Quantum Gravity Meets Statistical Physics

Jan de Boer, Amsterdam

Based on:
-Alex Belin, JdB, arXiv:2006.05499
-Alex Belin, JdB, Pranyal Nayak, Julian Sonner, arXiv:201207875

GGI, May 12, 2021

Also based on ongoing work with Alex Belin, Julian Sonner, Pranjal Nayak, Diego Liska, Tarek Anous, Igal Arav and Shira Chapman.

We normally view gravity as a low-energy effective field theory.

This would normally imply that gravity has no access to or possesses information about energies $E \gtrsim \Lambda_{\mathrm{UV}}$

But gravity is also very different from standard low-energy effective field theory.

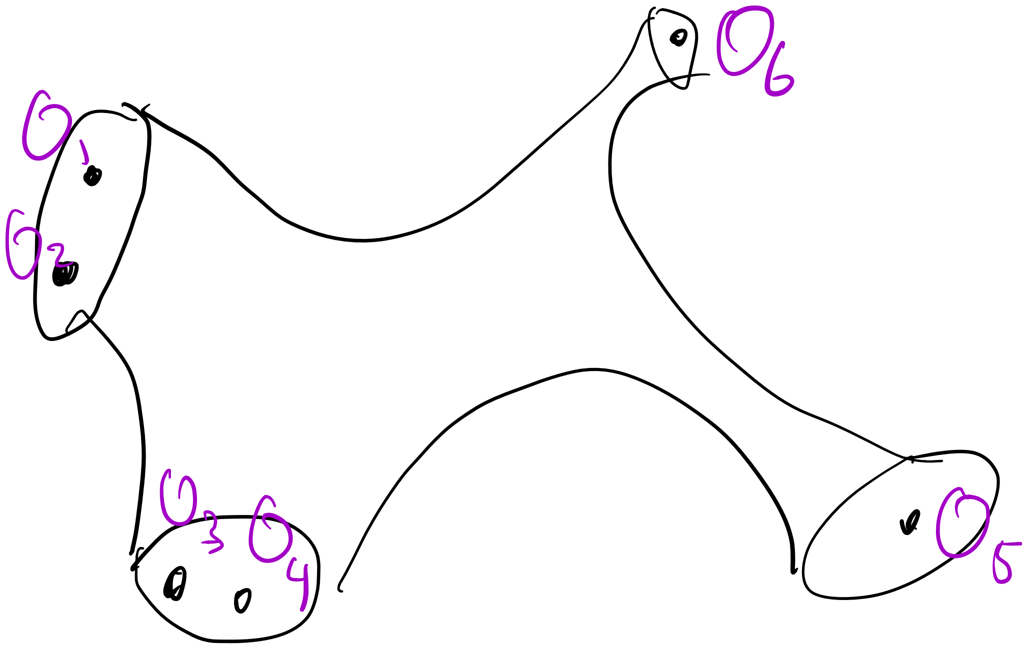
It knows for example about

- Black hole entropy – the high temperature partition function
- The partition function on various Euclidean manifolds in AdS/CFT (eg finite temperature correlators)
- The page curve (using island/replica wormholes)

Penington ’19
Almheiri, Engelhardt, Marolf, Maxfield ‘19
Penington, Shenker, Stanford, Yang ‘19
Almheiri, Hartman, Maldacena, Shaghoulian, Tajdini ‘19

From a field theory point of view, these seem to all be related to *coarse grained* UV data.

Gravity knows about certain Euclidean wormholes



Whether or not to include such wormholes or other configurations depends on what we mean by “low-energy effective field theory”

What do we mean by the low-energy effective field theory?

- Do we view it as a “complete” theory as in 2d JT gravity or as in 3d gravity?
- Should we take all semiclassical saddles seriously? In particular, how serious should we take wormhole solutions?
- Are correlators with exponentially long timescales part of the low-energy effective field theory?
- Does it include non-perturbative objects such as D-branes?

In this talk I will consider the question: what is the dual quantum mechanical description of a low-energy effective field theory which includes gravity?

The word “dual” refers to AdS/CFT, but the lessons learned are hopefully more generally applicable.

As we will see, the dual description will depend on what we mean by “low-energy effective field theory”.

This dual description can also have different UV completions which may be a single UV theory or it may be an ensemble of UV theories, which may once more depend on various choices that one can make.

The spectral density

From the entropy of a black hole we obtain an *approximate* expression for

$$Z(\beta) = \int dE \rho(E) e^{-\beta E}$$

Typically, we do not have the required exp(-S) accuracy to resolve the exact density of states

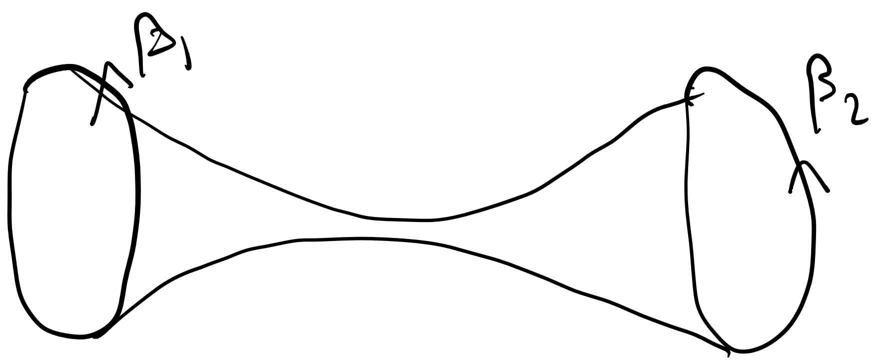
$$\rho(E) = \sum_i \delta(E - E_i)$$

We would then be able to see all the individual microstates of the black hole in LEEFT. Exceptions could be integrable models, topological theories, or BPS black holes.

What about

$$\langle Z(\beta_1) Z(\beta_2) \rangle_c \quad \text{or} \quad \langle \rho(E_1) \rho(E_2) \rangle_c$$

In a single microscopic theory such connected two point functions vanish. But the existence of wormholes suggests that LEEFT may yield a non-zero answer.



How can that be?

Consider a large set of N random phases $e^{i\phi_i}$

$$\left(\sum e^{i\phi_i}\right)\left(\sum e^{i\phi_i}\right)^* = \underbrace{\sum_{i=j} 1}_{\substack{\text{N} \\ \text{visible in} \\ \text{gravity}}} + \underbrace{\sum_{i\neq j} e^{i(\phi_i-\phi_j)}}_{\substack{\text{erratic} \\ \text{invisible in} \\ \text{gravity}}}$$

So LEEFT is sensitive to the average size of fluctuations
but not to the individual fluctuations themselves

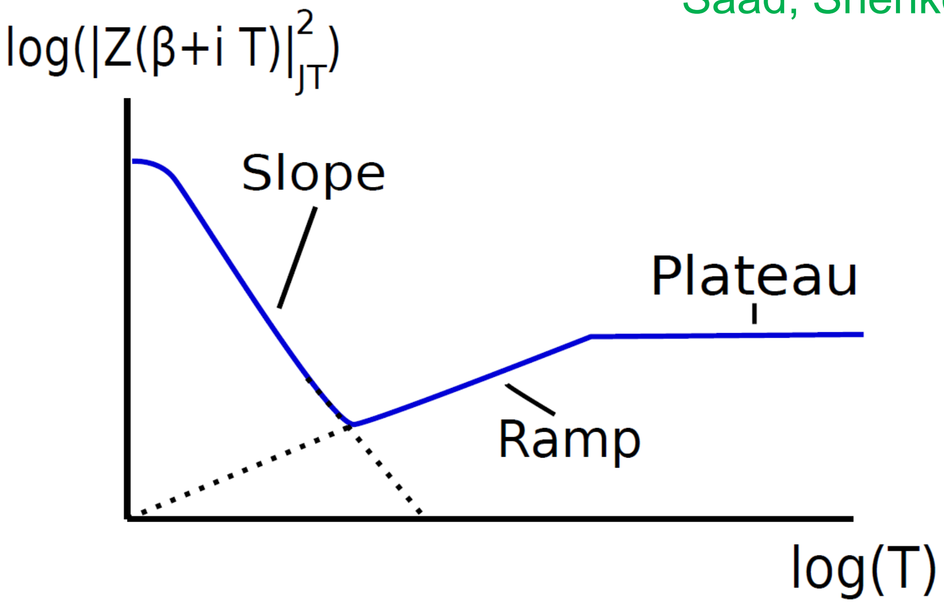
$$\left(\sum e^{i\phi_i}\right)\left(\sum e^{i\phi_i}\right)^* = \sum_{i=j} 1 + \sum_{i\neq j} e^{i(\phi_i - \phi_j)}$$

What happens in the UV? Possibilities:

- The relevant gravitational solution (eg wormhole) is unstable and factorization is restored (but solution remains as off-shell configuration)
- UV physics adds the fluctuating contributions $\sum_{i\neq j} e^{i(\phi_i - \phi_j)}$ and factorization is restored
- The UV theory is an average of theories, averaging makes the fluctuating term exactly zero, and factorization is not restored

Using wormholes in JT gravity, one can find the following picture for the so-called spectral form factor

Saad, Shenker, Stanford, '19



Knows about some discrete features of the spectrum (eigenvalue repulsion) - but spoils factorization

What is the right formalism to capture these features?

$$\rho_{\mathrm{micro}} \Longrightarrow \rho_{\mathrm{LEEFT}} = \int \mu[\rho] d\rho$$

This can arise from UV coarse graining but also through averaging over theories

$$\mu[\rho] = \int dJ \delta(\rho - Z[J]^{-1} e^{-\beta H[J]})$$

$$\bar{\rho} = \int \mu[\rho] d\rho \, \rho$$

$$\overline{\rho \otimes \rho} = \int \mu[\rho] d\rho \, \rho \otimes \rho \neq \bar{\rho} \otimes \bar{\rho}$$

Assume for example that

$$\overline{\rho \otimes \rho} = A \otimes A + SB \otimes B$$

with S the swap operator, then

$$\overline{\mathrm{Tr}(\rho O_1)\mathrm{Tr}(\rho O_2)} = \underbrace{\mathrm{Tr}(AO_1)\mathrm{Tr}(AO_2)}_{\text{disconnected}} + \underbrace{\mathrm{Tr}(BO_1 BO_2)}_{\text{connected}}$$

Related work:

Pollack, Rozali, Sully, Wakeham ‘20
Liu, Vardhan ‘20
Altland, Sonner ‘20
Janssen, Mirbabayi, Zograf ’21

What are the principles? What is the general structure? Can such a structure emerge from an RG flow?

Is this structure sufficient to capture all the features of LEEFT?

It is not clear how to capture finite temperature correlation functions using state averages only.

To capture those we replace the high-energy matrix elements of operators by statistical quantities.

The Eigenstate Thermalization Hypothesis (ETH) is a well-known hypothesis which connects precision low-energy data with statistical high-energy data:

$$\langle E_i|O_a|E_j\rangle = \delta_{ij} f_a(\bar{E}) + e^{-S(\bar{E})/2} g_a(\bar{E},\Delta E) R^a_{ij}$$

Deutsch '91
Srednicki '94
Foini, Kurchan '19

$f_a(\bar{E})$: one point functions of simple operators

$g_a(\bar{E},\Delta E)$: two point functions of simple operators

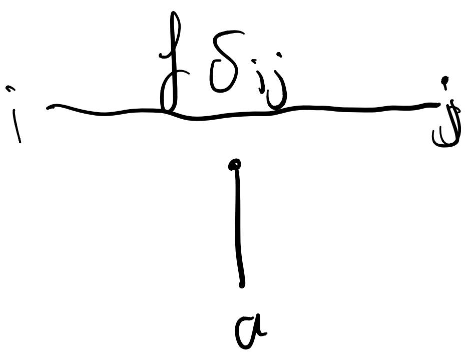
R^a_{ij} : Gaussian random variables

$$\langle R^a_{ij}\rangle = 0, \qquad \langle R^a_{ij} R^b_{kl}\rangle = \delta^{ab}\delta_{il}\delta_{jk}$$

Split the spectrum in low-energy (light) and high-energy (heavy).

Can represent ETH as a picture

$$\langle E_i | \mathcal{O}_a | E_j \rangle =$$



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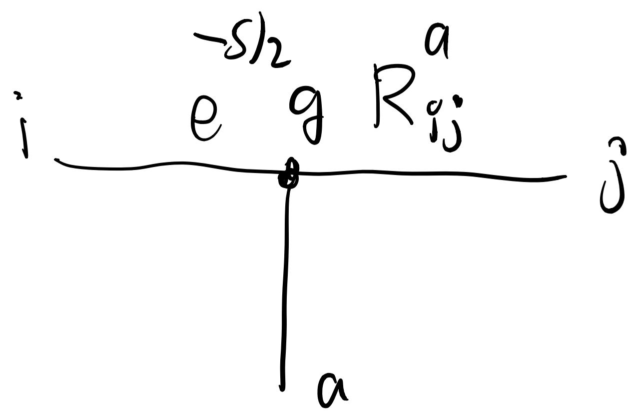
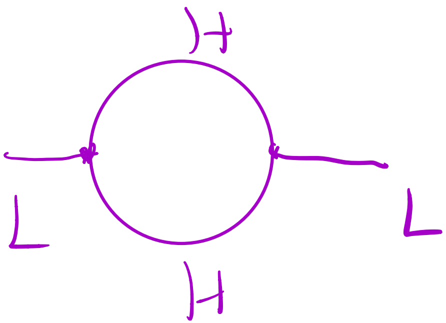
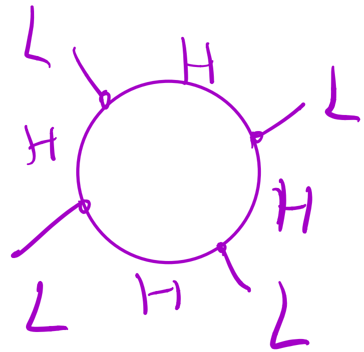
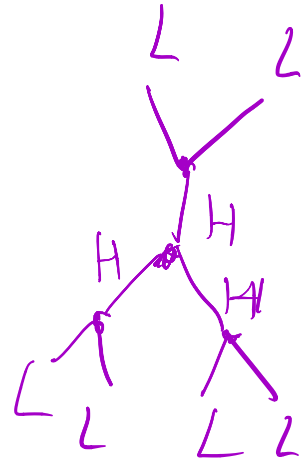
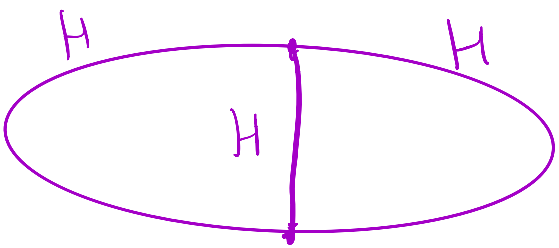


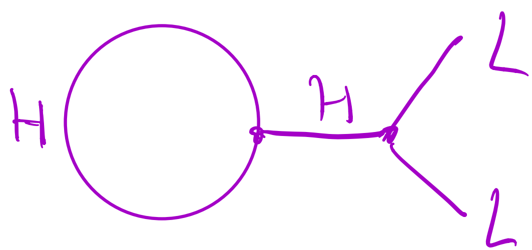
In AdS/CFT, the three-point vertices correspond to OPE coefficients. Can draw diagrams with loops but with only low-energy-light operators on the external legs

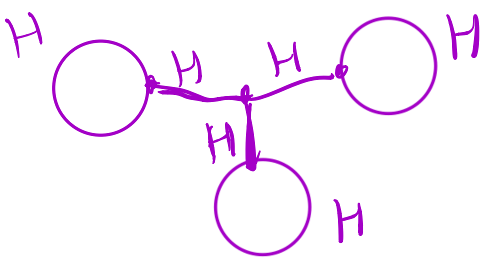












We propose that this generalizes to other OPE coefficients as well, leading to the OPE Randomness Hypothesis:

$C_{ijk}, C_{ija}, C_{iab}$ are random variables in the heavy Indices with an approximate Gaussian distribution

Intuition is based on the fact that it is very difficult to distinguish high-energy states.

Gravity (including wormholes) can resolve the moments of these randomly distributed variables but not their individual values.

(see purple pictures!)

Belin, JdB '20

On OPE coefficients:

- Distributional properties were first analyzed in Pappadopulo, Rychkov, Espin, Rattazzi ‘12
- In d>2 the diagonal term C_{iia} can be computed from the finite temperature one-point function Gobeil,Maloney,Ng,Wu ’16
- Off-diagonal terms $|C_{ija}|^2 \sim e^{-S}$ were recently obtained from hydrodynamics in d>2 by Delacrétaz ‘20
- In d=2 Collier, Maloney, Maxfield, Tsiares ’19 argued for an extension of ETH based on asymptotics for all cases that involve at least one heavy operator:

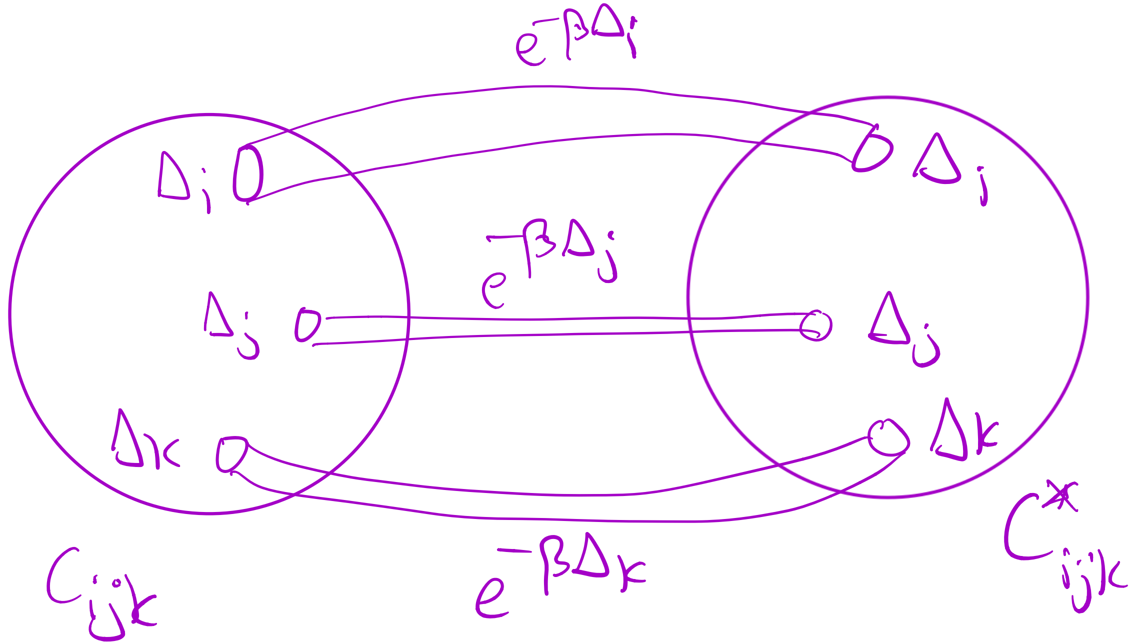
$$\overline{C^2_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}}} \approx 16^{-\Delta} e^{-2\pi\sqrt{\frac{c-1}{12}\Delta}} \Delta^{2(\Delta_1+\Delta_2)-\frac{c+1}{4}}, \qquad \Delta \gg c, J, \Delta_i, J_i$$

$$\overline{C^2_{\mathcal{O}_0\mathcal{O}_1\mathcal{O}_2}} \approx e^{-4\pi\sqrt{\frac{c-1}{12}\Delta_1}} \Delta_1^{\Delta_0}, \qquad \Delta_1, \Delta_2 \gg c, J_i, \Delta_0, J_0, |\Delta_1 - \Delta_2|$$

$$\overline{C^2_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_3}} \approx \left(\frac{27}{16}\right)^{3\Delta_1} e^{-6\pi\sqrt{\frac{c-1}{12}\Delta_1}} \Delta_1^{\frac{5c-11}{36}}, \qquad \Delta_1, \Delta_2, \Delta_3 \gg c, J_i, |\Delta_i - \Delta_j|$$

The square of the genus two partition function

$$Z_{g=2\times g=2} = \left(\sum_{i,j,k} C_{ijk}C^*_{ijk}e^{-3\beta\Delta}\right)\left(\sum_{l,m,n} C_{lmn}C^*_{lmn}e^{-3\beta\Delta}\right)$$



The square of the genus two partition function

$$Z_{g=2\times g=2} = \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta}\right)\left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta}\right)$$

C: Gaussian variables

Standard Wick contraction

Reproduces $\left(Z_{g=2}\right)^2 \sim e^{\frac{c}{2}\frac{\pi^2}{\beta}}$

The square of the genus two partition function

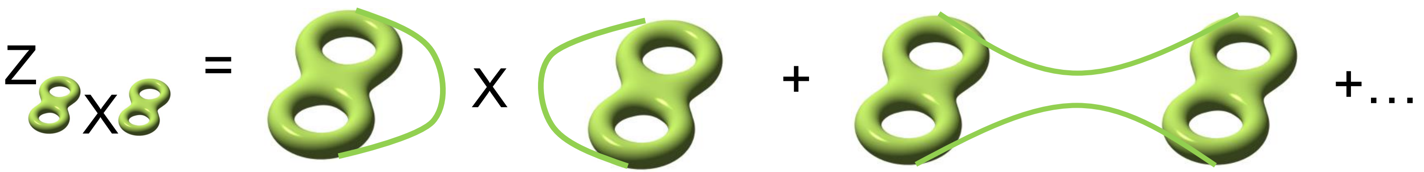
$$Z_{g=2\times g=2} = \left(\sum_{i,j,k} C_{ijk} C^*_{ijk} e^{-3\beta\Delta}\right)\left(\sum_{l,m,n} C_{lmn} C^*_{lmn} e^{-3\beta\Delta}\right)$$

C: Gaussian variables
Non-Standard Wick contraction

Result:

$$\begin{aligned} Z^{\text{nonstandard}}_{g=2\times g=2} &= \sum_{\Delta} \left(\frac{27}{16}\right)^{-6\Delta} \left(e^{2\pi\sqrt{\frac{c}{3}\Delta}}\right)^3 f(\Delta)^2 e^{-6\beta\Delta} \\ &\simeq \sum_{\Delta} e^{-6\beta\Delta} = \mathcal{O}(1) \end{aligned}$$

Agrees with the existence of a genus two wormhole



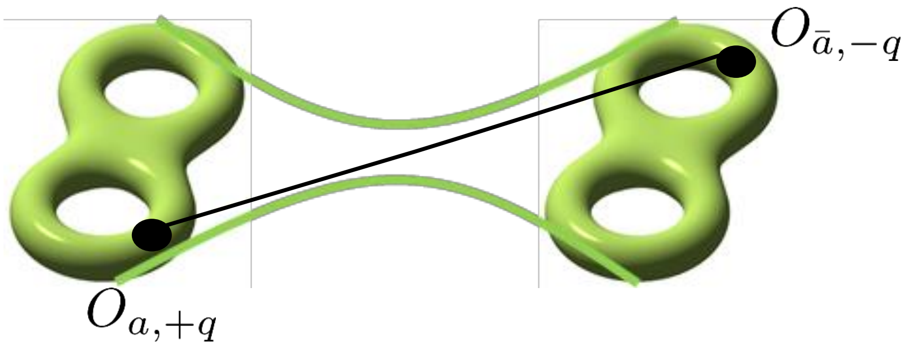
$$ds^2 = \ell_{\mathrm{ads}}^2 (d\tau^2 + \cosh^2 \tau d\Sigma_g^2)$$

Maldacena, Maoz ‘04

$$\left(1 + b e^{-\frac{c}{2}\frac{\pi^2}{\beta}}\right)$$

The wormhole does not compute a fluctuation but simply confirms the Gaussian statistics.

Another example: charged correlators in theories with global symmetries Belin, JdB, Nayak, Sonner '20



If the gravitational theory has a global symmetry then this two point function does not vanish.

But this is weird because the one-point functions are strictly zero in the CFT and there is no room for fluctuations.

Two possibilities:

1. The global symmetry in gravity is actually gauged. Then the wormhole contribution vanishes.

2. The global symmetry must be weakly broken with breaking of the order e^{-S}

   →then the magnitude of the one-point function will be of the order of the fluctuation as computed by the wormhole.

To reproduce the second possibility, we need a version of ETH which violates charge conservation

$$\langle E_i, q_i | O_{a,Q} | E_j, q_j \rangle = \delta_{ij}\delta_{Q,0} f_a(\bar{E}, \bar{q}) + e^{-S(\bar{E},\bar{q})/2} g_a(\bar{E}, \Delta E, \bar{q}, \Delta q) R^a_{ij}$$

This is different from the more conventional ETH which respects charge conservation and which is needed for the first possibility

$$\langle E_i, q_i | O_{a,Q} | E_j, q_j \rangle = \delta_{ij}\delta_{Q,0} f_a(\bar{E}, \bar{q}) + e^{-S_{q_i}(\bar{E})} g_a(\bar{E}, \Delta E, q_i, q_j) R^a_{ij} \delta_{q_i, Q+q_j}$$

Notice that both reproduce the correct finite temperature one- and two-point functions, but differ in their prediction for the wormhole.

DISCUSSION

- With OPE coefficients as random variables we can capture many aspects of a gravitational low-energy effective field theory.
- Gravity as LEEFT computes the moments of these random variables but not their individual values.
- It is an interesting question what the minimal statistical structure is that we need; is universal wave function statistics sufficient?
- Is all we need a version of random matrix theory?
- Is there a notion of averaging which produces the required type of OPE statistics with/without charge? (cf a lot of recent work on averaging)
- Do we need additional structure in d>2? (e.g. statistics of line operators)

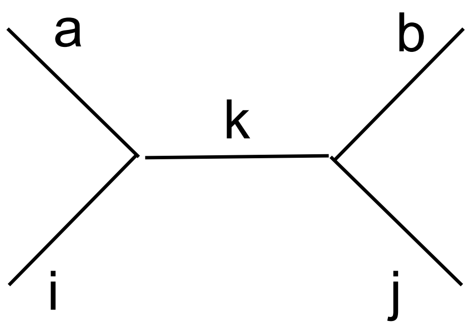
Can we combine the

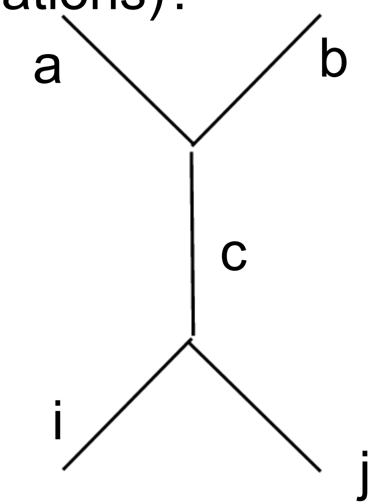
two pictures ?

DISCUSSION

- Are corrections to Gaussianity important? Can be explored by considering moduli dependence, correlation functions on various surfaces, the non-Cardy regime, etc: more details, less universality.

- What are the implications of crossing (is there a statistical solution of the bootstrap equations)?





$$R^{a}_{ij} R^{b*}_{kl} = \delta^{ab}(\delta_{ik}\delta_{jl}) + hC_{a}{}^{bc}\,(R^{c}_{ki}\delta_{jl} + \ldots)$$

DISCUSSION

- It would be interesting to reobtain the Page curve from this perspective or does that require additional input?
- We have focused on saddle points. The could also be off-shell configurations which spoil factorization and contribute, but beyond d=2,3 it seems hard to control such off-shell computations. It is also not clear we should even in principle allow such off-shell configurations in low-energy effective field theory. (cf Cotler, Jensen '20)
- Implications for cosmology?

- LEEFT BULK = LEEFT BOUNDARY + CROSSING SYMMETRY ?

+ MODULAR INVARIANCE