Gravitational Bremsstrahlung in the Post-Minkowskian Effective Field Theory

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Based on work with S. Mougiakakos and F. Vernizzi
[arXiv:2102.08339]

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Outline

Three phases, different approaches

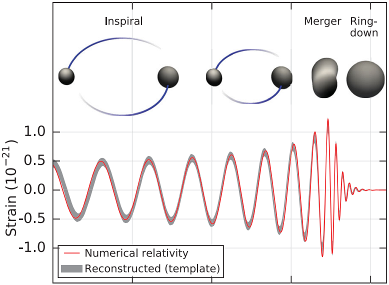


Figure: LIGO and VIRGO scientific collaboration, Phys. Rev. Lett. **116** 6 (2016).

- Inspiral and ringdown phases studied using perturbation theory
- Merger phase studied using numerical relativity
- Interplay between the different phases (EOB, Inspiral-merger-ringdown)

Inspiral phase

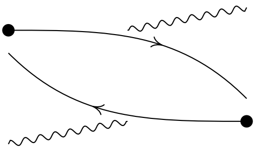
Inspiral phase contains most of the signal. Traditionally studied in the Post Newtonian (non-relativistic) regime $v \ll c$.

L. Blanchet *(2014) [1310.1528], (2019) [1812.07490]*

D. Bini and T. Damour *(2017) [1706.06877]*

Post-Minkowskian: a complementary approach

- Perturbative study in G while keeping the velocity fully relativistic
- One can still split in conservative + dissipative effects



- Traditional GR
  S. Kovacs and K. Thorne *Astrophys. J. 200 (1975) - 215, 217 (1977) - 224 (1978)* , K. Westpfahl and M. Goller *Lett. Nuovo Cim. 26 (1979) 573-576* .

- Scattering Amplitudes
  C. Cheung, I. Z. Rothstein, M. P. Solon *(2018) 1808.02489* , Z. Bern et al. *(2019) [1908.01493], (2021) [2101.07254]* , D. Kosower, B. Mayee, D. O'Connell *(2019) [1811.10950]* - E. Herrmann et al. *(2021) [2101.07255]*
- Eikonal
  P. Di Vecchia, C. Heissenberg, R. Russo, G. Veneziano *(2020) [2008.12743], (2021) [2101.05772], [2104.03256]*
- Worldline EFT
  G. Kälin and R. A. Porto *(2020) [2006.01184], [2007.04977]* , G. Mogull, J. Plefka, J. Steinhoff *(2021) [2010.02865]* , G. U. Jakobsen et al. *(2021)[2101.12688]*

PM Effective Field Theory

Setting up our EFT

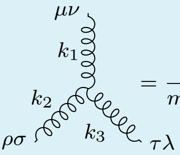
Expansion around Minkowski $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}/m_{\rm Pl}$

Spin and Finite-size effects

$$S = \underbrace{-2m_{\rm Pl}^2 \int d^4x \sqrt{-g} R}_{} - \sum_{a=1,2} \frac{m_a}{2} \int d\tau_a \left[g_{\mu\nu}(x_a) \mathcal{U}_a^\mu(\tau_a) \mathcal{U}_a^\nu(\tau_a) + 1 \right] + \dots$$

$$= \frac{i}{k^2} P_{\mu\nu;\rho\sigma}$$

$$P_{\mu\nu;\rho\sigma} = \frac{1}{2}(\eta_{\mu\rho}\eta_{\nu\sigma} + \eta_{\mu\sigma}\eta_{\nu\rho} - \eta_{\mu\nu}\eta_{\rho\sigma})$$



$$= \frac{i}{m_{\rm Pl}} V_{\mu\nu\rho\sigma\lambda\tau}(k_1, k_2, k_3)$$

Implicit splitting in potential+radiation modes, flexibility (and care) in the $i\varepsilon$ prescription

G. Kälin and R. A. Porto *(2020) [2006.01184]*

$$\hbar = 1\,, \quad c = 1\,, \quad m_{\rm Pl} = 1/\sqrt{32\pi G}\,, \quad \eta_{\mu\nu} = {\rm diag}(+,-,-,-)\,, \quad \int_q \equiv \int \frac{d^4q}{(2\pi)^4}$$

PM Effective Field Theory

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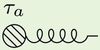
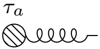
$$S = -2m_{\rm Pl}^2 \int d^4x \sqrt{-g} R - \sum_{a=1,2} \frac{m_a}{2} \int d\tau_a \left[g_{\mu\nu}(x_a) \mathcal{U}_a^\mu(\tau_a) \mathcal{U}_a^\nu(\tau_a) + 1 \right] + \cdots$$



$$= -\frac{i m_a}{2 m_{\rm Pl}} \int d\tau_a \int_q e^{-iq\cdot x_a} \mathcal{U}_a^\mu \mathcal{U}_a^\nu$$

Polyakov action reduces the point-particle vertices.

G. Kälin and R. A. Porto *(2020) [2006.01184]*



$$= -\frac{i m_a}{2 m_{\rm Pl}} \int d\tau_a \int_q e^{-iq\cdot x_a(\tau_a)} \mathcal{U}_a^\mu(\tau_a)\mathcal{U}_a^\nu(\tau_a)$$

Isolate the powers of G

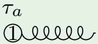
$$x_a^\mu(\tau_a) = b_a^\mu + u_a^\mu \tau_a + \delta^{(1)} x_a^\mu(\tau_a) + \dots$$

$$\mathcal{U}_a^\mu(\tau_a) = u_a^\mu + \delta^{(1)} u_a^\mu(\tau_a) + \dots$$

- $u_a = \lim_{\tau_a \to -\infty} \mathcal{U}_a^\mu(\tau_a)\,, \qquad b_a \cdot u_a = 0$
- $\delta^{(n)} x_a^\mu$, $\delta^{(n)} u_a^\mu$ deviations from the straight motion at order G^n containing **both** conservative and radiation effects

G. Kälin and R. A. Porto *(2020) [2006.01184]*

$$= -\frac{i m_a}{2 m_{\rm Pl}} u_a^\mu u_a^\nu \int d\tau_a \int_q e^{-iq\cdot(b_a + u_a \tau_a)}$$

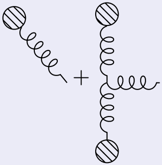


$$= -\frac{i m_a}{2 m_{\rm Pl}} \int d\tau_a \int_q e^{-iq\cdot(b_a + u_a \tau_a)}$$

$$\times \left(2\delta^{(1)} u_a^{(\mu}(\tau_a) u_a^{\nu)} - i(q\cdot\delta^{(1)} x_a(\tau_a)) u_a^\mu u_a^\nu\right)$$

Matching procedure

The pseudo Stress-Energy Tensor



$$+\ldots \equiv -\frac{i}{2m_{\rm Pl}}\int d^4x\, T^{\mu\nu}(x)h_{\mu\nu}(x) = -\frac{i}{2m_{\rm Pl}}\int_k \tilde{T}^{\mu\nu}(-k)\tilde{h}_{\mu\nu}(k)$$

Classical Amplitude and Asymptotic Waveform

$$\mathcal{A}_\lambda(k) = -\frac{1}{2m_{\rm Pl}}\epsilon^{*\lambda}_{\mu\nu}(\mathbf{k})\tilde{T}^{\mu\nu}(k)\,, \qquad \epsilon^\lambda_{0\nu} = 0\,,\; k^\mu\epsilon^\lambda_{\mu\nu} = 0\,,\; \eta^{\mu\nu}\epsilon^\lambda_{\mu\nu} = 0$$

$$h_{\mu\nu}(x) = -\frac{1}{4\pi r}\sum_{\lambda=\pm 2}\int \frac{dk^0}{2\pi} e^{-ik^0(t-r)}\epsilon^\lambda_{\mu\nu}(\mathbf{k})\mathcal{A}_\lambda(k)|_{k^\mu = k^0 n^\mu}$$

The Amplitude is the only thing we need to compute.

LO amplitude

$$\gamma \equiv u_1 \cdot u_2 \,, \qquad b \equiv b_1^\mu - b_2^\mu \,, \qquad \omega_a \equiv k \cdot u_a \,, \qquad \bar{\delta}^{(n)}(\omega_a) = (2\pi)^n \delta^{(n)}(\omega_a) \,.$$

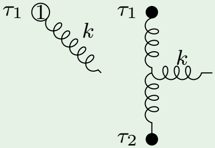
LO Amplitude

$$\tilde{T}^{\mu\nu}_{(1)}(k) = \sum_a m_a u_a^\mu u_a^\nu e^{ik\cdot b_a} \bar{\delta}(\omega_a)$$

$$\mathcal{A}^{(1)}_\lambda(k) = -\frac{1}{2m_{\rm Pl}} \sum_a m_a \, \epsilon^{*\lambda}_{\mu\nu}(\mathbf{n}) u_a^\mu u_a^\nu \; e^{ik\cdot b_a} \bar{\delta}(\omega_a)$$

$\bar{\delta}(\omega_a) \to$ non-radiating piece relevant to compute $J_{\rm rad}$ (see later).

NLO Amplitude



$$\mathcal{A}_\lambda^{(2)}(k) = -\frac{m_1 m_2}{8 m_{\rm Pl}^3}\epsilon^{*\lambda}_{\mu\nu}(\mathbf{n})\Bigg\{\Bigg[-2\left(\gamma I_{(0)} + \omega_1\omega_2 J_{(0)}\right)u_1^\mu u_2^\nu + \left(-\frac{2\gamma^2-1}{2}\frac{k\cdot I_{(1)}}{(\omega_1 + i\epsilon)^2} + \frac{2\gamma\omega_2}{\omega_1 + i\epsilon}I_{(0)} + 2\omega_2^2 J_{(0)}\right)u_1^\mu u_1^\nu + \left(\frac{2\gamma^2-1}{\omega_1 + i\epsilon}I^\mu_{(1)} + 4\gamma\omega_2 J^\mu_{(1)}\right)u_1^\nu + \frac{2\gamma^2-1}{2}J^{\mu\nu}_{(2)}\Bigg]e^{ik\cdot b_1}\Bigg\} + (1\leftrightarrow 2)$$

Retarded Boundary conditions

$$\frac{1}{x+i\epsilon} = P\left(\frac{1}{x}\right) - \frac{i}{2}\delta(x) \rightarrow \text{Other static contributions}$$

$$\mathcal{A}_\lambda^{(2)}(k) = -\frac{m_1 m_2}{8 m_{\rm Pl}^3}\epsilon^{*\lambda}_{\mu\nu}(\mathbf{n})\Bigg\{\Bigg[-2\left(\gamma I_{(0)}+\omega_1\omega_2 J_{(0)}\right)u_1^\mu u_2^\nu + \left(-\frac{2\gamma^2-1}{2}\frac{k\cdot I_{(1)}}{(\omega_1+i\epsilon)^2}+\frac{2\gamma\omega_2}{\omega_1+i\epsilon}I_{(0)}+2\omega_2^2 J_{(0)}\right)u_1^\mu u_1^\nu + \left(\frac{2\gamma^2-1}{\omega_1+i\epsilon}I^\mu_{(1)}+4\gamma\omega_2 J^\mu_{(1)}\right)u_1^\nu + \frac{2\gamma^2-1}{2}J^{\mu\nu}_{(2)}\Bigg]e^{ik\cdot b_1}\Bigg\} + (1\leftrightarrow 2)$$

Two sets of master integrals

$$I_{(n)}^{\mu_1\ldots\mu_n} \equiv \int_q \hat{\delta}\left(q\cdot u_1-\omega_1\right)\hat{\delta}\left(q\cdot u_2\right)\frac{e^{-iq\cdot b}}{q^2}q^{\mu_1}\ldots q^{\mu_n}$$

$$J_{(n)}^{\mu_1\ldots\mu_n} \equiv \int_q \hat{\delta}\left(q\cdot u_1-\omega_1\right)\hat{\delta}\left(q\cdot u_2\right)\frac{e^{-iq\cdot b}}{q^2(k-q)^2}q^{\mu_1}\ldots q^{\mu_n}$$

- The first set can be solved analytically.
- The second set can be express as a one dimensional integration over a Feynman parameter.

Master Integrals

Integral I_0

$$I_{(0)} \equiv \int_q \hat{\delta}\left(q \cdot u_1 - \omega_1\right) \hat{\delta}\left(q \cdot u_2\right) \frac{e^{-iq\cdot b}}{q^2}$$

$$= -\frac{1}{\gamma v} \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^2} \frac{e^{i\mathbf{q}_\perp\cdot\mathbf{b}}}{|\mathbf{q}_\perp|^2 + \frac{\omega_1^2}{\gamma^2 v^2}} = -\frac{1}{2\pi\gamma v} K_0\left(\frac{|\mathbf{b}|\omega_1}{\gamma v}\right)$$

Integral J_0

$$J_{(0)} \equiv \int_q \hat{\delta}\left(q \cdot u_1 - \omega_1\right) \hat{\delta}\left(q \cdot u_2\right) \frac{e^{-iq\cdot b}}{q^2 (k-q)^2}$$

$$= \frac{1}{\gamma v} \int_0^1 dy\, e^{-iyk\cdot b} \int \frac{d^2\boldsymbol{q}_\perp}{(2\pi)^2} \frac{e^{i\boldsymbol{q}_\perp\cdot\boldsymbol{b}_\perp}}{\left[\boldsymbol{q}_\perp^2 + \frac{s^2(y)}{\gamma^2 v^2}\right]^2} = \frac{|\mathbf{b}|^2}{4\pi\gamma v} \int_0^1 dy\, e^{-iyk\cdot b} \frac{K_1\left(zf(y)\right)}{zf(y)}$$

$$s(y) = \sqrt{(1-y)^2\omega_1^2 + 2\gamma y(1-y)\omega_1\omega_2 + y\omega_2^2}$$

Master Integrals

Integral I_0

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$$s(y) = \sqrt{(1-y)^2\omega_1^2 + 2\gamma y(1-y)\omega_1\omega_2 + y\omega_2^2}$$

$$b_2^\mu = 0\,, \quad b_1^\mu = b^\mu\,, \quad u_2^\mu = \delta_0^\mu\,, \quad k^\mu = \omega n^\mu\,, \quad u_1^\mu = \gamma v^\mu = \gamma(1, v\mathbf{e}_v)\,, \quad \mathbf{e}_v \equiv \mathbf{v}/v\,, \quad \mathbf{e}_b = \mathbf{b}/|\mathbf{b}|$$

$$\mathcal{A}_\lambda^{(1)}(k) = -\frac{m_1}{2m_{\rm Pl}}\,\frac{\gamma v^2}{n\cdot v}\epsilon_{ij}^{*\lambda}\mathbf{e}_v^i\mathbf{e}_v^j\,\hat{\delta}(\omega)e^{ik\cdot b}\,, \quad \mathcal{A}_\lambda^{(2)}(k) = -\frac{Gm_1m_2}{m_{\rm Pl}\gamma v}\epsilon_{ij}^{*\lambda}\mathbf{e}_I^i\mathbf{e}_J^j A_{IJ}(k)e^{ik\cdot b}$$

$$\begin{aligned}
A_{vv} &= c_1 K_0\big(z(n\cdot v)\big) + ic_2\Big[K_1\big(z(n\cdot v)\big) - i\pi\delta\big(z(n\cdot v)\big)\Big] \\
&\quad + \int_0^1 dy\, e^{iyzv\mathbf{n}\cdot\mathbf{e}_b}\Big[d_1(y)zK_1\big(zf(y)\big) + c_0K_0\big(zf(y)\big)\Big] \\
A_{vb} &= ic_0\Big[K_1\big(z(n\cdot v)\big) - i\pi\delta\big(z(n\cdot v)\big)\Big] + i\int_0^1 dy\, e^{iyzv\mathbf{n}\cdot\mathbf{e}_b} d_2(y)zK_0\big(zf(y)\big) \\
A_{bb} &= \int_0^1 dy\, e^{iyzv\mathbf{n}\cdot\mathbf{e}_b} d_0(y)zK_1\big(zf(y)\big)
\end{aligned}$$

$$z \equiv \frac{|\mathbf{b}|\omega}{v}\,, \qquad f(y) \equiv \sqrt{(1-y)^2(n\cdot v)^2 + 2y(1-y)(n\cdot v) + y^2/\gamma^2}$$

$$c_0 = 1 - 2\gamma^2\,, \qquad c_1 = -c_0 + \frac{3-2\gamma^2}{n\cdot v}\,, \quad c_2 = vc_0\frac{\mathbf{n}\cdot\mathbf{e}_b}{n\cdot v}$$

$$d_0(y) = f(y)c_0\,, \quad d_1(y) = \ldots$$

$$\mathcal{A}_\lambda^{(2)}(k) = -\frac{Gm_1m_2}{m_{\rm Pl}\gamma v}\epsilon_{ij}^{*\lambda}\mathbf{e}_I^i\mathbf{e}_J^j A_{IJ}(k)e^{ik\cdot b}$$

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A_{bb} &= \int_0^1 dy\, e^{iyzv\mathbf{n}\cdot\mathbf{e}_b} d_0(y)zK_1\big(zf(y)\big)
\end{aligned}$$

Consistency checks with S. Kovacs and K. Thorne *Astrophys. J. 224 (1978)*

- Coincides with the Forward limit, i. e. $\mathbf{k} \parallel \mathbf{v}$. In this limit $k\cdot b = 0$ and one can perform the y integral.
- Agreement in the small velocity limit i.e. $v \ll 1$

Waveform direct space

$$b_2^\mu = 0\,,\quad b_1^\mu = b^\mu\,,\quad u_2^\mu = \delta_0^\mu\,,\quad k^\mu = \omega n^\mu\,,\quad u_1^\mu = \gamma(1, v\mathbf{e}_v)\,,\quad \mathbf{e}_v \equiv \mathbf{v}/v\,,\quad \mathbf{e}_b = \mathbf{b}/|\mathbf{b}|$$

Master Integral time domain

$$\int \frac{d\omega}{2\pi}\Big\{I_{(0)}, J_{(0)}\Big\} e^{-i\omega(\mathbf{n}\cdot\mathbf{b}+t-r)} = \frac{1}{n\cdot u_1}\int_{\mathbf{q}} e^{i\mathbf{q}\cdot\tilde{\mathbf{b}}(t-r)} f\left(\tilde{\omega}, \mathbf{q}\right)$$

$$\tilde{\mathbf{b}}(t-r) \equiv \mathbf{b} + \frac{\gamma v}{n\cdot u_1}(t - r + \mathbf{b}\cdot\mathbf{n}) \qquad \tilde{\omega} \equiv -\frac{\gamma v}{n\cdot u_1}\mathbf{q}\cdot\mathbf{e}_v$$

Waveform G. U. Jakobsen et al. *(2021)[2101.12688]*

$$h_{\pm 2}^{(2)} = \frac{m_1 m_2 G}{8 m_{\rm Pl} r}\int_{\mathbf{q}} e^{i\mathbf{q}\cdot\tilde{\mathbf{b}}}\left[\frac{q^i\mathcal{N}_\pm^i}{\mathbf{q}^2\left(\mathbf{q}\cdot\mathbf{e}_v - i\epsilon\right)} + \frac{q^i q^j\mathcal{M}_\pm^{ij}}{\mathbf{q}^2\left(\mathbf{q}^2 + \mathbf{q}\cdot L\cdot\mathbf{q}\right)}\right]$$

$$L^{ij} \equiv 2\frac{v}{n\cdot v}\mathbf{e}_v^{(i}\mathbf{n}^{j)}$$

$$\mathcal{N}_\pm^i \equiv 4\frac{\gamma v}{(n\cdot v)^2}(\boldsymbol{\epsilon}^\pm\cdot\mathbf{e}_v)^2\left[(1+v^2)\,n^i - 4v\,e_v^i\right] + 8\frac{\gamma(1+v^2)}{n\cdot v}(\boldsymbol{\epsilon}^\pm\cdot\mathbf{e}_v)\epsilon_\pm^i$$

$$\mathcal{M}_\pm^{ij} \equiv 16\frac{\gamma v^4}{(n\cdot v)^3}(\boldsymbol{\epsilon}^\pm\cdot\mathbf{e}_v)^2\,e_v^i e_v^j + 8\frac{\gamma(1+v^2)}{n\cdot v}\,\epsilon_\pm^i\epsilon_\pm^j - 32\frac{\gamma v^2}{(n\cdot v)}(\boldsymbol{\epsilon}^\pm\cdot\mathbf{e}_v)\,e_v^{(i}\epsilon_\pm^{j)}$$

Radiated observables

$$u \equiv t - r$$

$$h_{\mu\nu}(x) = -\frac{1}{4\pi r}\sum_{\lambda=\pm 2}\int \frac{dk^0}{2\pi} e^{-ik^0 u}\epsilon^{\lambda}_{\mu\nu}(\mathbf{k})\mathcal{A}_\lambda(k)|_{k^\mu = k^0 n^\mu}$$

Linear and Angular momentum fluxes

$$P^\mu_{\rm rad} = \int d\Omega\, du\, r^2\, n^\mu\, \dot{h}_{ij}\dot{h}_{ij}$$

$$J^i_{\rm rad} = \epsilon^{ijk}\int d\Omega\, du\, r^2 \left(2h_{jl}\dot{h}_{lk} - x_j\partial_k h_{lm}\dot{h}_{lm}\right)$$

$$\dot{h}_{\mu\nu}(x) \propto \omega\mathcal{A}_\lambda(k) = \omega\mathcal{A}_\lambda(k)_{\rm finite}\,, \qquad \mathcal{A}_\lambda(k)_{\rm finite} = \mathcal{A}_\lambda(k) - \left(\substack{\text{static}\\ \text{contributions}}\right)$$

A time derivative removes all static contributions

Different scaling

$$\mathcal{A}^{(1)}_\lambda(k) \propto \hat{\delta}(\omega)$$

$$P^\mu_{\rm rad} = O\left(G^3\right)\,, \qquad J^i_{\rm rad} = O\left(G^2\right)$$

LO Radiated Linear Momentum

Momentum in terms of the Amplitude

$$P^\mu_{\rm rad} = \sum_\lambda \int_k \delta(k^2)\theta(k^0)k^\mu \, |\mathcal{A}_\lambda(k)_{\rm finite}|^2$$
$$= \frac{G^3 m_1^2 m_2^2}{|\mathbf{b}|^3} \frac{u_1^\mu + u_2^\mu}{\gamma+1} \mathcal{E}(\gamma) + \mathcal{O}(G^4)$$

Homogeneous mass dependence, result fixed by the probe limit
S. Kovacs and K. Thorne *Astrophys. J. 224 (1978)* .

$\mathcal{E}(\gamma)$ recently found in E. Herrmann et al. *(2021) [2101.07255]* and confirmed in P. Di Vecchia et al. *(2021)[2104.03256]*

$$\frac{\mathcal{E}(\gamma)}{\pi} = f_1 + f_2 \log\left(\frac{\gamma+1}{2}\right) + f_3 \frac{\gamma \,\mathrm{arcsinh}\sqrt{\frac{\gamma-1}{2}}}{\sqrt{\gamma^2-1}}$$

$$f_1 = \frac{210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151}{48\left(\gamma^2-1\right)^{3/2}}$$

$$f_2 = -\frac{35\gamma^4 + 60\gamma^3 - 150\gamma^2 + 76\gamma - 5}{8\sqrt{\gamma^2-1}}$$

$$f_3 = \frac{\left(2\gamma^2-3\right)\left(35\gamma^4 - 30\gamma^2 + 11\right)}{8(\gamma^2-1)^{3/2}}$$

LO Radiated Linear Momentum

Momentum in terms of the Amplitude

$$P^\mu_{\rm rad} = \sum_\lambda \int_k \delta(k^2)\theta(k^0)k^\mu \, |\mathcal{A}_\lambda(k)_{\rm finite}|^2$$
$$= \frac{G^3 m_1^2 m_2^2}{|\mathbf{b}|^3} \frac{u_1^\mu + u_2^\mu}{\gamma+1} \mathcal{E}(\gamma) + \mathcal{O}(G^4)$$

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$$\frac{\mathcal{E}(\gamma)}{\pi} = f_1 + f_2 \, \log\left(\frac{\gamma+1}{2}\right) + f_3 \, \frac{\gamma \, \text{arcsinh}\sqrt{\frac{\gamma-1}{2}}}{\sqrt{\gamma^2-1}}$$

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$$f_3 = \frac{(2\gamma^2-3)\,(35\gamma^4 - 30\gamma^2 + 11)}{8(\gamma^2-1)^{3/2}}$$

$$P^\mu_{\rm rad} = \frac{G^3 m_1^2 m_2^2}{|\mathbf{b}|^3}\frac{u_1^\mu + u_2^\mu}{\gamma+1}\mathcal{E}(\gamma) + \mathcal{O}(G^4)$$

$$\mathcal{E}(\gamma) = \frac{2|\mathbf{b}|^3}{\pi^2(\gamma^2-1)}\sum_\lambda \int d\Omega \int_0^\infty \omega^2 d\omega \left|\epsilon^{*\lambda}_{ij}\mathbf{e}^i_I \mathbf{e}^j_J A_{IJ}(k)\right|^2$$

- Analytic result for $\mathcal{E}(\gamma)$ cannot (yet) be found due to the involved integration in y.
- The computation is possible at virtually any PN order

$$\frac{\mathcal{E}}{\pi} = \frac{37}{15}v + \frac{2393}{840}v^3 + \frac{61703}{10080}v^5 + \frac{3131839}{354816}v^7 + \mathcal{O}(v^9)$$

  This is in perfect agreement with `E. Herrmann et al.` *(2021) [2101.07255]*

- Agreement with known results at 2PN once written in the CoM frame.
  `L. Blanchet and G. Schaefer` *Mon. Not. Roy. Astron. Soc. (1989)* .

Spectral dependence

$$P^{\mu}_{\rm rad} = \frac{G^3 m_1^2 m_2^2}{|\mathbf{b}|^3} \frac{u_1^\mu + u_2^\mu}{\gamma + 1} \mathcal{E}(\gamma) + \mathcal{O}(G^4)$$

Spectral dependence

$$\mathcal{E}(\gamma) = \frac{2v^3}{\pi^2(\gamma^2 - 1)} \int d\Omega \int_0^\infty z^2 dz f(z, \Omega) \,, \qquad z \equiv \frac{|\mathbf{b}|\omega}{v}$$

Spectrum $f(z, \Omega)$ depends only on

$$\left\{ I_i^{(s)}, I_i^{(c)} \right\}(z, \Omega) \equiv \int_0^1 dy \left\{ \sin(yzv\mathbf{n} \cdot \mathbf{e}_b), \cos(yzv\mathbf{n} \cdot \mathbf{e}_b) \right\} g_i(z, \Omega; y)$$

$$g_0(z, \Omega; y) \equiv d_0(y) z K_1 \left(z f(y) \right)$$

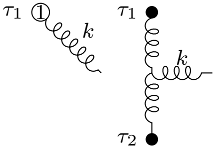
$$g_1(z, \Omega; y) \equiv c_0 K_0 \left(z f(y) \right) + d_1(y) z K_1 \left(z f(y) \right)$$

$$g_2(z, \Omega; y) \equiv d_2(y) z K_0 \left(z f(y) \right)$$

$$a_{IJ} \equiv [(\mathbf{e}_\theta \cdot \mathbf{e}_I)(\mathbf{e}_\theta \cdot \mathbf{e}_J) + (\mathbf{e}_\phi \cdot \mathbf{e}_I)(\mathbf{e}_\phi \cdot \mathbf{e}_J)]/2$$

$$\mathbf{e}_I = \{\mathbf{e}_v, \mathbf{e}_b\} \,, \qquad \mathbf{e}_\theta = (\cos\theta \cos\phi, \cos\theta \sin\phi, -\sin\theta) \,, \qquad \mathbf{e}_\phi = (-\sin\phi, \cos\phi, 0)$$

Soft Limit



$$\mathcal{A}_\lambda^{(2)}(k) = -\frac{m_1 m_2}{8 m_{\rm Pl}^3}\epsilon^{*\lambda}_{\mu\nu}(\mathbf{n})\Big\{\Big[-2\left(\gamma I_{(0)} + \omega_1\omega_2 J_{(0)}\right)u_1^\mu u_2^\nu + \left(-\frac{2\gamma^2-1}{2}\frac{k\cdot I_{(1)}}{(\omega_1+i\epsilon)^2} + \frac{2\gamma\omega_2}{\omega_1+i\epsilon}I_{(0)} + 2\omega_2^2 J_{(0)}\right)u_1^\mu u_1^\nu + \left(\frac{2\gamma^2-1}{\omega_1+i\epsilon}I^\mu_{(1)} + 4\gamma\omega_2 J^\mu_{(1)}\right)u_1^\nu + \frac{2\gamma^2-1}{2}J^{\mu\nu}_{(2)}\Big]e^{ik\cdot b_1}\Big\} + (1\leftrightarrow 2)$$

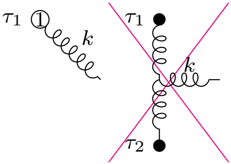
Small frequency limit, i.e. $|\mathbf{b}|\omega/v \ll 1$

Amplitude in the soft limit

The amplitude in the small frequency limit is insensitive to the gravitational self-interactions.

$$\mathcal{A}_\lambda^{(2)}(k)_{\omega\to 0} = -\frac{G m_1 m_2}{m_{\rm Pl}|\mathbf{b}|}\frac{i}{\gamma\omega n\cdot v}\epsilon^{*\lambda}_{ij}(c_2 \mathbf{e}_v^i\mathbf{e}_v^j + 2c_0\mathbf{e}_v^i\mathbf{e}_b^i)$$

Soft Limit



$$\mathcal{A}^{(2)}_\lambda(k) = -\frac{m_1 m_2}{8m_{\rm Pl}^3}\epsilon^{*\lambda}_{\mu\nu}(\mathbf{n})\Bigg\{\Bigg[-\xcancel{2\left(\gamma I_{(0)} + \omega_1\omega_2 J_{(0)}\right)}\, u_1^\mu u_2^\nu$$
$$+\left(-\frac{2\gamma^2-1}{2}\frac{k\cdot I_{(1)}}{(\omega_1+i\epsilon)^2} + \frac{2\gamma\omega_2}{\omega_1+i\epsilon} I_{(0)} + \xcancel{2\omega_2^2 J_{(0)}}\right) u_1^\mu u_1^\nu$$
$$+\left(\frac{2\gamma^2-1}{\omega_1+i\epsilon} I^\mu_{(1)} + \xcancel{4\gamma\omega_2 J^\mu_{(1)}}\right) u_1^\nu + \xcancel{\frac{2\gamma^2-1}{2} J^{\mu\nu}_{(2)}}\Bigg] e^{ik\cdot b_1}\Bigg\} + (1\leftrightarrow 2)$$

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$$\mathcal{A}_\lambda^{(2)}(k)_{\omega\to 0} = -\frac{G m_1 m_2}{m_{\mathrm{Pl}}|\mathbf{b}|}\frac{i}{\gamma\omega n\cdot v}\epsilon_{ij}^{*\lambda}(c_2 \mathbf{e}_v^i \mathbf{e}_v^j + 2c_0 \mathbf{e}_v^i \mathbf{e}_b^i)$$

Spectrum in the Small frequency limit

$$\left.\frac{dE_{\mathrm{rad}}}{d\omega}\right|_{\omega\to 0} = \frac{1}{2(2\pi)^3}\sum_\lambda \int d\Omega |\omega \mathcal{A}_\lambda(k)_{\omega\to 0}|^2$$

$$= \frac{4}{\pi}\frac{(2\gamma^2-1)^2}{\gamma^2 v^2}\frac{G^3 m_1^2 m_2^2}{|\mathbf{b}|^2}\mathcal{I}(v) + \mathcal{O}(G^4)$$

$$\mathcal{I}(v) \equiv -\frac{16}{3} + \frac{2}{v^2} + \frac{2(3v^2-1)}{v^3}\mathrm{arctanh}(v)$$

L. Smarr *Phys. Rev. D 15 (1977)* , P. Di Vecchia et al. *(2021) [2101.05772]* .

LO Radiated Angular Momentum

$$J^i_{\text{rad}} = \epsilon^{ijk} \int d\Omega \, du \, r^2 \left(2 h_{jl} \dot{h}_{lk} - x_j \partial_k h_{lm} \dot{h}_{lm} \right)$$
$$= \epsilon^{ijk} \int d\Omega \, r^2 \left(2 h^{(1)}_{jl} \delta_{mk} - x_j \partial_k h^{(1)}_{lm} \right) \int du \, \dot{h}^{(2)}_{lm} + O\left(G^3\right)$$

Gravitational wave memory

Wave memory determined by the Soft Limit

$$\int du \, \dot{h}_{ij} = \frac{i}{4\pi r} \sum_{\lambda} \int \frac{d\omega}{2\pi} \epsilon^{\lambda}_{ij} \, \hat{\delta}(\omega)\omega \, \mathcal{A}_{\lambda}(k)_{\omega \to 0}$$

LO Angular Momentum does not depend on gravitational self-interactions.

`T. Damour` *(2020) [2010.01641]* .

$$J^i_{\rm rad} = \epsilon^{ijk} \int d\Omega\, r^2 \left(2h^{(1)}_{jl}\delta_{mk} - x_j \partial_k h^{(1)}_{lm}\right) \int du\, \dot{h}^{(2)}_{lm} + O\left(G^3\right)$$

LO Angular momentum

Polar coordinates $\mathbf{e}_\theta = (\cos\theta\cos\phi, \cos\theta\sin\phi, -\sin\theta)\,, \mathbf{e}_\phi = (-\sin\phi, \cos\phi, 0)$, $\mathbf{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$.

$$\mathbf{J}_{\rm rad} = \sum_\lambda \int \frac{d\Omega}{(4\pi)^2}\, \omega \mathcal{A}^{(2)*}_\lambda(k)_{\omega\to 0}\, \hat{\mathbf{J}}\, a^{(1)}_\lambda + \mathcal{O}(G^3)$$

$$\hat{\mathbf{J}} \equiv \lambda(\mathbf{n} + \cot\theta\,\mathbf{e}_\theta) + \hat{\mathbf{L}}$$

$$a^{(1)}_\lambda = -\frac{m_1}{2m_{\rm Pl}} \frac{\gamma v^2}{n\cdot v} \epsilon^{*\lambda}_{ij} \mathbf{e}^i_v \mathbf{e}^j_v$$

$$\hat{L}^x = i(\sin\phi\,\partial_\theta + \cot\theta\cos\phi\,\partial_\phi)$$

$$\hat{L}^y = -i(\cos\phi\,\partial_\theta - \cot\theta\sin\phi\,\partial_\phi)$$

$$\hat{L}^z = -i\partial_\phi$$

$$\mathbf{J}_{\rm rad} = \frac{2(2\gamma^2-1)}{\gamma v} \frac{G^2 m_1 m_2 J}{|\mathbf{b}|^2} \mathcal{I}(v)(\mathbf{e}_b \times \mathbf{e}_v) + \mathcal{O}(G^3)$$

$$\mathcal{I}(v) \equiv -\frac{16}{3} + \frac{2}{v^2} + \frac{2(3v^2-1)}{v^3}\,{\rm arctanh}(v)$$

T. Damour *(2020) [2010.01641]* .

Conclusions and Future directions

- Worldline EFT methods prove to be efficient and useful also in the PM study of the binary inspiral problem (unbound case)
- Small number of topologies thanks to the Polyakov action
- First stepping stone for a derivation of P^{μ}_{rad} and $\mathbf{J}_{\text{rad}}$ alternative to other methods

- New way of approaching integrals of the form $J^{\mu_1 \cdots \mu_n}_{(n)}$
- Include Spin and finite-size effects in the radiative sector
- Build a map to connect unbound and bound quantities

Thank you for your attention!