String Interactions and Memory Effects

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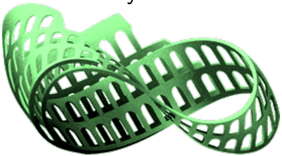


A. Addazi, M. Bianchi, MF, A. Marcianò Nucl. Phys. B **965** (2021), 115356
A. Aldi, M. Bianchi, MF Phys. Lett. B **813** (2021), 136037
A. Aldi, M. Bianchi, MF arXiv:2101.07054

Plan

STRING MEMORY EFFECTS IN GRAVITATIONAL WAVES

- general idea
- string memory from scattering processes (NP α' corrections)
- scattering regimes and phenomenological implications

STRING MEMORY EFFECTS IN ELECTRO-MAGNETIC WAVES

- general Idea
- NP α' corrections
- spinning corrections (NP in α')
- phenomenological implications

Gravitational Memory

In General Relativity the GW profile $h_{\mu\nu}$ produced by a source $S_{\mu\nu}$ obeys

$$\Box h_{\mu\nu}(t,\vec{x}) = -16\pi G_N \, \mathcal{S}_{\mu\nu}(t,\vec{x})$$

the causal retarded wave, emitted from $\vec{x}'$

$$h_{\mu\nu}(t,\vec{x}) = 4G_N \int d^3x' \, \frac{\mathcal{S}_{\mu\nu}(t-|\vec{x}-\vec{x'}|,\vec{x'})}{|\vec{x}-\vec{x'}|}$$

In (GR) high energy collisions the produced GW observed at large distance, $|\vec{x}|>>|\vec{x}'|$ (dipole approx.), has a plane-wave behavior

$$h^{(\omega)}_{\mu\nu}(t,\vec{x}) = \tilde{e}_{\mu\nu}(\omega,\vec{x})\, e^{-\imath\omega t} + \tilde{e}^*_{\mu\nu}(\omega,\vec{x})\, e^{\imath\omega t}$$

the structure of the polarization tensor is dictated by the Weinberg's soft theorem

$$\tilde{e}^{\mu\nu}(\omega,\vec{x}) = \frac{4G_N}{\omega|\vec{x}|}\sum_a \frac{p_a^{(\mu}p_a^{\nu)|}}{n_\omega\cdot p_a}\, e^{\imath\omega|\vec{x}|}$$

p_a momentum of the a^{th} particle involved in the process, $\sum_{a\in out} p_a = \sum_{a\in in} \tilde{p}_a$

integrating over the frequency ω, with causality prescription for $t-|\vec{x}|>0$, using

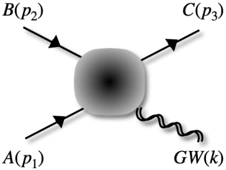
$$\frac{1}{\omega - i\varepsilon} - \frac{1}{\omega + i\varepsilon} = 2\pi i\delta(\omega)$$

one finds a constant term, which is the gravitational memory

$$e^{\mu\nu}(t,\vec{x}) = -\frac{4G_N}{R}\left\{\sum_{a\in out}\frac{p_a^{(\mu}p_a^{\nu)|}}{np_a} - \sum_{a\in in}\frac{\tilde{p}_a^{(\mu}\tilde{p}_a^{\nu)|}}{n\tilde{p}_a}\right\}$$

String Gravitational Memory

String scattering amplitude SOURCE of GW



$$\mathscr{M}_{3+1} \Rightarrow S^{\mu\nu} = \frac{\delta \mathscr{M}_{3+1}}{\delta h_{\mu\nu}}$$ (String source)

$$\Box\, e_{\mu\nu}(t, \vec{x}) = -\frac{\delta \mathscr{M}_{3+1}}{\delta h_{\mu\nu}}$$ GW emission from string scattering amplitude

Using the amputated amplitude, the GW polarization reads

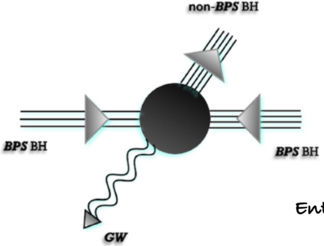
$$\tilde{e}^{\mu\nu}(\omega, \vec{x}) = \int d^3y \frac{e^{\imath\omega|\vec{x}-\vec{y}|}}{4\pi|\vec{x}-\vec{y}|} \widetilde{\mathcal{M}}_{3+1}^{(\mu\nu)|}(\omega, \vec{y}; p_a) \underset{|\vec{x}|>>1}{\Longrightarrow} \frac{e^{\imath\omega R}}{4\pi R} \mathcal{M}_{3+1}^{(\mu\nu)|}(\omega, \vec{k} = \omega\vec{n}; p_a)$$

anti-Fourier transforming to time, the GW profile is given by

$$e^{\mu\nu}(t, \vec{x}) = \frac{4G_N}{R} \sum_a \frac{p_a^{(\mu} p_a^{\nu)|}}{n \cdot p_a} \int_{-\infty}^{+\infty} \frac{d\omega}{2\pi\omega}\, e^{-\imath\omega u}\, \mathcal{F}(\omega, n \cdot p_a)$$

where $u = t - R$

Heterotic String Setup



Heterotic string: $SUSY_L \otimes BOSE_R$

Masses of Stringy BH states

$$M^2_{BPS} = |\mathbf{P}_L|^2 = |\mathbf{P}_R|^2 + \frac{4}{\alpha'}(N_R - 1)\,,$$
$$M^2_{non-BPS} = |\mathbf{P}_L|^2 + \frac{4}{\alpha'}N'_L = |\mathbf{P}_R|^2 + \frac{4}{\alpha'}(N_R - 1)$$

Entropy of the states: Hardy-Ramanujan-Hagedorn formula

$$d(\mathbf{P}_L, \mathbf{P}_R) = e^{S_{BH}/\kappa_B} \approx e^{2\pi\sqrt{\frac{\alpha'}{4}(|\mathbf{P}_L|^2 - |\mathbf{P}_R|^2)}} = e^{2\pi\sqrt{N_R}}\,.$$

massive states are chosen to be spinless $J=0$ and compact $R \sim G_N M$

in the perturbative regime ($g_s << 1$)

- mass eigenstates are not compact $R \sim \alpha' M > G_N M$
- Coherent states are compact

From the tree-level process **BPS BH + BPS BH → non-BPS BH + GW**

$$\mathcal{M}_{3+1}(h, k; p_a, \zeta_a) = 16\pi G_N \mathcal{M}_{3BH}(p_a, \zeta_a) \sum_a \frac{p_a \cdot h \cdot p_a}{k \cdot p_a} \prod_b \frac{\Gamma(1 + k \cdot p_b)}{\Gamma(1 - k \cdot p_b)}$$

where $G_N \sim g_s^2 \alpha' \frac{\alpha'^3}{\mathcal{V}_6}$ and $\mathcal{M}_{3BH}(p_a, \zeta_a)$ is physical and non zero also when $k \to 0$

The amplitude results factorized as the product of the soft factor with the Shaphiro-Virasoro factor but the graviton momentum k is not soft ! ! !

String contributions from the amplitude

the characteristic string scales are $\alpha' k \cdot p_a = \alpha' \omega n \cdot p_a$

- choosing string states with masses $M \simeq 20\, M_\odot$
- for a VIRGO/LIGO frequency of 10 - 100 Hz

$$\alpha' k \cdot p_a \simeq O(1) \Longleftrightarrow e^{-iu/\alpha' n \cdot p_a}$$

not so small compared with GR contributions, even when $T_s \sim 1/\alpha'$ is around the GUT scale or beyond

String Gravitational Memory from BPS-BH's Merging

From **BPS BH + BPS BH → non-BPS BH + GW** the GW profile reads

$$e^{\mu\nu}(t,x) = \frac{4G_N}{R}\sum_a \frac{p_a^{(\mu}p_a^{\nu)|}}{\ell_a}\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\,e^{-i\omega u}\,\frac{\mathcal{F}(\omega,\ell_a)}{\omega}$$

with scattering length $\ell_a = \alpha' n{\cdot}p_a = \alpha'(E_a - \vec{n}{\cdot}\vec{p}_a)$, and $\sum_a \ell_a = 0$

$\mathcal{F}(\omega,\ell_a)/\omega$ admit a Mittag-Leffler expansion in the infinite number of ω poles

$$\frac{\mathcal{F}(\omega,\ell_a)}{\omega} = \frac{1}{\omega}\prod_{a=1}^{3}\frac{\Gamma(1+\omega\ell_a)}{\Gamma(1-\omega\ell_a)} = \frac{1}{\omega} + \sum_{a=1}^{3}\sum_{n_a=1}^{\infty}\frac{R_{n_a}(\lambda_{b,a})}{\omega + n_a/\ell_a}$$

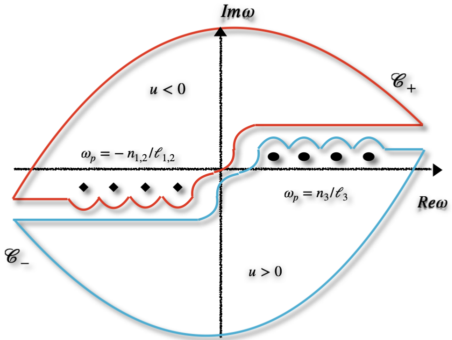
where the residues have the form

$$R_{n_a}(\lambda_{b,a}) = \frac{(-)^{n_a}}{(n_a!)^2}\prod_{b\neq a}\frac{\Gamma(1-n_a\lambda_{b,a})}{\Gamma(1+n_a\lambda_{b,a})}, \quad \lambda_{b,a} = \frac{\ell_b}{\ell_a}$$

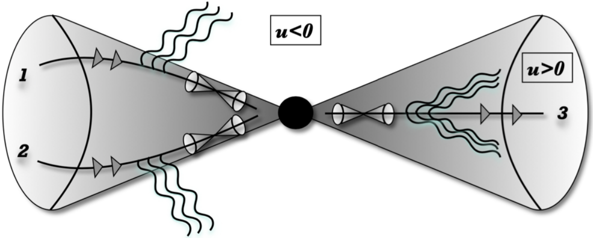
The first term of the pole expansion (massless pole) reproduces the GW memory

while the rest of the expansion produces an infinite sum of corrections due to the infinite tower of massive string excitations

$$\Delta_s e^{\mu\nu}(t,\vec{x}) = \frac{4G_N}{R}\sum_{b=1}^{3}\frac{p_b^{(\mu}p_b^{\nu)|}}{\ell_b}\sum_{a=1}^{3}\sum_{n_a=1}^{\infty}R_{n_a}(\lambda_{b,a})\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\frac{e^{-i\omega u}}{\omega+n_a/\ell_a+i\epsilon}$$



the prescription produces a causality structure of the form



$$\Delta e^{\mu\nu}(t,\vec{x}) = \frac{4G_N}{R}\sum_{b=1}^{3}\frac{p_b^{(\mu}p_b^{\nu)|}}{n\cdot p_b}\left(\theta(-u)\sum_{a=1}^{2}\delta_a(\lambda_{b,a};u) - \theta(u)\,\delta_3(\lambda_{b,3};u)\right)$$

where

$$\delta_a(\lambda_{b,a};u) = \sum_{n_a=1}^{\infty} R_{n_a}(\lambda_{b,a})\,e^{in_a u/\ell_a}$$

The GW profile from the stringy source turns out to be

$$e^{\mu\nu}(t,\vec{x}) = e^{\mu\nu}_{\rm const}(t,\vec{x}) + \Delta_s e^{\mu\nu}(t,\vec{x})$$

there is a modulation of the signal in u/ℓ_a that is non-perturbative in α'

Rational Kinematics

In CoM fame of the non-BPS state (3), $\widetilde{M}_3 = E_3 + \omega = \omega + \sqrt{M_3^2 + \omega^2}$

$$E_1 = \frac{\widetilde{M}_3^2 + M_1^2 - M_2^2}{2\widetilde{M}_3}\,, \quad E_2 = \frac{\widetilde{M}_3^2 + M_2^2 - M_1^2}{2\widetilde{M}_3}\,, \quad |\vec{p}| = \frac{\sqrt{\mathcal{F}(M_1^2, M_2^2, \widetilde{M}_3^2)}}{2\widetilde{M}_3}$$

The physical region is spanned by $\mathcal{F}(M_1^2, M_2^2, \widetilde{M}_3^2) > 0$, so much so that

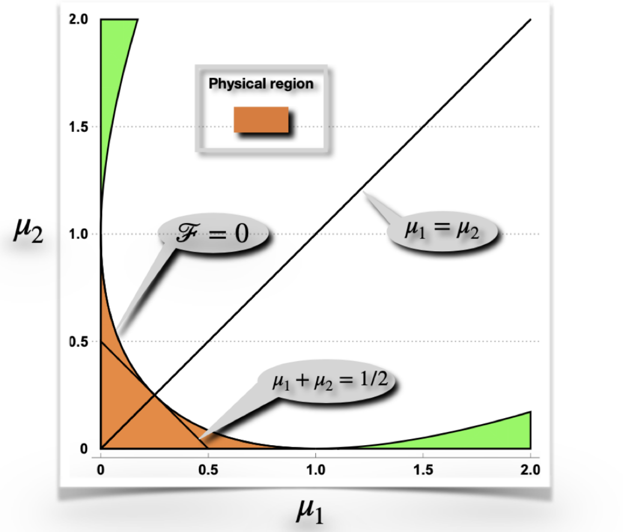
$$\mu_{1,2} = \frac{M_{1,2}^2}{\widetilde{M}_3^2}; \quad 0 < \mu_{1,2} < 1\,, \quad (\mu_1 - \mu_2)^2 - 2(\mu_1 + \mu_2) + 1 > 0$$

if $\mu_1 = \mu_2 = \mu$ one has simple expressions for $\lambda_{b,a}$

$$-1 < -\frac{1}{2}\left(1 + \sqrt{1 - 4\mu}\right) \leq \lambda_{13,23} \leq \frac{1}{2}\left(-1 + \sqrt{1 - 4\mu}\right) < 0$$

and by momentum conservation

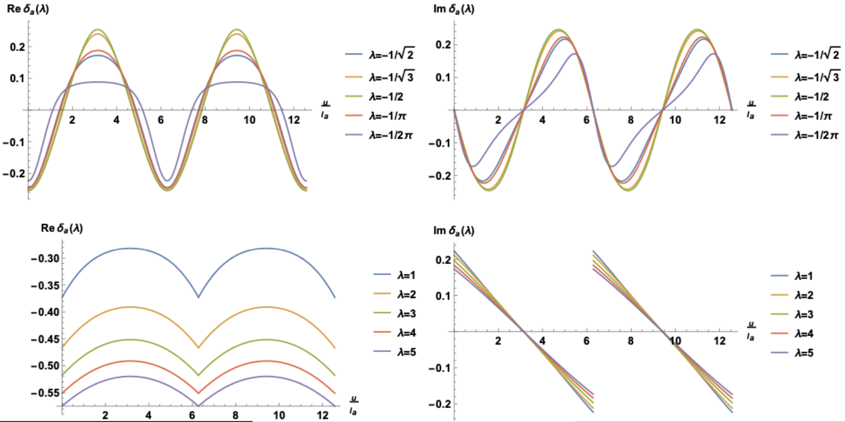
$$\lambda_{a+1,a} + \lambda_{a+2,a} = -1$$


Physical region
$\mathscr{F} = 0$
$\mu_1 = \mu_2$
$\mu_1 + \mu_2 = 1/2$
μ_2
μ_1
2.0
1.5
1.0
0.5
0
0
0.5
1.0
1.5
2.0

$$\delta_a(\lambda_{b,a}; u) = \sum_{n_a=1}^{\infty} R_{n_a}(\lambda_{b,a})\, e^{i n_a u/\ell_a}$$

$$\delta_3(\lambda_{13} = -1/2) = \sum_{n=1}^{\infty} \frac{(-)^n}{(n!)^2} \left(\frac{\Gamma(1+n/2)}{\Gamma(1-n/2)}\right)^2 e^{-inu/\ell_3} = -\frac{e^{iu/\ell_3}}{2\pi}\mathcal{K}\left(\frac{e^{-2iu/\ell_3}}{16}\right)$$

$$\delta_1(\lambda_{31} = 1) = \sum_{n=1}^{\infty} \frac{(-)^n}{(n!)^2} \frac{\Gamma(1+2n)\Gamma(1-n)}{\Gamma(1-2n)\Gamma(1+n)} e^{-inu/\ell_1} = -\frac{1}{2} + \frac{1}{\pi}\mathcal{K}(16\, e^{-iu/\ell_1})$$



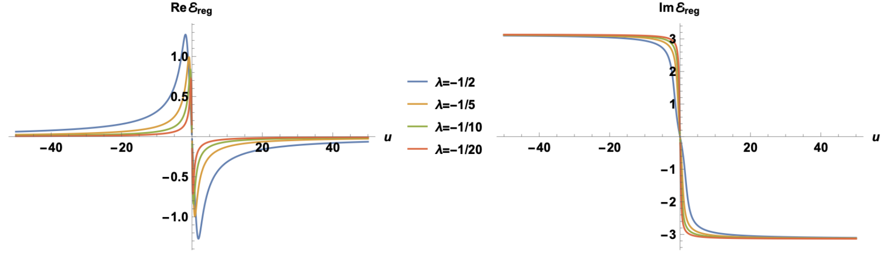
High Energy Regime

In the regime in which $\omega\ell_a >> 1$ the SV factor behaves as

$$\mathcal{F}_{SV} \simeq e^{-\beta\omega} \text{ with } -\beta = \ell_3(\lambda_{13}\log\lambda_{13} + \lambda_{23}\log\lambda_{23})$$

the string correction to GW profile becomes

$$\delta_{reg}(\lambda_{b,3}; u) = \int_{-\infty}^{\infty} d\omega\, \frac{e^{-i\omega u}}{\omega}(e^{-\beta|\omega|} - 1) = 2i \arctan\frac{u}{\beta}$$

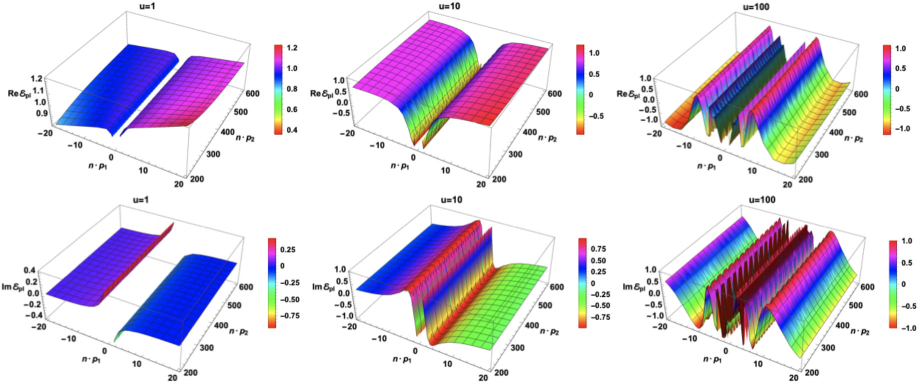


Regge Behaviour (Plunge)

$M_1 << M_2 \simeq M_3$ so that $\epsilon = \ell_1 << \ell_2 \simeq -\ell_3 = \ell$ and the SV factor reduces to

$$\mathcal{F}_{SV} \simeq (-\omega\ell)^{-2\omega\epsilon}$$

by saddle point estimation : $\delta_{pl}(\ell_a; u) = \exp\left(-\imath\frac{u}{2\ell_1} + \frac{2\ell_1}{\ell_2}\exp\frac{\imath u}{2\ell_1}\right)$

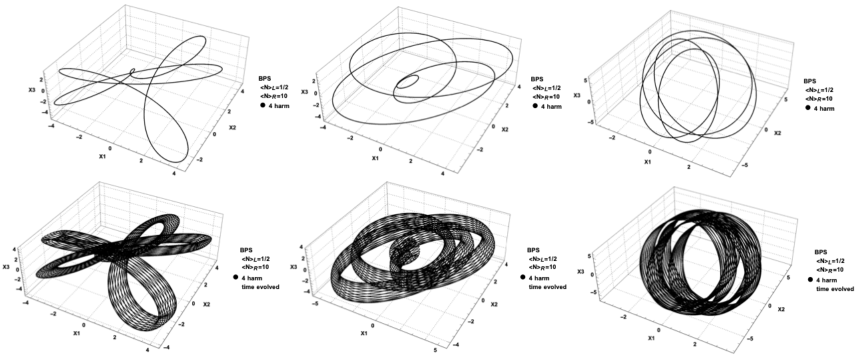


Coherent states picture

The averaged mass and gyration radius of a coherent state is given by

$$M^2 = \frac{1}{\alpha'} \sum_{n=1}^{\infty} |\lambda_n|^2 \,, \qquad R^2 = \alpha' \sum_{n=1}^{\infty} \frac{|\lambda_n|^2}{n^2}$$

where λ_n^μ is the polarization of the n^{th} harmonic. Choosing $\lambda_n^\mu = V^\mu e^{-\alpha n} n^\beta$

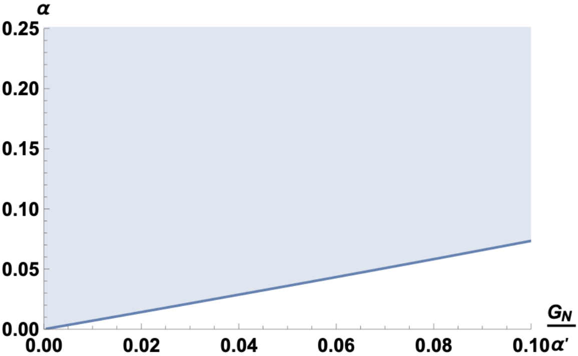


Coherent states can be compact in the perturbative regime $g_s << 1$

$$G_N = \frac{R}{M} \Longleftrightarrow \frac{g_s^2}{\mathcal{V}_{(6)}} = \sqrt{\frac{\sum_n \frac{|\lambda_n|^2}{n^2}}{\sum_n |\lambda_n|^2}} \approx 10^{-3} << 1 \quad (\text{no } \alpha' \text{ dependence})$$

in the specific case of $\lambda_n^\mu = V^\mu e^{-\alpha n} n^\beta$ one has

$$\frac{G_N}{\alpha'} = \sqrt{\frac{Li_{2-2\beta}(e^{-2\alpha})}{Li_{-2\beta}(e^{-2\alpha})}}\Bigg|_{\beta=1} = \sqrt{2}\sinh\alpha\sqrt{1-\tanh\alpha}$$



SECOND PART

Electro-Magnetic Memory

EM potential A^μ generated from the source J^μ at large distance $R = |\vec{x}| >> |\vec{x}'|$

$$\widetilde{A}^\mu(\omega, \vec{x}) = \int d^3x' \frac{e^{i\omega|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|} \widetilde{J}^\mu(\omega, \vec{x}') \approx \frac{e^{i\omega R}}{4\pi R} \widehat{J}^\mu(\omega, \vec{k} = \omega \vec{n})$$

QED amplitude with a soft photon and n hard particles in its leading contribution

$$\mathcal{A}_{n+1}^{QED}(a, k; p_j) = g \sum_{j=1}^{n} \frac{Q_j a \cdot p_j}{k \cdot p_j} \mathcal{A}_n^{QED}(p_j) + \ldots, \ (Q_j \text{ charge of the } j^{th} \text{ particle})$$

Classical EM potential at large distance (EM memory)

$$\widetilde{A}^\mu(\omega, \vec{x}) = g \frac{e^{i\omega R}}{\omega R} \sum_j \frac{Q_j p_j^\mu}{n p_j} \underset{(\omega)\text{integral}}{\Longrightarrow} \frac{g}{R} \left\{ \sum_{a \in out} \frac{Q_a p_a^\mu}{n p_a} - \sum_{a \in in} \frac{Q_a \tilde{p}_a^\mu}{n \tilde{p}_a} \right\}$$

Classical EM potential from spinning hard particles

$$\widetilde{A}^\mu(\omega, \vec{x}) = g \frac{e^{i\omega R}}{\omega R} \sum_j Q_j \frac{p_j^\mu + J_j^{\mu\nu} k_\nu}{n p_j} + \ldots \text{ where } J_j^{\mu\nu} = p_j^\mu \partial_{p_j}^\nu - p_j^\nu \partial_{p_j}^\mu + S^{\mu\nu}$$

String EM memory

Open string amplitudes as a source of EM potential

$$\widehat{J}^{ST}_{\mu}(k;p_j) = \frac{\delta \mathcal{A}^{ST}_{n+1}(a,k;p_j)}{\delta a^{\mu}(k)}$$

universal Veneziano's factor (n-pt generalizations)

the EM potential produced

$$\widetilde{A}^{\mu}_{ST}(\omega,\vec{x}) = \int d^3x' \frac{e^{i\omega|\vec{x}-\vec{x}'|}}{4\pi|\vec{x}-\vec{x}'|}\widetilde{J}^{\mu}_{ST}(\omega,\vec{x}';p_j) \approx \frac{e^{i\omega R}}{4\pi R}\frac{\delta \mathcal{A}^{ST}_{n+1}(a,k;p_j)}{\delta a_{\mu}(k)}\bigg|_{k^{\mu}=\omega(1,\vec{n})}$$

gives the QED behavior plus a universal correction (effect of infinite poles)

$$A^{ST}_{\mu}(t,\vec{x}) = A^{QED}_{\mu}(t,\vec{x}) + \Delta_s A_{\mu}(t,\vec{x})$$

with causal structure

$$\Delta_s A_{\mu} = \theta(u)\Delta^{(>)}_s A_{\mu} + \theta(-u)\Delta^{(<)}_s A_{\mu}\,, \quad u = t-R$$

From planar duality properties : $\Delta^{(>)}_s A_{\mu} + \Delta^{(<)}_s A_{\mu} = 0$

Some analytic results

Unoriented bosonic string

$$\Delta_s^{>} A^{\mu}(t,\vec{x}) \approx -\theta(u)\,\frac{g}{4\pi R}\sum_a \frac{p_a^{\mu}}{n\cdot p_a}\sum_{r=1}^{\infty}\frac{(-)^r}{r!}\,\frac{\Gamma(1-r\lambda_{a,3})}{\Gamma(1+r\lambda_{a+1,3})}\,e^{iru/\ell_3}$$

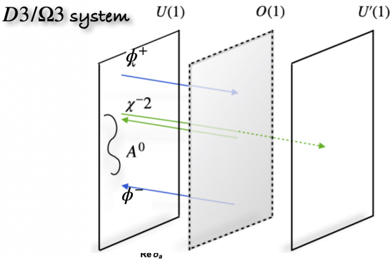
Type I string (with spin 1 particles)

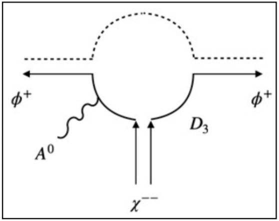
$$\Delta_s^{>} A^{\mu}(t,\vec{x}) \approx -\theta(u)\,\frac{g}{4\pi R}\sum_a \frac{1}{n\cdot p_a}\sum_{r=1}^{\infty}\frac{(-)^r}{r!}\left(p_a^{\mu} - n\cdot J_a^{\mu}\,\frac{r}{\ell_3}\right)\frac{\Gamma(1-r\lambda_{a,3})}{\Gamma(1+r\lambda_{a+1,3})}\,e^{iru/\ell_3}$$

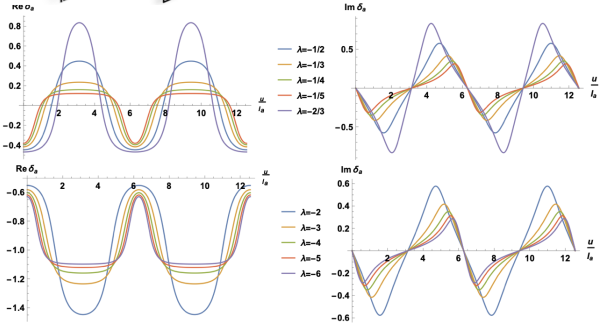
spinning correction with a general higher spin state of spin N

$$\Delta_{(N)}^{>} A^{\mu}(t,\vec{x}) \approx \sum_{h=1}^{N}\frac{d^h}{du^h}\Delta_s^{>} A^{\mu}(t,\vec{x})$$

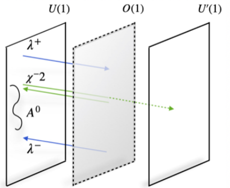
Corrections From Unoriented Bosonic String

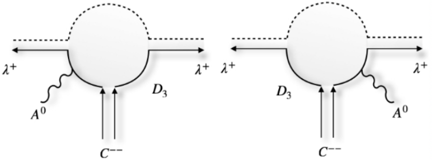


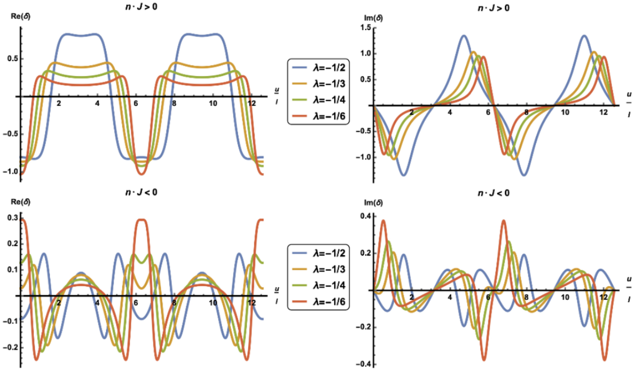




EM Memory in Type I

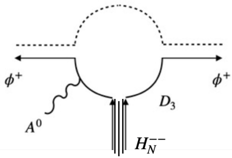






Spinning Correction to EM Memory

Higher spin states
of general spin N



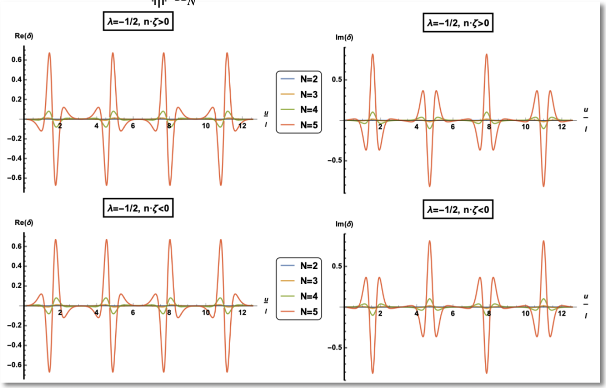
String modulation

$$A^\mu + \Delta^{(>)} A^\mu + \Delta_N^{(>)} A^\mu$$

Classical | Spinless | Spinning

Spinning corrections to the photon profile for different values of spin N

$$\Delta_N^{(>)} A^\mu$$



Conclusion

- Gravitational wave context
  - GW emitted from string interactions : $h_{\mu\nu}(t,\vec{x})\Big|_{\text{GW memo}} + \Delta_s h_{\mu\nu}(t,\vec{x})\Big|_{\text{St memo}}$
  - compact expressions and profiles of $\Delta_s h_{\mu\nu}(t,\vec{x})$ for special kinematics
  - $\Delta_s h_{\mu\nu}(t,\vec{x})$ in different scattering regimes, High Energy and Regge regime
  - coherent states picture, very massive and compact objects

- Electro-Magnetic wave context
  - EM-W emitted from string interactions : $A_{\mu}(t,\vec{x})\Big|_{\text{EM memo}} + \Delta_s A_{\mu}(t,\vec{x})\Big|_{\text{St memo}}$
  - compact expressions and profiles of $\Delta_s A_{\mu}(t,\vec{x})$ for special kinematics
  - spinnig corrections to the string EM memory
  - analysis done in both bosonic string and superstring theories