Collider physics tools for dissipative effects in binary dynamics

based on: EH, J. Parra-Martinez, M. Ruf, M. Zeng
• arXiv: 2101.07255
2104.03957

• Kosower, Maybee, O'Connell (KMOC) arXiv: 1811.10950

• basic idea: ∘ derive classical (gravitational) observables from quantum scattering amplitudes (fixed order in G, all orders in v)

∘ exploit on-shell simplification + mature collider physics tools

• discuss 2-body dynamics for hyperbolic orbits

• 2 key observables: 1) gravitational impulse → χ

2) radiated momentum → E_{rad}

KMOC observables look like cross-sections

[simple (inclusive) observables ↔ few kinematic scales]

Outline

EH 1) Review of KMOC & relation to scattering amplitudes

- gravitational impulse
- radiative momentum

EH 2) Integrands from generalized unitarity

MR 3) reverse unitarity

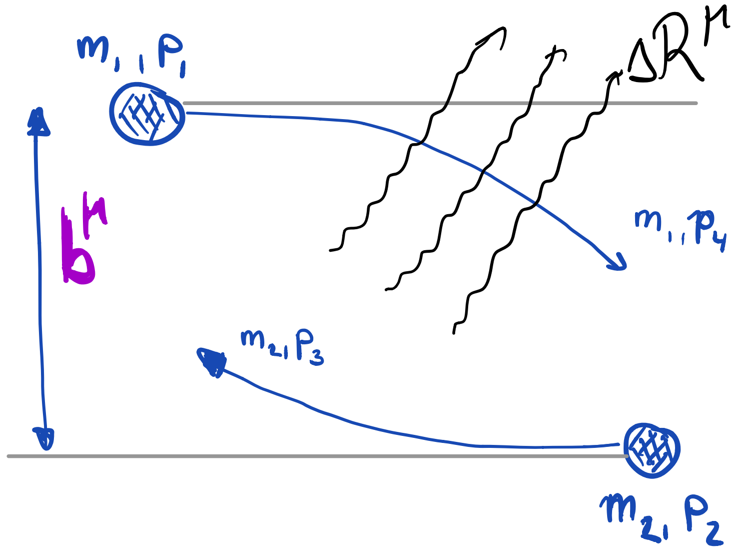
MR 4) cutting rules & phase-space integrals

MR 5) assembly of observables & results

N.B.: today focus on spinless black holes!

L2

1) Review of KMOC & relation to scattering amplitudes



- details, see also:
  - conference talk by D. Kosower
  - talk by R. Gonzo

- scattering of wave packets separated by impact parameter b^μ

- presence of gravitational interactions has 2 key effects:

  - deflection of trajectories $\leftrightarrow$ momentum shift
    $= \text{impuls } \Delta p_1^\mu \implies \sin^2\frac{\chi}{2} = \frac{-\Delta p^2}{4 p_\infty^2}$

  - gravitational Bremsstrahlung $\leftrightarrow \Delta R^\mu \implies \Delta E = \frac{(p_1+p_2)\cdot\Delta R}{|p_1+p_2|}$

    c.m. frame

• observables $\longleftrightarrow$ quantum operators

$$\Delta \mathcal{O} = \langle \text{out} | \mathbb{O} | \text{out} \rangle - \langle \text{in} | \mathbb{O} | \text{in} \rangle$$

asymptotic future asymptotic past

• key feature: $|\text{out}\rangle = S\,|\text{in}\rangle = (\mathbb{1} + iT)|\text{in}\rangle$

scattering amplitudes

$$\Delta \mathcal{O} = i \langle \text{in} | [\mathbb{O}, T] | \text{in} \rangle + \langle \text{in} | T^{\dagger} [\mathbb{O}, T] | \text{in} \rangle$$

"virtual matrix elements" "real contributions"

$$|\text{in}\rangle = \int d\Phi(p_1)\, d\Phi(p_2)\; \varphi(p_1)\, \varphi(p_2)\; e^{\frac{i p_1 \cdot b}{\hbar}}\; |p_1, p_2\rangle$$

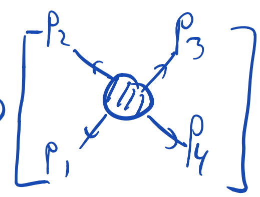
spatial separation

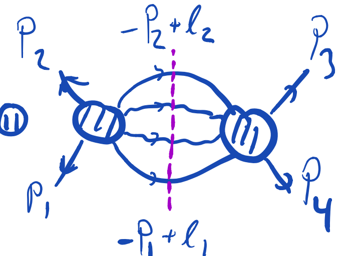
"phase-space integral" wave-functions momentum eigenstates

• subtle classical limit [see KMOC: arXiv:1811.10950]

$$\Delta\mathcal{O} = \underbrace{i\int \hat{d}^D q\; \hat{\delta}(-2p_1\cdot q)\,\hat{\delta}(2p_2\cdot q)\, e^{iq\cdot b}}_{\text{"transverse Fourier-transform"}}\;\underbrace{\left(\mathcal{I}_{\mathcal{O},v}+\mathcal{I}_{\mathcal{O},r}\right)}_{\text{KMOC kernels}}$$

$q = p_1 + p_4$

∘ $\mathcal{I}_{\mathcal{O},v} = \langle p_3, p_4 | [\mathbb{O}, T] | p_1, p_2 \rangle \quad = \Delta\mathbb{O}\,\Big[$  $\Big]$

∘ $\mathcal{I}_{\mathcal{O},r} = -i\langle p_3, p_4 | T^\dagger [\mathbb{O}, T] | p_1 p_2 \rangle = -i\sum_X \int d\Phi_{2+|X|}\; \Delta\mathbb{O}$ 

Σ exchanged "messengers"

on-shell phase-space integral (inclusive)

measurement fct. (depends on observable)

looks like cross-section

∘ expand amplitudes in small coupling G

$\hbar$-counting [KMOC]:

massive momenta	:	$p_i \sim \mathcal{O}(1)$
mom-transfer	:	$q \sim \mathcal{O}(\hbar)$
graviton loop-momenta	:	$\ell_i \sim \mathcal{O}(\hbar)$

"soft region" [Beneke, Smirnov]

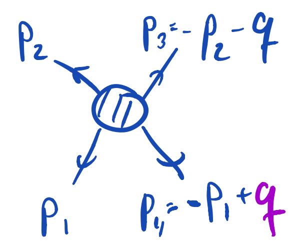
• classical expansion $\hat{=}$ small q-expansion

1.1 Gravitational impuls

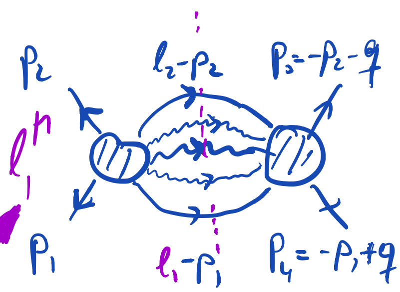
- observable : momentum shift on particle "i" during scattering event

$$\Delta p_1^\mu = \langle in | S^\dagger \mathbb{P}_1^\mu S | in \rangle - \langle in | \mathbb{P}_1^\mu | in \rangle$$

$$\mathcal{I}^\mu_{p_1} = q^\mu$$



$$- i \sum_X \int d\tilde{\Phi}_{2+|X|} \; l_1^\mu$$



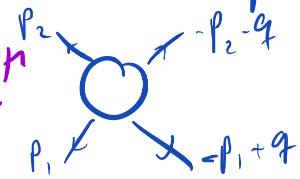
"virtual" impulse kernel

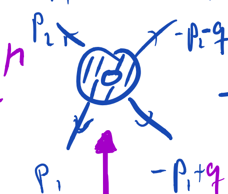
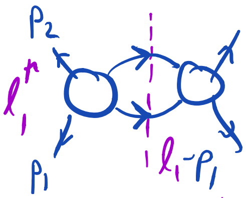
"real" impulse kernel

measurement function $\leftrightarrow \mathbb{P}_1^\mu$

Perturbative expansion of impulse

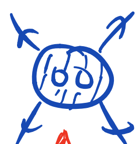
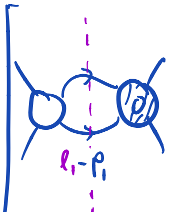
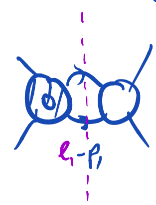
- **LO:** only "virtual" tree amplitude

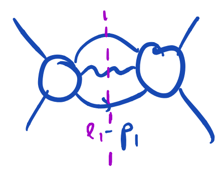
$I^{\mu(0)}_{p_1} = q^\mu$ 

- **NLO:** $I^{\mu(1)}_{p_1} = q^\mu$  $- i \int d\tilde{\Phi}_2 \, \ell_1^\mu$ 

has ∘ imaginary parts
∘ superclassical contributions

cancels iterations and imaginary parts

- **NNLO:** $I^{\mu(2)}_{p_1} = q^\mu$  $- i \int d\tilde{\Phi}_2 \, \ell_1^\mu \Big[$  $+$  $\Big]$

$- i \int d\tilde{\Phi}_3 \, \ell_1^\mu$ 

N.B.: full 2-loop amplitude is building block! [2104.03957]

cancellation of superclassical terms and imaginary parts more nontrivial

cutting rules & unitarity $\longrightarrow$ simplified formulae for kernels

(see MR talk)

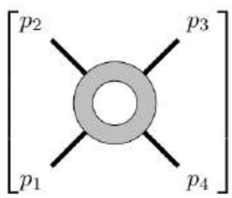
- decompose impulse into transverse and longitudinal comp.

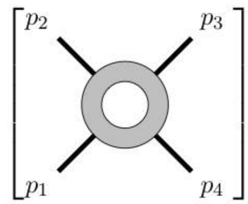
$$\boxed{\mathcal{I}^\mu = \mathcal{I}_\perp\, q^\mu + \mathcal{I}_{u_1}\, \check{u}_1^{\,\mu} + \mathcal{I}_{u_2}\, \check{u}_2^{\,\mu}}$$

$$\check{u}_1^{\,\mu} = \frac{y\, u_2^\mu - u_1^\mu}{y^2-1} \qquad \check{u}_2^{\,\mu} = \frac{y\, u_1^\mu - u_2^\mu}{y^2-1}$$

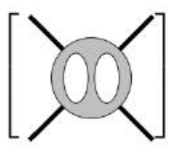
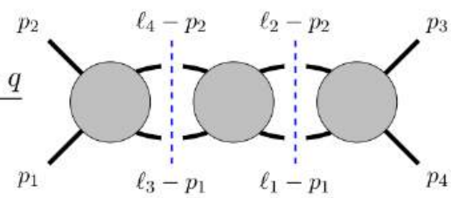
$$u_i^\mu \simeq \frac{p_i^\mu}{m_i} + \mathcal{O}(q^2), \quad u_i^2 = 1, \quad u_1\cdot u_2 = \underset{(\gamma)}{y} \simeq \sigma + \mathcal{O}(q^2), \quad u_i \cdot \check{u}_j = \delta_{ij}$$

- simplified KMOC kernels:

◦ NLO: $\mathcal{I}_\perp = \mathrm{Re}\Big[$ 

$\Big]$ $\qquad \mathcal{I}_{u_1} = \dfrac{-q^2}{2m_1}\, i\,\mathrm{Im}\Big[$ 

$\Big]$,

◦ NNLO : $\mathcal{I}_\perp^{(2)} = \mathrm{Re}\Big[$  $\Big] - i\displaystyle\int d\tilde{\Phi}_2^2\, \frac{(\ell_1-\ell_3)\cdot q}{2q^2}$ 

GR-result:

$$\mathcal{I}_\perp^{(2)} = 4\pi \frac{(-q^2)^{-2\epsilon}}{\epsilon} G^3 m_1^2 m_2^2 \Bigg[s\left(8\sigma^2 - \frac{1}{2\left(\sigma^2-1\right)^2}\right)$$

$$+\, m_1 m_2 \left(f_1^{\mathrm{LS}}(\sigma) - \frac{2}{3}\sigma(14\sigma^2+25)\right)$$

$$+\, m_1 m_2 \left(\sigma f_3^{\mathrm{LS}}(\sigma) - 4\left(4\sigma^4 - 12\sigma^2 - 3\right)\right) \frac{\operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}} \Bigg],$$

$$f_1^{\mathrm{LS}}(\sigma) = -\frac{(2\sigma^2-1)^2(5\sigma^2-8)}{3(\sigma^2-1)^{3/2}}, \qquad f_3^{\mathrm{LS}}(\sigma) = \frac{2(2\sigma^2-1)^2(2\sigma^2-3)}{(\sigma^2-1)^{3/2}},$$

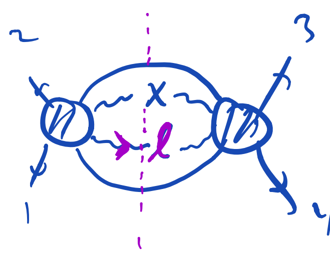
1.2. Radiated momentum

- momentum carried to infinity by GW

$$\Delta R^\mu = \langle in | S^\dagger \mathbb{R}^\mu S | in \rangle - \cancel{\langle in | \mathbb{R}^\mu | in \rangle}$$

no graviton in initial state

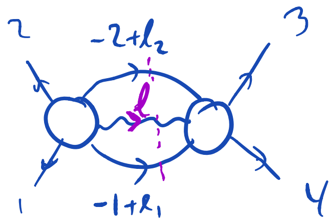
$$\mathcal{I}_R^\mu = -i \sum_X \int d\tilde{\Phi}_{3+X} \; \ell^\mu$$



- at least 1-graviton exchanged
- momentum measured on graviton line

- LO: starts @ $\mathcal{O}(G^3)$: [EH, J. Parra-Martinez, M. Ruf, M. Zeng arXiv: 2101.07255]

$$\mathcal{I}_R^{\mu(0)} = -i \int d\tilde{\Phi}_3 \; \ell^\mu$$



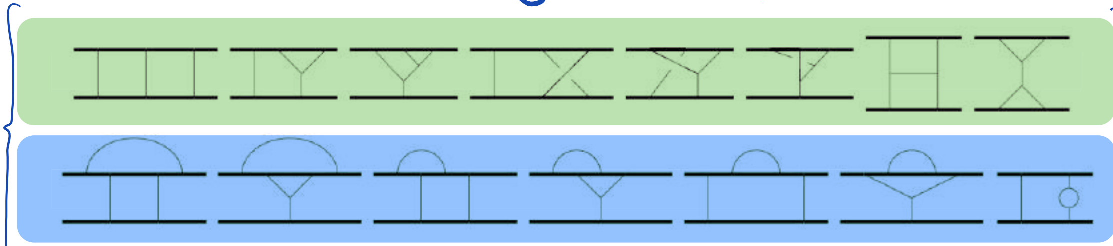
2. Integrands from generalized unitarity

- key ingredients in KMOC:
  - virtual scattering amplitudes
  - unitarity cuts of amplitudes

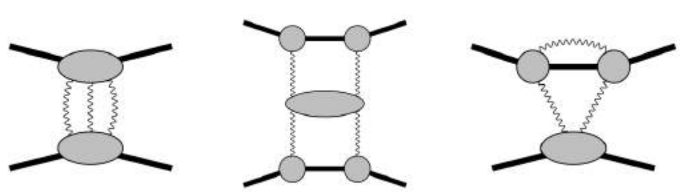
[see also talk by R. Roiban]

[Bern, Dixon, Dunbar, Kosower]

- can derive amplitudes by generalized unitarity (match residues of an ansatz of diagrams against field theory cuts $\left(=\prod_i \mathcal{M}_i^{\text{tree}}\right)$)

$\Gamma^{(2)}_{\text{cl.}} = \Big\{$  $\Big\}$

$$N_{\Gamma_i} = a_{i,1}(p_1\cdot p_2)^6 + a_{i,2}(p_1\cdot p_2)^5(\ell_1\cdot p_1) + \cdots + a_{i,n}(\ell_1\cdot\ell_2)^6\,.$$



→ linear system of eqs. for $a_{i,j}$

$$\mathcal{M}^{(L)}(p_1,p_2,p_3,p_4) = \sum_{\Gamma_i^{(L)}\in\{\Gamma^{(L)}_{\text{cl.}}\}} \int \prod_j^L \hat{d}^D\ell_j \frac{N_{\Gamma_i}(\ell,p)}{\prod_{\alpha_i} P_{\alpha_i}(\ell,p)}\,,$$

- to do : ◦ evaluate the integrals
  - ◦ assemble results
  - ◦ checks & analysis

Questions?

→ Michael