Conservative Potential from Scattering Amplitudes for Binary Systems with Conformal Coupling to Galileons

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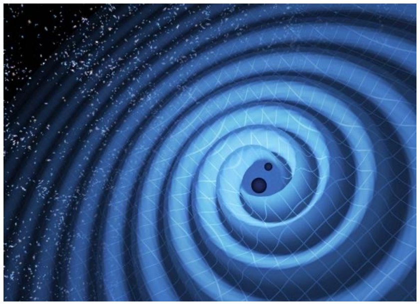
Work in progress with Claudia de Rham and Andrew Tolley

GGI Gravitational scattering, inspiral, and radiation Workshop

Imperial College London

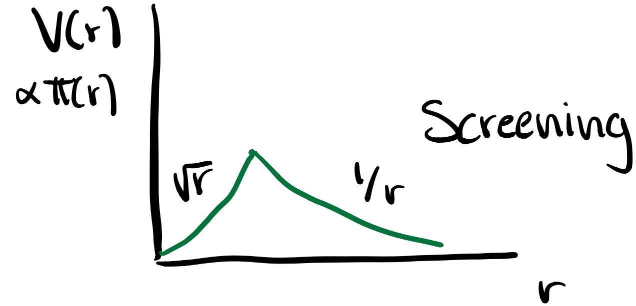
Outline

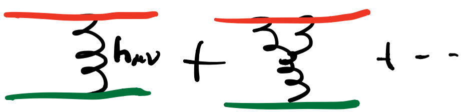
- Motivation:



Beyond GR

5th Force



- 

$\Updownarrow$

$V(\bar{p}, \bar{r}, \bar{S}_a)$ m_1 m_2

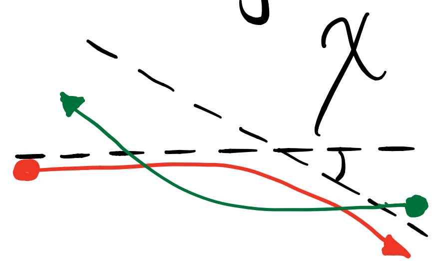
- $S_{p.p.}[A^2(\pi) g_{\mu\nu}, x^\mu]$

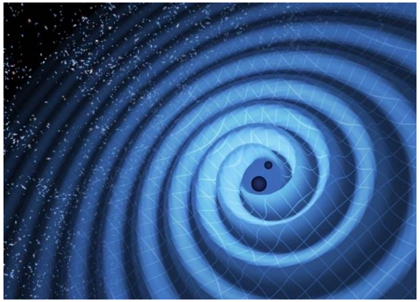
$\Longleftrightarrow$ $V(\bar{p}, \bar{r})$

match

subtle for conformal coupling

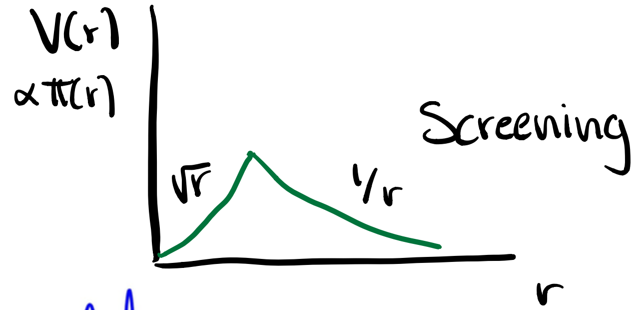
- Scattering angle

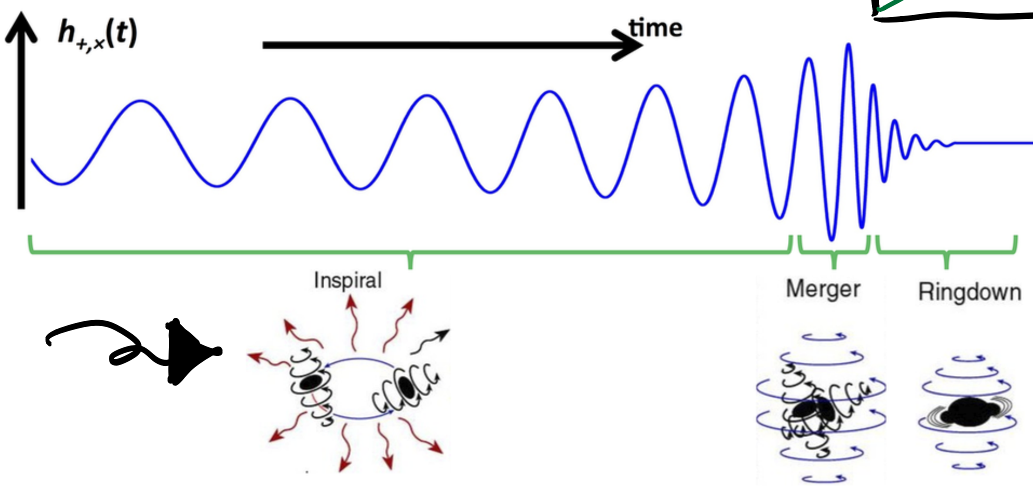




Motivation

5th Force





perturbative Gravity

Can test GR extensions

Classical info. can be extracted from $\mathcal{A}$

HERE: Proof of principle for cubic galileon

Why go beyond GR?

Missing explanation for Dark Energy

⇒ Scalar-tensor theories

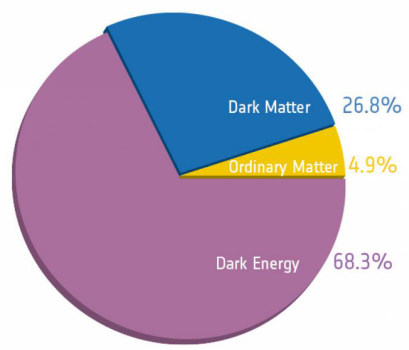
$$S = \int d^4x \sqrt{-g}\, \mathcal{L}[\phi, \nabla\phi, \nabla^2\phi; g_{\mu\nu}]$$

2+1 DOF ↔ no ghosts

i.e. Horndeski

→ B. Horndeski

DHOST



Can give accelerated expansion

Solar system tests

No deviation from GR observed

$\rightarrow$ Screening mechanism, IR modifications

Constraints from c_{GW}

GW170817 / GRB 170817 A   LIGO COLLABORATION

DE

$H_0 \simeq 10^{-42}\,\mathrm{GeV}$

LIGO

$P_{LIGO} \simeq 10^{-22}\,\mathrm{GeV}$

$$\boxed{10^{-15} \lesssim \frac{c_{GW}}{c} - 1 \lesssim 10^{-16}}$$

Gravitational Cherenkov radiation   MOORE, NELSON

COSMIC RAY

$E \sim 10^{10}\,\mathrm{GeV}$

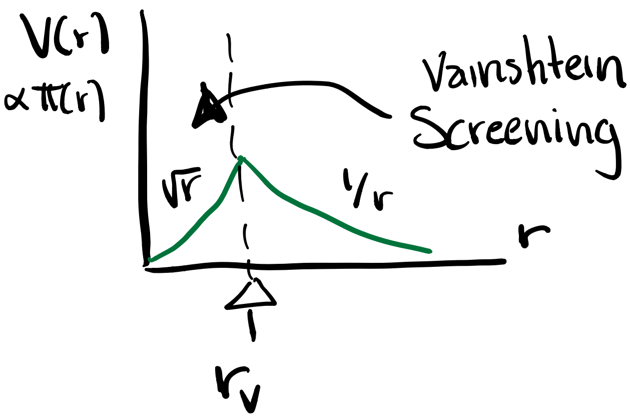
$$\boxed{10^{-15} \lesssim \frac{c_{GW}}{c} - 1}$$

CUBIC GALILEON

$$\mathcal{L} = -\frac{1}{2}(\partial\pi)^2 - \frac{1}{\Lambda^3}\Box\pi(\partial\pi)^2$$

- Gives rise to screening
  Understood in few scenarios

5th Force

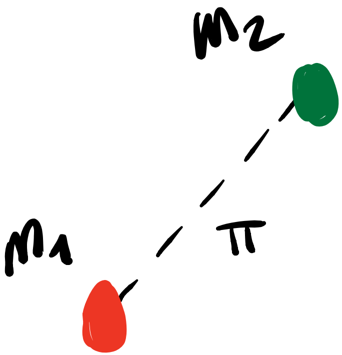


Galileons

- shift symmetry / soft limit
  $$\delta\pi = c + b_\mu x^\mu \Rightarrow A \sim p^2$$
- bending modes of branes
- longitudinal mode massive gravity
- decoupling limit of DGP

Can Amplitude methods give new insights?

Test simple scenario:



Conformally coupled matter

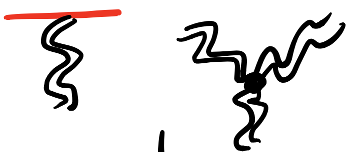
Spinless (described by ϕ)

Cubic Galileon interactions

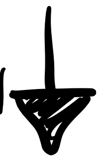
First: Review GR case: Minimally coupled scalars.

Gravity from Scattering Amplitudes

3-point $\mathcal{A}$

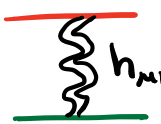


unitarity

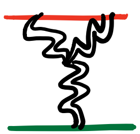


* Also double copy

Φ_1 $h_{\mu\nu}$ Φ_2

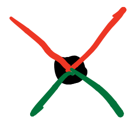


classical

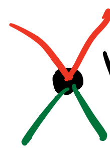


$S_{eff}[\Phi]$

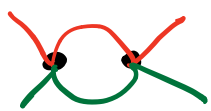
$V\,\Phi_{\bar{p}}\,\Phi_{\bar{p}'}\,\Phi_{-\bar{p}}\,\Phi_{-\bar{p}'}$



$V_G(\bar{p},\bar{q},\bar{S})$



V_{G^2}



$V_G \quad V_G$

$S_{p.p.}[x,S]$

$\lim m_1 \gg m_2$

$V(\bar{p},r,\bar{S})$

$\vdots$

Black Hole spacetime

Bern, Luna, Roiban, Chen, Zeng, Cheung, Rothstein, Solon, Vaidya; ...

Duff; Neil, Rothstein; Donoghue, Holstein, Gabrecht, Konstandin; Chung, Huang, Kim, Lee; ..

1 Identify Classical Contributions

$$\frac{1}{b} \sim \underset{\text{transfered}}{q} \ll \underset{\text{external}}{p}, m$$

Restore $\hbar$:

$$G \to \frac{G}{\hbar} \qquad q \to \hbar q$$

Classical non-linearities

$$\epsilon^{NL} = \frac{h}{M_{pl}} \propto \underbrace{MGq}_{\sim r_s} \hbar^0$$

Quantum corrections

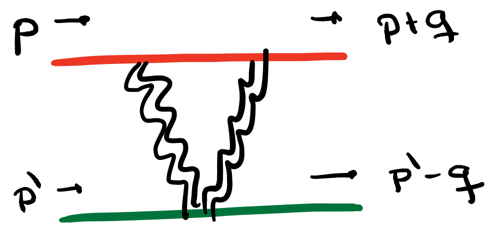
$$\epsilon^{Q} = \frac{\partial^2}{M_{pl}^2} \propto q^2 G \hbar$$

$$\epsilon^{Q} \ll \epsilon^{NL}$$

Dimensional analysis $\Rightarrow$

$$V = \frac{GM^2}{q^2} \sum_{a,b=0}^{\infty} c_{a,b} \underset{\epsilon^{NL}}{(MqG)^a} \underset{\epsilon^{NR}}{\left(\frac{p}{M}\right)^b} \hbar^0$$

2 Classical physics from loops



$v \ll 1$

$p = (E_1, \bar{p})$

$p' = (E_2, -\bar{p})$

$q = (0, \bar{q})$

Regions

hard	$p \sim m(1, \bar{v})$
q.soft	$l \sim q(1, 1)$
$\boxed{\text{potential} \quad l \sim q(1, \bar{v})}$	
radiation	$l \sim q(\bar{v}, \bar{v})$

$$\underbrace{\left(G^2 m_1^4 m_2^2\right)}_{\substack{3 \\ \text{matter vertices}}} \underbrace{\left(\frac{1}{q^2}\right)^2}_{\substack{2 \\ \text{graviton prop.}}} \underbrace{\left(\frac{1}{mq}\right)}_{\substack{2 \\ \text{matter prop.}}} \underbrace{q^4}_{\substack{1 \\ \text{loop int.}}} \sim \frac{G m_1^2 m_2^2}{q^2}(G m_1 q)$$

2PM

non analiticity

Simpler non-relativistic integration.

3 Matching $\longleftrightarrow$ Finding V

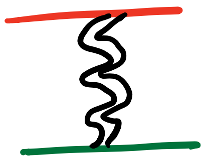
Cheung, Rothstein, Solon

on-shell
lim $p \ll m$

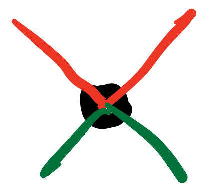
$$\frac{A^{clas}}{4E_1E_2} = A^{EFT}$$

$V\,\Phi_{\bar{p}}\,\Phi_{\bar{p}'}\,\Phi_{-\bar{p}}\,\Phi_{-\bar{p}'}$

$\mathcal{O}(G)$

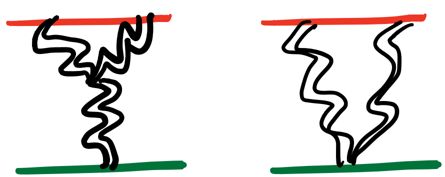


$\longleftrightarrow$

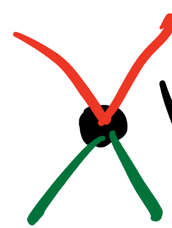


V_G

$\mathcal{O}(G^2)$

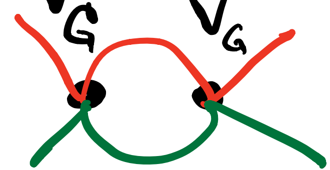


$\longleftrightarrow$



V_{G^2}

V_G V_G



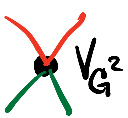
3' Lippmann-Schwinger Eq.

Cristofoli, Bjerrum-Bohr, Damgaard, Vanhove

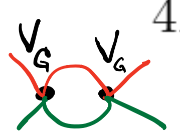
$$\tilde{\mathcal{M}}_4(\mathbf{p},\mathbf{p}+\mathbf{q}) = -V(\mathbf{p},\mathbf{p}+\mathbf{q}) + \int_{\boldsymbol{k}} \frac{V(\mathbf{p},\mathbf{k})\tilde{\mathcal{M}}_4^{(}\mathbf{k},\mathbf{p}+\mathbf{q})}{(E_p - E_k + i\epsilon)} \;;$$

Equivalent to EFT matching:

$$V(\mathbf{p},\mathbf{p}+\mathbf{q})\big|_{\mathcal{O}(g^n)} = -\frac{1}{4E_1(\mathbf{p})E_2(\mathbf{p})}\mathcal{M}_4(\mathbf{p},\mathbf{p}+\mathbf{q})\Big|_{\mathcal{O}(g^n)}$$



$$-\frac{1}{4E_1(\mathbf{p})E_2(\mathbf{p})}\int_{\boldsymbol{k}} \frac{\mathcal{M}_4(\mathbf{p},\mathbf{k})\mathcal{M}_4(\mathbf{k},\mathbf{p}+\mathbf{q})}{4E_1(k)E_2(k)(E_p - E_k + i\epsilon)}\Big|_{\mathcal{O}(g^n)}$$



4 Probe particle limit

$$m_1 \gg m_2$$

$$S_{P.P.} = -m \int d\tau \sqrt{-(\eta_{\mu\nu} + h_{\mu\nu}) \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau}}$$

Write ansatz given symmetries

$$\Longrightarrow V(\bar{p}, r)$$

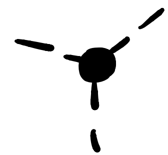
$$\Updownarrow$$

Match Wilson coef. $S^{eff.}[\Phi]$

→ Reproduce $h_{\mu\nu}$ of Black Hole

Cubic Galileon

$$\boxed{S_\pi = \int d^4x \left(-\frac{1}{2}(\partial\pi)^2 - \frac{1}{\Lambda^3}\Box\pi\,(\partial\pi)^2 \right)}$$

 $\sim p^4$

General coupling to matter

$$\boxed{S_\phi = \int d^4x \left(-\frac{1}{2}\sum_{i=1}^{2}(\partial\phi_i)^2 + A(\pi)\, m_i^2\phi_i^2 \right)}$$

$$A(\pi) = 1 + c_1\frac{g\pi}{M_{Pl}} + c_2\frac{g^2\pi^2}{M_{Pl}^2} + \dots$$

1 Scaling of classical V

$$\boxed{r_V = \frac{1}{\Lambda}\left(\frac{c_1 g m}{M_{pl}}\right)^{1/3}\frac{1}{\hbar}}$$

VALID for $\frac{r_s}{r}, \frac{r_V}{r} \ll 1$

Classical non-linearities

$$\epsilon^{NL} = \frac{\partial\partial\pi}{\Lambda^3} = r_V\, q\, \hbar^0$$

quantum corrections

$$\epsilon^{Q} = \frac{\partial^2}{\Lambda^2} = \frac{q^2}{\Lambda^2}\hbar^{1/3}$$

$$V = \frac{-8\pi G g^2 M^2}{q^2} \sum_{\substack{a<2n \\ n,c\in\mathbb{Z}}} (g^2 r_s q)^{n-a} (r_V q)^{3a} \left(\frac{p}{M}\right)^c \log q^2$$

Matter coupling

Galileon interactions

NR

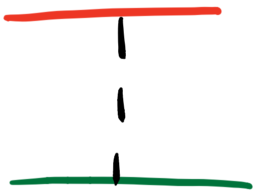
For n even

2 Classical Contributions

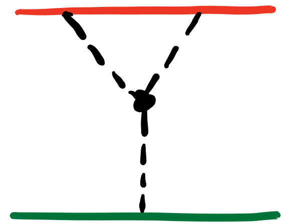
up to 2-loops

$\mathcal{O}(G)$

$\mathcal{O}(r_v^0)$

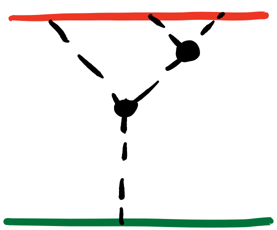


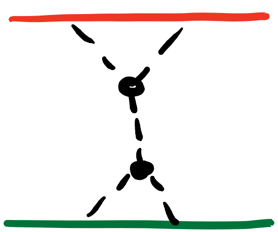
$\mathcal{O}(r_v^3)$

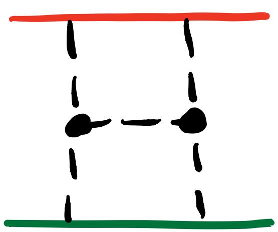


$C_1 \phi^2 \pi$

$\mathcal{O}(r_v^6)$

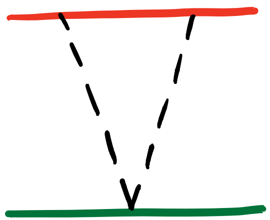


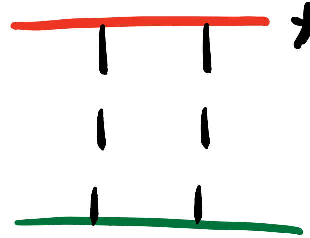




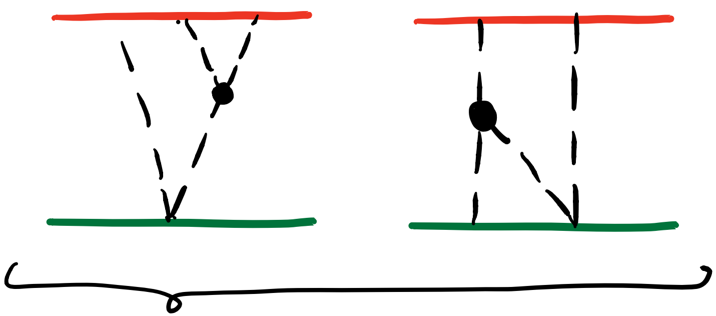
$\mathcal{O}(G^2)$

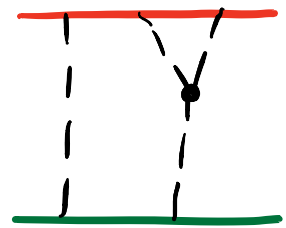
$\mathcal{O}(v_r^0)$



 *

$\mathcal{O}(v_r^3)$



 *

Only arise if $C_2 \neq 0$

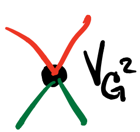
* Superclassical

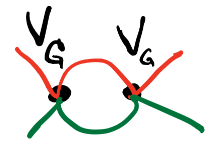
3 Matching $\leftrightarrow$ Finding V

- At order G $\quad V^G = -\mathcal{M}^G$

- At order G^2 include lower order V vertices in EFT matching / Born substraction

$$V^{G^2} = -\mathcal{M}^{G^2} - \int \frac{\mathcal{M}^G \mathcal{M}^G}{E_p - E_k + i\epsilon}$$





CONSERVATIVE POTENTIAL

(For $r_s/r,\ r_V/r \ll 1$)

Isotropic gauge

$$V^G = -\frac{2C_1^2 g^2 G m_a^2 m_b^2}{E_a E_b r}\Bigg(1 + \frac{r_{V_a}^3 + r_{V_b}^3}{4\pi r^3} \leftarrow \Delta$$

$$+ \frac{2}{7\pi^2 r^6}\left(r_{V_a}^6 + r_{V_b}^6 + \frac{399}{32} r_{V_a}^3 r_{V_b}^3\left(1 - \frac{25}{42}\frac{(3E_a^2 + 3E_b^2 + 4E_aE_b)}{E_a^2 E_b^2}|\bar{p}|^2 + \dots\right)\right)\Bigg)$$

$\Delta\Delta$

H

$$
V^{G^2} = -\frac{4 c_1^2 g^4 G^2 m_a^2 m_b^2}{E_a E_b r^2} \Bigg(c_2 (m_a+m_b) + \frac{c_1^2}{2}\,\underline{\frac{m_a^2 m_b^2}{E_a^2 E_b^2}\,\frac{E_a^2+E_aE_b+E_b^2}{E_a+E_b}}
$$

$$
+ \frac{c_2}{2\pi r^3}\left(m_a r_{v_a}^3 + m_b r_{v_b}^3 + \frac{3\, m_a^{1/2} m_b^{1/2} r_{v_a}^{3/2} r_{v_b}^{3/2}}{E_a E_b}\left(1 - \frac{31E_a^2+8E_aE_b+31E_b^2}{40E_a^2E_b^2}|\bar{p}|^2 + \dots \right)\right)
$$

$$
+ \frac{c_1^2}{4\pi r^3}\,\underline{\frac{m_a^2 m_b^2}{E_a^2 E_b^2}\,\frac{E_a^2+E_aE_b+E_b^2}{E_a+E_b}}\,\left(r_{v_a}^3 + r_{v_b}^3\right) + \mathcal{O}(r_v^6) \Bigg)
$$

from $\mathcal{M}$

from EFT matching / Born substr.

4 Match to p.p.

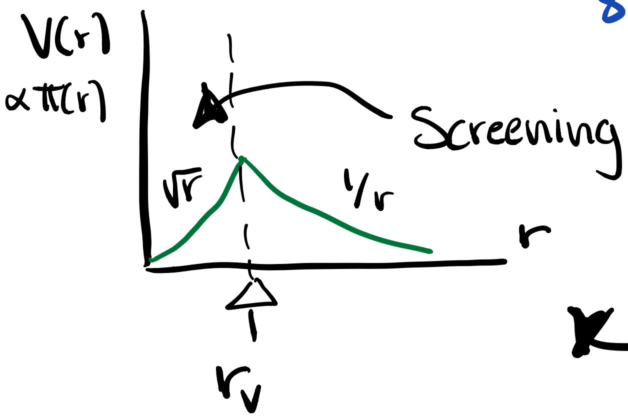
$m_a \gg m_b$

$$S_{p.p.} = -\sum_{i=1}^{2} m_i \int d\tau \sqrt{-\tilde{g}_{\mu\nu}\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}} \qquad \tilde{g}_{\mu\nu} = A^2(\pi)\,\eta_{\mu\nu}$$

→ $\frac{A(\pi)\, T^\mu_\mu}{M_{pl}}$ coupling. Known exact solution

$$\pi'(r) = \frac{\Lambda r^3}{8}\left(-1+\sqrt{1+\frac{4 r_{r_a}^3}{\pi r^3}}\right)$$

← Sourced by m_a (at rest)



$$\mathcal{H}_b = \sqrt{|\bar{p}_b|^2 + m_b^2 A^2(\pi)}$$

← felt by m_b

Amplitudes Side $m_a \gg m_b$

Conformal coupling

$$S_\phi = \int d^4x \sqrt{-\tilde{g}} \left(-\frac{1}{2} \sum_i (\partial \phi_i)^2 + m_i^2 \phi_i^2 \right) \quad \tilde{g} = A^2(\pi)\eta$$

! Careful matching correct scattering states

$$A(\pi) = 1 + c_g \frac{\pi}{M_{Pl}} \qquad m^2 \phi^2 A$$

$$\tilde{\phi} = A(\pi)\,\phi$$

We want states:

$$\phi(\bar{x})|0\rangle = \int_{\bar{p}} \frac{1}{2E_{\bar{p}}} |p\rangle e^{-i\bar{x}\cdot\bar{p}}$$

$$|p\rangle = \sqrt{2E_{\bar{p}}}\, a^\dagger_{\bar{p}} |0\rangle$$

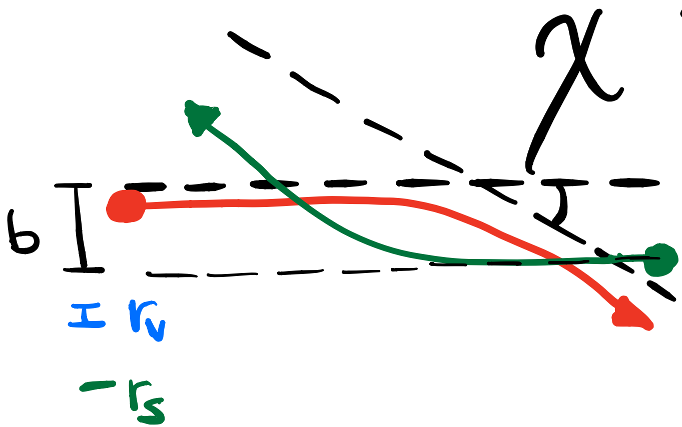
$$\Rightarrow \mathcal{L} \supset -\frac{1}{2} m_i^2 \tilde{\phi}_i^2 A^2(\pi)$$

AGREES WITH P.P.

Scattering Angle

Bjerrum-Bohr, Cristofoli, Damgaard; Källin, Porto; Damour; ...

$$\mathcal{H} = E$$



$$|\bar{p}|^2 = |\bar{p}_\infty|^2 - V_{eff}(E,r)$$

$$= |\bar{p}_\infty|^2 - \frac{1}{2E} \mathcal{M}_{cl}(|p_\infty|, r)$$

$$\chi = \sum_{k=1}^{\infty} \frac{2b}{k!} \int_0^{\infty} du \left(\frac{d}{db^2}\right)^k \frac{V_{eff}^k \, (u^2+b^2)^{k-1}}{|p_\infty|^{2k}}$$

$$
\begin{aligned}
\chi_{\epsilon_{Gb}} = {} & \frac{4c_1^2 g^2 G m_a^2 m_b^2}{Eb|\mathbf{p}_\infty|^2}\left(1 - \frac{3}{16}\left(\frac{r_{V_a}^3}{b^3} + \frac{r_{V_b}^3}{b^3}\right)\right. \\
& \left. + \frac{32}{35}\left(\frac{r_{V_a}^6}{b^6} + \frac{399 m_a m_b r_{V_a}^3 r_{V_b}^3}{32 E_a E_b b^6}\left(1 - \frac{25}{42}\frac{(3E_a^2 + 4E_aE_b + 3E_b^2)}{E_a^2 E_b^2}|\mathbf{p}_\infty|^2 + \mathcal{O}(||\mathbf{p}_\infty|^4)\right) + \frac{r_{V_b}^6}{b^6}\right)\right)
\end{aligned}
$$

$\chi^{k=1}$

$$
\begin{aligned}
\chi_{\epsilon_{Gb}^2} = {} & -\frac{16 c_1^4 g^4 G^2 m_a^4 m_b^4}{\pi b^2 {M_{\rm pl}}^2 |\mathbf{p}_\infty|^4 E^2}\left(\left(\frac{r_{V_a}^3}{b^3} + \frac{r_{V_b}^3}{b^3}\right)\right. \\
& \left. - \frac{1065}{1024}\left(\frac{r_{V_a}^6}{b^6} + \frac{399 r_{V_a}^3 r_{V_b}^3}{32 b^6} + \frac{r_{V_b}^6}{b^6}\right)\right)
\end{aligned}
$$

$\chi^{k=2}$

$$
\begin{aligned}
& - \frac{c_1^2(c_1^2 + 2c_2) g^4 m_a^2 m_b^2}{32\pi E {M_{\rm pl}}^4 b^2 |\mathbf{p}_\infty|^2}\Bigg((m_a + m_b) \\
& - \frac{8}{3\pi^2}\left(\frac{m_a r_{V_a}^3}{b^3} + \frac{2(m_a m_b r_{V_a} r_{V_b})^{3/2}}{E_a E_b b^3}\left(1 + \frac{(31E_a^2 + 8E_aE_b + 31E_b^2)}{40E_a^2E_b^2}|\mathbf{p}_\infty|^2 + \mathcal{O}(|\mathbf{p}_\infty|^4)\right) + \frac{m_b r_{V_b}^3}{b^3}\right)\Bigg)
\end{aligned}
$$

$\chi^{k=1}$

Probe particle limit

$\epsilon_{Gb} = g^2 \frac{r_s}{b}$ $\epsilon_{Vb} = \frac{r_{V_a}}{b}$ $\epsilon_{pm} = \frac{P_\infty}{m_a} \simeq 2 \frac{\overbrace{m_b}^{\epsilon_M}}{m_a}(E - m_a)$

χ^{LO} χ^{NLO}

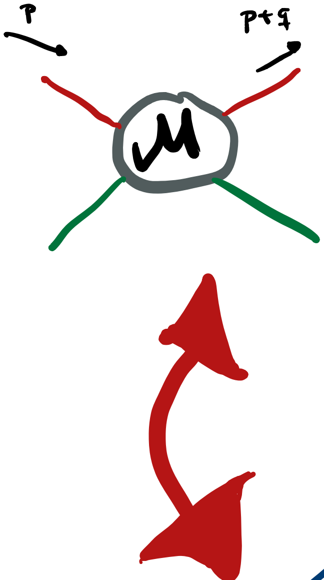
$$\chi_{\epsilon_{Gb}} = \frac{\epsilon_{Gb}\epsilon_M}{\epsilon_{pm}}\left(1 - \frac{3}{16}\epsilon_{Vb}^3 + \frac{32}{35\pi^2}\epsilon_{Vb}^6 + \cdots\right)$$

$$\chi_{\epsilon_{Gb}^2} = -\frac{\epsilon_{Gb}^2\epsilon_M^2\epsilon_{Vb}^3\left(1 - \frac{1065}{1024}\epsilon_{Vb}^3\right)}{\pi\epsilon_{pm}^2} - \frac{\epsilon_{Gb}^2\epsilon_M\left(1 + 2C_2/C_1^2\right)\pi\left(1 - \frac{8}{3\pi^2}\epsilon_{Vb}^3\right)}{4\epsilon_{pm}}$$

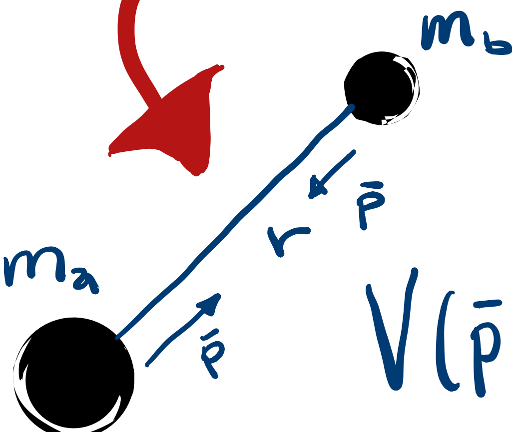
Amplitudes calculation matches p.p. calculation

* Valid as long as $\frac{r_{min}}{r_V} \sim \frac{\epsilon_{pm}}{\epsilon_M \epsilon_{Gb} \epsilon_{Vb}} \gg 1$

Conclusions



- Amplitudes methods apply beyond minimal coupling
- Careful matching: consider correct scattering states



$$V(\bar{p}, r)$$

$$\chi(E, b)$$

- Add Spin
- Is resummation possible?
  - → Access screened region
- Add Gravity