Grazing collisions and eccentric inspirals of black holes

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Gravitational scattering, inspiral and radiation

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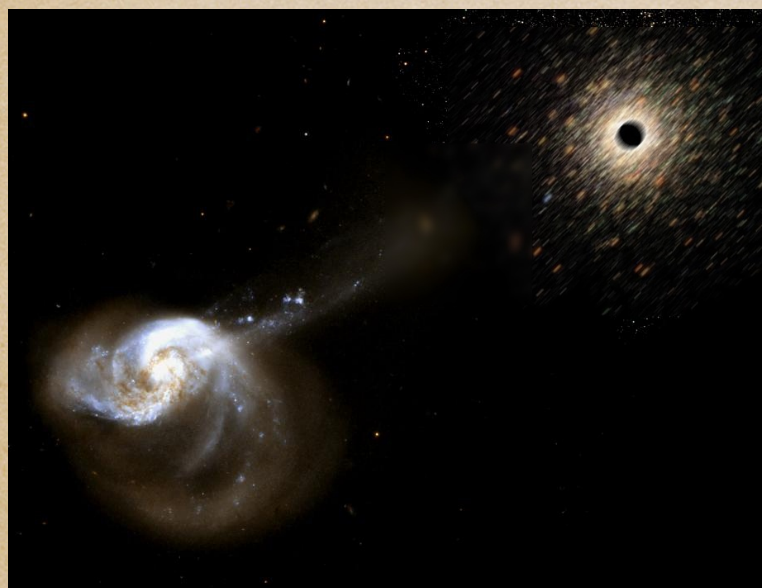
Overview

- Introduction
- High-energy head-on collisions of black holes
- Grazing collisions of black holes
- Black-hole collisions in higher dimensions
- GW emission and recoil from unequal-mass eccentric BBHs
- Conclusions

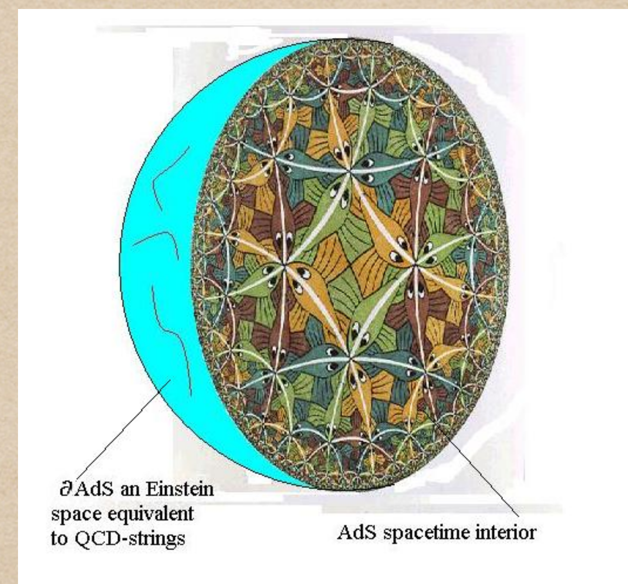
1. Introduction and motivation

Black hole research areas

Astrophysics



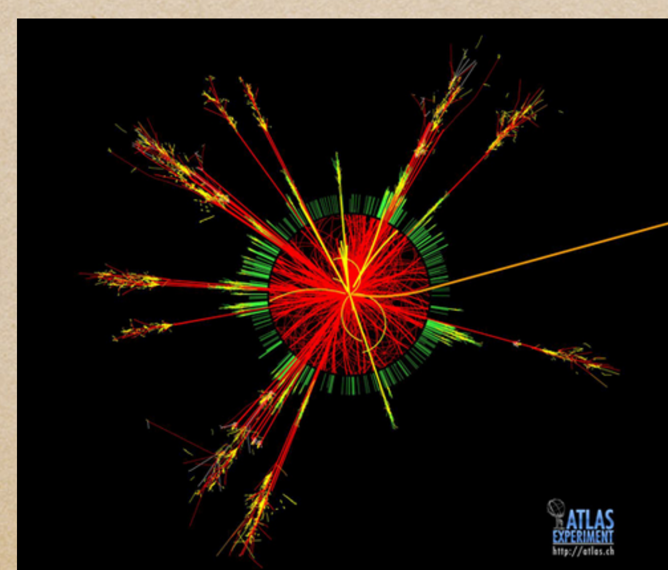
Holography



BH properties

GW Physics

High-Energy Physics



Fluid Analogies

Major goals in BH studies

- Model BHs in astrophysical environments: Kicks, Accretion etc.
- GW source modeling, template banks for LIGO, Virgo, KAGRA, LISA
- Model asymptotically AdS Black Holes
- Scattering threshold, GW emission in high-energy collisions
- Properties of BHs: Stability, Entropy, Ringdown etc.
- Probe environments of BHs: Scalar Fields, Modified Gravity

Methodology

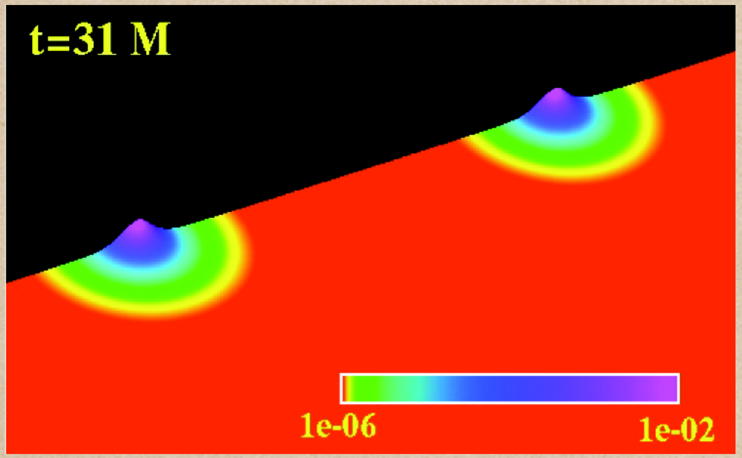
- Analytic studies
- Perturbation theory, post-Newtonian, etc.
- **Numerical Relativity**: 3+1, BSSNOK, CCZ4 formulations etc.

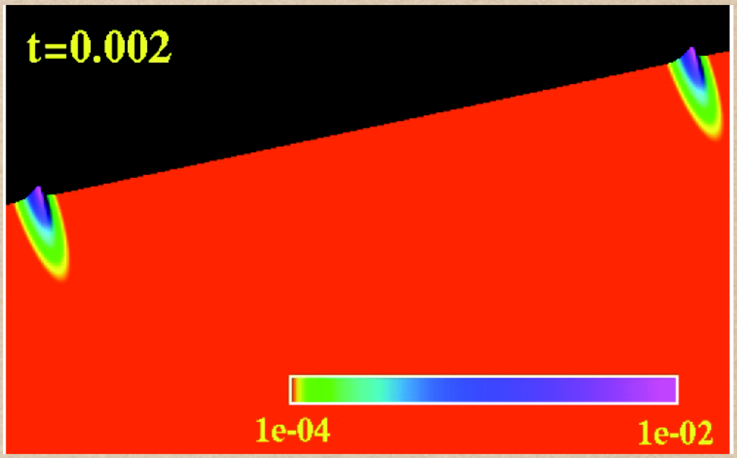
2. High-energy head-on collisions of BHs

Does matter matter?

- Hoop conjecture $\Rightarrow$ kinetic energy triggers BH formation
- Einstein + minimally coupled, massive complex scalar field

  "Boson stars" Pretorius & Choptuik '10





- BH formation threshold $\gamma_{\rm thr} = 2.9 \pm 10\,\% \sim 1/3\ \gamma_{\rm hoop}$
- Similar results for collisions of perfect fluid balls

  East & Pretorius '13, Rezzolla & Tanaki '13

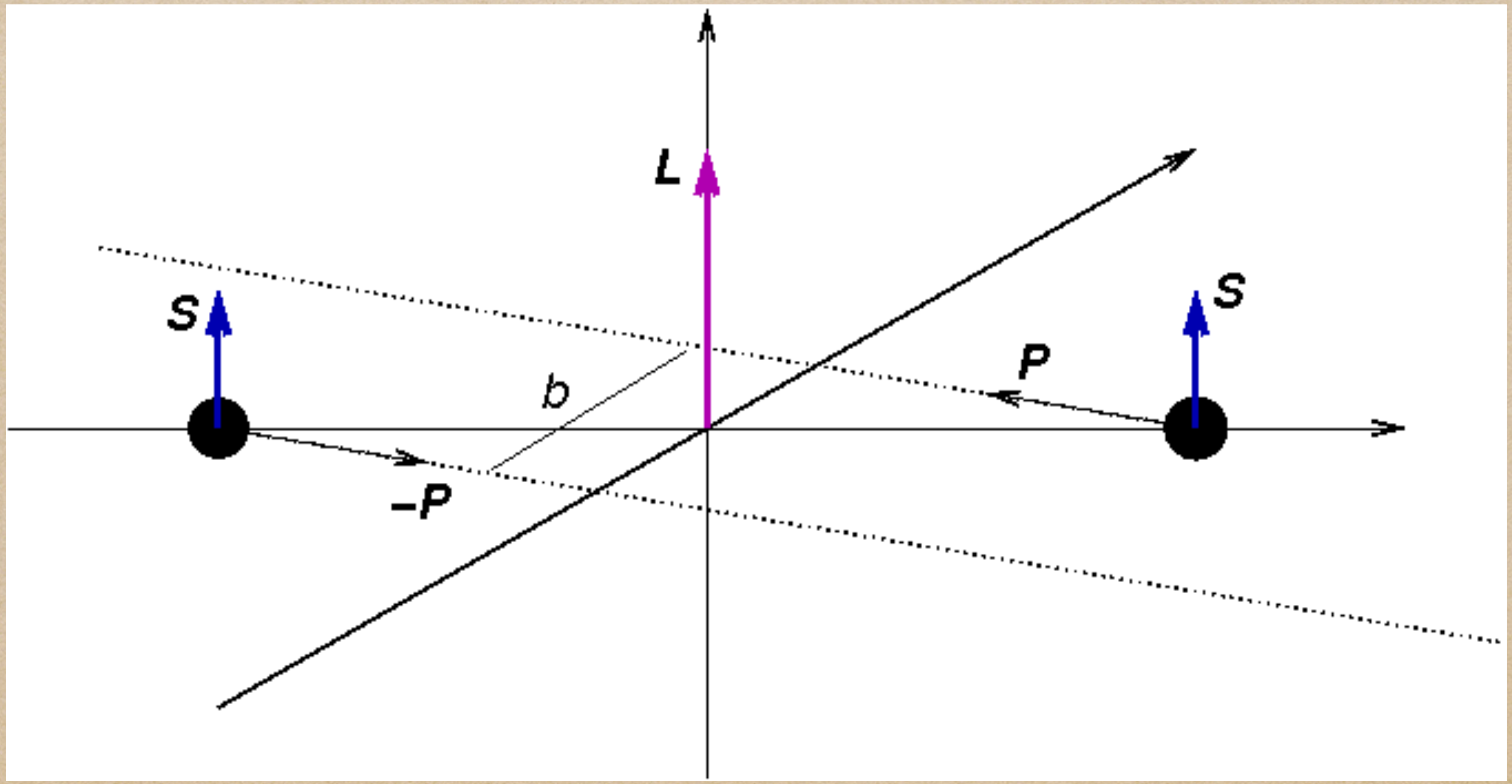
Collisions of BHs in D=4

- Orbital hang-up: Campanelli et al, gr-qc/0604012
- Free paramerers: Mass ratio $q = m_2/m_1$

  Boost $\gamma = 1/\sqrt{1-v^2}$

  Impact parameter $b = L/P$

  Spin (aligned only) S



- How are scattering threshold and radiated GW energy affected?

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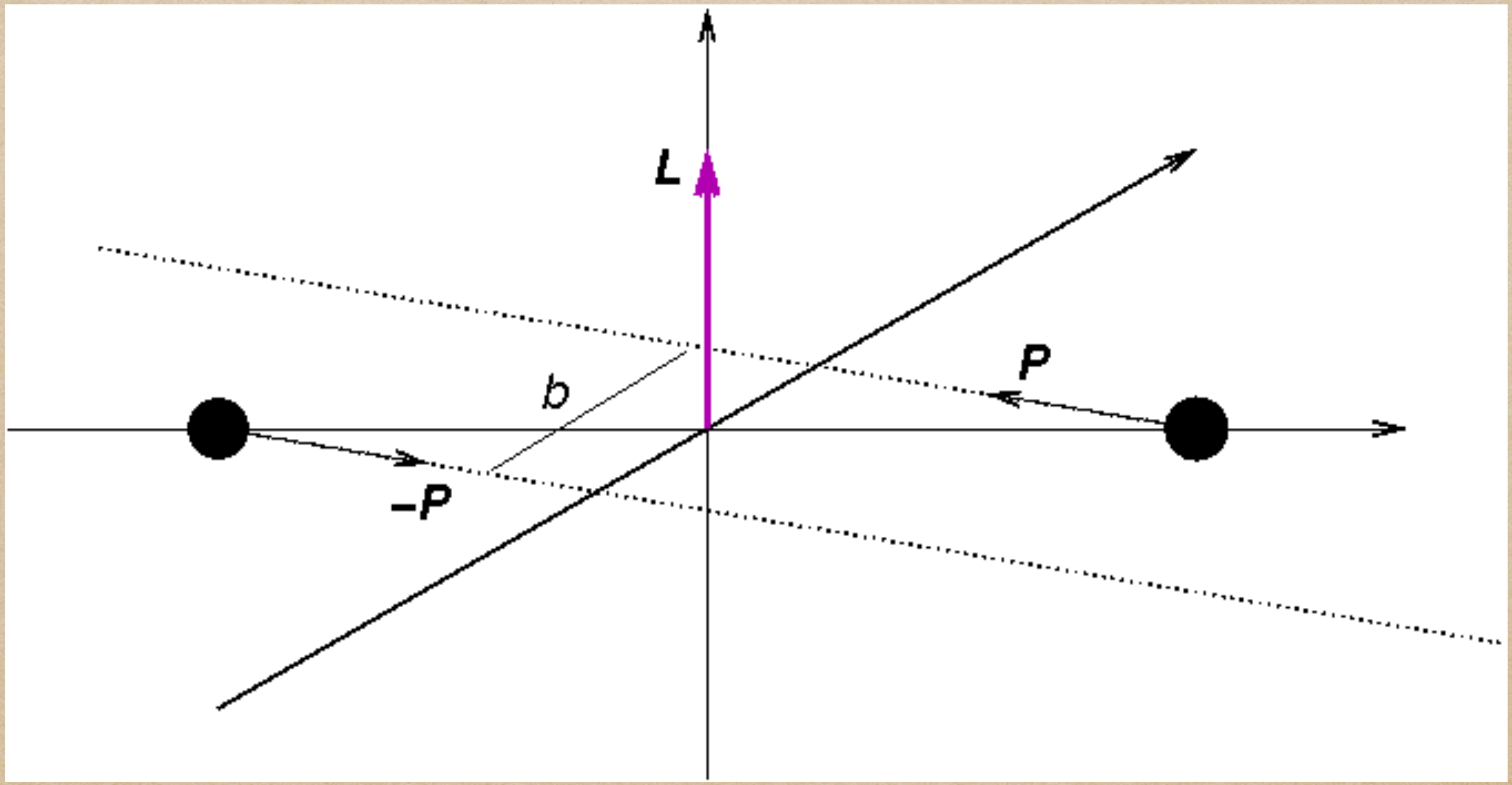

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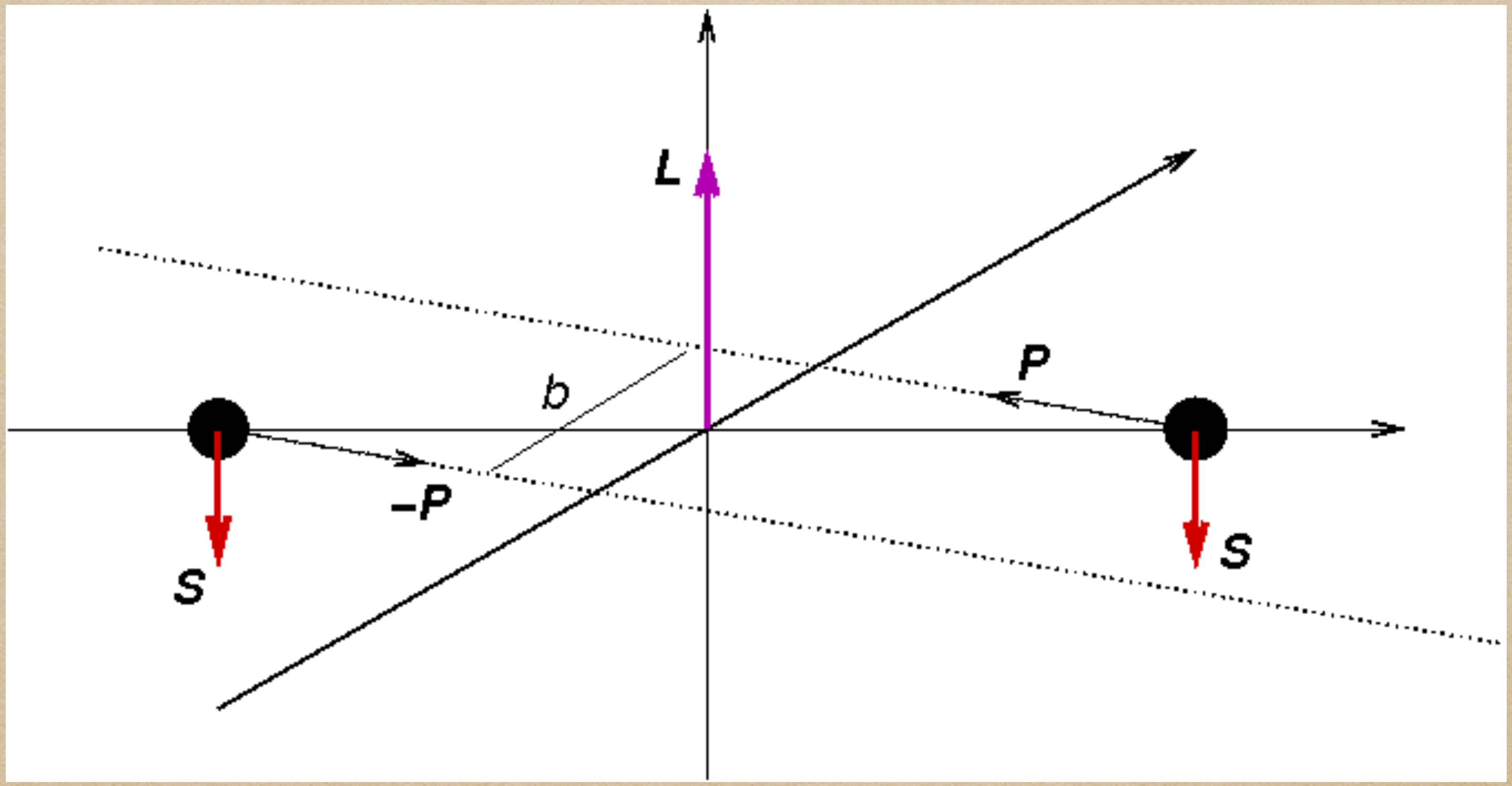
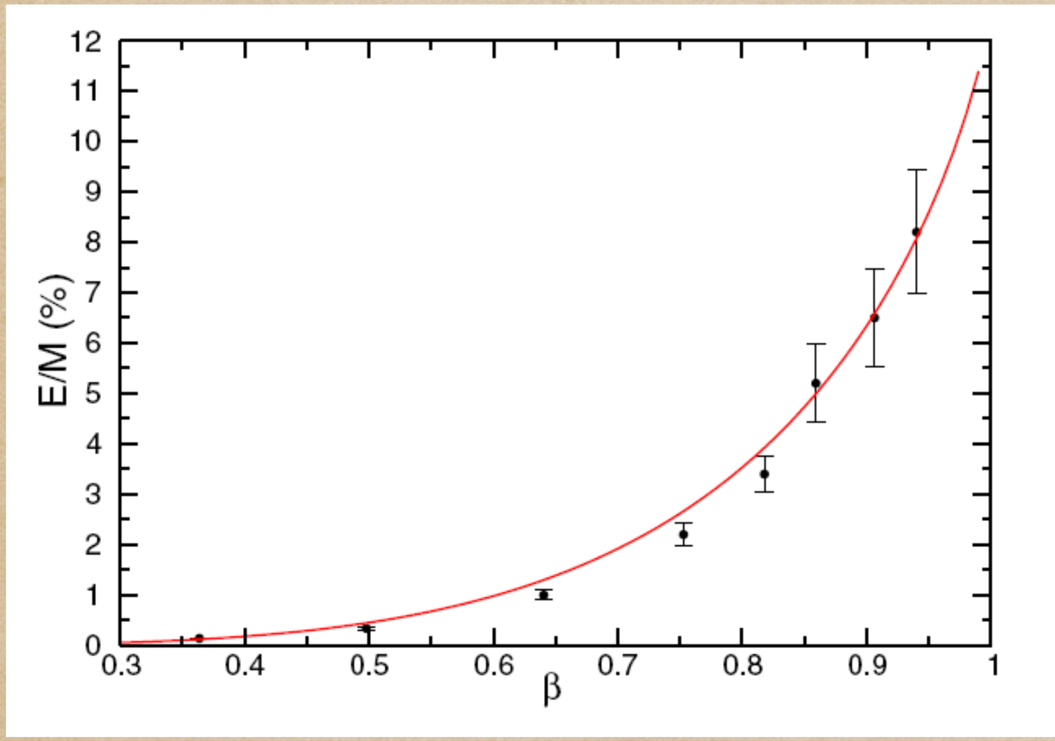


- How are scattering threshold and radiated GW energy affected?

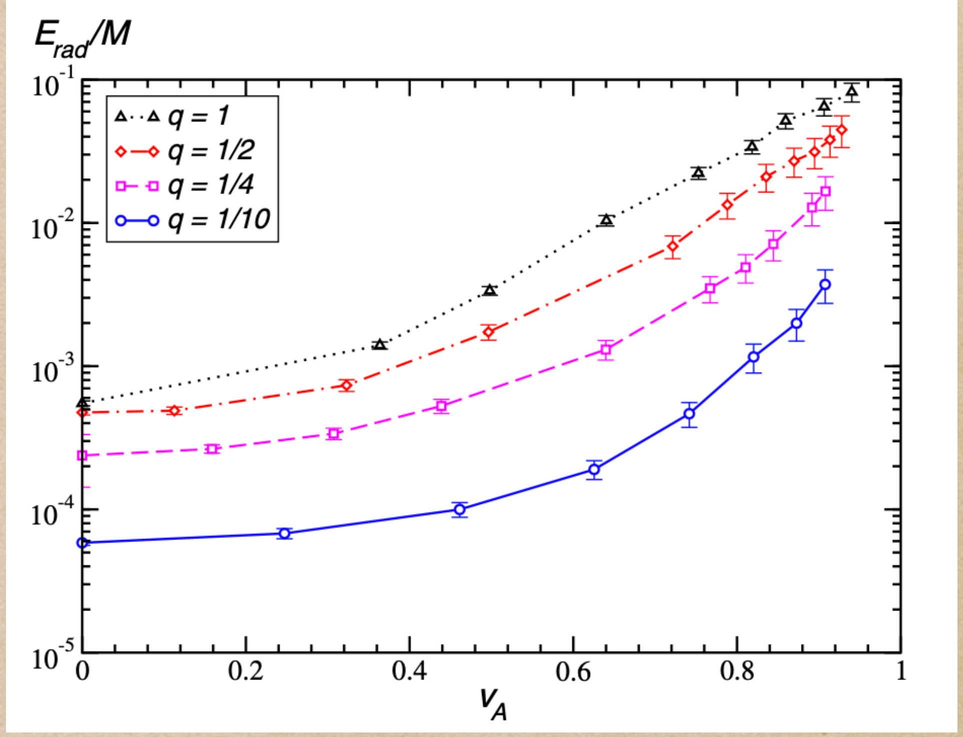
Head-on collisions of BHs in 4D

- For $v \to c$ kinetic energy dominates: Structure irrelevant
- Model particle collisions through BH collisions.

  Determine energy loss in GWs



Sperhake et al, 0806.1738
Healy et al, 1506.06153



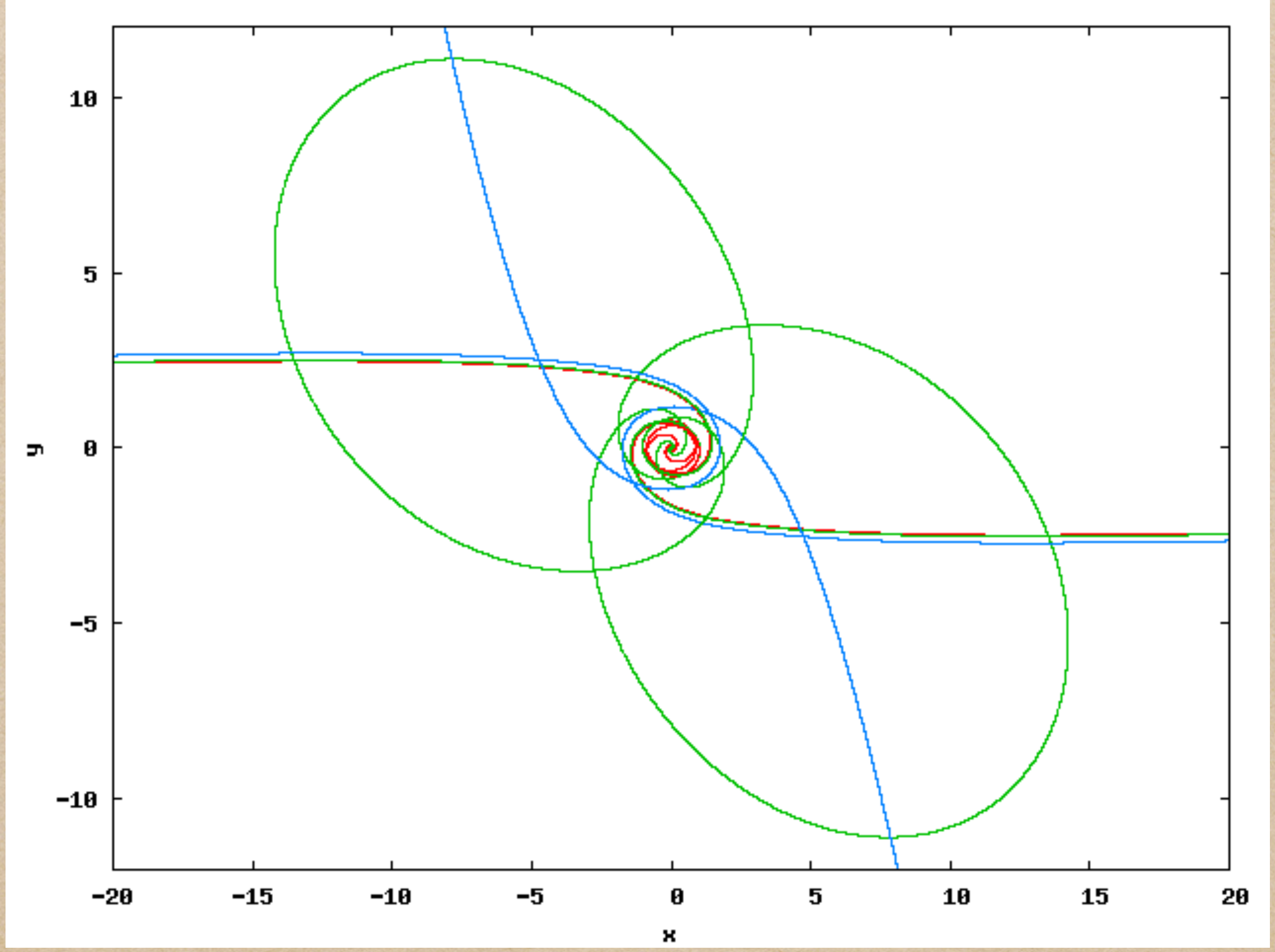
Sperhake et al, 1511.08209

$$q = 1 \qquad E_{\rm rad}(c)/M = 12.7 \pm 1.5\,\%$$
$$q = 1/2 \qquad E_{\rm rad}(c)/M = 11.2 \pm 2.7\,\%$$
$$q = 1/4 \qquad E_{\rm rad}(c)/M = 11.6 \pm 3.0\,\%$$
$$q = 1/10 \qquad E_{\rm rad}(c)/M = 12.0 \pm 3.0\,\%$$

3. Grazing collisions of BHs

D=4 grazing collisions: $b = 0\,, \quad \vec{S} = 0\,, \quad \gamma = 1.52$

- Radiated energy up to at least $\approx 35\ \%\ M$
- Immediate vs. Delayed vs. No merger



Sperhake et al, 0907.1252

Scattering threshold

- $b < b_{\rm scat} \quad \Rightarrow \quad$ Merger
  $b > b_{\rm scat} \quad \Rightarrow \quad$ Scattering
- Numerical study: $b_{\rm scat} = \dfrac{2.5 \pm 0.05}{v} M$

  Shibata et al PRD 0810.4735

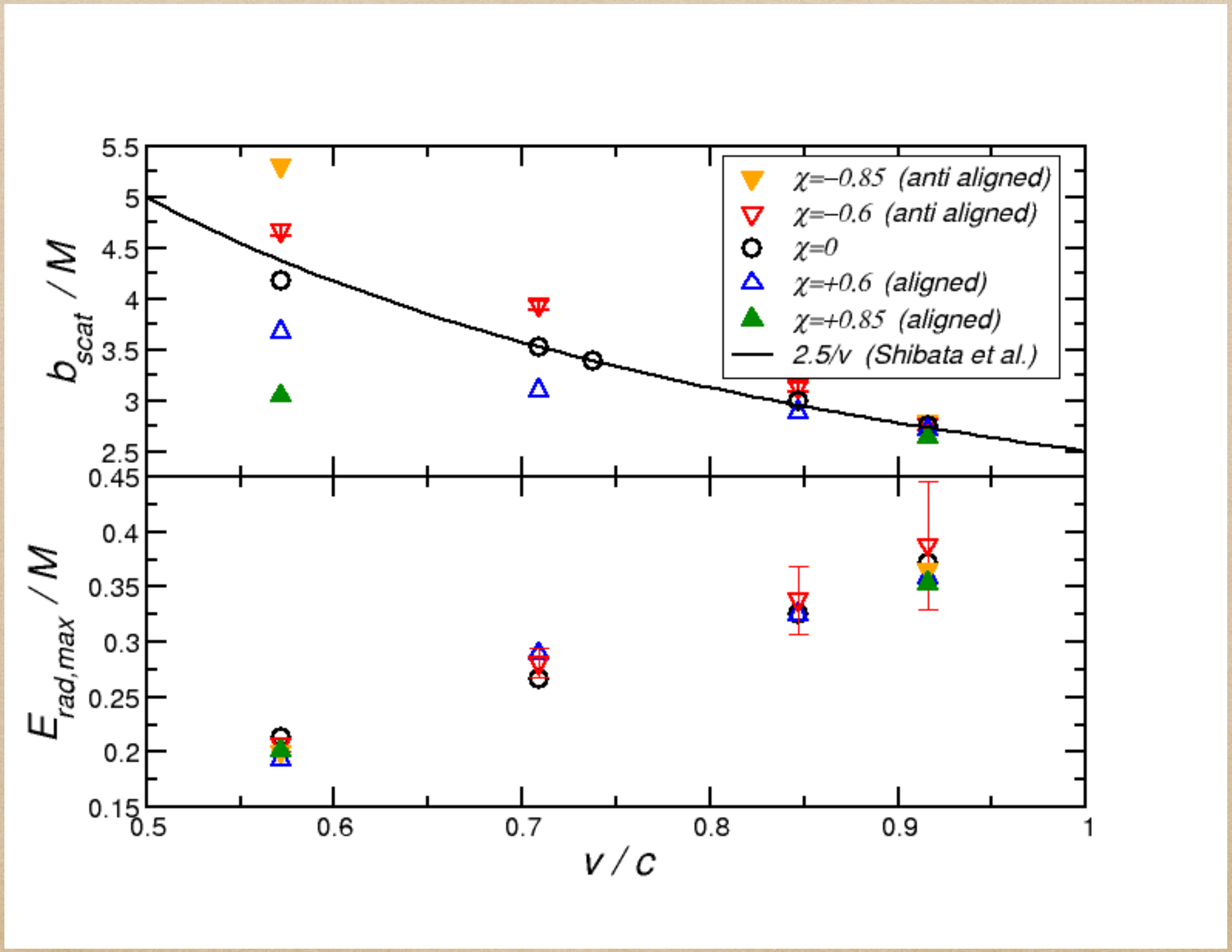
- Limit from Penrose construction: $b_{\rm scat} = 1.685\ M$

  Yoshino & Rychkov PRD hep-th/0503171

- Impact of structure of the colliding BHs?
  $\rightarrow$ Collide spinning BHs

Grazing collisions in D=4

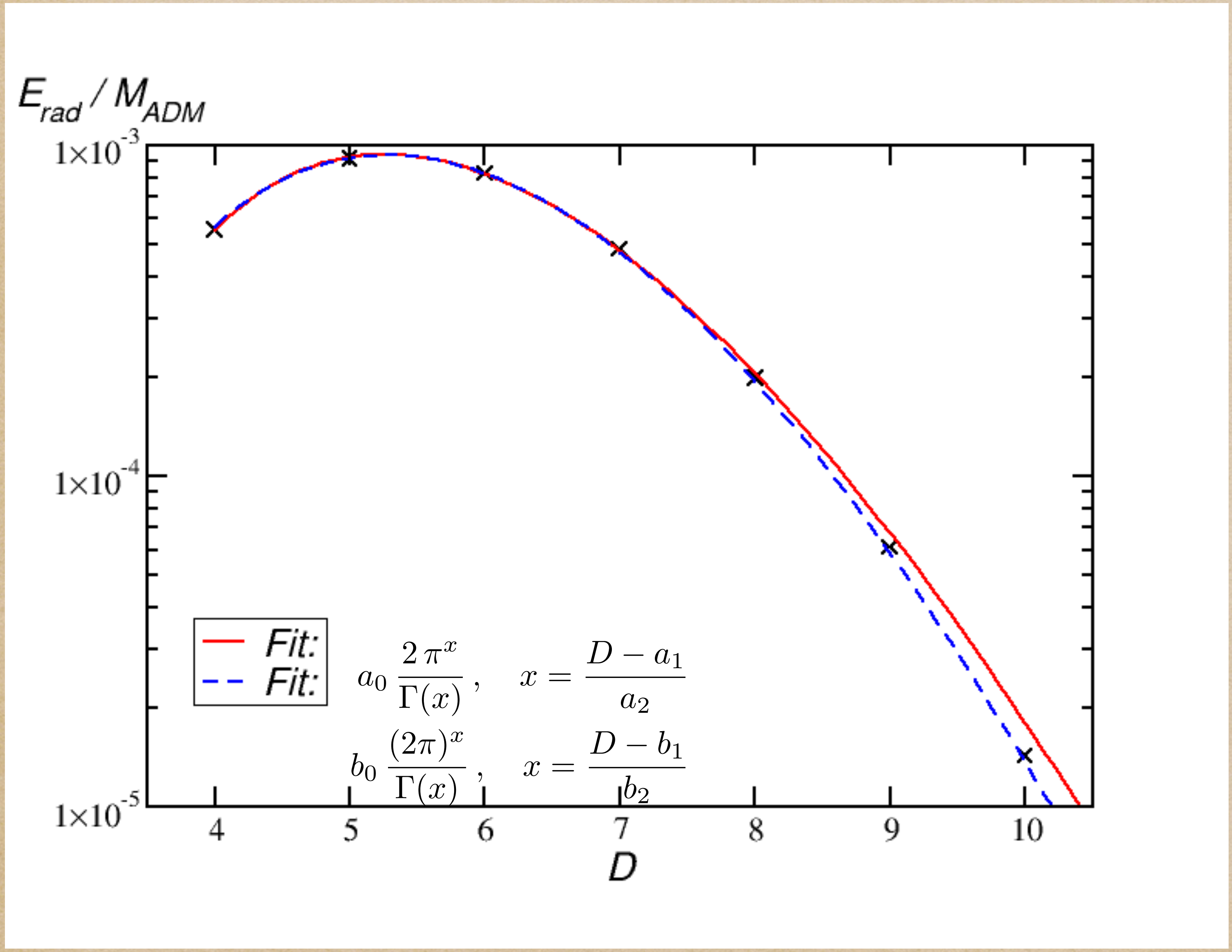
- Spins: aligned, zero, anti aligned   Sperhake et al PRL 1211.6114
- $b_{\rm scat}$, $E_{\rm rad}$: spin effects washed out as $v \to c$



4. Black-hole collisions in D>4

Head-on collisions from rest: $q = 1$

Recall: $\mathcal{A}_{D-2} = \dfrac{2\pi^{(D-1)/2}}{\Gamma\left(\frac{D-1}{2}\right)}$ (Thanks to Chris Moore)



$a_0 = 2.8236 \times 10^{-5}$

$a_1 = -2.47721$

$a_2 = 0.767106$

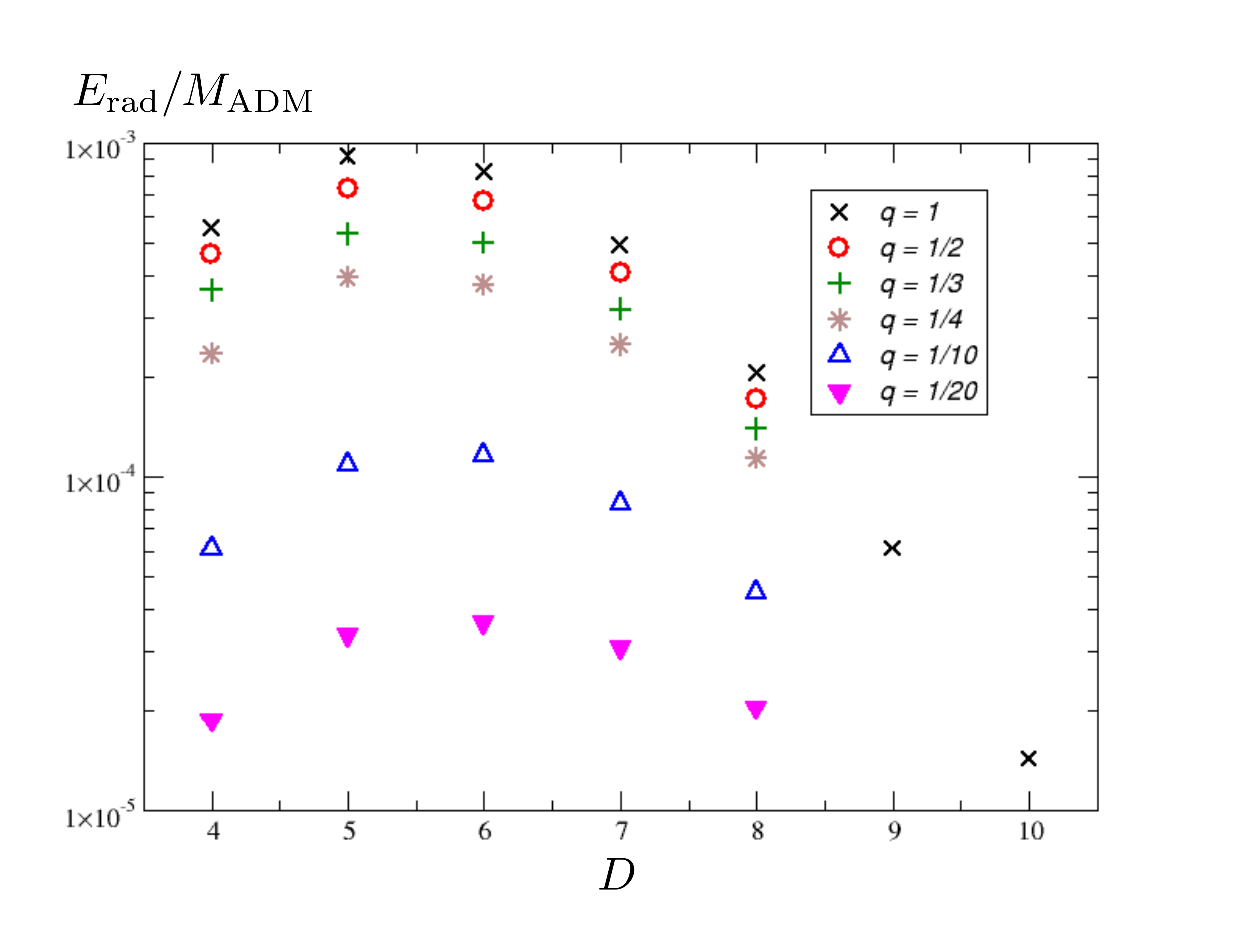
$b_0 = 1.72877 \times 10^{-6}$

$b_1 = -1.57712$

$b_2 = 0.549696$

Cook et al, 1709.10514

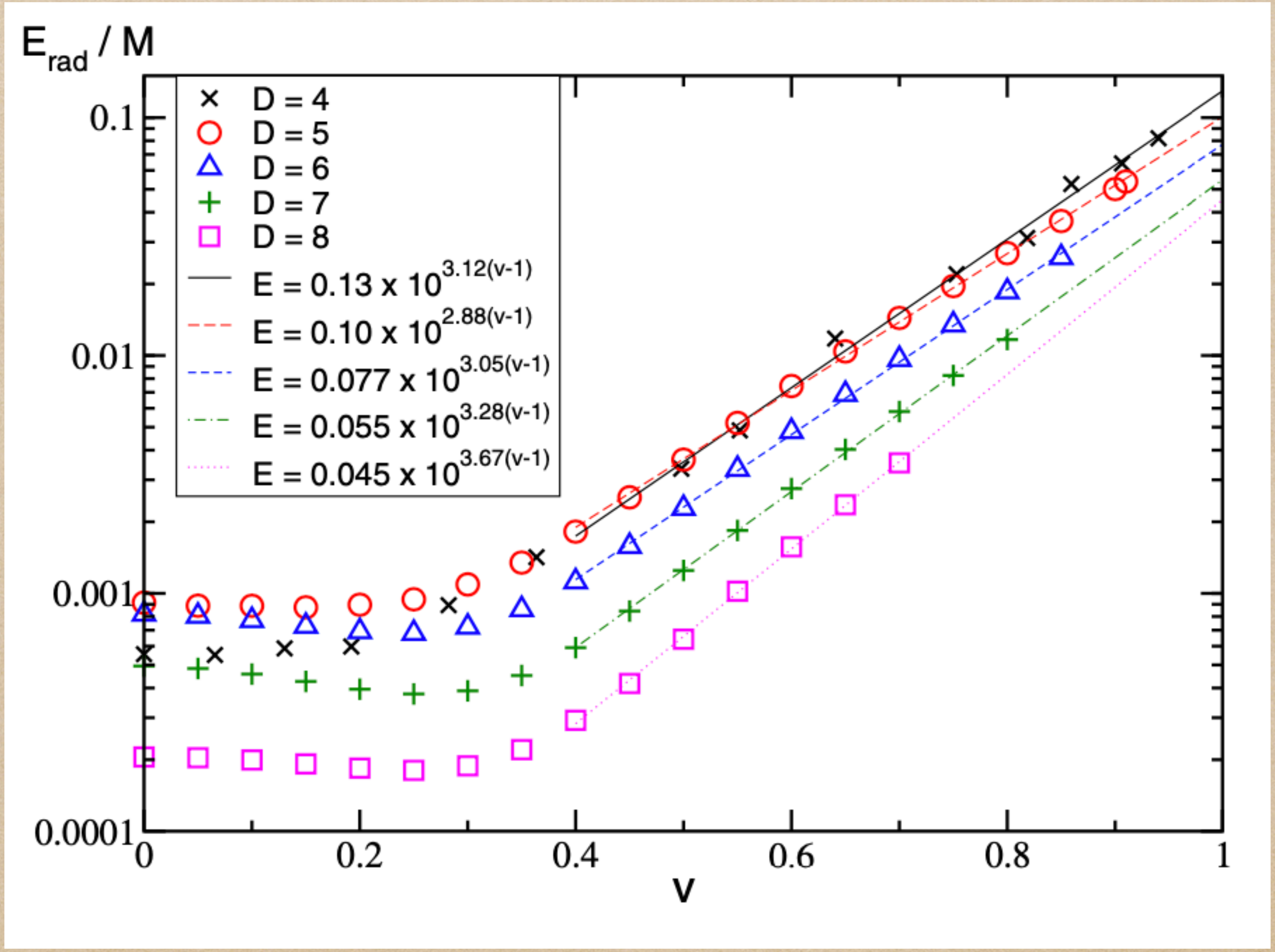
Head-on collisions from rest: $q \leq 1$



Cook et al, 1709.10514

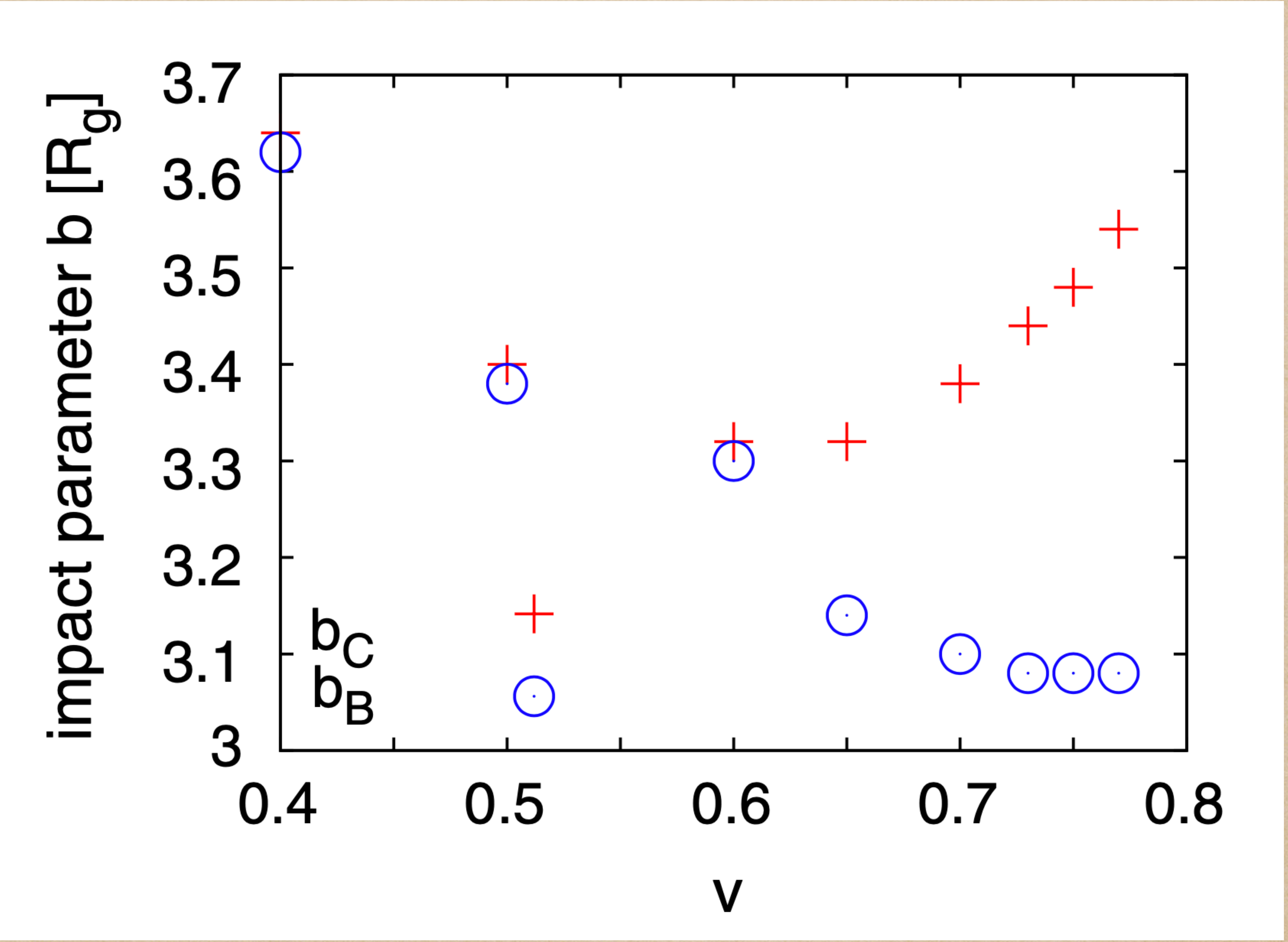
Head-on collisions of BHs in $D \geq 4$

- Boosted, equal-mass, non-spinning BHs



Sperhake et al, 1909.02997

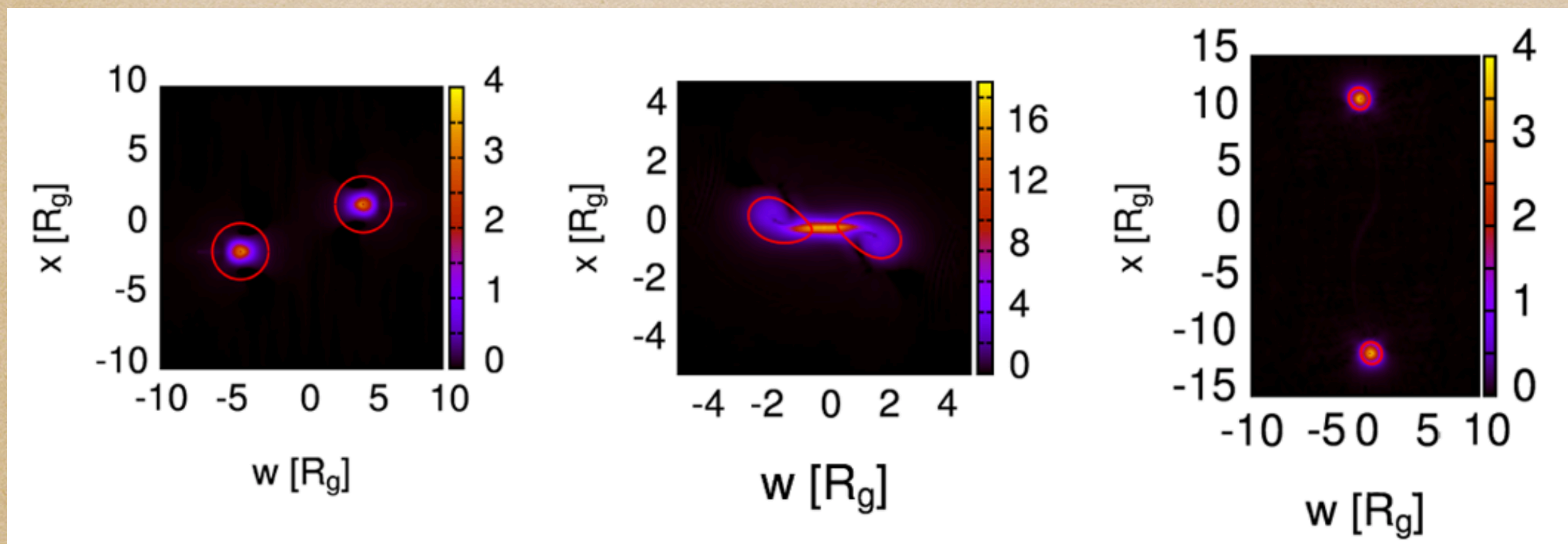
Scattering theshold in D=5



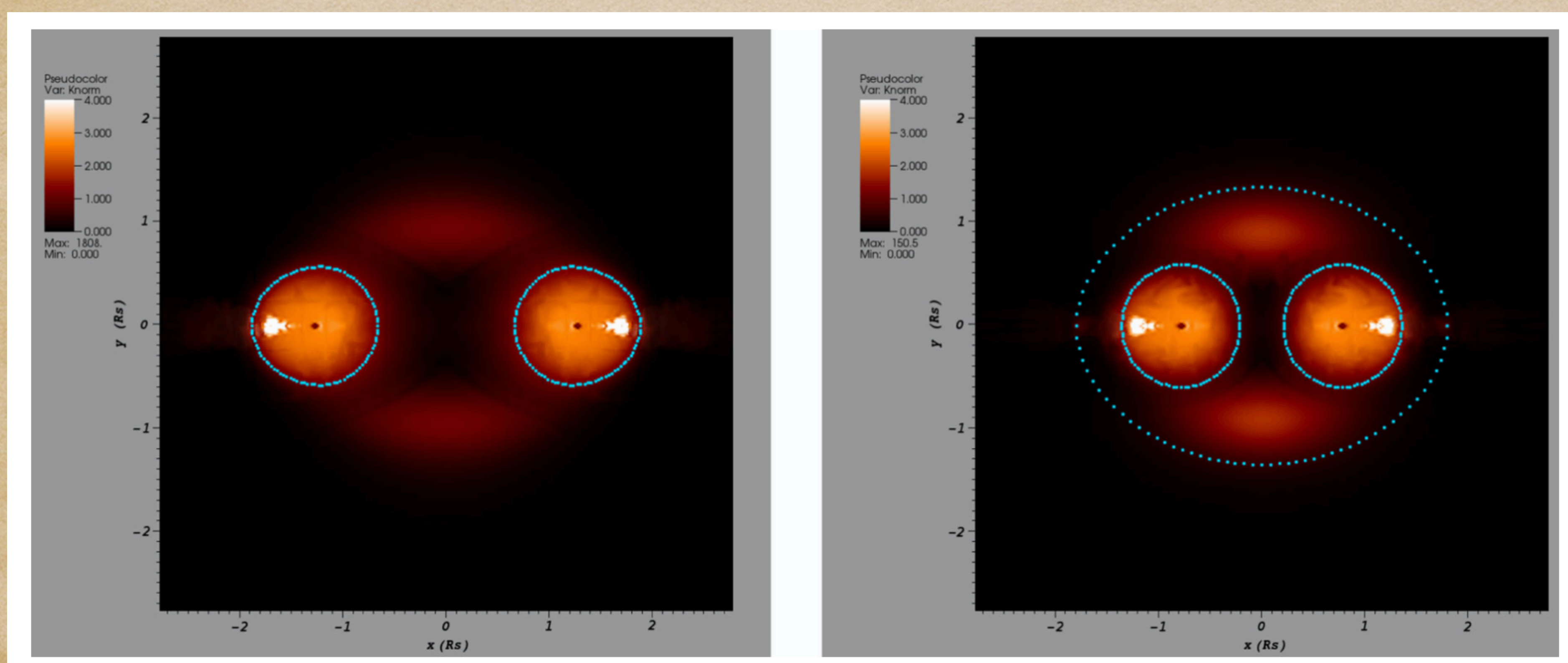
Okawa et al, 1105.3331

Super-Planckian regimes in BH collisions

- Kretschmann scalar: exceeds AH value outside AH



Okawa et al, 1105.3331



Sperhake et al, 1909.02997

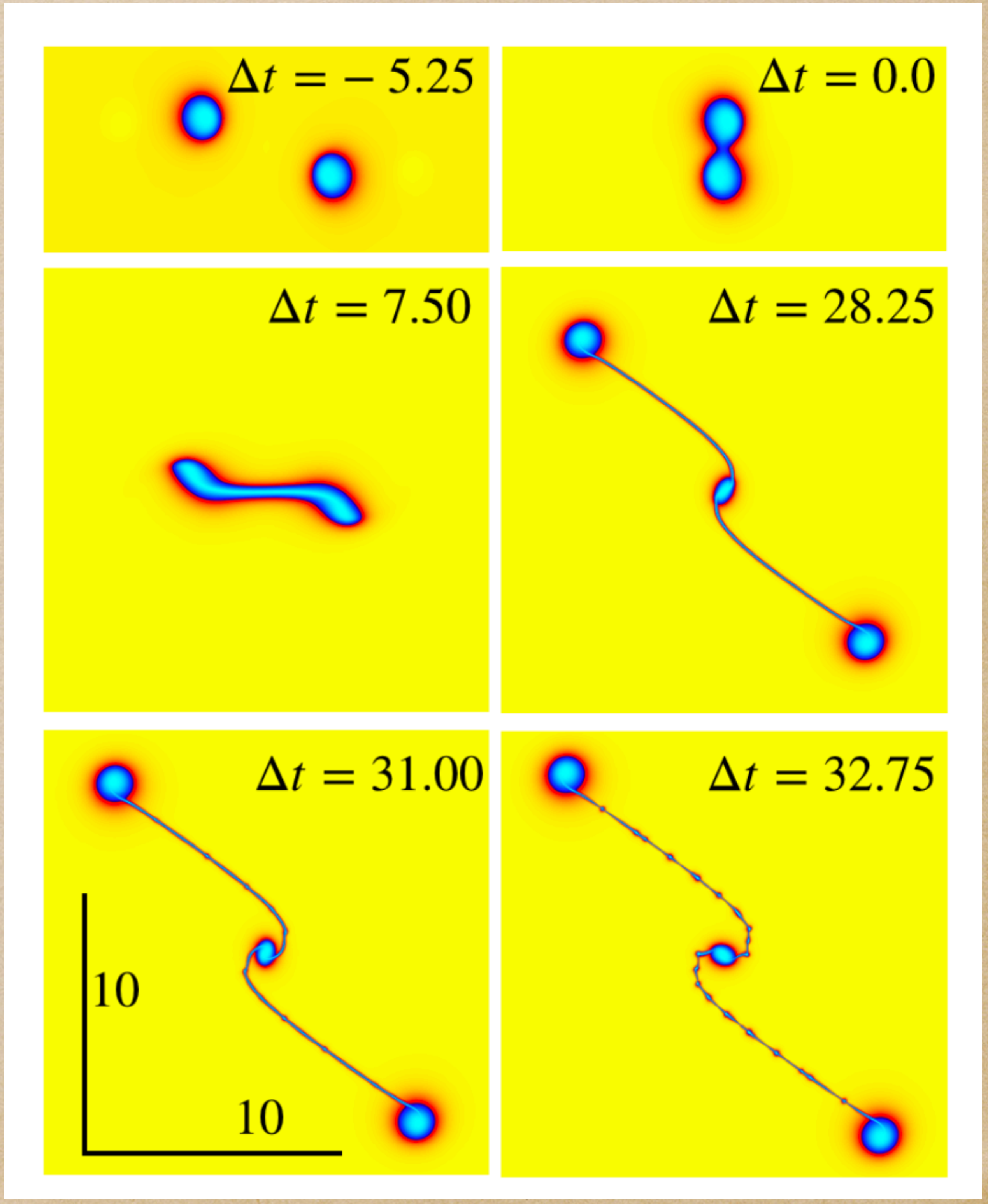
Gregory-Laflamme instability in D=7

- Collide 2 Myers-Perry BHs near threshold of merger

  Andrade et al, 2011.03049

- Like GL in black strings

  Lehner & Pretorius, 1006.5960

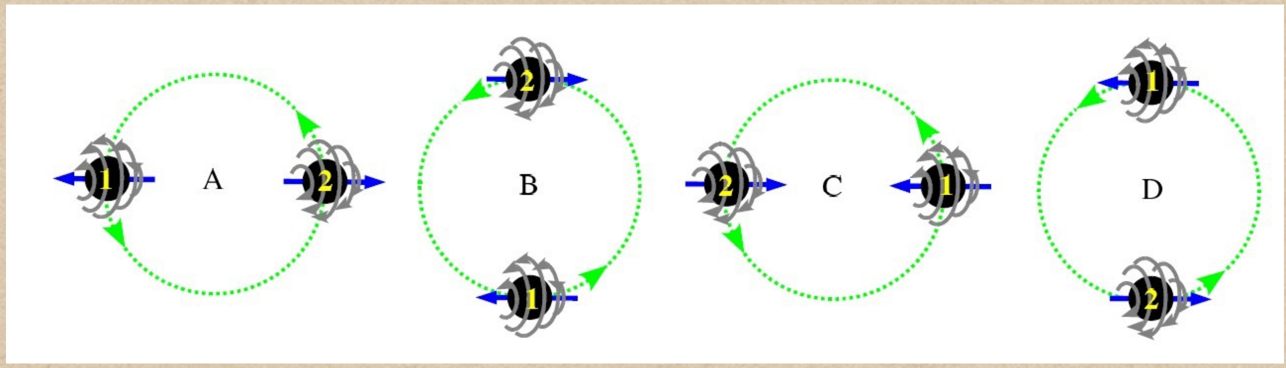


5. Eccentric, unequal-mass BH binaries

Original Motivation: Black-hole kicks

- Anisotropic GW emission $\Rightarrow$ recoil of remnant BH
  - Asymmetry through spin; super kick

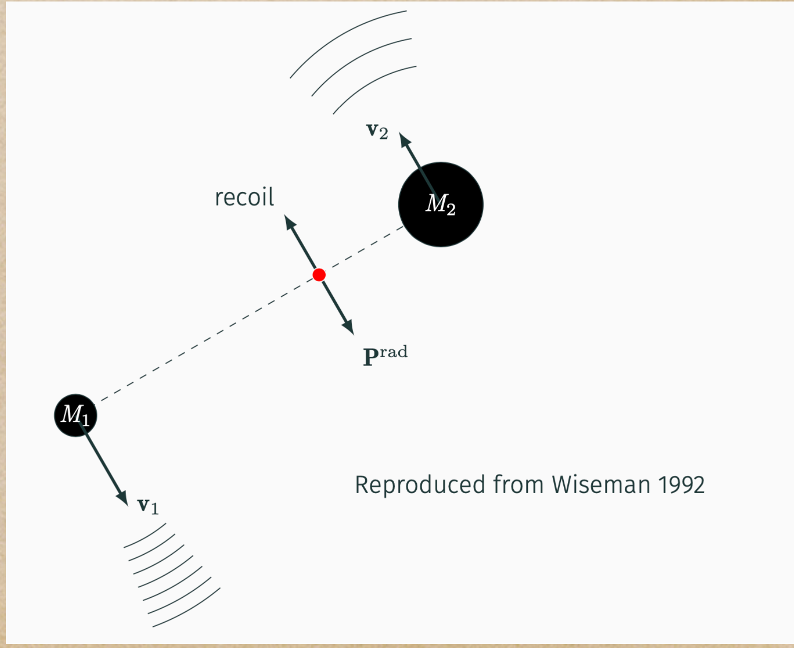
    González et al gr-qc/0702052, Campanelli et al gr-qc/0702133



Pretorius 0710.1338

  - Asymmetry through unequal masses

    González et al gr-qc/0610154



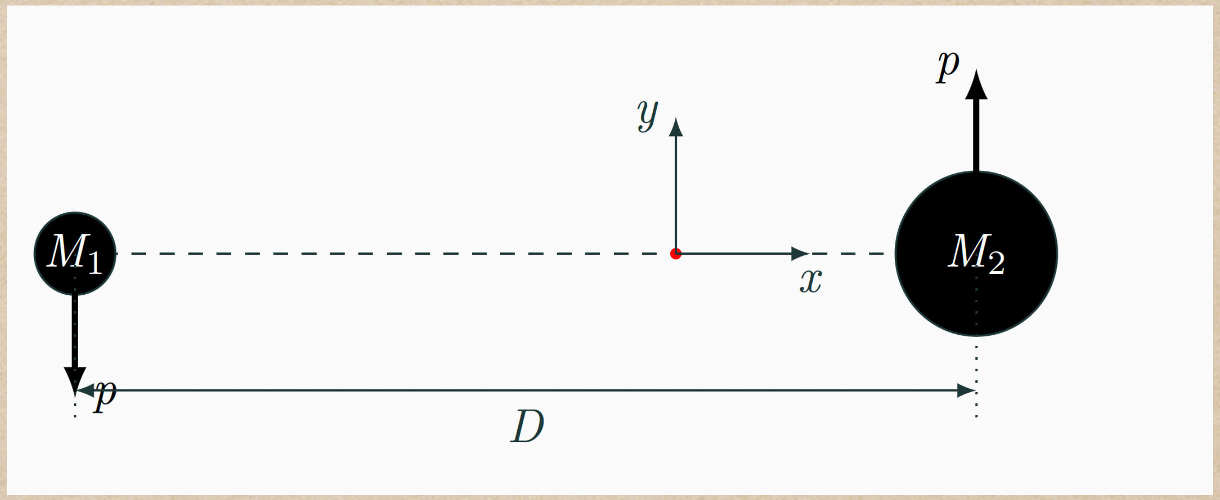
- Kick important for SMBH formation, BH populations, galaxy structure,…
- Eccentricity enhances super kicks

  US et al 1910.01598

  Check the same for unequal-mass kicks

Setup

- Non-spinning BH binaries with masses $M_1 \leq M_2\,, \quad q := \frac{M_1}{M_2}$



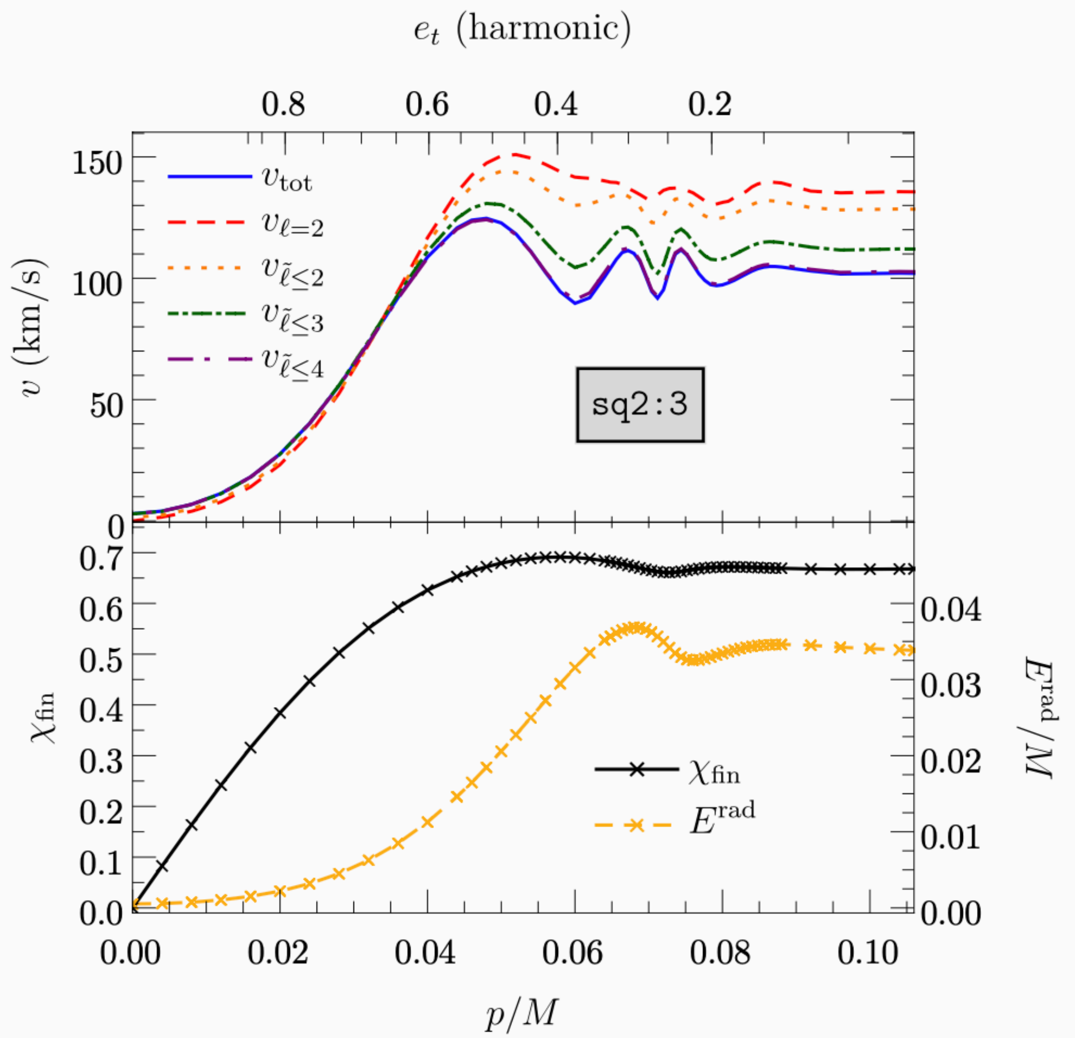
- Vary $D,\ p$ at const. binding energy $E_b = M_{\rm ADM} - M_1 - M_2 = {\rm const}$
- Four sequences: `sq2:3`, `sq1:2`, `lq1:2`, `sq1:3`

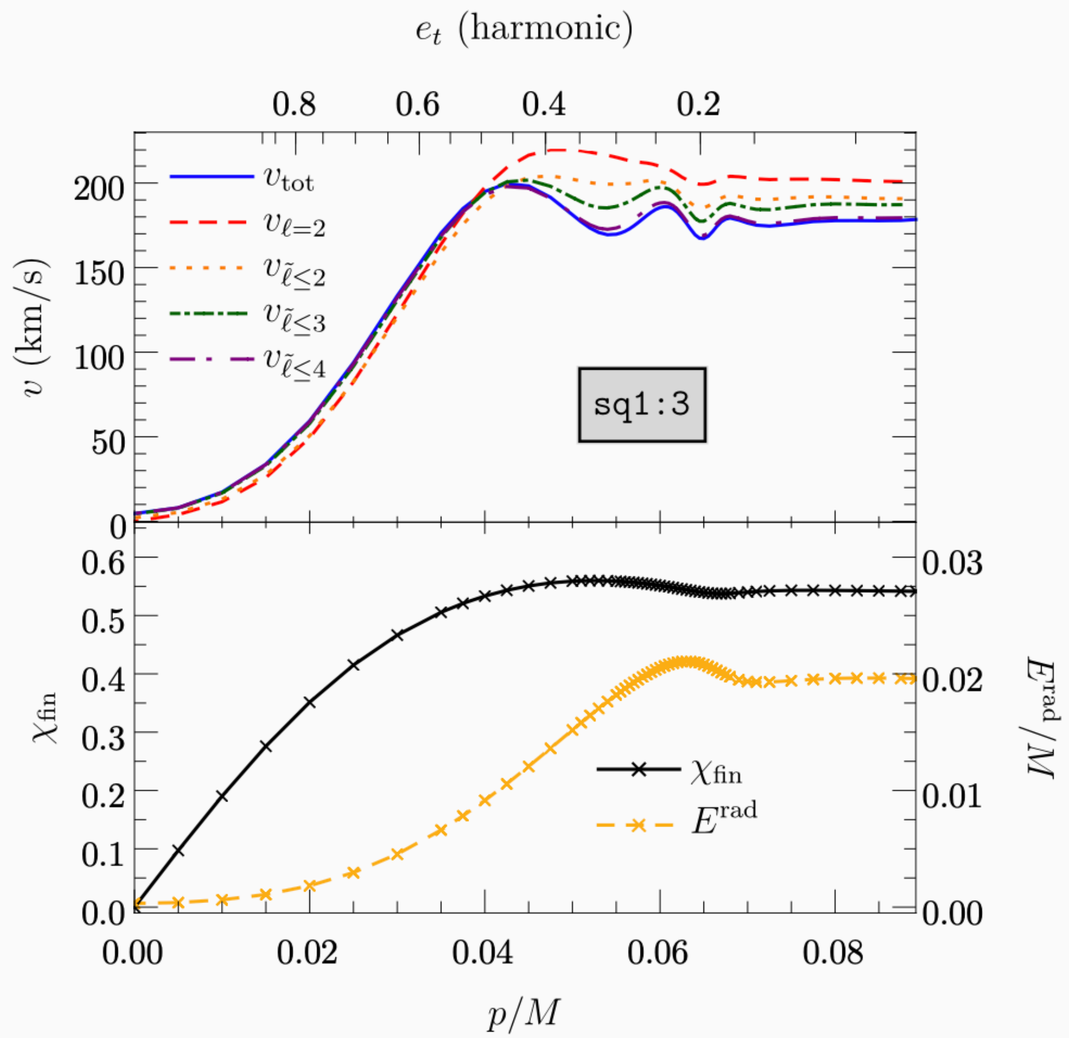
  With 3 mass ratios $q = 2/3, \quad q = 1/2, \quad q = 1/3$
- s = "short" (~ 3 orbits in qc limit), l = "long" (~ 6 Orbits)

  Long sequence `lq1:2` to check for artefacts from short inspiral.
- No rigorous eccentricity estimate in GR

  Use 3PN (harmonic gauge) e_t Memmesheimer at al gr-qc/0407049

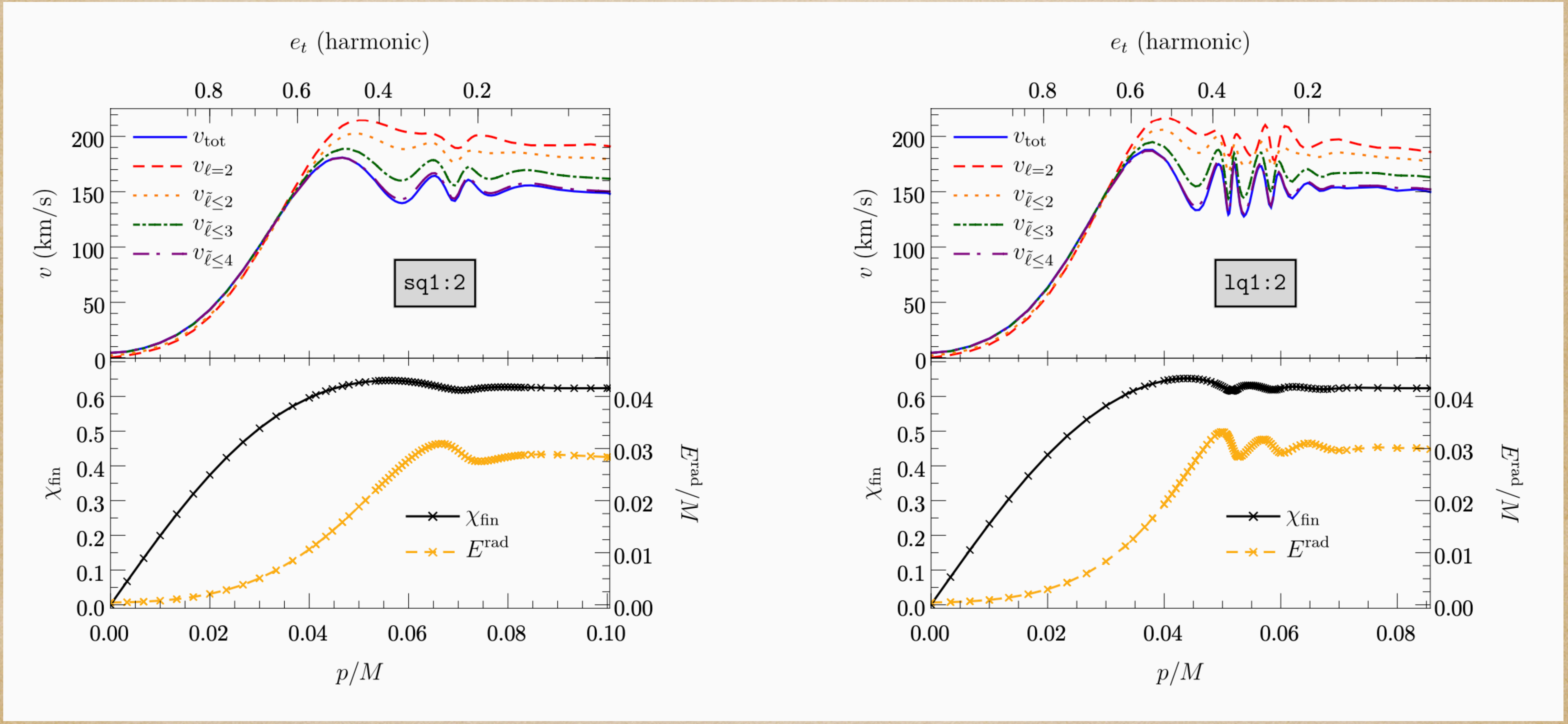
Results: sq2:3, sq1:3





- Oscillatory dependence on eccentricity!

Results: sq1:2, lq1:2



- Oscillatory dependence on eccentricity!

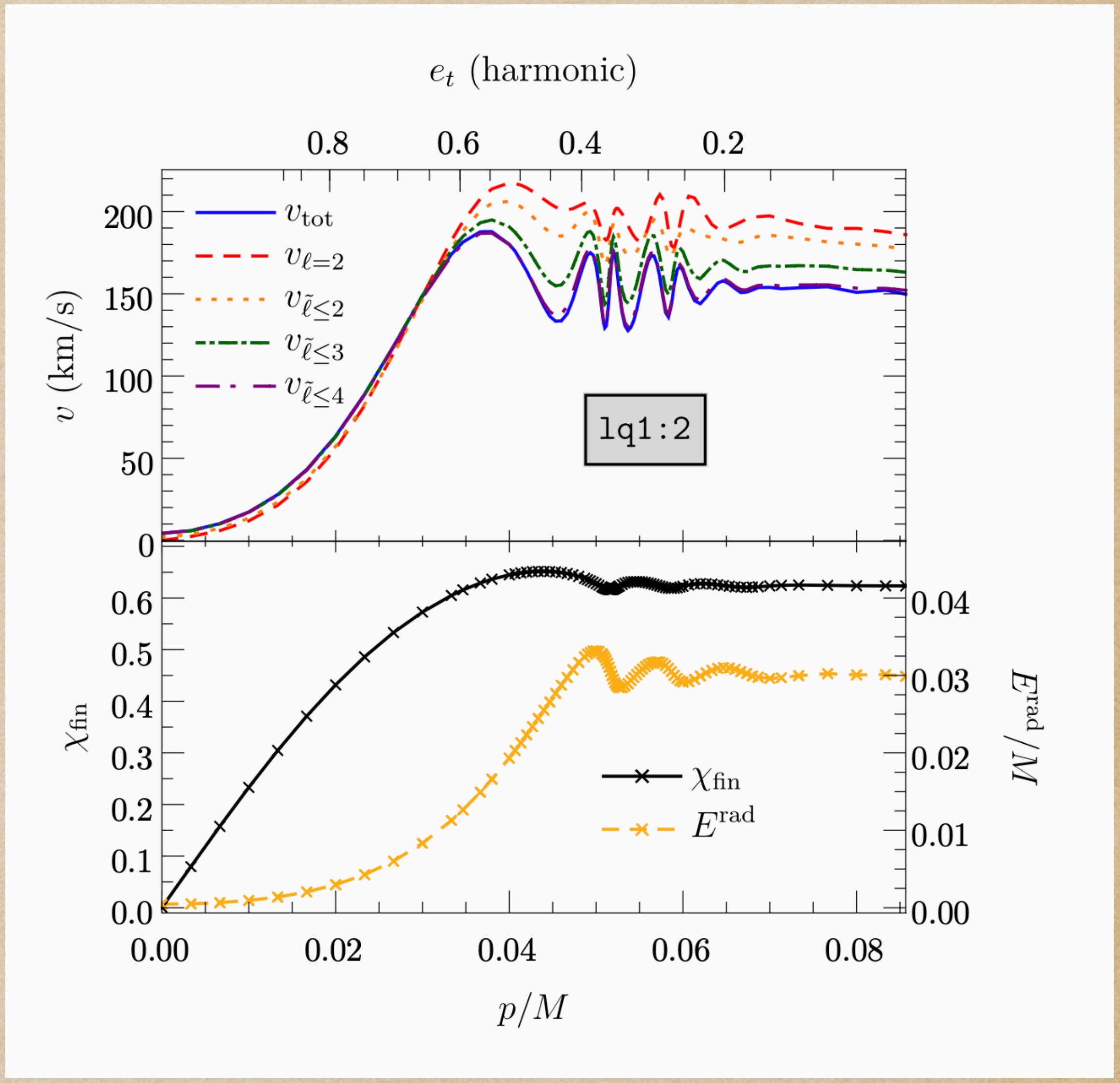
Summary of observations

- Max kick at $e_t \sim 0.5$

  Exceeds qc kick by

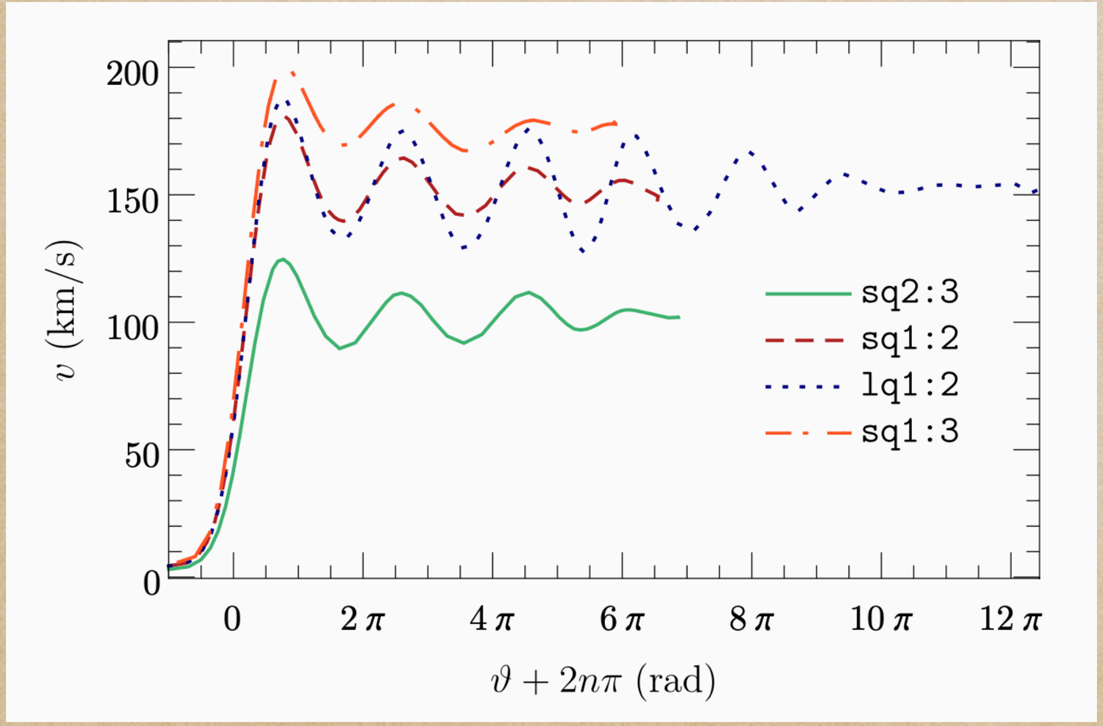
sq2:3	sq1:2	lq1:2	sq1:3
$22\,\%$	$22\,\%$	$25\,\%$	$12\,\%$

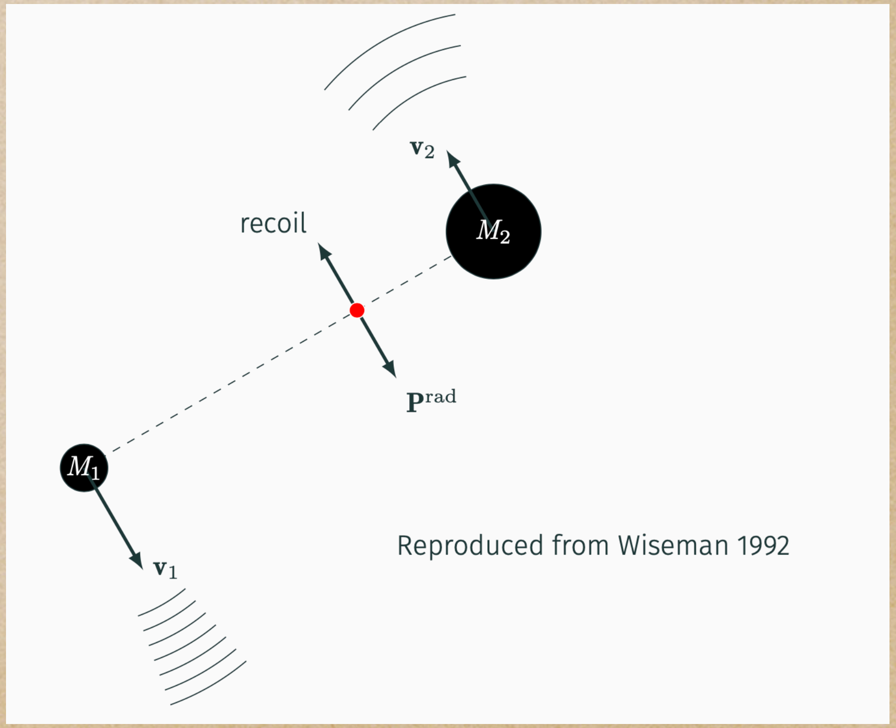
- Oscillatory variation increases in frequency, magnitude for longer inspiral; high sensitivity for long inspirals?
- Fewer/less pronounced Oscillations in $E_{\rm rad}$, $\chi_{\rm fin}$; Extrema not aligned.
- Oscillations in all partial recoils $v_{\tilde{\ell}\leq\tilde{\ell}_0}$
- Higher-order terms $v_{\tilde{\ell}>2}$ systematically reduce kick



Interpretation

- Only "special" direction: apoapsis
- Goal: Measure BH infall direction relative to apoapsis
- Problem: neither rigorously defined. So approximate
  apoapsis $\approx x$ Axis; infall $\sim$ kick direction (beaming!)
- Ideally expect 2π periodicity of $v_{\rm kick}(\vartheta)$
- Must not forget apsidal precession (Mercury!), so only $\lesssim 2\pi$



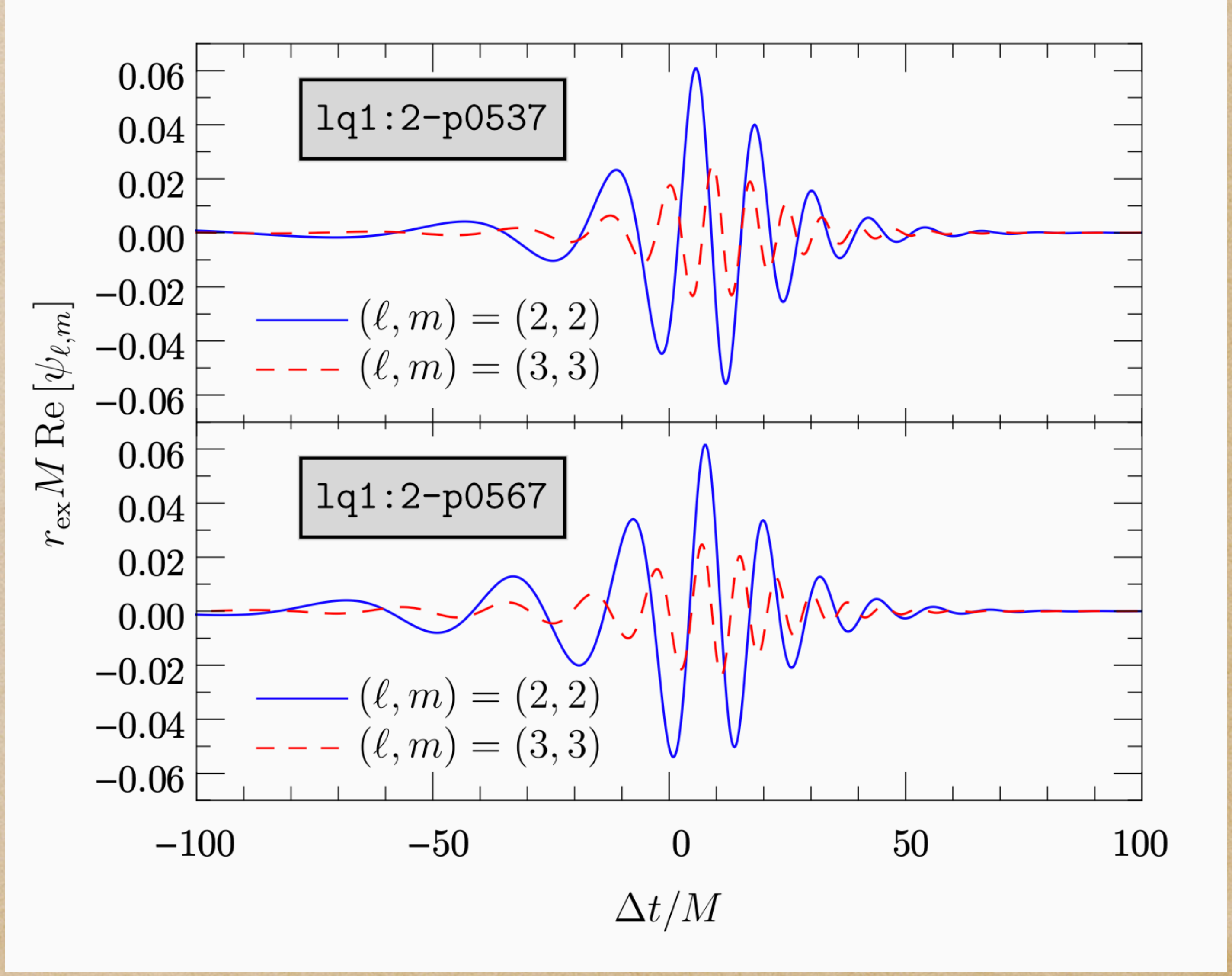


First glimpse at waveform features

- Kick arises from overlap of multipoles!
- So expect features in $\psi_{\ell' m'}$ **relative** to $\psi_{\ell m}$
- Example: ψ_{33} versus ψ_{22} for:

lq1:2-p0537	lq1:2-p0567
$v = 128\,\mathrm{km/s}$	$v = 173\,\mathrm{km/s}$

- Better alignment for large kick
- Similar pattern for other $v_{\min} - v_{\max}$ configurations and other multipoles



Summary

- High-energy head-on collisions in D=4: $E_{\rm rad}/M$ up to $13\,\%$
- Grazing collisions in D=4: $E_{\rm rad}/M$ up to $\sim 50\,\%$

  Inner structure of BHs washed out for large velocities

  Scattering threshold $b_{\rm scat} = \frac{2.5 \pm 0.05}{v} M$
- GW emission in general weaker in higher dimensions
- Trans Planckian regions may occur outside horizons in D>4
- Max kick at $e_t \sim 0.5$; $\approx 12 - 25\,\%$ larger than qc result.
- Additional oscillations in $v_{\rm kick}(e_t)$
- Oscillations stronger and more rapid in long inspirals.

  Suggests that kick and GW sensitively depend on e_t i.e.

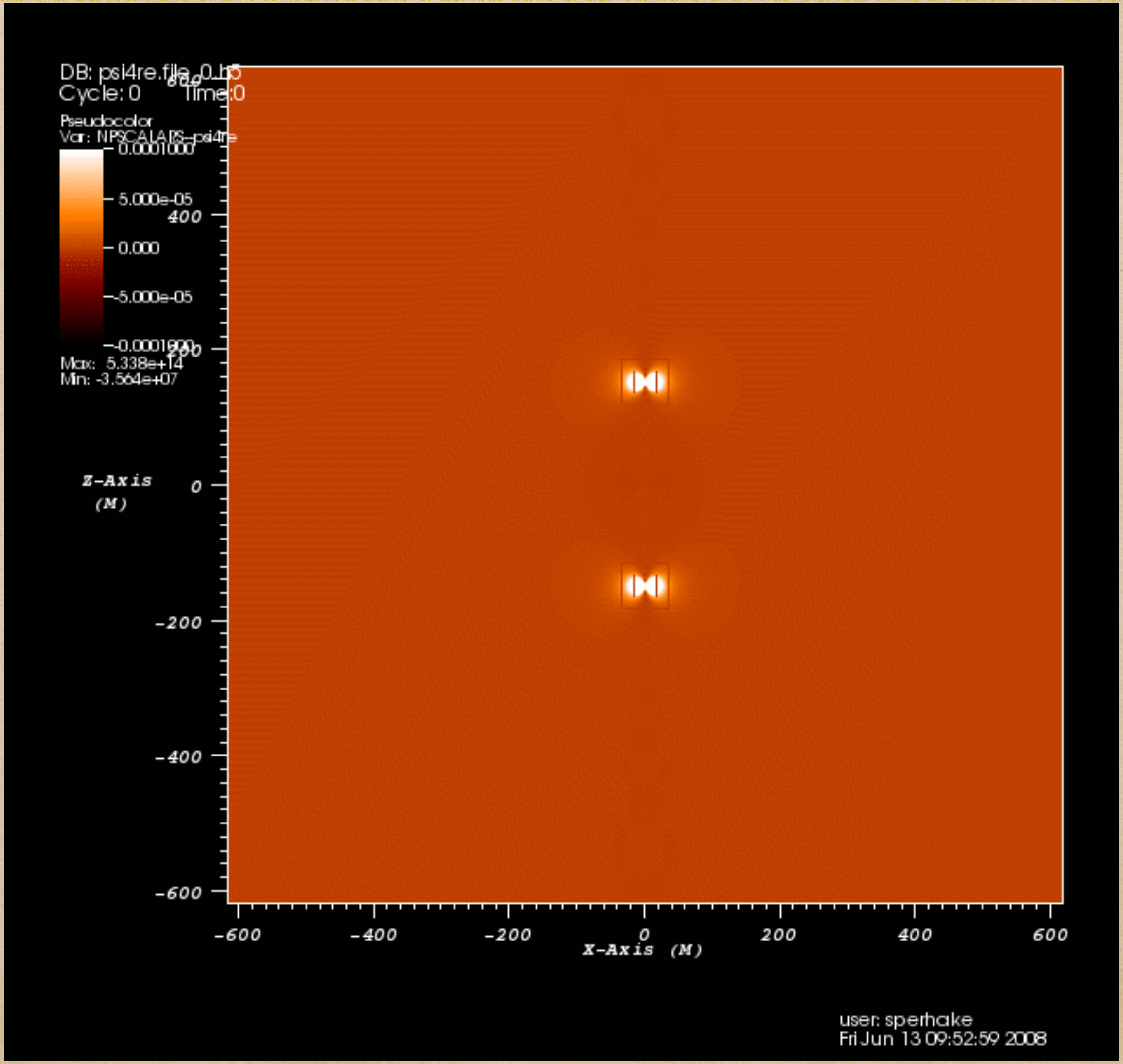
  Infinitesimal ${\rm d}e_t$ may cause finite change in $v_{\rm kick}$, $\psi_{\ell m}$
- Kick variation due to angle of infall vs. apsidal direction

2. High-energy head-on collisions of BHs

Head-on collisions of BHs in 4D

$v = 0.94$

$\gamma = 2.93$



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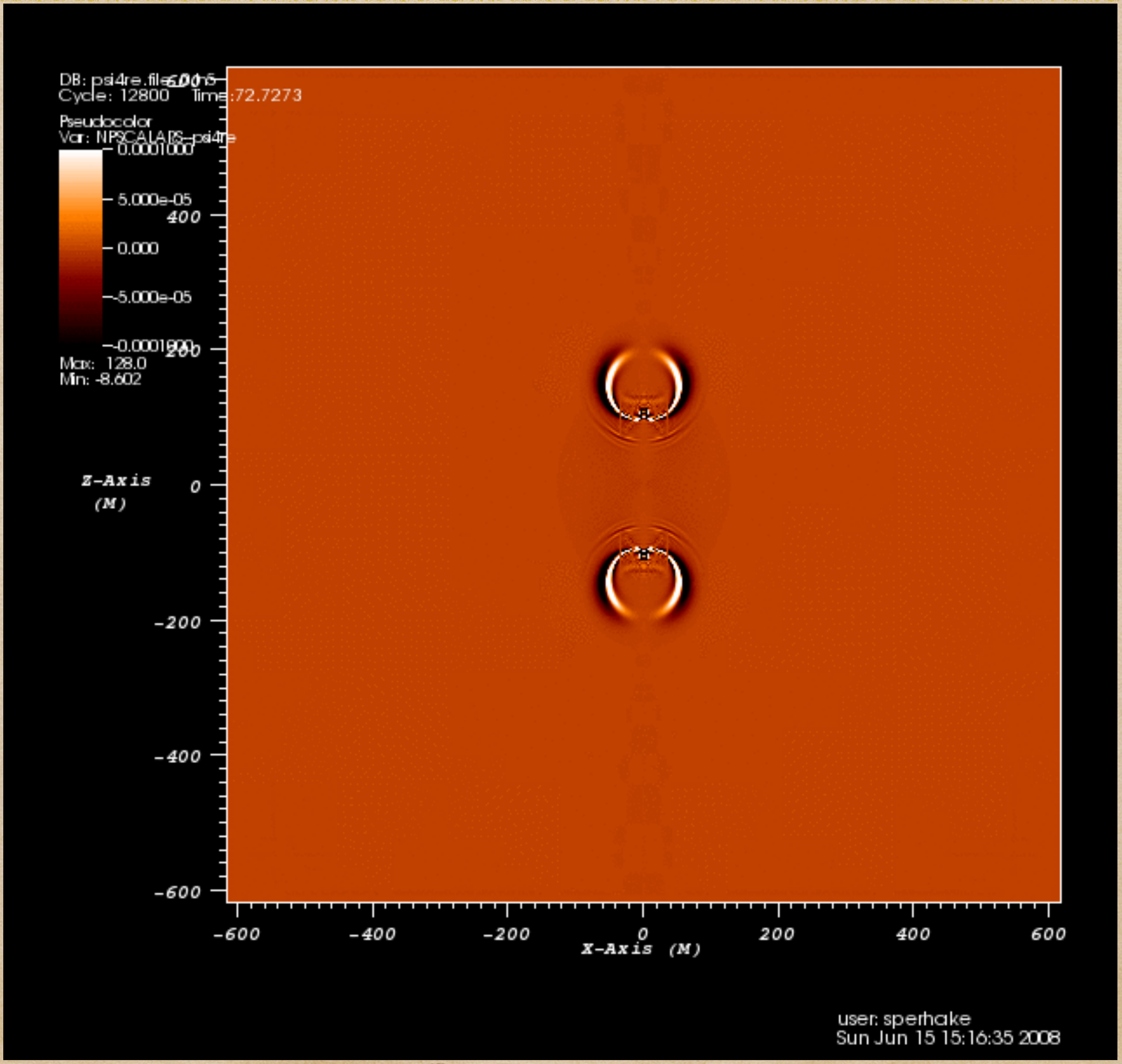

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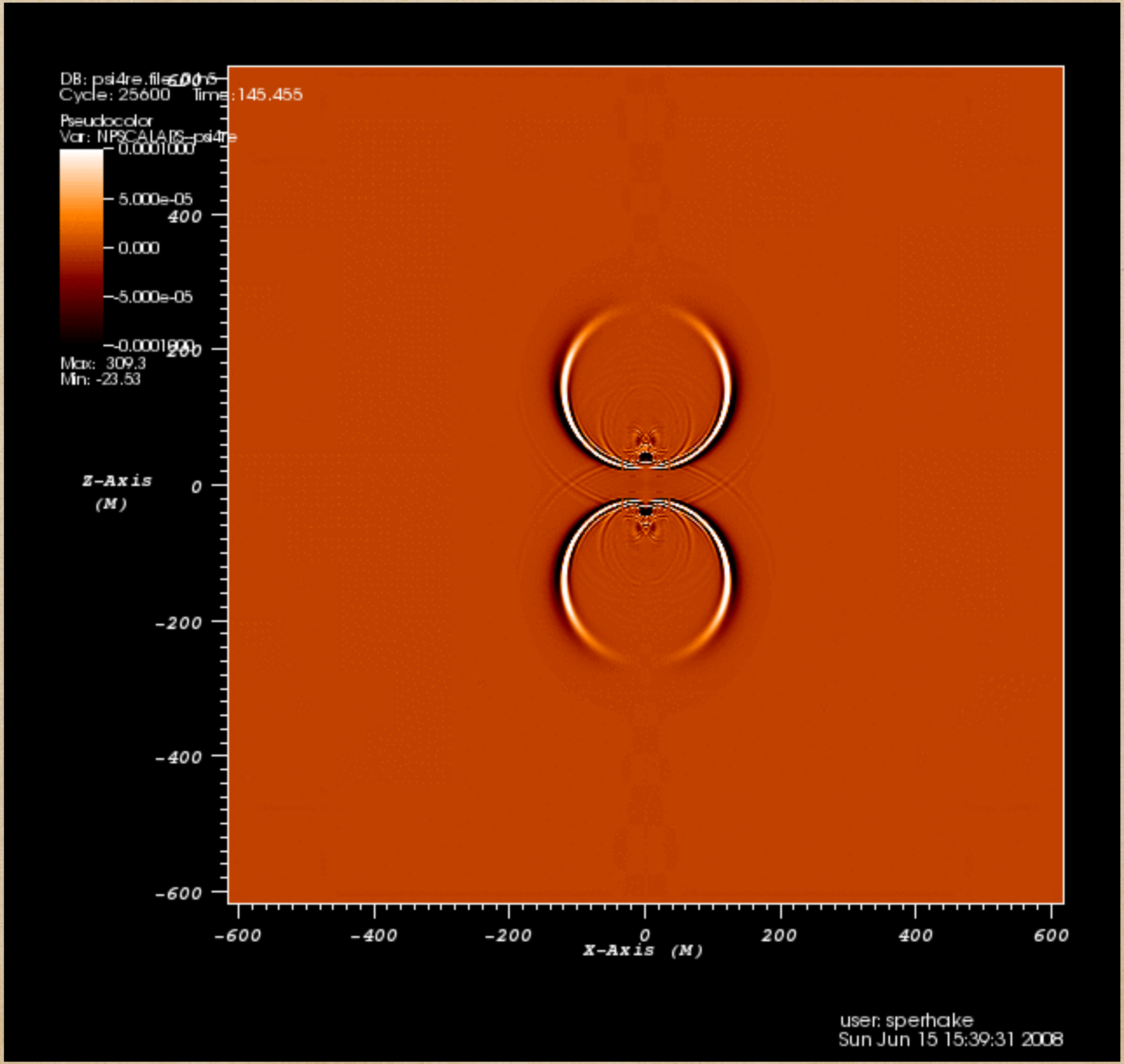

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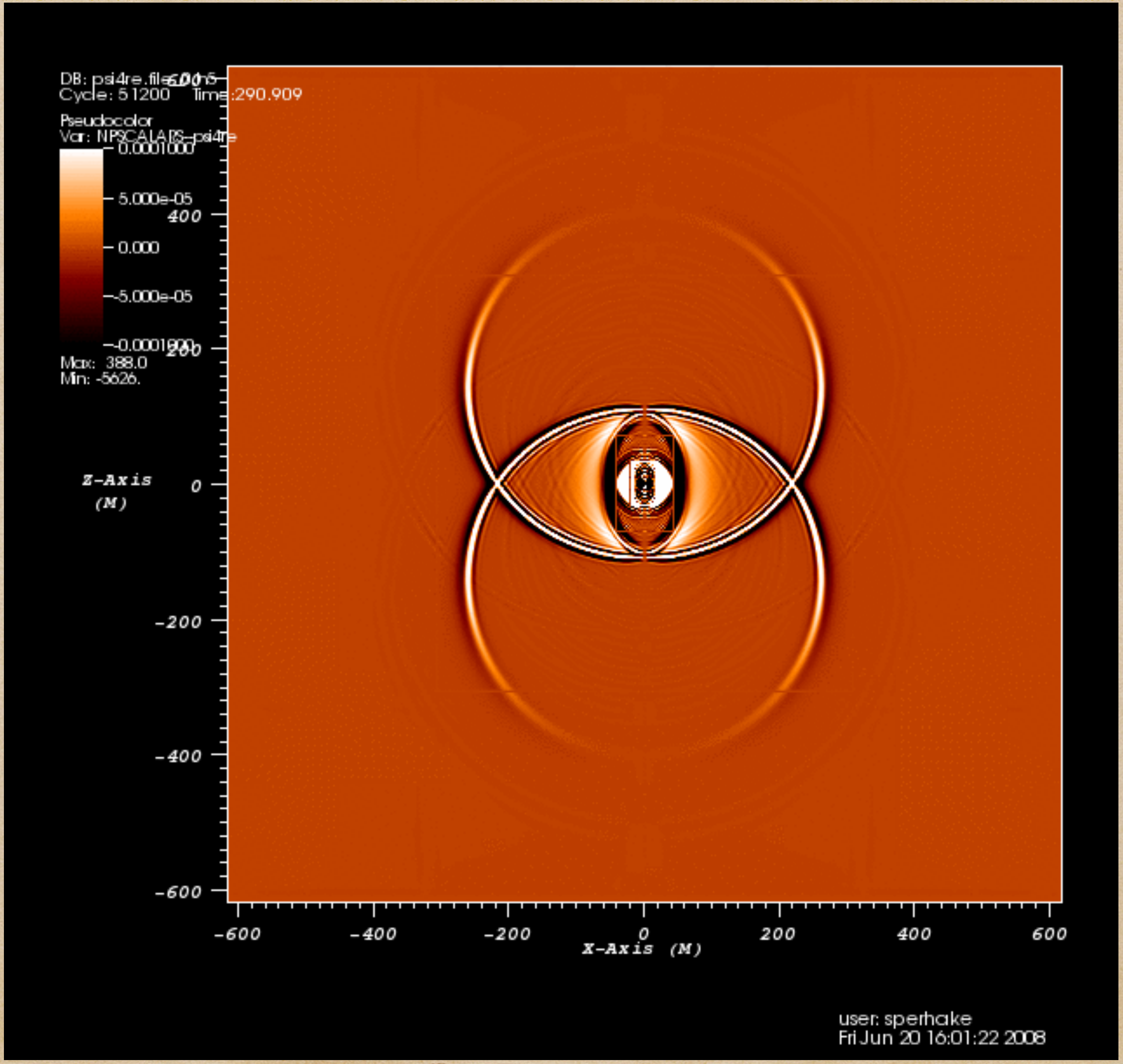

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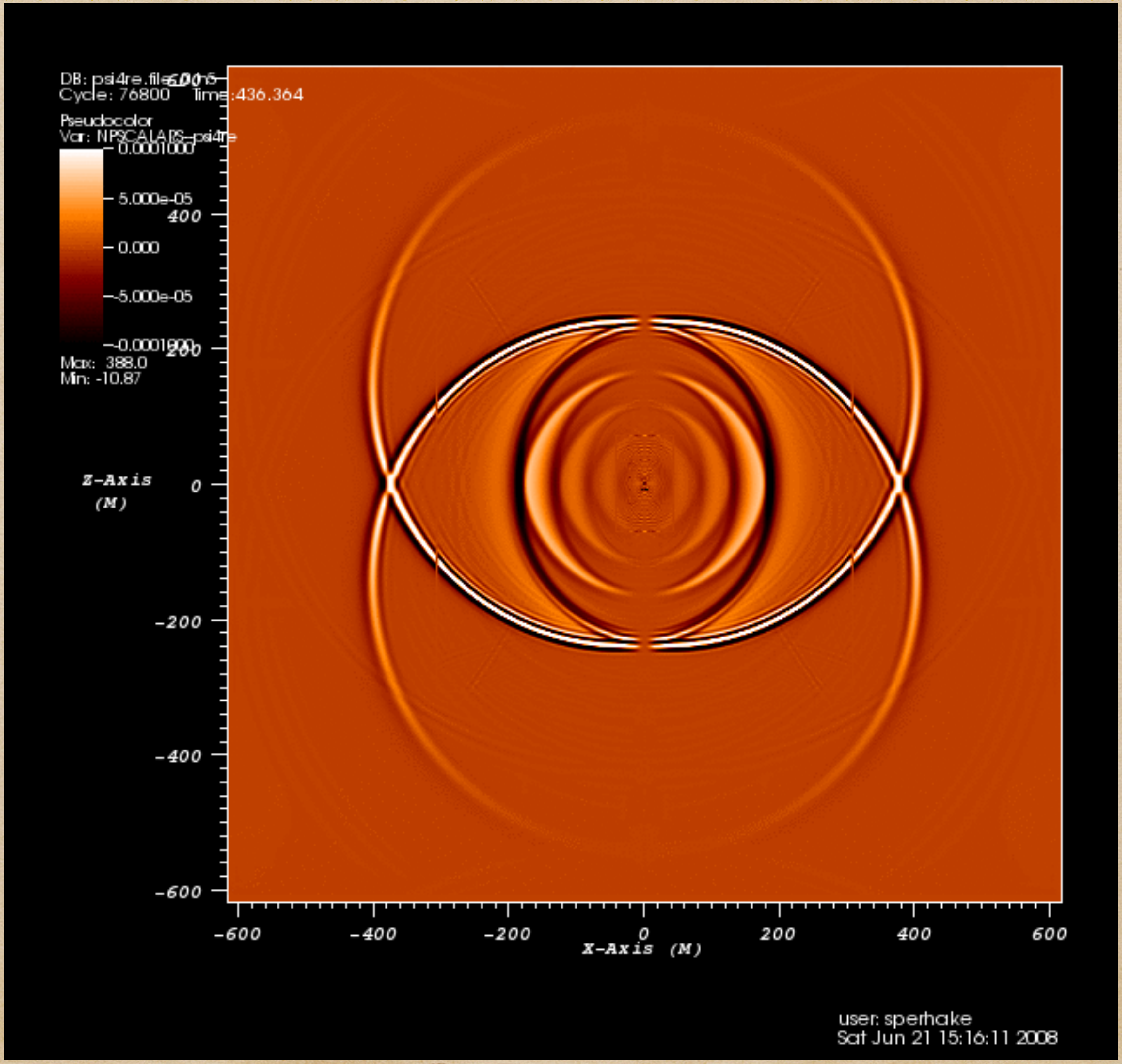

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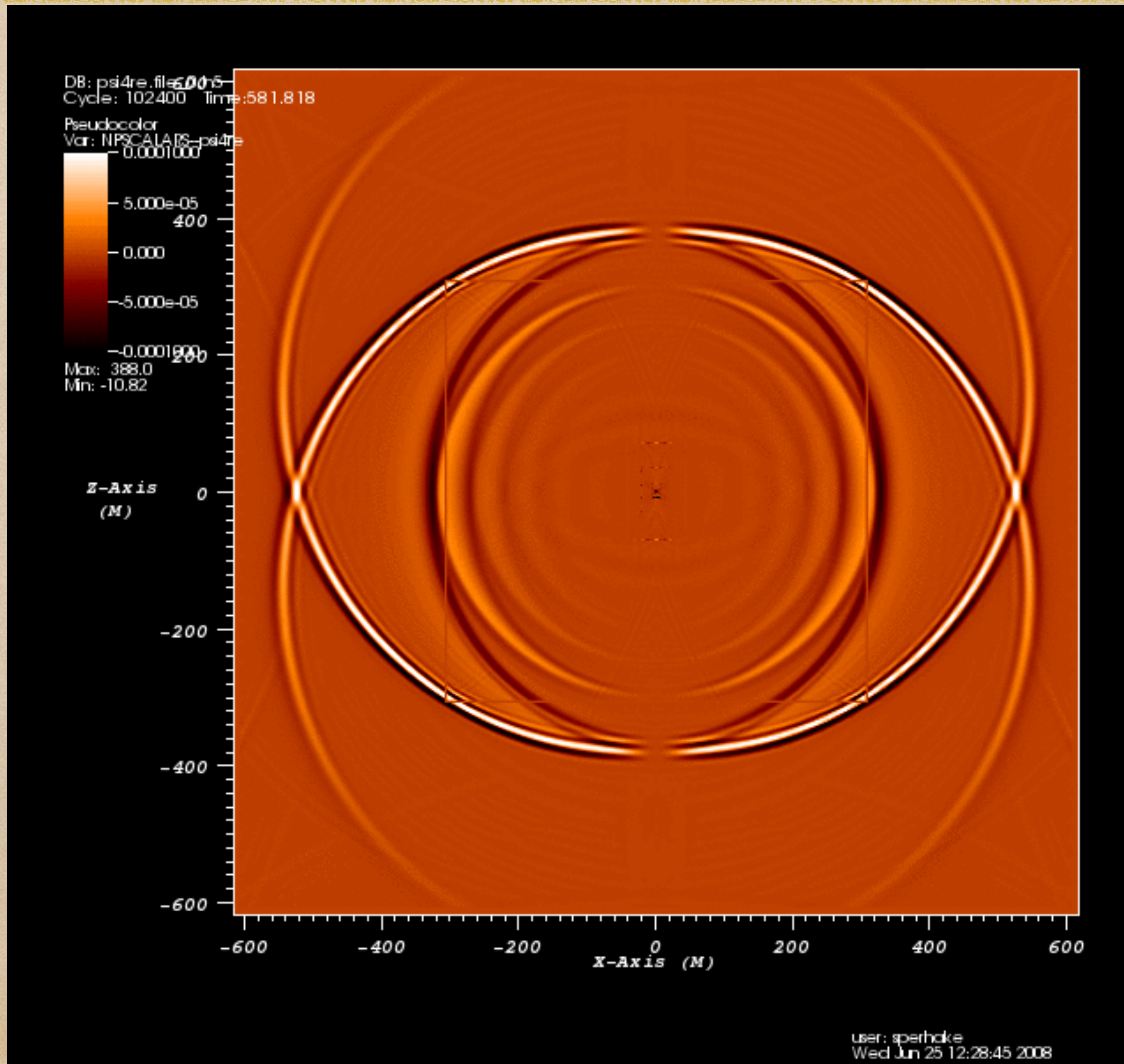
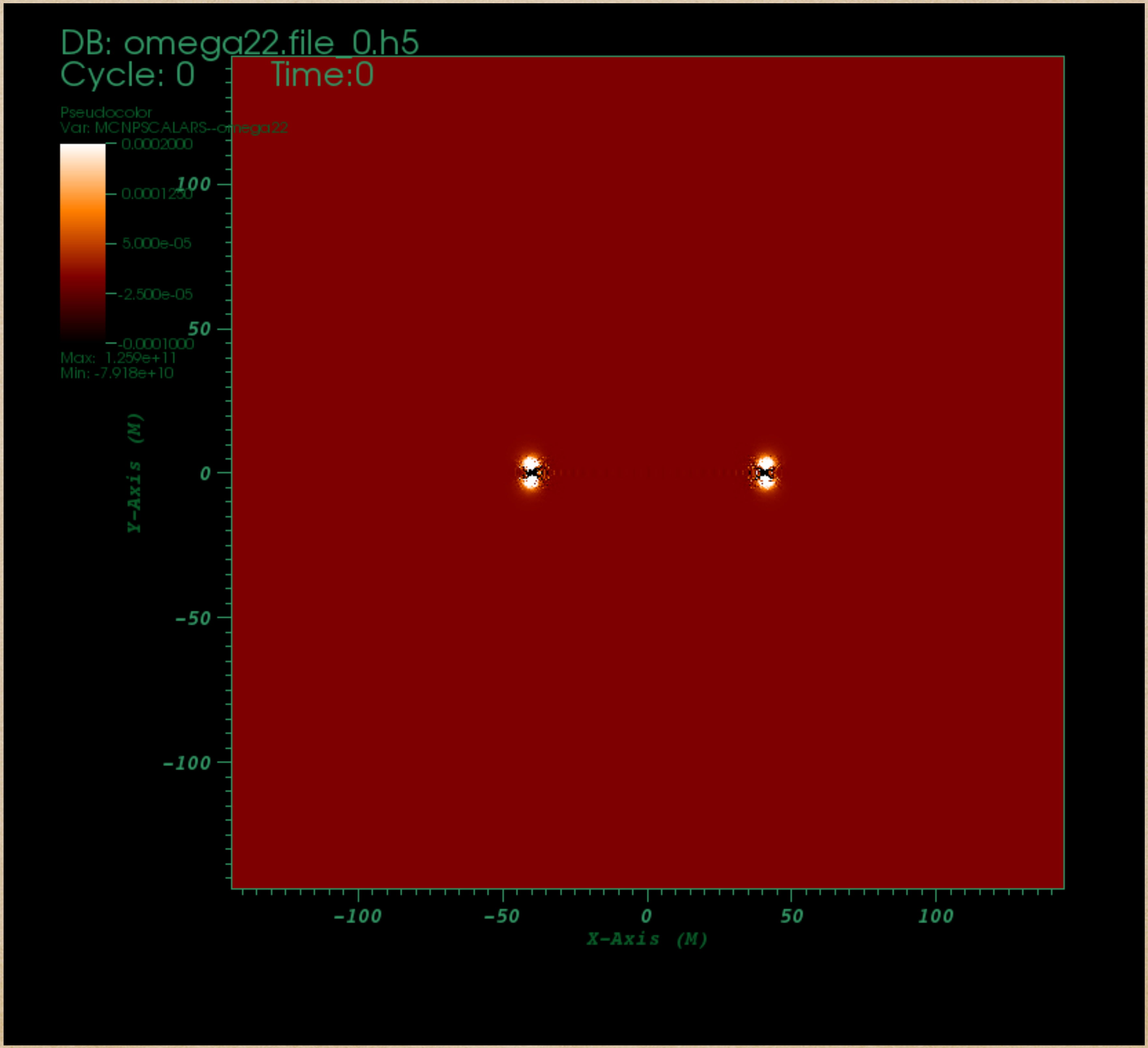


4. Black-hole collisions in $D>4$

Boosted head-on collision $D = 6\,, \quad v \approx 0.54\,c$



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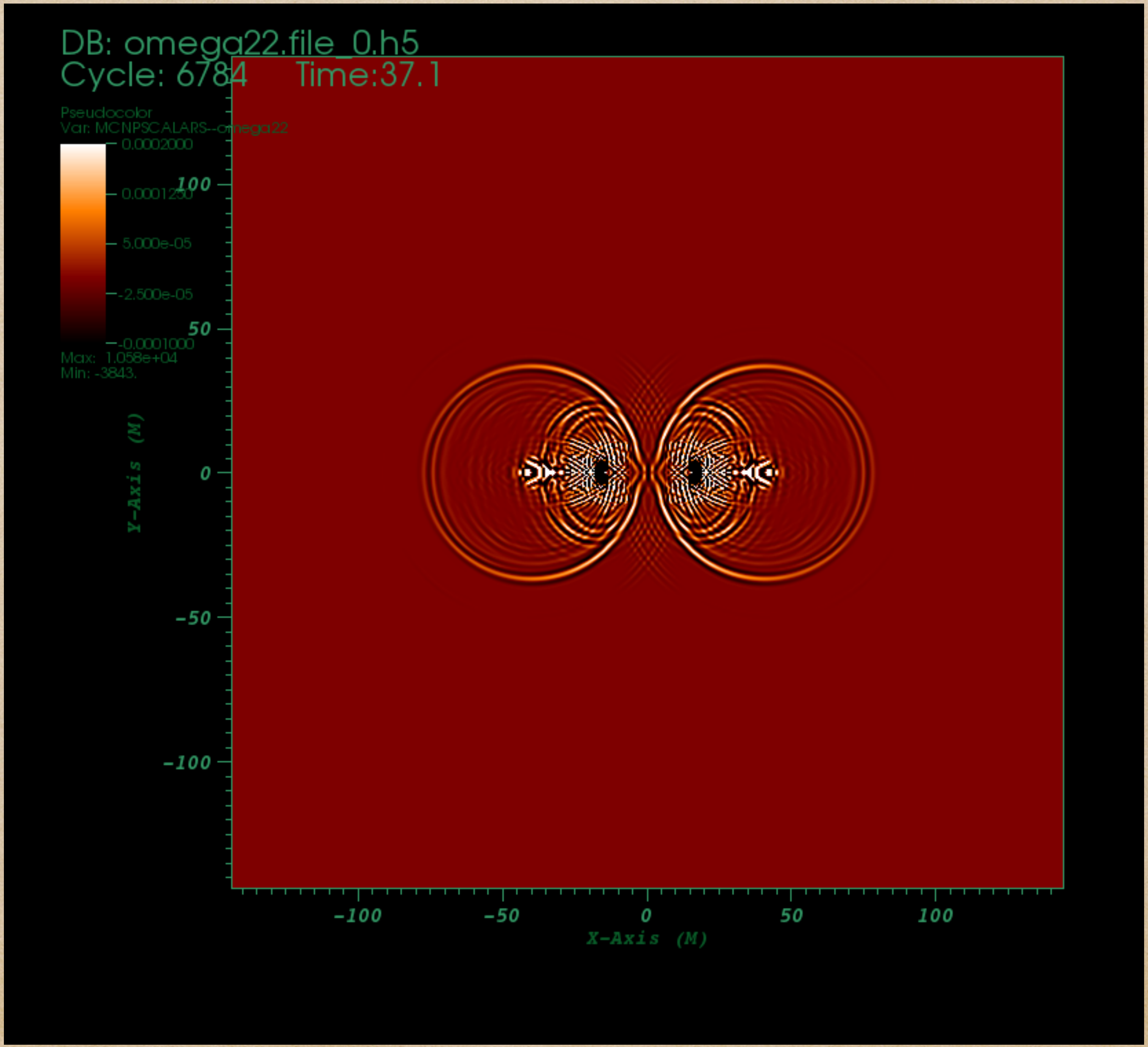

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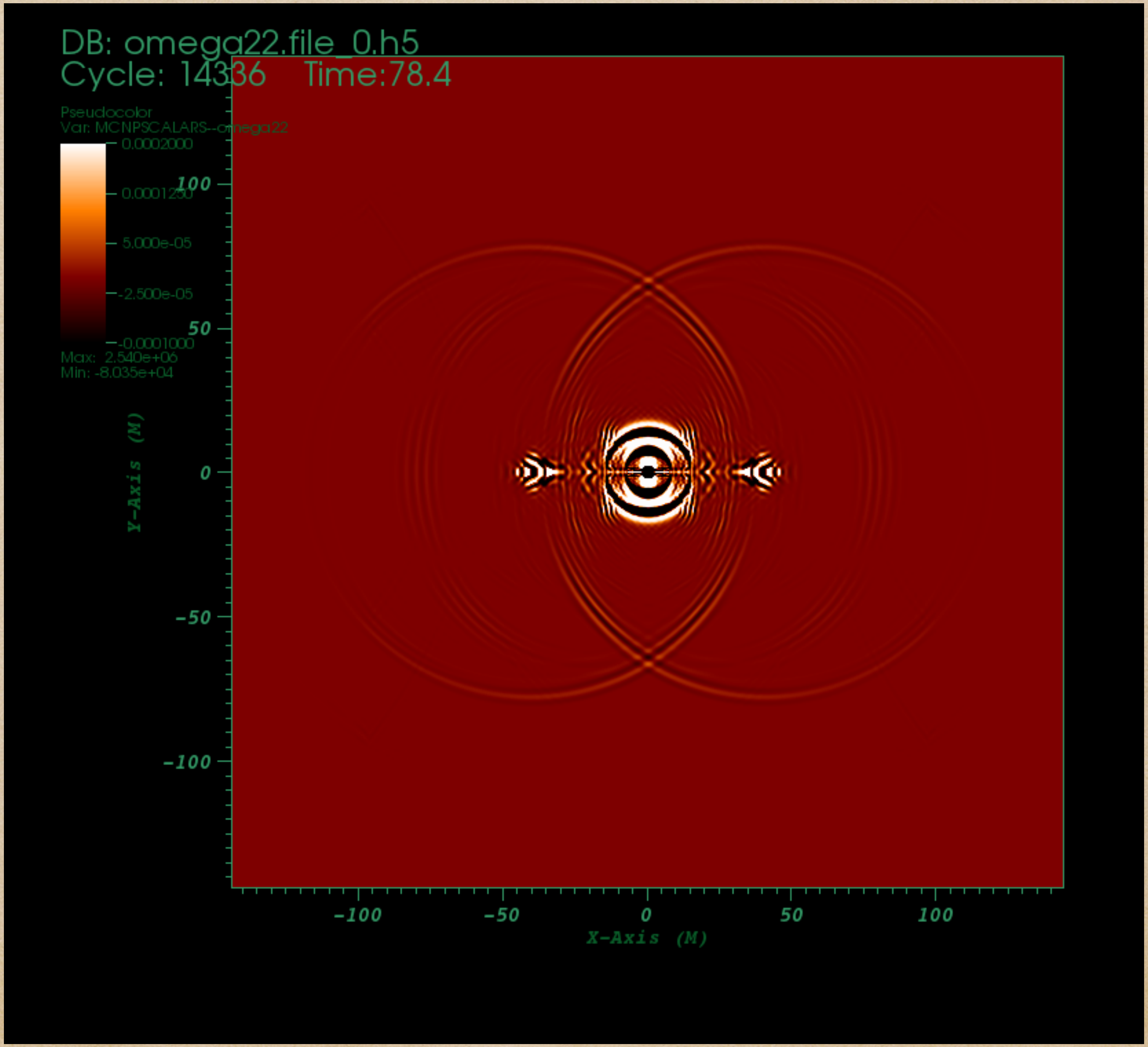

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