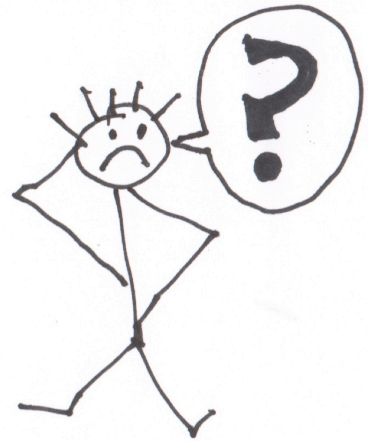
NOT THE END!

The story so far follows from antipodal matching of the leading $\frac{1}{r}$ component of the metric & leading soft thm.

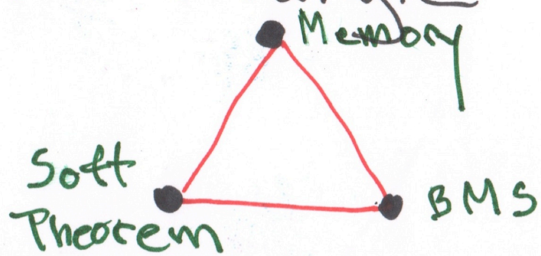
$\exists$ an ∞ hierarchy of subleading, subsubleading ..... structures.

More later.

Now I will turn to what seems a completely different topic.

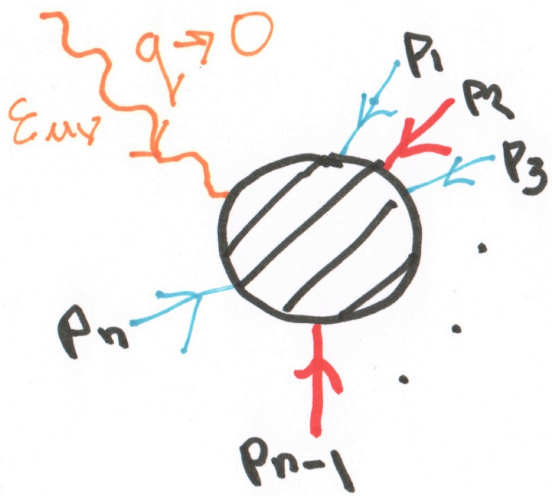


In fact we will see shortly that it is exactly the same topic in different mathematical language: a corner of the triangle

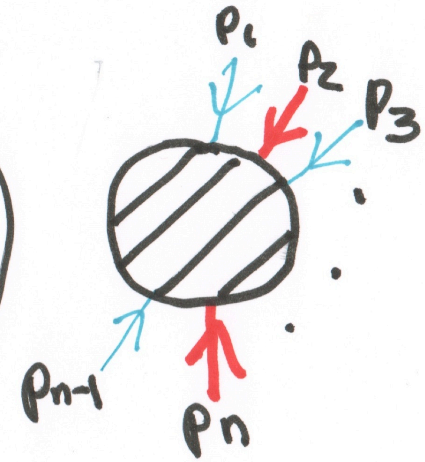


Soft Graviton Theorem

Weinberg 1965

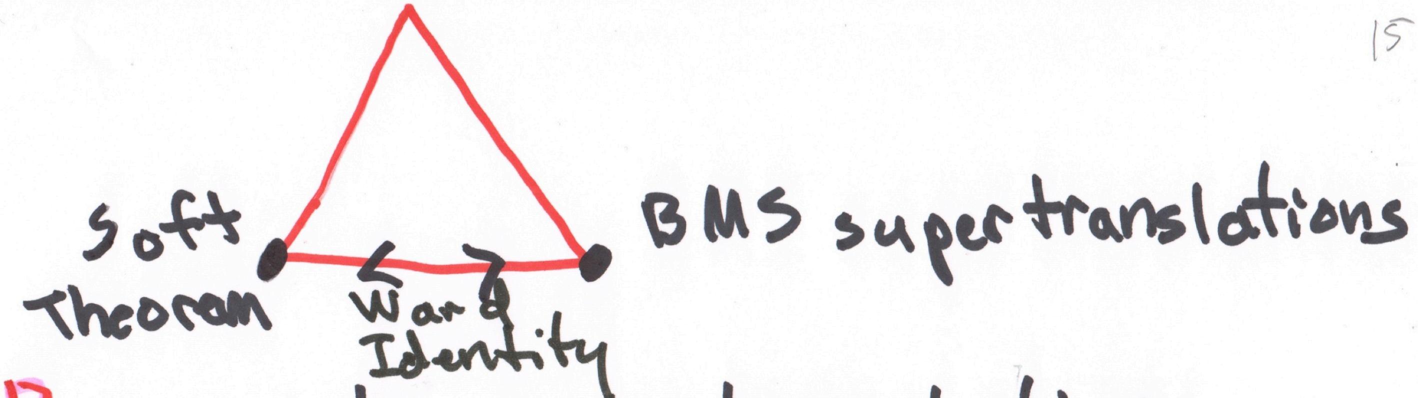


$$= \left(\sum_{k=1}^{n} \frac{\varepsilon_{\mu\nu} p_k^\mu p_k^\nu}{q \cdot p_k} \right)$$



$$+ \mathcal{O}(q^0)$$

Exact & universal formula with simple diagrammatic derivation. Valid in any 4D consistent classical or quantum theory of gravity.



Let **B** generate supertranslations.
Then

$$B\delta - \delta B = 0$$

moreover

$$\langle out | B\Delta - \Delta B | in \rangle$$

is precisely Weinberg's soft graviton theorem. ∞ of associated conserved quantities is total energy at each incoming angle.

AS 1312.2229
He, Lysov, Mitra AS 1401.7026

GR Vacuum is ∞-Degenerate

Classical

$g_{\mu\nu} = \eta_{\mu\nu}$ NOT preserved by stranslations

changes angular momentum!!

"~~GR problem of ang. momentum~~"

Quantum

$[H, a^{+}_{\substack{soft\\grav}}] = 0; \quad [J, a^{+}_{\substack{soft\\grav}}] \neq 0$

$\Rightarrow \quad a^{+}_{\substack{soft\\grav}} |0\rangle \neq |0\rangle$

= new vacuum

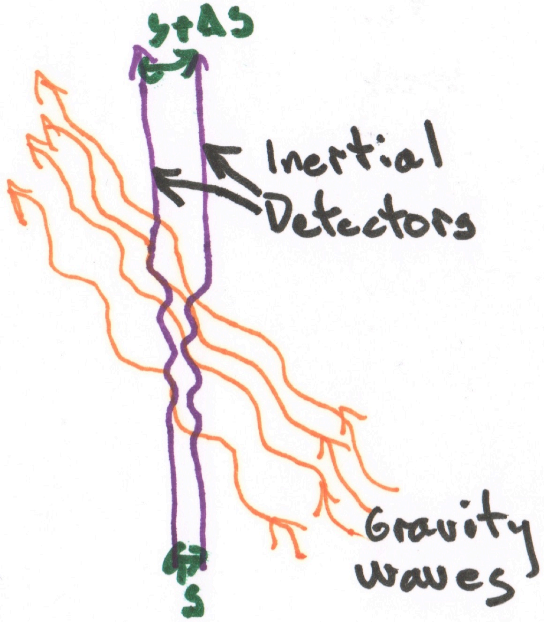
Supertranslation symmetry is "spontaneously broken".

Its fascinating that these two essentially equivalent discoveries were made 3 years apart.... perhaps an example of how ideas percolate unrecognized through our community...

Now for a second apparent topic change!

Gravitational Memory

Zeldovich & Polnarev '74
Braginsky & Thorne '87
Christodoulou '91



AF Minkowski retarded metric:

$$ds^2 = -du^2 - 2\,du\,dr + r^2\gamma_{z\bar z}\,dz\,d\bar z + rC_{zz}\,dz^2 + rC_{\bar z\bar z}\,d\bar z^2 + \dots$$

(gravity waves)

Flat iff $C_{zz} = -2D_z^2 C(z,\bar z)$

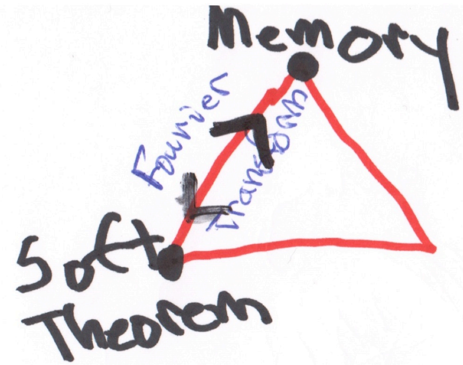
Integrated geodesic deviation equation:

$$\Delta s^{\bar z} = \frac{1}{2r}\gamma^{z\bar z}\Delta C_{zz}\, s^z$$

final − initial = change in C_{zz}

$\Delta C_{zz} = -\frac{1}{2}D_z^2 f$ = BMS transf. A. S. Zhiboedov

DC effect. Quite possibly measured in coming decades. Pulsar timing array? LIGO? eLISA?



"Infrared Photons & Gravitons"
S. Weinberg, 1965

The dominance of the $1/(p\cdot q)$ pole in (2.5) implies that the effect of attaching one soft-graviton line to an arbitrary diagram is to supply a factor equal to the sum of (2.5) over all external lines in the diagram

$$\int d\omega\, e^{i\omega u} \quad (8\pi G)^{1/2} \sum_n \eta_n p_n{}^\mu p_n{}^\nu / [p_n \cdot q - i\eta_n \epsilon]. \qquad (2.7)$$

=

"Gravitational Wave-Burst with Memory and Experimental Prospects"
V. Braginsky and K.S. Thorne 1987

permanent change in the gravitational-wave field (the burst's memory) $\delta h_{ij}^{\mathrm{TT}}$ is equal to the 'transverse, traceless (TT) part'[36] of the time-independent, Coulomb-type, $1/r$ field of the final system minus that of the initial system. If $\mathbf{P}^A$ is the 4-momentum of mass A of the system and P_i^A is a spatial component of that 4-momentum in the rest frame of the distant observer, and if $\mathbf{k}$ is the past-directed null 4-vector from observer to source, then $\delta h_{ij}^{\mathrm{TT}}$ has the following form:

$$\delta h_{ij}^{\mathrm{TT}} = \delta \left(\Sigma_A \frac{4 P_i^A P_j^A}{\mathbf{k}\cdot \mathbf{P}^A} \right)^{\mathrm{TT}} \qquad (1)$$

Here we use units with $G=c=1$. In the observer's local Car-

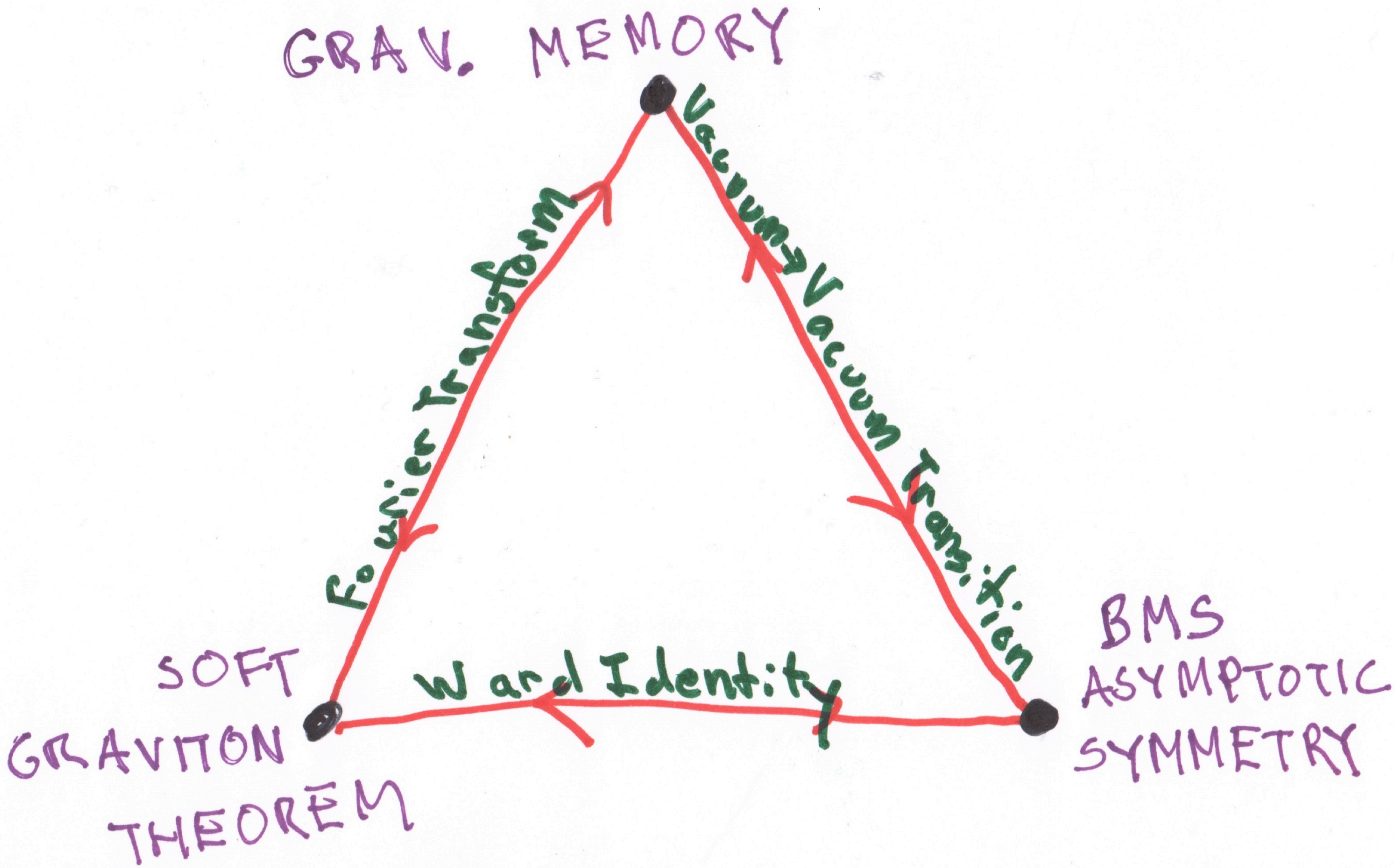
!!!

Changing notations, using $\Theta(u) = \int \frac{d\omega}{\omega} \frac{e^{i\omega u}}{2\pi i}$ and elementary particles → black holes, stars!

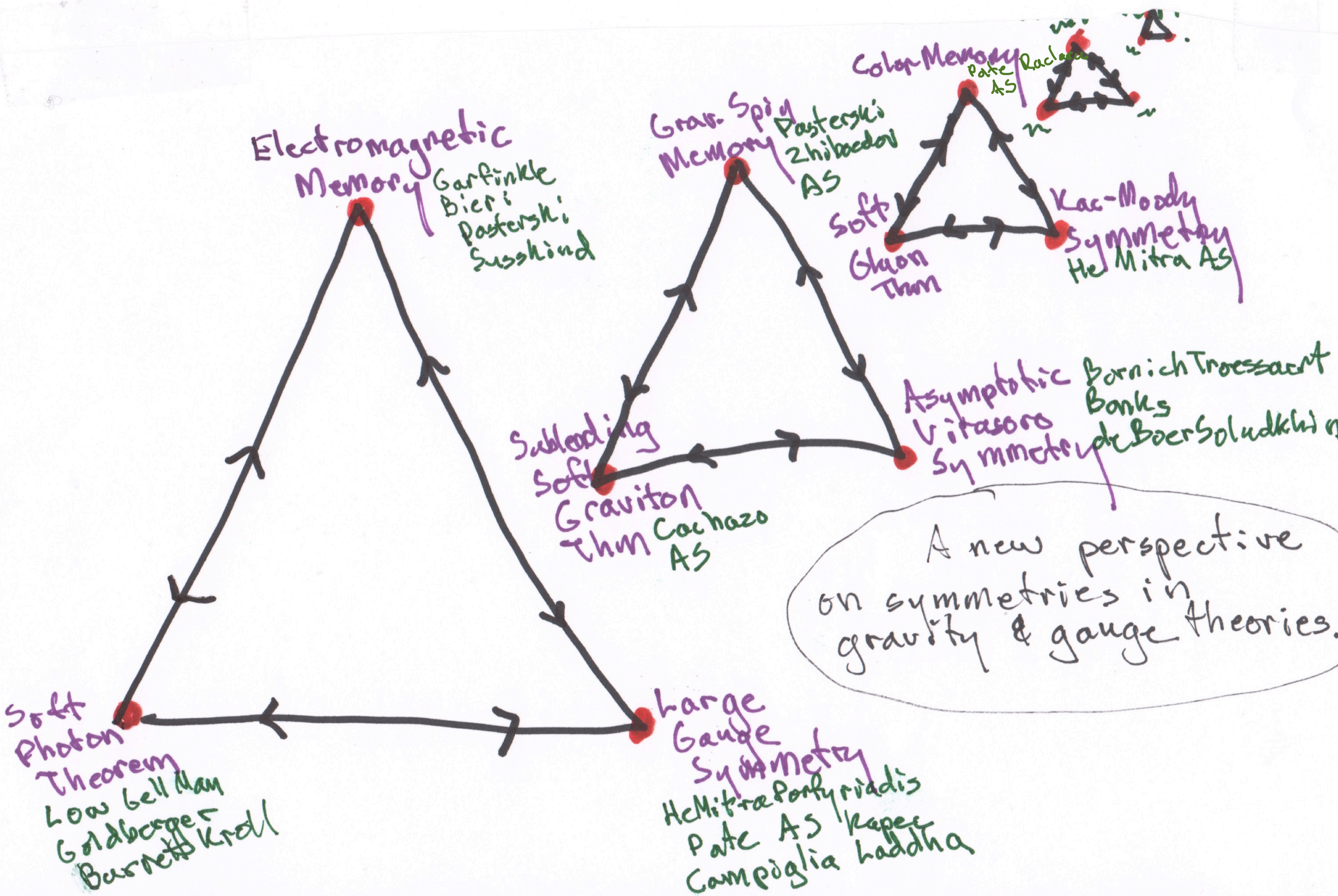
Memory effect provides physical method to measure soft gravitons.

AS + Zhiboedov 2014

SUMMARY:
IR TRIANGLE



REVERBERATING TRIANGLES



A new perspective on symmetries in gravity & gauge theories.

What are the fundamental symmetries of SM + GR?

i) Consider AF approximation.
ii) Diffeos mostly trivial.
iii) Non-trivial includes diagonal subgroup of $BMS^+ \otimes BMS^-$
iv) Certainly more, at present no proposal for complete answer

Important basic question w/ observational content. Surprising not yet answered!

∃ many possible formalisms

Canonical, covariant S-matrix, Lagrangian, soft thms, asymp. analysis ...

Should all be equivalent. Esp. efficient

"Generalized currents on celestial sphere"

~~Gauge choice boundary conditions~~

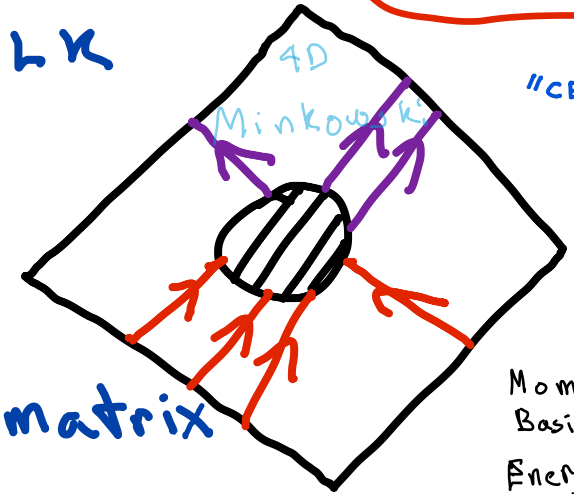
Recent progress

A. Guevara, M. Himwich, M. Pate & AS

hep-th 2103.03961

Basic Idea

BULK

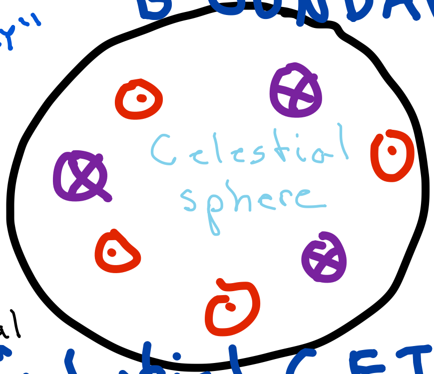


S-matrix

"CELESTIAL HOLOGRAPHY"

=

BOUNDARY



Celestial CFT correlator

Momentum Basis ⟺ Conformal Basis

Energy ⟺ Boost weight

SL(2,C) Lorentz = global 2D conformal

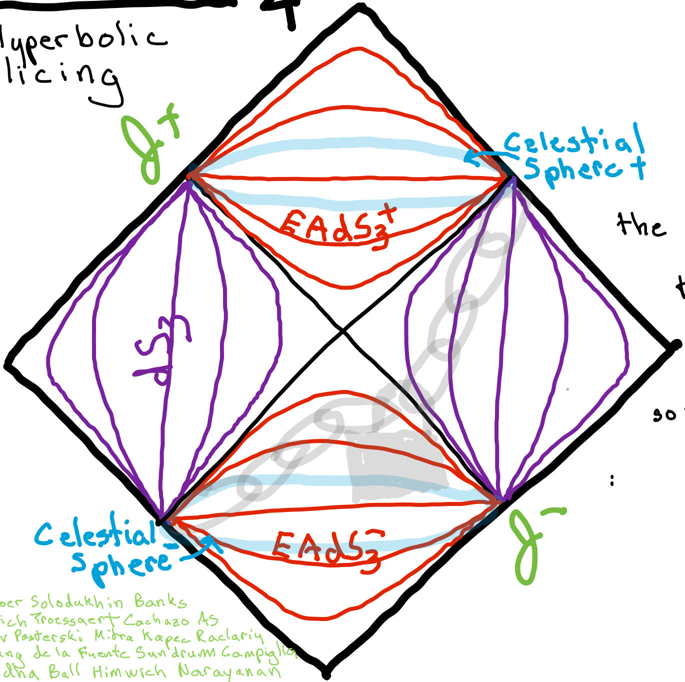
Superrotations = local 2D conformal

Supertranslations $\mathcal{O}_{\Delta}(z,\bar{z}) \to \mathcal{O}_{\Delta+1}(z,\bar{z})$!

.... more ...

Minkowski$_4$

Hyperbolic slicing



Thanks to the subleading soft graviton thm, we now know the ∞-dimensional 2D conformal symmetry is uplifted to the celestial sphere. But there are ∞-ly many more from supertranslations, subsubleading soft gravitons, ...

deBoer Solodukhin Banks
Barnich Troessaert Cachazo AS
Lysov Pasterski Mitra Kapec Raclariu
Cheung de la Fuente Sundrum Campiglia
Laddha Ball Himwich Narayanan
. . . .

Why is celestial holography of interest?

1. Flat space version of AdS/CFT, but bottom-up (c.f. Brown-Henneaux), not (so-far) top-down (c.f. Maldacena) and applies to real world.

2. Computationally effective, can apply powerful constraints/methods of 2D CFT to 4D (classical and quantum) gravity. V useful for symmetry analysis.

3. . . . .

HOW TO REPRESENT A 4D SYMMETRY ON THE CELESTIAL SPHERE

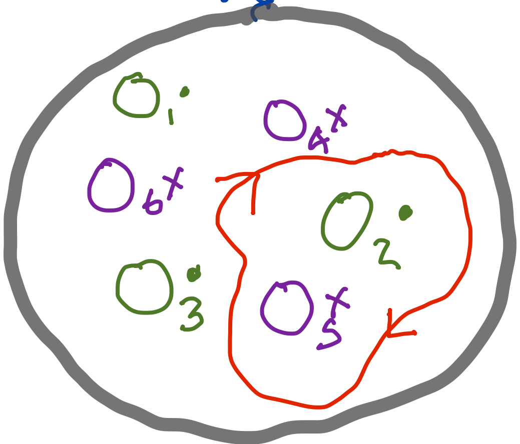
$$ds^2 = -du^2 - 2\,du\,dr + r^2 \gamma_{z\bar{z}}\, dz\, d\bar{z} + r C_{zz}\, dz^2 + r C_{\bar{z}\bar{z}}\, d\bar{z}^2 + \dots$$

$N_{zz} \equiv \partial_u C_{zz}$ Bondi News

$$P_z^- \equiv \frac{1}{4G}\int_{-\infty}^{\infty} du\, D^z N_{zz} - [\partial^-], \quad \partial_{\bar{z}} P_z^- = 0$$

Asymptotic constraint away from fluxes/operators

$\mathcal{I}^+$



$$\oint_C f(z) \langle P^-(z)\, \mathcal{O}_1(z_1) \dots \mathcal{O}_6(z_6) \rangle = \sum_{k=2,5} \frac{f(z_k) E_k}{z - z_k} \langle \mathcal{O}_1(z_1) \dots \mathcal{O}_6(z_6) \rangle$$

← shifts boost weight by one

$f(z) = 1$ global translation

SUPERTRANSLATIONS

Under $\bar{z} \to \frac{a\bar{z}+b}{c\bar{z}+d}$, $\exists$ SL2R doublet of currents $P^{\pm}(z)$. Moreover $\exists$ SL2R triplet starting from superrotation generator $\partial_{\bar{z}} \int_{-\infty}^{\infty} du\, u\, D^{\bar{z}} N_{\bar{z}\bar{z}} \doteq 0$ Continuing infinite tower from $\int du\, u^{1-n} D^{\bar{z}} N_{\bar{z}\bar{z}}$ denoted $H_n^k(z)$, $k = 1, 0, -1 \ldots$, $\frac{k-2}{2} \leq n \leq \frac{2-k}{2}$ of currents.

Highly reminiscent of Newman-Penrose conserved quantities from 60s.

SYMMETRY ALGEBRA

2103.03961

Guevara, Himwich, Pate AS

We found an infinite tower of left-moving currents in finite-dimensional $Sl2_R$ representations

$$\left[H^k_n, H^l_{n'}\right] = -\frac{\kappa}{2}\left[n'(2-k) - n(2-l)\right] \frac{(\frac{2-k}{2} - n + \frac{2-l}{2} - n' - 1)!\,(\frac{2-k}{2} + n + \frac{2-l}{2} + n' - 1)!}{(\frac{2-k}{2} - n)!(\frac{2-l}{2} - n')!\,(\frac{2-k}{2} + n)!(\frac{2-l}{2} + n')!} H^{k+l}_{n+n'},$$

$\frac{k-2}{2} \le n \le \frac{2-k}{2}$

$k, l = 1, 0, -1, \ldots$

(o) Exhibits power of celestial formulation

(i) Obeys Jacobi, Poincare - fully determined by?

(ii) Infinite tower of soft thms, symmetries & memories.

(iii) $k = 1, 0$ generate all gauge, $k = 1, 0, -1$ all gravity

(iv) These are symmetries of nature w/ in principle observable memory effects. $k = 1, 0$ known.

Conclusion

Gravitational scattering is a rich problem theoretically & exp. Asymptotic symmetries, soft theorems and the celestial reformulation are leading to new insights. There is much yet to be understood.


Thank you for listening!