The perturbative Regge limit

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Gravitational scattering, inspiral and radiation Workshop,
Galileo Galilei Institute
May 10^{th}, 2021

The perturbative Regge limit

1. Phenomenology
2. Supersymmetry
3. Gravity

The perturbative Regge limit

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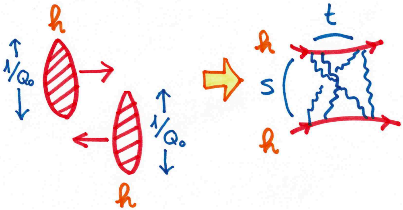
Hadron collisions governed by the strong interaction



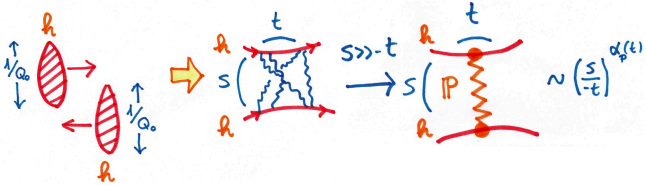
Non-perturbative system dominated by long distancies

Analyticity, unitarity and crossing symmetry in scattering amplitudes

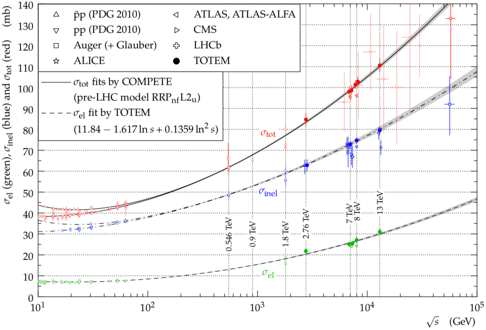
$$\mathcal{A}(s,t) \simeq \int d\omega \left(\frac{s}{-t}\right)^{\omega} f_\omega$$

$s \gg -t$: rightmost singularity in f_ω dominates

Vacuum singularity or Pomeron

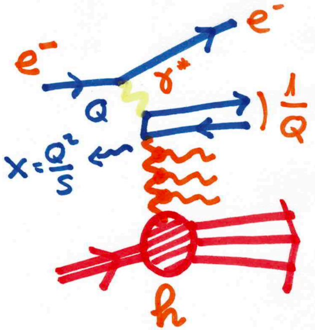


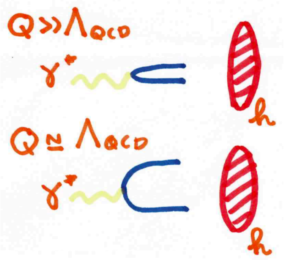
Growth of total cross section $\sigma_{\rm tot}(pp) \sim s^{\alpha_P(0)-1} \sim s^{0.1}$



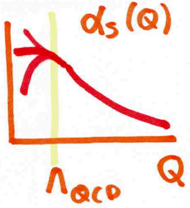
1. Phenomenology

Deep Inelastic Scattering is similar: $e^- -$ proton
Photon virtuality fixes the resolved distances in the hadron

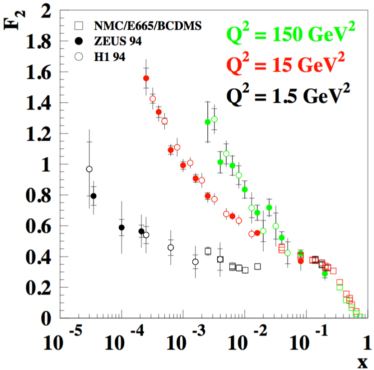


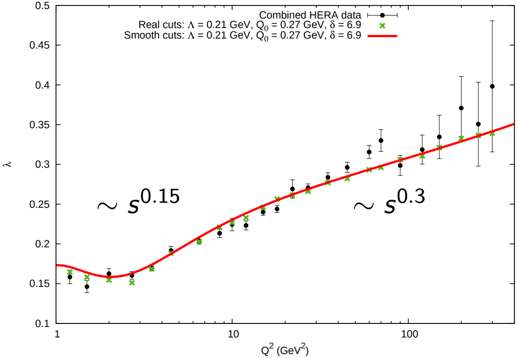


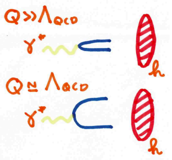
$s \gg -t \Longrightarrow$ Small Bjorken $x = \frac{Q^2}{s}$
Q large $\Longrightarrow$ perturbative process
Q small $\Longrightarrow$ non-perturbative



Fast growth of $F_2(x, Q^2) \simeq x^{-\lambda(Q)}$ if $x \to 0$



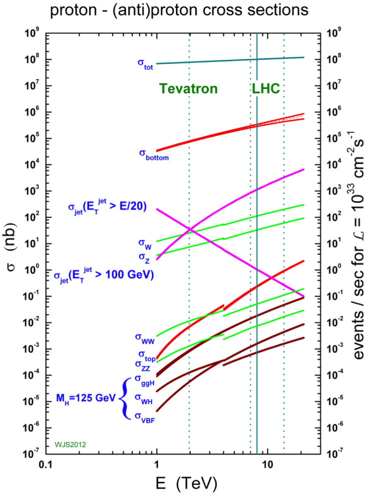


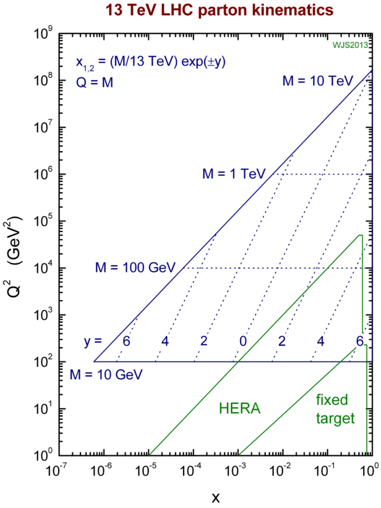


Perturbative: Hard Pomeron $\sim s^{0.3}$

Non-perturbative: Soft Pomeron $\sim s^{0.15} \simeq \sigma_{\rm total}(pp)$

$\sigma_{\mathrm{tot}}(pp)$ dominated by QCD processes to be understood
Big theoretical uncertainty in parton distribution functions at low x (LHC)

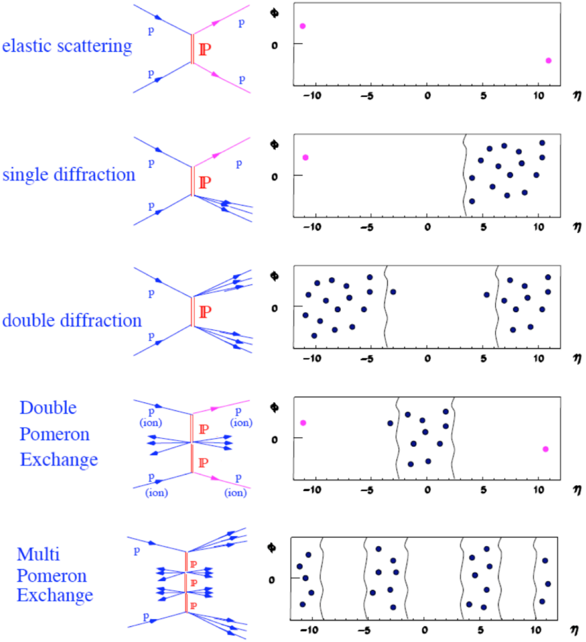




1. Phenomenology

Regge theory describes processes with low and high multiplicity

Pre-QCD. Description in terms of quarks and gluons?



1. Phenomenology

Identify scale $Q > \Lambda_{\rm QCD}$ to use perturbation theory $\alpha_s(Q) \ll 1$

$\alpha_s(Q) \log\left(\frac{s}{t}\right) \sim \mathcal{O}(1)$ in the limit $s \gg -t, Q^2$ & dominate amplitudes

BFKL resums $\alpha_s^n \log^n(s) \sim \alpha_s^n (y_A - y_B)^n$ at LL $[\text{BFKL}]^{3729}_{1978}$

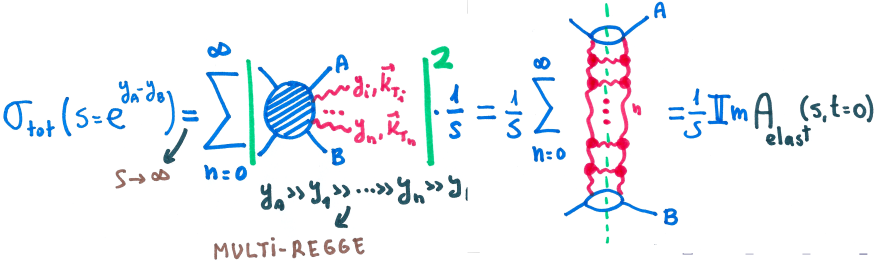
$\sim \alpha_s^n (y_A - y_B)^{n-1}$ at NLL $[\text{Fadin-Lipatov}]^{1099}_{1998}$

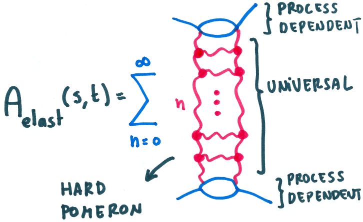
$\sim \alpha_s^n (y_A - y_B)^{n-2}$ at NNLL

$y_A - y_B$ = rapidity distance between particles A and B

$$\sigma^{\rm LL}_{\rm tot} = \sum_{n=0}^{\infty} \mathcal{C}^{\rm LL}_n \alpha_s^n \int_{y_B}^{y_A} dy_1 \int_{y_B}^{y_1} dy_2 \cdots \int_{y_B}^{y_{n-1}} dy_n = \sum_{n=0}^{\infty} \frac{\mathcal{C}^{\rm LL}_n}{n!} \underbrace{\alpha_s^n (y_A - y_B)^n}_{\rm LL}$$

Kinematical origin. Linked to elastic amplitudes via optical theorem:





Vacuum singularity corresponds to bound state of two reggeized gluons
$SL(2, C)$ invariant Hamiltonian
Based on bootstrap in QCD
Similar in SUSY, electroweak and gravity

Associated rich phenomenology for any collider

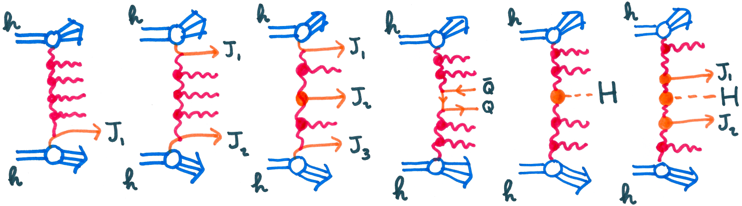
- lepton-lepton (LEP, $\sigma_{\gamma^*\gamma^*}$)
- lepton-hadron (DIS at small x)
- hadron-hadron (Tevatron, LHC)
- heavy ion collisiones (RHIC, LHC)

Key ingredients:

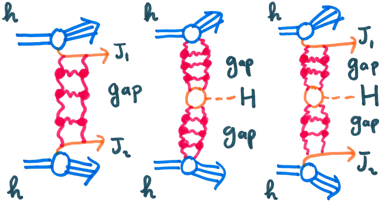
1. Universal interaction: Monte Carlo BFKLex [Chachamis,SV]
2. Coupling to external particles: Lipatov effective action [Hentschinski,SV]

Phenomenology at LHC:

Producción Jets, W, Z, Drell-Yan, quarks, Higgs . . . in different topologies:

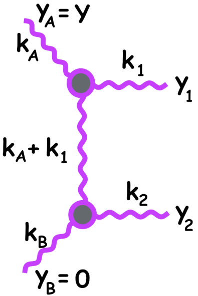


Diffractive production (rapidity gaps, non active regions in detectors):



WG on Forward Physics and Diffraction, LHC Physics Centre at CERN: https://lpcc.web.cern.ch/lhc-wg-forward-physics-and-diffraction

Monte Carlo event generator `BFKLex`: with Grigorios Chachamis (Lisbon)
Effective Feynman rules: basis of Lipatov's High Energy effective action



Gluon Regge trajectory: $\omega\left(\vec{q}\right) = -\frac{\alpha_s N_c}{\pi} \log \frac{q^2}{\lambda^2}$
Non-IR finite

Modified propagators in the t-channel:

$$\left(\frac{s_i}{s_0}\right)^{\omega(t_i)} = e^{\omega(t_i)(y_i - y_{i+1})}$$

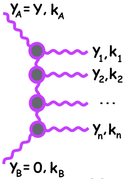
$$\left(\frac{\alpha_s N_c}{\pi}\right)^2 \int d^2\vec{k}_1 \frac{\theta\left(k_1^2 - \lambda^2\right)}{\pi k_1^2} \int d^2\vec{k}_2 \frac{\theta\left(k_2^2 - \lambda^2\right)}{\pi k_2^2} \delta^{(2)}\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2 - \vec{k}_B\right)$$
$$\times \int_0^Y dy_1 \int_0^{y_1} dy_2 e^{\omega\left(\vec{k}_A\right)(Y - y_1)} e^{\omega\left(\vec{k}_A + \vec{k}_1\right)(y_1 - y_2)} e^{\omega\left(\vec{k}_A + \vec{k}_1 + \vec{k}_2\right) y_2}$$

Each diagram is not IR finite when $\lambda \to 0$.

Only after summation over all possible final states we get IR finiteness

1. Phenomenology

$$\sigma(Q_1, Q_2, Y) = \int d^2\vec{k}_A d^2\vec{k}_B \underbrace{\phi_A(Q_1, \vec{k}_A)\,\phi_B(Q_2, \vec{k}_B)}_{\text{PROCESS-DEPENDENT}} \underbrace{f(\vec{k}_A, \vec{k}_B, Y)}_{\text{UNIVERSAL}}$$

BFKLex $= f\left(\vec{k}_A, \vec{k}_B, Y\right) = \sum_n \Big|$  $\Big|^2$

$$= \quad e^{\omega(\vec{k}_A)Y}\left\{\delta^{(2)}\left(\vec{k}_A - \vec{k}_B\right) + \sum_{n=1}^{\infty}\prod_{i=1}^{n}\frac{\alpha_s N_c}{\pi}\int d^2\vec{k}_i \frac{\theta\left(k_i^2 - \lambda^2\right)}{\pi k_i^2}\right.$$

$$\left.\times \int_0^{y_{i-1}} dy_i e^{\left(\omega\left(\vec{k}_A + \sum_{l=1}^{i}\vec{k}_l\right) - \omega\left(\vec{k}_A + \sum_{l=1}^{i-1}\vec{k}_l\right)\right)y_i}\delta^{(2)}\left(\vec{k}_A + \sum_{l=1}^{n}\vec{k}_l - \vec{k}_B\right)\right\}$$

Analytic representation:

$$f\left(\vec{k}_A, \vec{k}_B, Y\right) = \sum_{n=-\infty}^{\infty}\frac{e^{in(\theta_A - \theta_B)}}{\pi|\vec{k}_A||\vec{k}_B|}\int_{a-i\infty}^{a+\infty}\frac{d\gamma}{2\pi i}\left(\frac{|\vec{k}_A|^2}{|\vec{k}_B|^2}\right)^{\gamma-\frac{1}{2}} e^{\frac{\alpha_s N_c}{\pi}Y\chi_0(\gamma,n)}$$

$$\chi_0(\gamma, n) \quad = \quad 2\Psi(1) - \Psi\left(\gamma + \frac{|n|}{2}\right) - \Psi\left(1 - \gamma + \frac{|n|}{2}\right)$$

$SL(2, C)$: $\gamma = \frac{1}{2} + i\nu$ anomalous dimension, n conformal spin.

When $s \to \infty$, $n = 0, \gamma \simeq \frac{1}{2}$ dominate:

$\chi_0(\gamma) \simeq 4 \log 2 + 14\zeta(3) \left(\gamma - \frac{1}{2}\right)^2 + \ldots$

Too fast growth ($> \ln^2 s$ Froissart bound):

$$f\left(\vec{k}_A, \vec{k}_B, Y\right) \simeq \frac{e^{\frac{\alpha_s N_c}{\pi} Y 4 \log 2}}{\pi |\vec{k}_A||\vec{k}_B|} \frac{e^{\frac{-t^2}{56\zeta(3)\frac{\alpha_s N_c}{\pi} Y}}}{\sqrt{14\pi\zeta(3)\frac{\alpha_s N_c}{\pi} Y}} \simeq s^{0.5} > s^{0.3}$$

NLL is more complicated, sensitive to the running & choice of energy scale:

$$\sigma_{\rm tot}^{\rm NLL} = \sum_{n=1}^{\infty} \frac{\mathcal{C}_n^{\rm LL}(\mathbf{k}_i)}{n!} \left(\alpha_s - \mathcal{A}\alpha_s^2\right)^n \left(y_A - y_B - \mathcal{B}\right)^n$$

$$= \sigma_{\rm tot}^{\rm LL} - \sum_{n=1}^{\infty} \frac{\left(\mathcal{B}\,\mathcal{C}_n^{\rm LL}(\mathbf{k}_i) + (n-1)\,\mathcal{A}\,\mathcal{C}_{n-1}^{\rm LL}(\mathbf{k}_i)\right)}{(n-1)!} \underbrace{\alpha_s^n \left(y_A - y_B\right)^{n-1}}_{\rm NLL}$$

besides, quarks enter the game ...

All of this is captured by Quasi-Multi-Regge kinematics.

Important Bootstrap relations also at NLL (Fadin et al)

1. Phenomenology

- $n = 0$ Large correction

Origin in collinear contributions

Extend formalism beyond Regge limit [Ciafaloni, Colferai, Salam, Stasto]

$$\omega = \bar{\alpha}_s \left(2\psi(1) - \psi\left(\gamma + \frac{\omega}{2}\right) - \psi\left(1 - \gamma + \frac{\omega}{2}\right)\right)$$

Resummation in terms of a Bessel funcion [SV]

$$\omega \simeq \int_0^1 \frac{dx}{1-x} \left\{ \left(x^{\gamma-1} + x^{-\gamma}\right) \sqrt{\frac{2\bar{\alpha}_s}{\ln^2 x}} J_1\left(\sqrt{2\bar{\alpha}_s \ln^2 x}\right) - 2\bar{\alpha}_s \right\}$$

in transverse momentum space

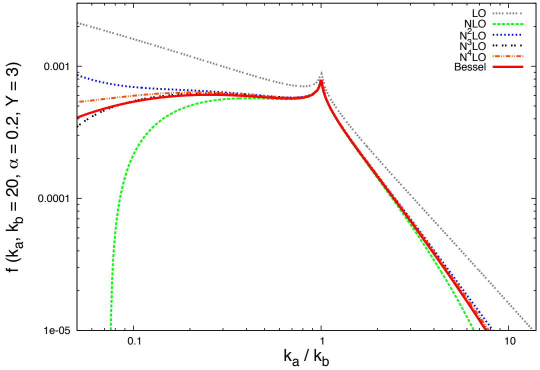
$$\sum_{n=1}^{\infty} \frac{(-\bar{\alpha}_s)^n}{2^n n!(n+1)!} \ln^{2n} \frac{\vec{k}_A^2}{\left(\vec{k}_A + \vec{k}_i\right)^2}$$

Investigating origin in HE effective action [SV]

Implemented in BFKLex

Impact factors control the kinematical region:

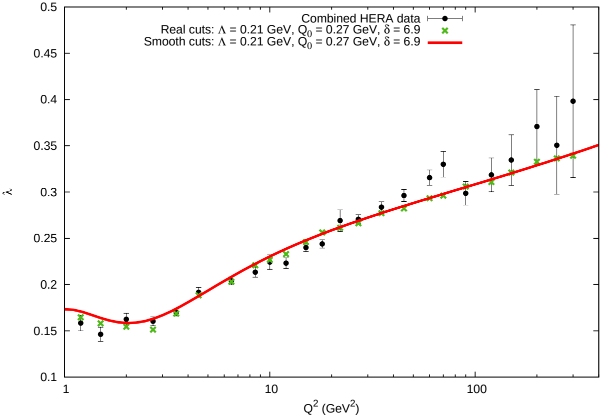
$$\sigma = \int d^2\vec{k}_A d^2\vec{k}_B \underbrace{\phi_A(Q_1,\vec{k}_A)\,\phi_B(Q_2,\vec{k}_B)}_{\text{PROCESS-DEPENDENT}} \underbrace{f(\vec{k}_A,\vec{k}_B,Y)}_{\text{UNIVERSAL}}$$

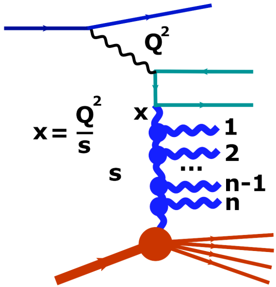


Important to go beyond the MRK limit.

For original BFKL we need "δ-like" impact factors $\phi_{A,B}$ & $Q_1 \simeq Q_2$

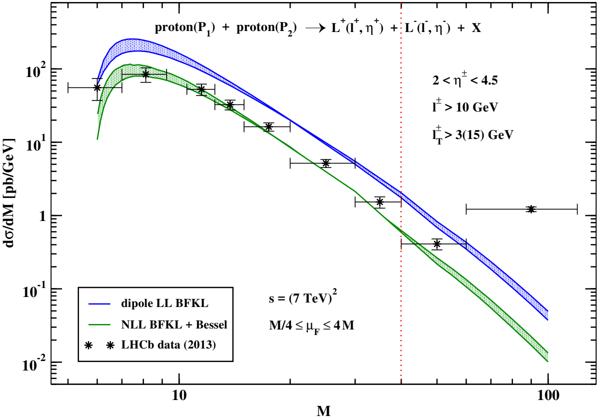
How far can we get analytically?:
Small x DIS: $F_2(x, Q^2) \simeq x^{-\lambda(Q^2)}$
A NLL Multi-Regge approach fits data well [Hentschinski,Salas,SV]

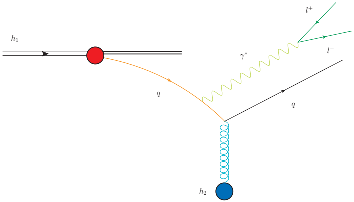




Transition from a perturbative to a non-perturbative Pomeron not well understood. Need more exclusive observables: LHC is the playground now.

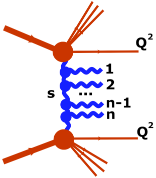
Forward Drell-Yan production at LHC [Celiberto,Gordo,SV]
The same unintegrated gluon density works well for current data





Previous analysis by [Brzeminski,Motyka,Sadzikowski,Stebel]
We work with BFKL at NLL plus collinear corrections

LHC observable proposed to pin down original BFKL,
without collinear contamination: remove χ_0

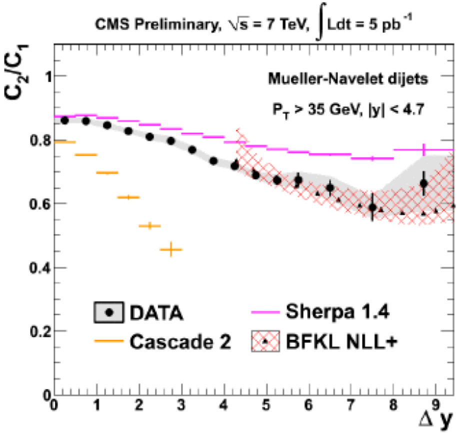


[Orr-Stirling] [Del-Duca]

$$< \cos{(m\,\theta)} > \simeq e^{\alpha Y (\chi_m - \chi_0)}$$

[SV] [SV,Schwennsen]

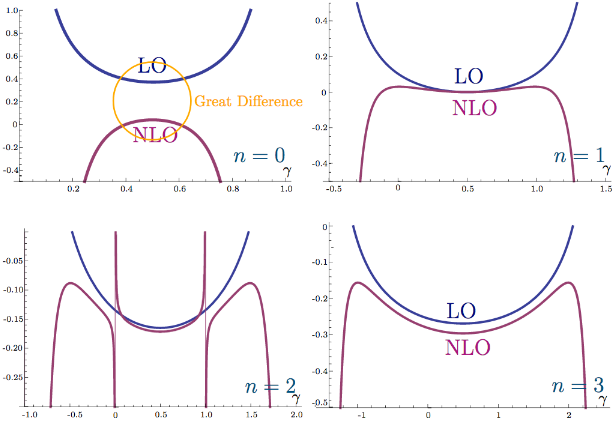
$$\mathcal{R}_{m,n} = \frac{<\cos{(m\,\theta)}>}{<\cos{(n\,\theta)}>} \simeq e^{\alpha Y (\chi_m - \chi_n)}$$



Confirmed by [Wallon et al] [Colferai et al] [Papa et al]

1. Phenomenology

Why? $n > 0$ stable under radiative corrections
$n = 0$ only one sensitive to collinear radiation [SV]



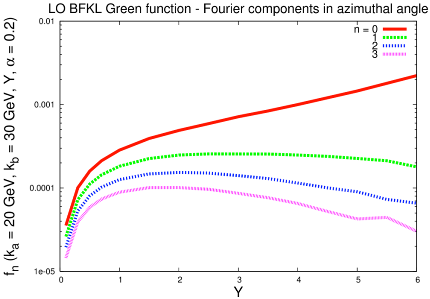
Plots for $\alpha\chi_n^{(LO)}(\gamma) + \alpha^2\chi_n^{(NLO)}(\gamma)$ + Collinear corrections

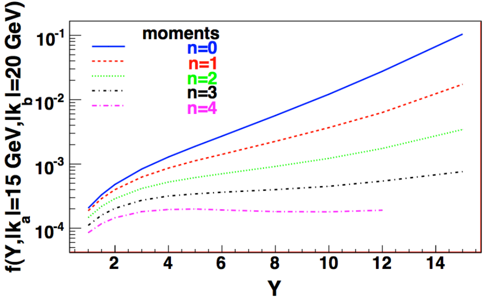
Growth with energy? Depends on the azimuthal angle Fourier component:

$$f_n\left(|\vec{k}_A|, |\vec{k}_B|, Y\right) = \int_0^{2\pi} \frac{d\theta}{2\pi} f\left(\vec{k}_A, \vec{k}_B, Y\right) \cos(n\,\theta)$$

BFKL

CCFM

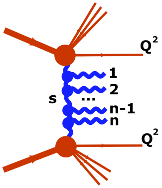


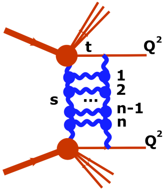


All [Catani,Ciafaloni,Fiorani,Marchesini] projections grow with energy, not in BFKL, distinct feature [Chachamis,Stephens,SV]

Find observables only sensitive to $n > 0$ to single out original BFKL

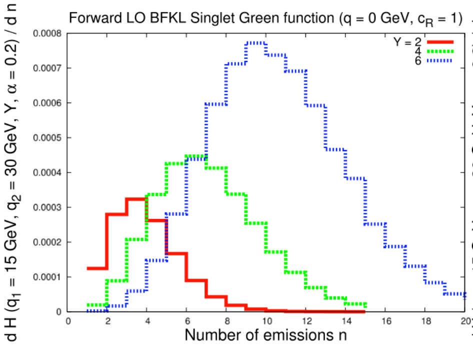
Interesting: Asymptotic Charges & Coherent States [Gonzo, Mc Loughlin, Medrano, Spiering]

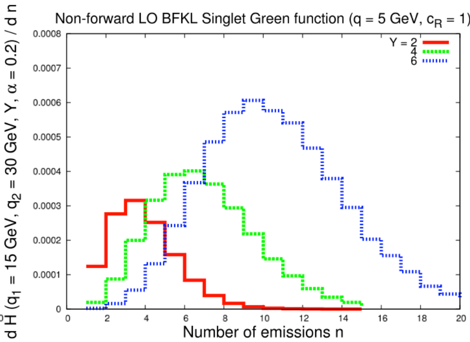




Cut Pomeron: Number of emissions?

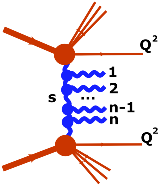
Pomeron: Number of rungs?



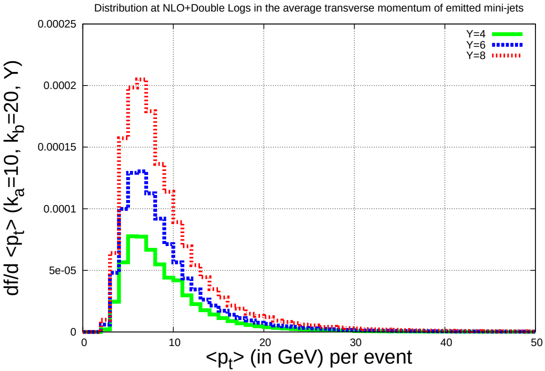


Implementation in BFKLex

Average transverse momentum of emitted mini-jets?



$$\langle p_t \rangle \;=\; \frac{1}{n}\sum_{i=1}^{n} |k_i|$$

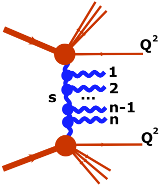


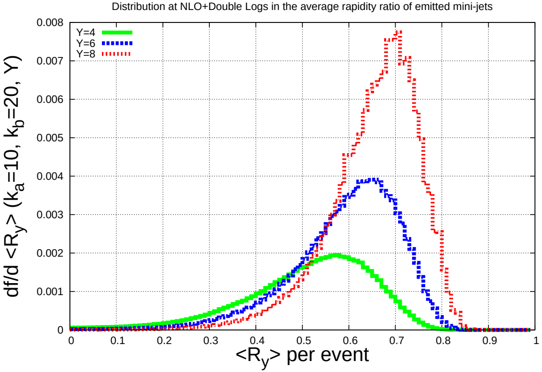
Mini-jet $\langle p_t \rangle_{\max}$ independent of rapidity separation of tagged forward jets

Average rapidity separation among emitted mini-jets?

$$\langle \mathcal{R}_y \rangle = \frac{1}{n+1} \sum_{i=1}^{n+1} \frac{y_i}{y_{i-1}} \quad \simeq 1 + \frac{\Delta}{Y} \ln \frac{\Delta}{Y}$$

if $Y \simeq N\Delta$ in MRK and $Y \gg \Delta$



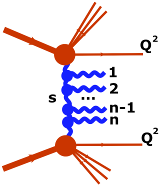


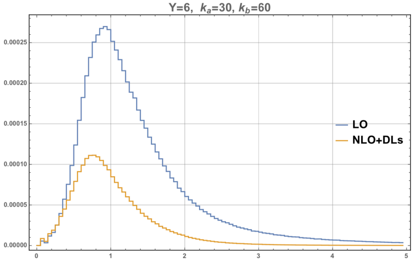
Higher $\langle \mathcal{R}_y \rangle_{\rm max}$ for higher energies: $\Delta_{\rm LO} \simeq 0.62$, $\Delta_{\rm LO+DLs} \simeq 0.81$

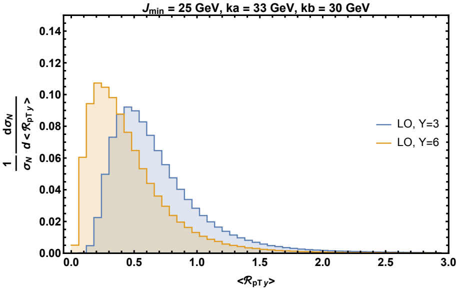
Lower mini-jet multiplicity when including higher order corrections

New observable mixes rapidity and p_T

$$\langle \mathcal{R}_{kY} \rangle = \frac{1}{n+1} \sum_{i=1}^{n+1} \frac{k_i e^{y_i}}{k_{i-1} e^{y_{i-1}}}$$







Lower cutoff in p_T reduces the BFKL effects

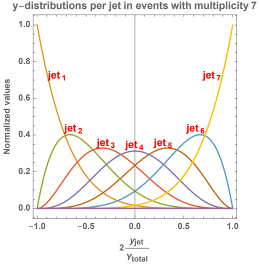
Multiperipheral models

[1968] Chew-Pignotti [1971] DeTar [2020] Bethencourt-Chachamis-SV

$$\sigma_{N+2} = \alpha^{N+2} \int_0^Y \prod_{i=1}^{N+1} dz_i \, \delta \left(Y - \sum_{s=1}^{N+1} z_s \right) = \alpha^2 \frac{(\alpha Y)^N}{N!}$$

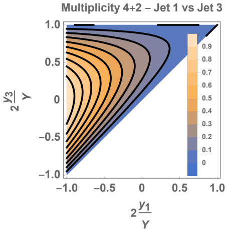
$$\frac{d\sigma^{(l)}_{N+2}}{dy_l} = \alpha^{N+2} \int_0^Y \prod_{i=1}^{N+1} dz_i \, \delta \left(Y - \sum_{s=1}^{N+1} z_s \right) \delta \left(y_l + \frac{Y}{2} - \sum_{j=1}^{l} z_j \right)$$

$$= \alpha^{N+2} \frac{\left(\frac{Y}{2} - y_l\right)^{N-l}}{(N-l)!} \frac{\left(y_l + \frac{Y}{2}\right)^{l-1}}{(l-1)!}$$



Correlations in rapidity

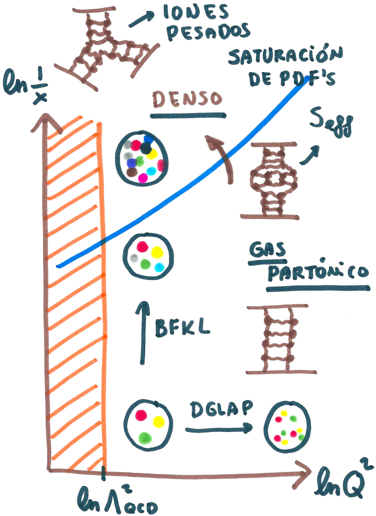
$$\frac{d^2\sigma^{(l,m)}_{N+2}}{dy_l dy_m} = \alpha^{N+2} \frac{\left(\frac{Y}{2} - y_l\right)^{N-l}}{(N-l)!} \frac{(y_l - y_m)^{l-m-1}}{(l-m-1)!} \frac{\left(y_m + \frac{Y}{2}\right)^{m-1}}{(m-1)!}$$



BFKLex generates the same distributions

Important to investigate fixed multiplicity final states

Correlations in rapidity, transverse momentum and azimuthal angle under study



Madrid node for European Network “Small-x physics” for STRONG-2020

The perturbative Regge limit

1. Phenomenology
2. Supersymmetry
3. Gravity

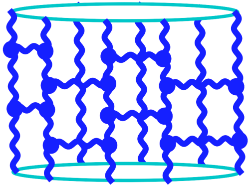
2. Supersymmetry

- $N = 4$ SYM important part of QCD amplitudes
- Experience in HE QCD very useful in SYM
- Principle of maximal transcendentality [Kotikov, Lipatov] [Dixon]
- Steinman relations [Bartels, Lipatov, SV] [Dixon, Caron-Huot ...]
- Integrability [Lipatov] [Faddeev, Korchemsky ...] [Kazakov, Gromov, Sizov ...]
- Work to calculate BFKL in QCD and SUSY at higher orders

  [Fadin] [Balitsky] [Caron-Huot, Gromov ...] [Del Duca, Gardi, Magnea ...]

2. Supersymmetry

Amplitudes in Generalized LLA
interesting structure in QCD
and $N = 4$ SYM at high energies



Equivalent to closed spin chain
(Heisenberg ferromagnet)

[Lipatov] [Faddeev, Korchemsky] [Janik, Wosiek, Kotanski, Derkachov, Manashov ...]

Monte Carlo approach to solve the system [Chachamis,SV]

Work to map this to the integrability structure

3 reggeized gluons in t-channel color singlet:
ODDERON

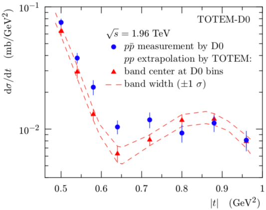


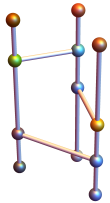
FIG. 6: Comparison between the D0 $p\bar{p}$ measurement at 1.96 TeV and the extrapolated TOTEM pp cross section, rescaled to match the OP of the D0 measurement. The dashed lines show the 1σ uncertainty band.

Drives the difference in
pp and $p\bar{p}$ total cross sections

[DØ and TOTEM Collaborations 2020 (arXiv:2012.03981)].

Bartels-Kwiecinski-Praszalowicz (BKP) equation:
Asymptotic intercept one [Bartels, Lipatov, Vacca]

$$\begin{aligned}(\omega - \omega(\mathbf{p}_1) - \omega(\mathbf{p}_2) - \omega(\mathbf{p}_3))\, f_\omega(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \\ \delta^{(2)}(\mathbf{p}_1 - \mathbf{p}_4)\,\delta^{(2)}(\mathbf{p}_2 - \mathbf{p}_5)\,\delta^{(2)}(\mathbf{p}_3 - \mathbf{p}_6) \\ + \int d^2\mathbf{k}\,\xi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{k})\, f_\omega(\mathbf{p}_1 + \mathbf{k}, \mathbf{p}_2 - \mathbf{k}, \mathbf{p}_3) \\ + \int d^2\mathbf{k}\,\xi(\mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_1, \mathbf{k})\, f_\omega(\mathbf{p}_1, \mathbf{p}_2 + \mathbf{k}, \mathbf{p}_3 - \mathbf{k}) \\ + \int d^2\mathbf{k}\,\xi(\mathbf{p}_1, \mathbf{p}_3, \mathbf{p}_2, \mathbf{k})\, f_\omega(\mathbf{p}_1 + \mathbf{k}, \mathbf{p}_2, \mathbf{p}_3 - \mathbf{k})\end{aligned}$$

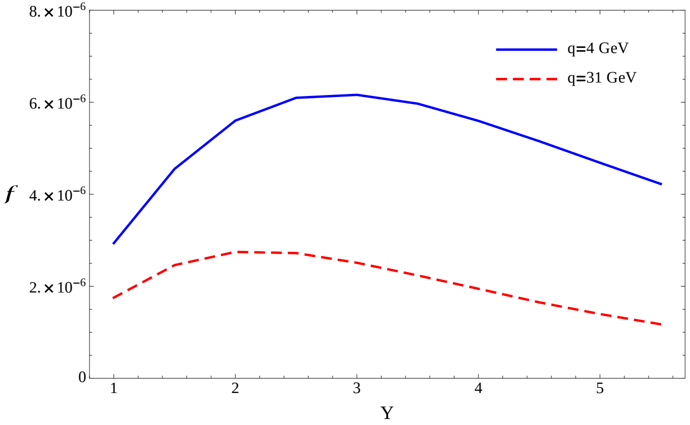


Squared Lipatov's emission vertex:

$$\xi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{k}) = \frac{\alpha_s N_c}{4}\frac{\theta(\mathbf{k}^2 - \lambda^2)}{\pi^2 \mathbf{k}^2}\left(1 + \frac{(\mathbf{p}_1 + \mathbf{k})^2 \mathbf{p}_2^2 - (\mathbf{p}_1 + \mathbf{p}_2)^2 \mathbf{k}^2}{\mathbf{p}_1^2 (\mathbf{k} - \mathbf{p}_2)^2}\right)$$

Gluon Regge trajectory: $\omega(\mathbf{p}) = -\frac{\bar{\alpha}_s}{2} \ln \frac{\mathbf{p}^2}{\lambda^2}$

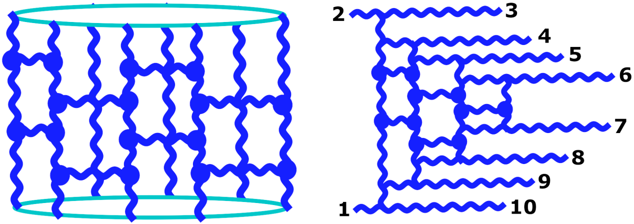
Our solution contains all the previous ones [Chachamis,SV]
which are projected out when integrating over different impact factors



Work in pheno and formal applications

Similar diagrams with Regge cuts in $N = 4$ SYM MHV [Bartels,Lipatov,SV]

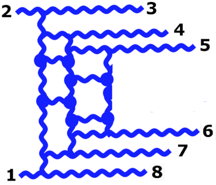
Complicated contributions at higher orders and more external particles:



Equivalent to an open spin chain [Lipatov]
MC methods also work here [Chachamis,SV]
Consider octet exchange with 3 reggeized gluons:

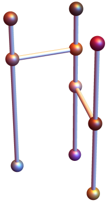
MHV 8-point amplitude

Open spin chain with 3 reggeized gluons



Similar equation to BKP but now IR divergent:

$$
\begin{aligned}
&(\omega - \omega(\mathbf{p}_1) - \omega(\mathbf{p}_2) - \omega(\mathbf{p}_3))\, f_\omega(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3) = \\
&\quad \delta^{(2)}(\mathbf{p}_1 - \mathbf{p}_4)\, \delta^{(2)}(\mathbf{p}_2 - \mathbf{p}_5)\, \delta^{(2)}(\mathbf{p}_3 - \mathbf{p}_6) \\
&\quad + \int d^2\mathbf{k}\, \xi(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{k})\, f_\omega(\mathbf{p}_1 + \mathbf{k}, \mathbf{p}_2 - \mathbf{k}, \mathbf{p}_3) \\
&\quad + \int d^2\mathbf{k}\, \xi(\mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_1, \mathbf{k})\, f_\omega(\mathbf{p}_1, \mathbf{p}_2 + \mathbf{k}, \mathbf{p}_3 - \mathbf{k})
\end{aligned}
$$



IR divergencies factorize:

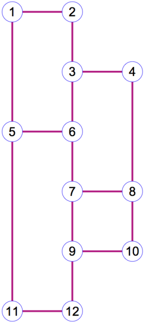
$$
f_{\mathrm{BKP}}^{\mathrm{adjoint}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathrm{Y}) = \left(\frac{\lambda^2}{\sqrt{\mathbf{p}_1^2 \mathbf{p}_3^2}}\right)^{\frac{\bar{\alpha}_s \mathrm{Y}}{2}} \underbrace{\widehat{f}_{\mathrm{BKP}}^{\mathrm{adjoint}}(\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathrm{Y})}_{\mathrm{IR-FINITE}}
$$

$\widehat{f}_{\mathrm{BKP}}^{\mathrm{adjoint}}$ is finite when $\lambda \to 0$ after sum over all diagrams

Graph complexity Number of possible spanning trees in the graph. (Cross all nodes without loops)

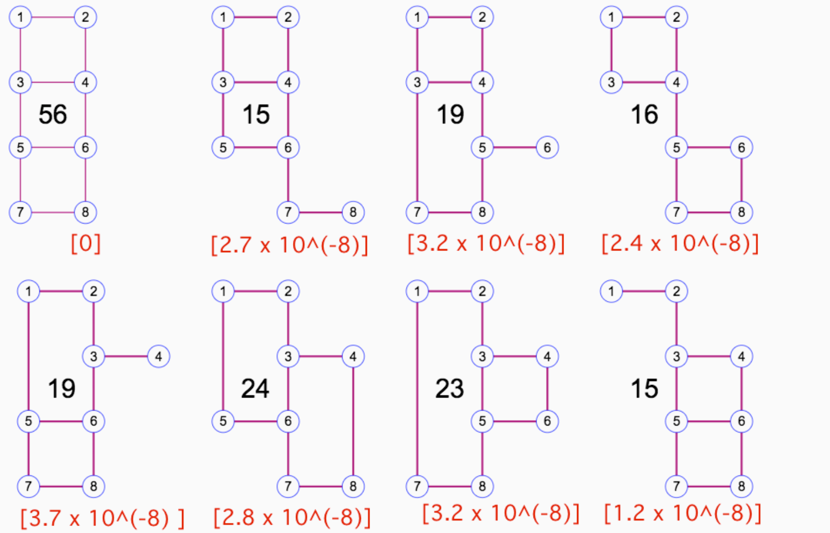
Consider Laplacian matrix $\boxed{L}$ of a graph with 6 rungs

$$
\boxed{L} = \begin{pmatrix}
2 & -1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 3 & -1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 2 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 3 & -1 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & -1 & 0 & -1 & 3 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 3 & -1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & -1 & 3 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 3 & -1 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -1 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & -1 & 2
\end{pmatrix}
$$



Matrix Tree theorem, Kirchoff:
The determinant of any principal minor of $\boxed{L}$ = Complexity of the graph

For fixed external momenta, each 4-rung topology with an associated complexity contributes to the GGF (amplitude) as



Number of rungs = 4

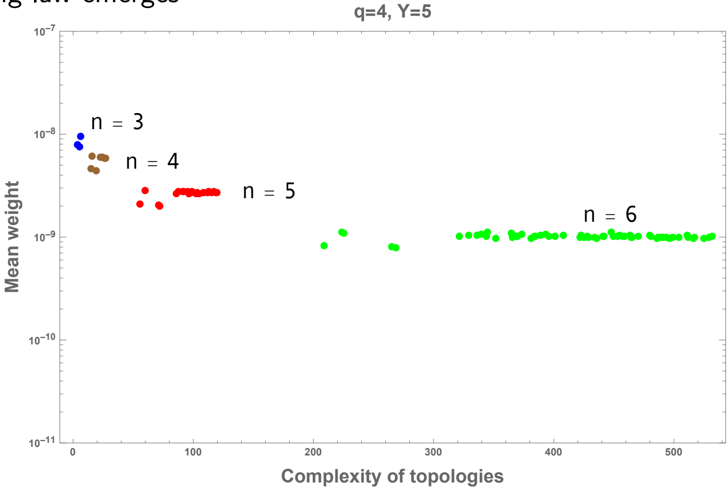
Average contribution to the amplitude for each Complexity Class:

Complexity	# diagrams	Average contribution
15	4	2.6×10^{-8}
16	2	3.3×10^{-8}
19	4	2.6×10^{-8}
23	2	3.2×10^{-8}
24	2	3.3×10^{-8}
56	2	0

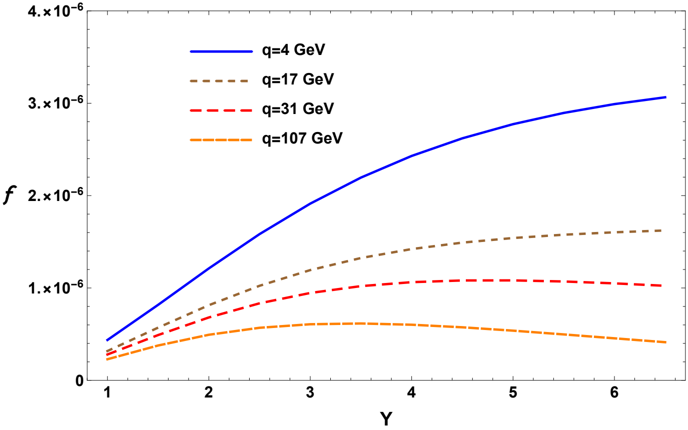
A "Complexity Democracy" arises ...

Mean weight per Complexity Class in the reggeized gluon net

A scaling law emerges



Origin in the underlying integrability?



This is the solution to the "Adjoint Pomeron" (open spin chain)
Important in the all-order structure of $N = 4$ SYM amplitudes

Interpretation within integrability?

Work to obtain BFKL eqn to all orders

First step in this direction in $N = 4$ SYM [Bartels,Lipatov,SV]

Use BDS [Bern,Dixon,Smirnov] ansatz for planar amplitudes
Proposed for all loops and number of external legs
It needs to be improved

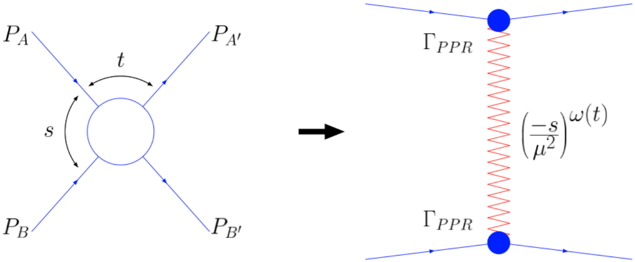
MHV amplitudes, color ordered and normalized by Born:

$$\log \mathcal{M}_n = \sum_{l=1}^{\infty} a^l \left(f^{(l)}(\epsilon) M_n^{(1)}(l\epsilon) + C^{(l)} + E_n^{(l)}(\epsilon) \right)$$

$d = 4 - 2\epsilon$, $l =$ number of loops, $n =$ number of legs

4-point amplitude is exact in the Regge limit:

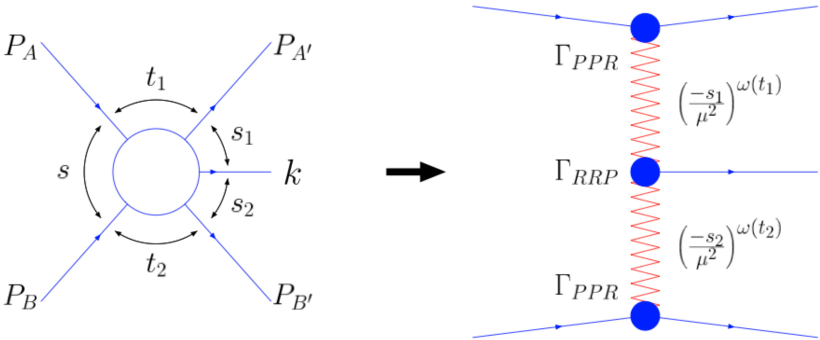
$$\log \mathcal{M}_4 \quad = \quad \omega(t) \log \frac{-s}{\mu^2} + 2 \log \Gamma(t) + \mathcal{O}(\epsilon)$$



Correct Regge gluon trajectory at NLL from BDS:

$$\begin{aligned}\omega(t) &= \int_0^a \frac{d\alpha}{\alpha}\left(\frac{\gamma(\alpha)}{4\epsilon} + \beta(\alpha)\right) - \frac{\gamma(a)}{4} \log \frac{-t}{\mu^2} \\ &= a\left(\frac{1}{\epsilon} - \log \frac{-t}{\mu^2}\right) + a^2\left[\zeta_2\left(-\frac{1}{2\epsilon} + \log \frac{-t}{\mu^2}\right) - \frac{\zeta_3}{2}\right] + \ldots\end{aligned}$$

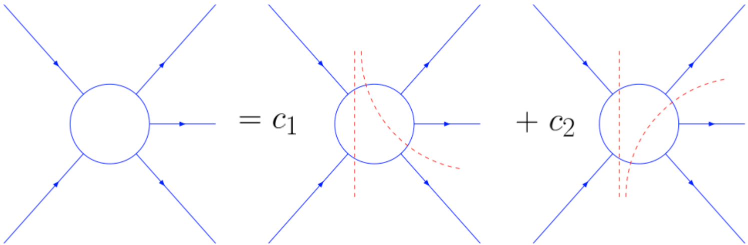
Using $\omega(t), \Gamma(t)$, the 5-point amplitude also coincides with its limit:



Consistentt with Regge theory if it fulfills the "dispersion relation":

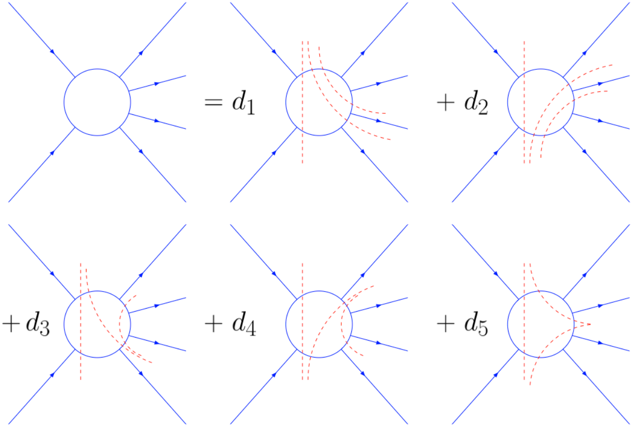
$$\frac{M_{2\to 3}}{\Gamma(t_1)\Gamma(t_2)} = c_1 \left(\frac{-s_1}{\mu^2}\right)^{\omega(t_1)-\omega(t_2)} \left(\frac{s\vec{k}_\perp^2}{-\mu^4}\right)^{\omega(t_2)} + c_2 \left(\frac{-s_2}{\mu^2}\right)^{\omega(t_2)-\omega(t_1)} \left(\frac{s\vec{k}_\perp^2}{-\mu^4}\right)^{\omega(t_1)}$$

where c_1, c_2 must be real numbers.



If so, the analytic properties are correct in all physical regions.

No such dispersion relation is posible for the BDS amplitude at 6 points:



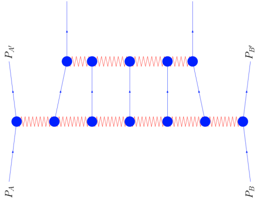
Incorrect analytic properties in certain regions $(s, s_2 > 0)$
which can be found only from 2 loops
It does not follow the Steinman relations

2. Supersymmetry

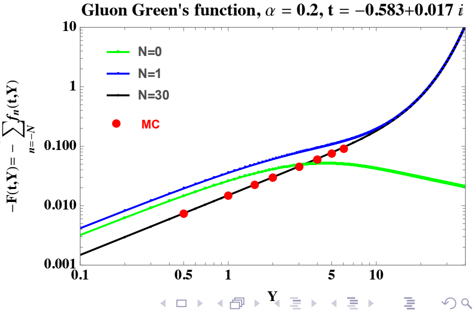
Origin: presence of a phase well known in HE QCD [Bartels,Lipatov,SV]

$$\frac{M_{2\to 4}}{\Gamma(t_1)\Gamma(t_3)} = \underbrace{e^{i\pi \frac{\gamma_K(a)}{4}\left(-\frac{1}{\epsilon}+\log\Omega\right)}}_{\text{Regge cut}} \left(\frac{-s_1}{\mu^2}\right)^{\omega(t_1)} \Gamma_{RRP} \left(\frac{-s_2}{\mu^4}\right)^{\omega(t_2)} \Gamma_{RRP} \left(\frac{-s_3}{\mu^2}\right)^{\omega(t_3)}$$

Solution: complete BDS with Regge cuts from 2 loops onwards



Calculated at LL [Bartels,Lipatov,SV] and NLL [Fadin,Lipatov]



Work to relate it to integrability

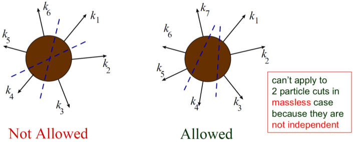
conformal blocks [Chachamis,SV]

Our conditions for the correct analytic structure
are important to solve them to all orders
and arbitrary number of external legs

Steinmann relations

Steinmann, Helv. Phys. Acta (1960) Bartels, Lipatov, Sabio Vera, 0802.2065

- Amplitudes should not have overlapping branch cuts:



$$\mathrm{Disc}_{s_{234}}\left[\mathrm{Disc}_{s_{123}}\mathcal{E}(u,v,w)\right]=0$$ ← Violated by ABDK and BDS ansatz!

L. Dixon From 2 to 7 Loops in planar N=4 SYM QMUL - 2019.11.07 35

Master Table

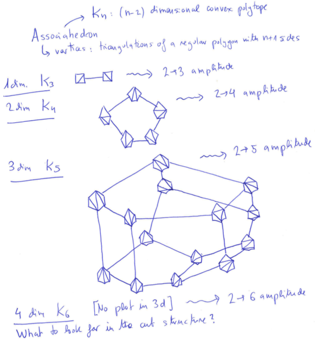
(MHV,NMHV): parameters left in $(\mathcal{E}^{(L)},\ E^{(L)} \& \tilde{E}^{(L)})$

Constraint	$L=1$	$L=2$	$L=3$	$L=4$	$L=5$	$L=6$
1. All functions	(6,6)	(25,27)	(92,105)	(313,372)	(991,1214)	(2951,3692?)
2. Symmetry	(2,4)	(7,16)	(22,56)	(66,190)	(197,602)	(567,1795?)
3. Final entry	(1,1)	(4,3)	(11,6)	(30,16)	(85,39)	(236,102)
4. Collinear limit	(0,0)	(0,0)	$(0^*,0^*)$	$(0^*,2^*)$	$(1^{*3},5^{*3})$	$(6^{*2},17^{*2})$
5. LL MRK	(0,0)	(0,0)	(0,0)	(0,0)	$(0^*,0^*)$	$(1^{*2},2^{*2})$
6. NLL MRK	(0,0)	(0,0)	(0,0)	(0,0)	$(0^*,0^*)$	$(1^*,0^*)$
7. NNLL MRK	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)
8. N^3LL MRK	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)
9. all MRK	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)
10. T^1 OPE	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(1,0)
11. $T^2F^2\ln^4 T$ OPE	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)
12. all T^2F^2 OPE	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)	(0,0)

(0,0) → amplitude uniquely determined Also have MHV at $L=7$

L. Dixon From 2 to 7 Loops in planar N=4 SYM QMUL - 2019.11.07 38

This is only the beginning. Steinman relations connected to associahedron



Out[ ]//MatrixForm=

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14	21	9	1	0	0	0	0	0	0
42	84	56	14	1	0	0	0	0	0
132	330	300	120	20	1	0	0	0	0
429	1287	1485	825	225	27	1	0	0	0
1430	5005	7007	5005	1925	385	35	1	0	0
4862	19 448	32 032	28 028	14 014	4004	616	44	1	0
16 796	75 582	143 208	148 512	91 728	34 398	7644	936	54	1
58 786	293 930	629 850	755 820	556 920	259 896	76 440	13 650	1365	65
208 012	1 144 066	2 735 810	3 730 650	3 197 700	1 790 712	659 736	157 080	23 100	1925
742 900	4 457 400	11 767 536	17 978 180	17 587 350	11 511 720	5 116 320	1 534 896	302 940	37 400
2 674 440	17 383 860	50 220 040	84 987 760	93 486 536	70 114 902	36 581 688	13 302 432	3 325 608	554 268
9 694 845	67 863 915	212 952 285	395 482 815	483 367 885	409 003 595	245 402 157	105 172 353	32 008 977	6 789 783
35 357 670	265 182 525	898 198 875	1 816 357 725	2 442 687 975	2 303 105 805	1 563 837 275	773 326 125	278 397 405	72 177 105
129 644 790	1 037 158 320	3 771 484 800	8 250 123 000	12 109 051 500	12 593 413 560	9 553 624 080	5 361 727 800	2 234 053 250	687 401 000
477 638 700	4 059 928 950	15 775 723 920	37 119 350 400	59 053 512 000	67 173 369 900	56 338 955 400	35 413 057 680	16 790 673 900	5 996 669 250
1 767 263 190	15 905 368 710	65 770 848 990	165 645 101 160	283 963 030 560	350 777 861 280	322 432 175 520	224 550 979 380	119 519 069 670	48 692 954 310
6 564 120 420	62 359 143 990	273 420 862 110	733 919 156 190	1 348 824 395 160	1 798 432 526 880	1 798 432 526 880	1 375 271 932 320	812 660 687 280	372 469 481 670

0	0	0	0	0	0	0	0	0	0
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0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0
77	1	0	0	0	0	0	0	0	0
2640	90	1	0	0	0	0	0	0	0
58 344	3536	104	1	0	0	0	0	0	0
969 969	88 179	4641	119	1	0	0	0	0	0
13 180 167	1 633 905	129 675	5985	135	1	0	0	0	0
153 977 824	24 496 472	2 662 660	186 200	7600	152	1	0	0	0
1 599 111 800	313 112 800	43 835 792	4 214 980	261 800	9520	170	1	0	0
15 111 606 510	3 532 583 340	610 569 960	75 869 640	6 503 112	361 284	11 781	189	1	0
132 166 590 270	36 045 433 710	7 457 675 940	1 147 334 760	127 481 640	9 806 280	490 314	14 421	209	1

Work in progress [SV]

Cluster algebras [Arkani-Hamed, Drummond, Gürdogan, He, Lam, Papathanasiou, Spradlin, Volovich ...]

The perturbative Regge limit

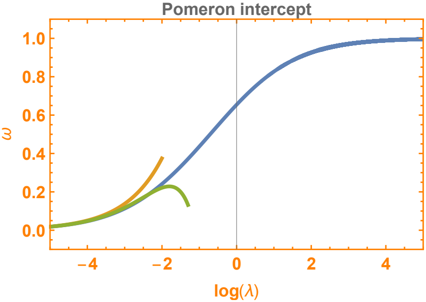
1. Phenomenology
2. Supersymmetry
3. Gravity

- Links between QCD and gravity: AdS/CFT & BCJ color-kinematics
- Both can be studied in the high energy limit
  Graviton = dual of pomeron at strong coupling [Polchinski-Strassler]

$$j - 1 = \omega = \lambda \left(2\psi(1) - \psi\left(\gamma + \frac{\omega}{2}\right) - \psi\left(1 - \gamma + \frac{\omega}{2}\right)\right) \text{ [\textit{Ciafaloni, Colferai, Salam, Stasto}]}$$

$$\lambda \ll 1 \;\Rightarrow\; \omega = \lambda \chi_0(\gamma) + \left(\sum_{m=0}^{\infty} \sum_{n=1}^{\infty} \frac{(-1)^n (2n)!}{2^n n! (n+1)!} \frac{\lambda^{n+1}}{(\gamma+m)^{2n+1}} + \gamma \to 1-\gamma \right) \text{ [\textit{SV}]}$$

$$\lambda \gg 1, \gamma = \frac{1}{2}, \text{fixed } \omega \;\Rightarrow\; \frac{\omega}{\lambda} \to 0 = 2\left(\psi(1) - \psi\left(\frac{1+\omega}{2}\right)\right) \;\Rightarrow\; \omega \to 1, j \to 2 \text{ [\textit{Stasto}]}$$



[Costa,Goncalves,Penedones]

[Kotikov,Lipatov]

[Bartels,Schomerus,Sprenger]

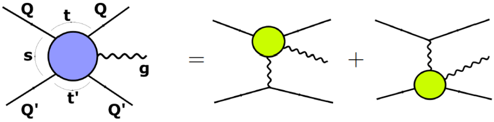
[Brower, Costa, Djuric, Raben, Tan]

[Gromov, Levkovich-Maslyuk,Sizov]

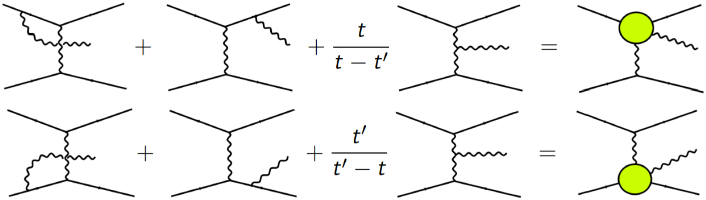
Lipatov high energy effective action is useful

3. Gravity

We redid Lev's calculation using standard Feynman rules [Serna,Vazquez-Mozo,SV]
In QCD we have



where the amplitude is the sum of two gauge invariant parts

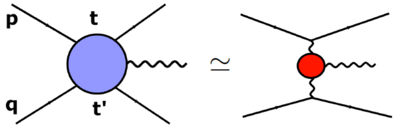


$t = k_1^2,\, t' = k_2^2$ Sudakov expansion

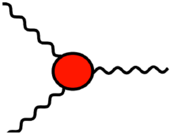
$$k_1 = \alpha_1 p + \beta_1 q + k_1^\perp \qquad k_2 = \alpha_2 p + \beta_2 q + k_2^\perp .$$

Multi-Regge kinematics (MRK):

$$1 \gg |\alpha_1| \gg |\alpha_2| = -\frac{t'}{s} \qquad 1 \gg |\beta_2| \gg |\beta_1| = -\frac{t}{s}$$

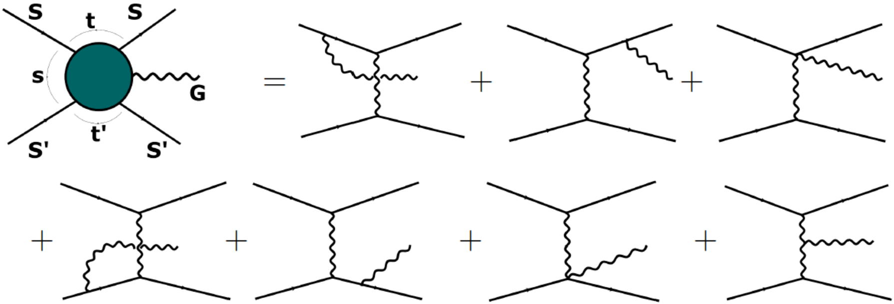


Universal Reggeized g - Reggeized g - g Effective Vertex (Lipatov) in MRK:



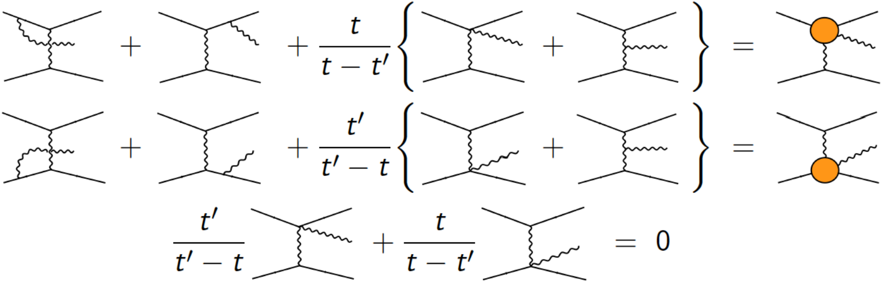
$$= ig\eta_{\mu\sigma}\left\{\left(\alpha_1 - \frac{2t}{s\beta_2}\right)p^\nu + \left(\beta_2 - \frac{2t'}{s\alpha_1}\right)q^\nu - \left(k_1^\perp + k_2^\perp\right)^2\right\}$$

The Closest Calculation in Einstein-Hilbert Gravity:

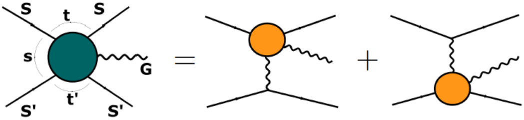


3. Gravity

Nice Trick is Still Nice but More Tricky:

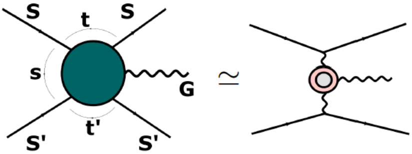


Exact Amplitude is the Sum of Two Gauge Invariant Sub-Amplitudes:

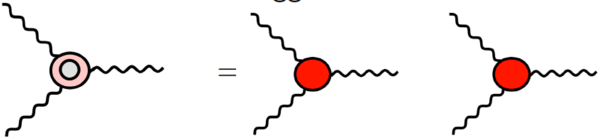


3. Gravity

Using the same Sudakov expansion and Multi-Regge kinematics:



Universal Reggeized G - Reggeized G - G Effective Vertex (Lipatov):



$$+ \quad 4\beta_1\alpha_2\left\{\frac{p^\mu p^\nu}{\beta_2^2} + \frac{q^\mu q^\nu}{\alpha_1^2} + \frac{p^\mu q^\nu + q^\mu p^\nu}{\alpha_1\beta_2}\right\}$$

Subtraction Term to Fullfil Steinman Relations (no simultaneous singularities in overlapping channels).

3. Gravity

Four-graviton amplitudes in N-SUGRA $\boxed{\mathcal{A}_{4,(N)} = \mathcal{A}_4^{\rm Born} \mathcal{M}_{4,(N)}}$
(N = number of gravitinos)

At 1-loop three contributions

$$\begin{aligned}
\mathcal{M}^{(1)}_{4,(N=8)} &= \underbrace{\alpha\, t \ln\left(\frac{-s}{-t}\right) \ln\left(\frac{-u}{-t}\right)}_{\rm Double\ \ Logs} \\
&+ \underbrace{\alpha\, \frac{t}{2} \ln\left(\frac{-t}{\lambda^2}\right)\left(\ln\left(\frac{-s}{-t}\right) + \ln\left(\frac{-u}{-t}\right)\right)}_{\rm Trajectory} \\
&- \underbrace{\alpha \frac{(s-u)}{2} \ln\left(\frac{-t}{\lambda^2}\right) \ln\left(\frac{-s}{-u}\right)}_{\rm Eikonal}
\end{aligned}$$

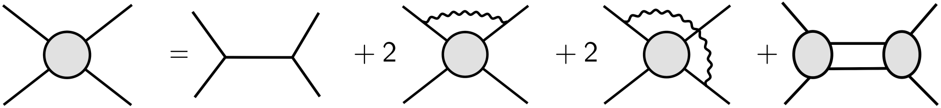
Regge limit $u \simeq -s$

$$\mathcal{M}^{(1)}_{4,(N=8)} \simeq \underbrace{(\alpha\, t) \ln^2\left(\frac{s}{-t}\right)}_{\text{Double Logs}} + \underbrace{(\alpha\, t) \ln\left(\frac{-t}{\lambda^2}\right) \ln\left(\frac{s}{-t}\right)}_{\text{Trajectory}} + i\,\pi\, \underbrace{(\alpha\, s) \ln\left(\frac{-t}{\lambda^2}\right)}_{\text{Eikonal}}$$

Evaluate Double Logs to all orders? Use

$$\mathcal{A}_{4,(N)} = \mathcal{A}_4^{\text{Born}} \left(\frac{s}{-t}\right)^{\alpha\, t\, \ln\left(\frac{-t}{\lambda^2}\right)} \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\,\omega}{2\pi i} \left(\frac{s}{-t}\right)^{\omega} \frac{f_\omega^{(N)}}{\omega}$$

Origin in the diagrams



with (1) virtual gravitons with lowest energy
(2) t-channel graviton/gravitino pairs with lowest energy

Associated equation

$$f_\omega^{(N)} = 1 - (\alpha\, t) \frac{d}{d\,\omega}\left(\frac{f_\omega^{(N)}}{\omega}\right) + (\alpha\, t)\left(\frac{N-6}{2}\right)\left(\frac{f_\omega^{(N)}}{\omega}\right)^2$$

Perturbative solution

$$\begin{aligned} f_\omega^{(N)} &= 1 + (\alpha\, t)\frac{(N-4)}{2\omega^2} + (\alpha\, t)^2 \frac{(N-4)(N-3)}{2\omega^4} \\ &- (\alpha\, t)^3 \frac{(N-4)\left(5N^2 - 26N + 36\right)}{8\omega^6} + \dots \end{aligned}$$

2-loop agreement in $N = 4, \ldots, 8$ SUGRA with BCJ [Boucher-Veronneau, Dixon].
3-loop agreement in $N = 8$ SUGRA [Henn,Mistlberger].

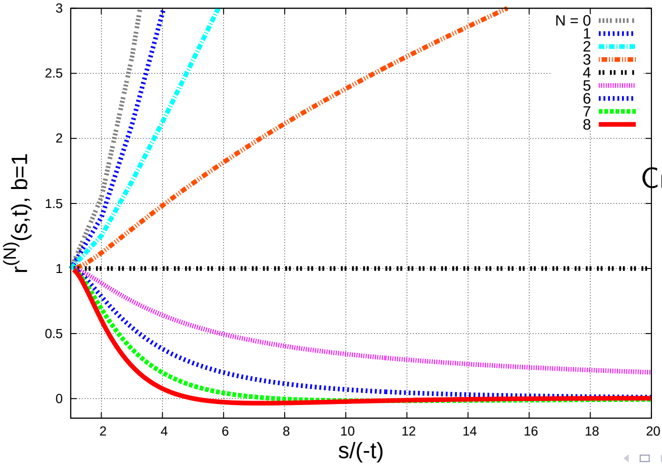
All-orders predictions *e.g.* $N = 8$:

$$\begin{aligned}
\mathcal{A}_{4,(N=8)} &= \mathcal{A}_4^{\rm Born} \left(\frac{-t}{\lambda^2}\right)^{\alpha t \left(\ln\left(\frac{s}{-t}\right) + i\pi\left(\frac{s}{t}\right)\right)} \\
&\times \left\{ 1 + 2\left(\frac{\alpha\, t}{2}\right) \ln^2\left(\frac{s}{-t}\right) + \frac{5}{3}\left(\frac{\alpha\, t}{2}\right)^2 \ln^4\left(\frac{s}{-t}\right) \right. \\
&+ \frac{37}{45}\left(\frac{\alpha\, t}{2}\right)^3 \ln^6\left(\frac{s}{-t}\right) + \frac{353}{1260}\left(\frac{\alpha\, t}{2}\right)^4 \ln^8\left(\frac{s}{-t}\right) \\
&\left. + \frac{583}{8100}\left(\frac{\alpha\, t}{2}\right)^5 \ln^{10}\left(\frac{s}{-t}\right) + \ldots \right\}
\end{aligned}$$

to be compared with other calculations in the future.

Resummation [Bartels,Lipatov,SV] [SV]:

Solution to a Schrödinger equation in terms of parabolic cylinder functions



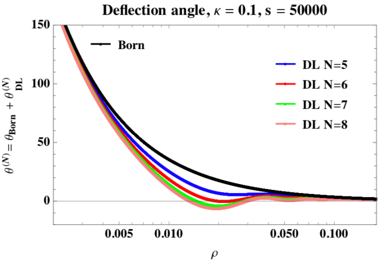
$N = 4$ SUGRA:
Critical boundary theory
Finite ($N > 4$)
Non-finite ($N < 4$)
at high energies

Subleading correction to N-SUGRA eikonal phase [SV]

DLs reduce gravitational strength: "Graviton Deflection Angle"

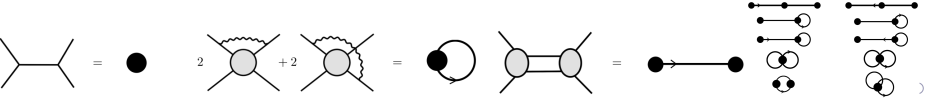
$$\begin{aligned}
\mathcal{C}^{(4)} &= (1,0,0,0,0,0,0,\dots)\,,\\
\mathcal{C}^{(5)} &= \left(1,\frac{1}{2},1,\frac{31}{8},\frac{91}{4},\frac{2873}{16},\frac{14243}{8},\dots\right),\\
\mathcal{C}^{(6)} &= (1,1,3,15,105,945,10395,\dots)\,,\\
\mathcal{C}^{(7)} &= \left(1,\frac{3}{2},6,\frac{297}{8},306,\frac{50139}{16},38286,\dots\right),\\
\mathcal{C}^{(8)} &= (1,2,10,74,706,8162,110410\dots)\,.
\end{aligned}$$

$$\theta_{\rm DL}^{(N)}(\rho,s) = -\theta_{\rm Born}(\rho,s)\sum_{m=2}^{\infty}\sum_{n=1}^{m-1}\frac{n(-\alpha s)^m\mathcal{C}^{(N)}_{m-n}}{(n!)^2 m^{2(m-n)+1}}\left(\frac{\rho^2}{4\alpha}\right)^n$$



N-SUGRA: DLs resummation for inelastic amplitudes

N=8 calculate full amplitude to all orders?



The perturbative Regge limit

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https://doi.org/10.1142/12127 | July 2021

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This book has been designed to honor Lev Nikolaevich Lipatov, as a person and as one of the leading scientists in theoretical high energy physics.

The book begins with three articles on Lev as a person, written endearingly by family members, a very close friend and Physics professor, Eugene Levin, and another outstanding scientist, Alfred Mueller. The book further collects 18 articles by several scientists who closely knew and/or collaborated with Lev.

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The perturbative Regge limit

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