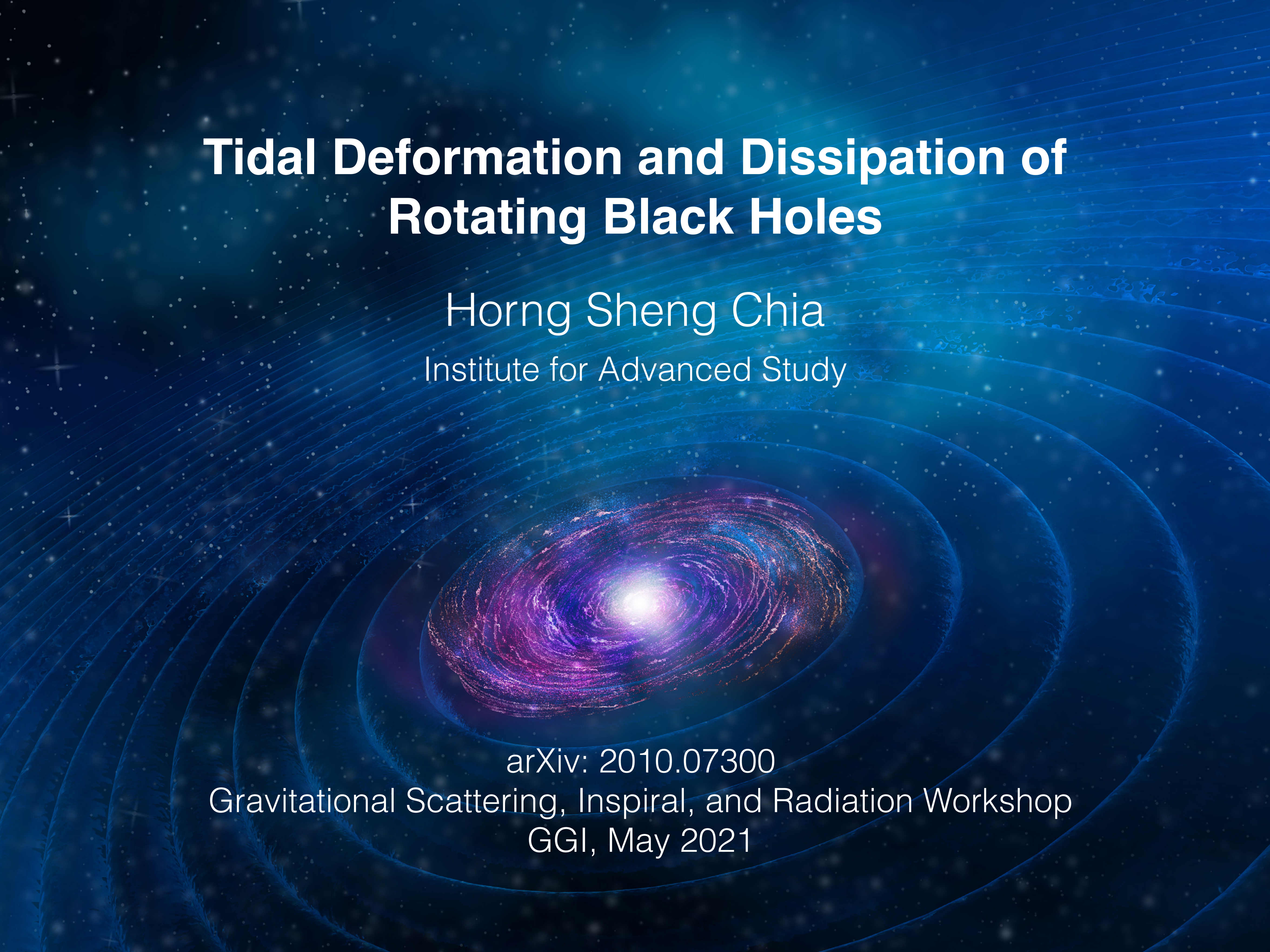


Tidal Deformation and Dissipation of Rotating Black Holes

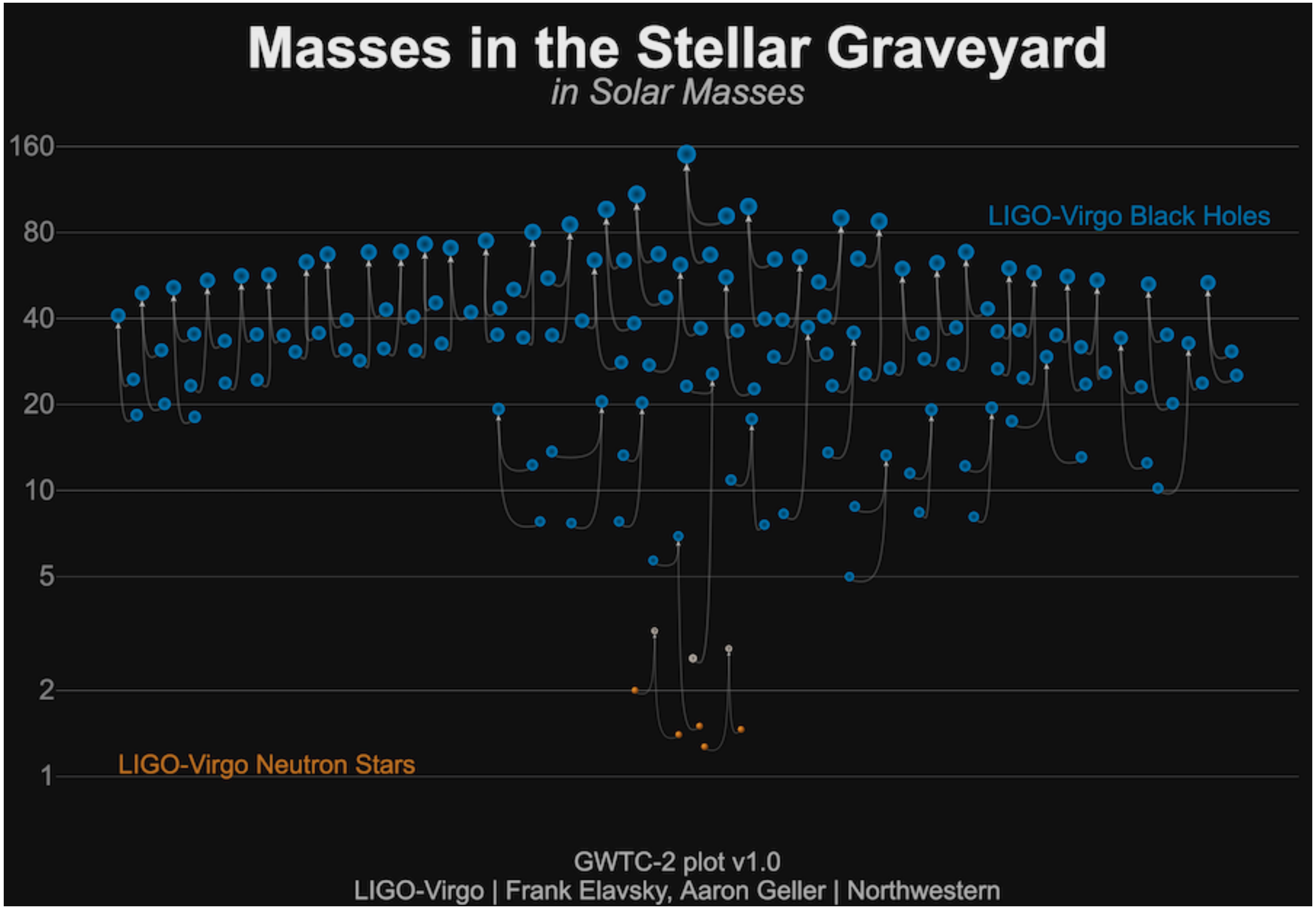
Horng Sheng Chia

Institute for Advanced Study

arXiv: 2010.07300

Gravitational Scattering, Inspiral, and Radiation Workshop
GGI, May 2021

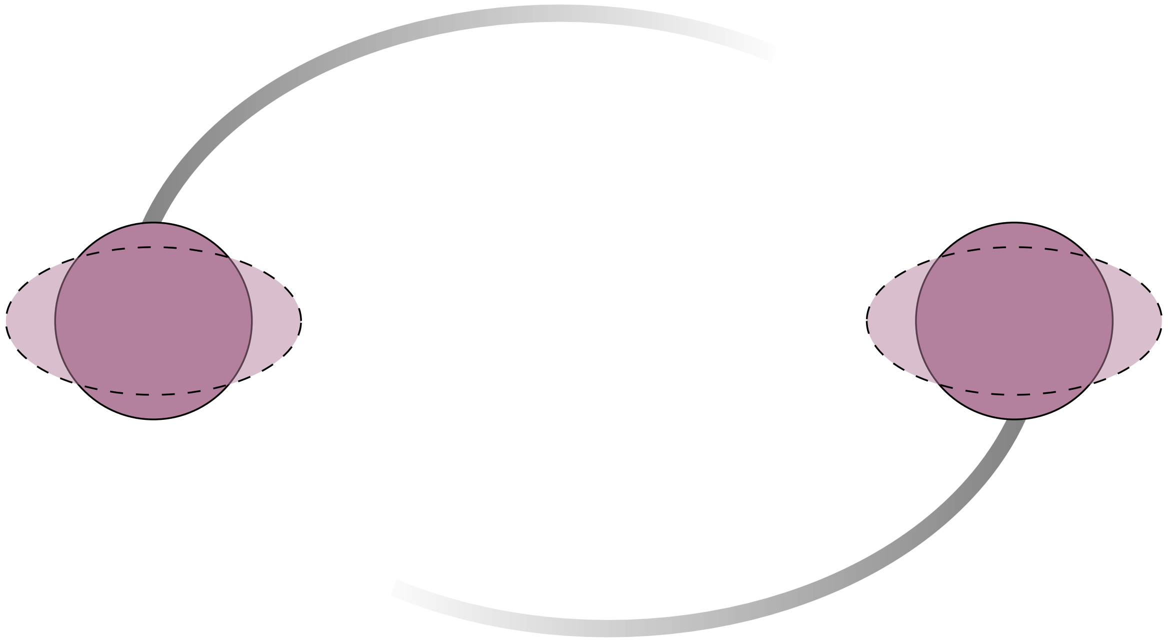
Golden Era of Black Hole Astronomy



LIGO Virgo Collaboration

Tidal Deformation

Astrophysical objects would deform when they are perturbed by external tidal fields.



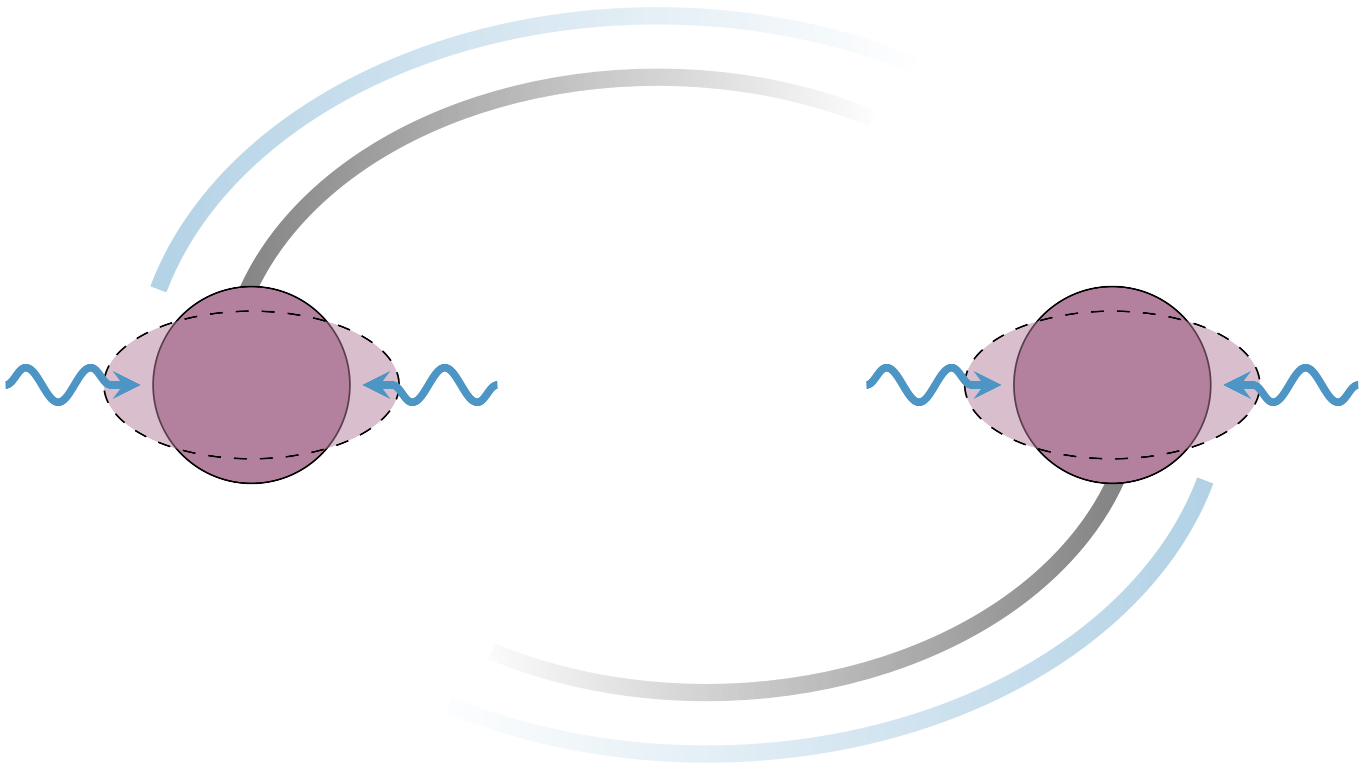
The tidal deformability of an object is quantified by its Love numbers, whose values depend on the internal structure of the object.

Love (1912)

Poisson, Will (Gravity Textbook)

Tidal Dissipation

The viscosity of the object would result in a transfer of energy and angular momentum between the external tidal field and the object.

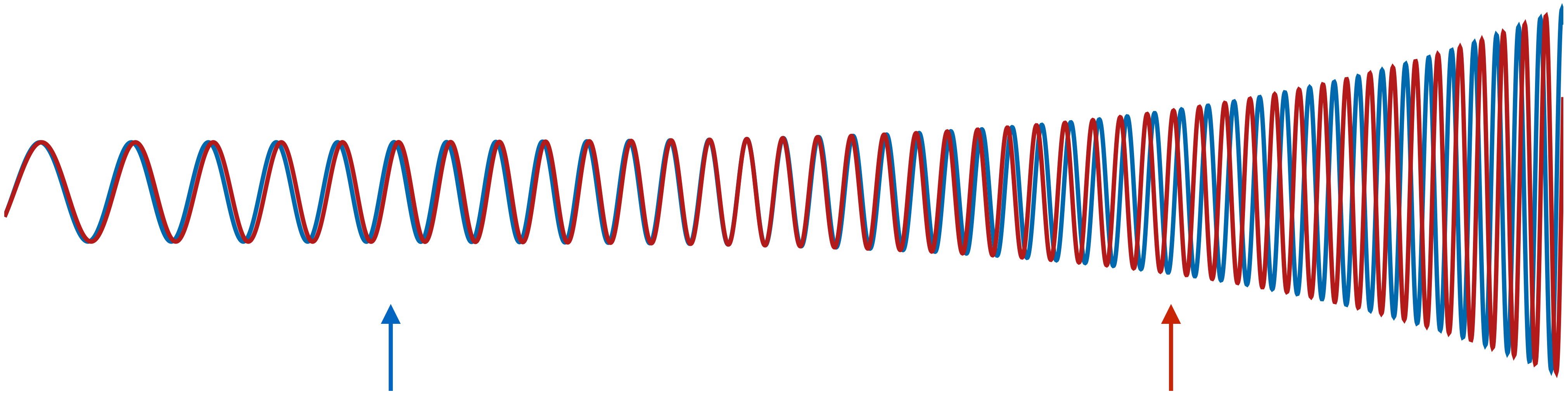


This tidal effect leads to tidal heating/torquing/acceleration of two-body systems. E.g. the gradual spin down of Earth along with the recession of the Moon.

Poisson, Will (Gravity Textbook)

Tidal Effects in Waveforms

These responses leave distinct imprints in the phase of the gravitational waves emitted by binary systems.



Tidal dissipation
(2.5PN order onwards)

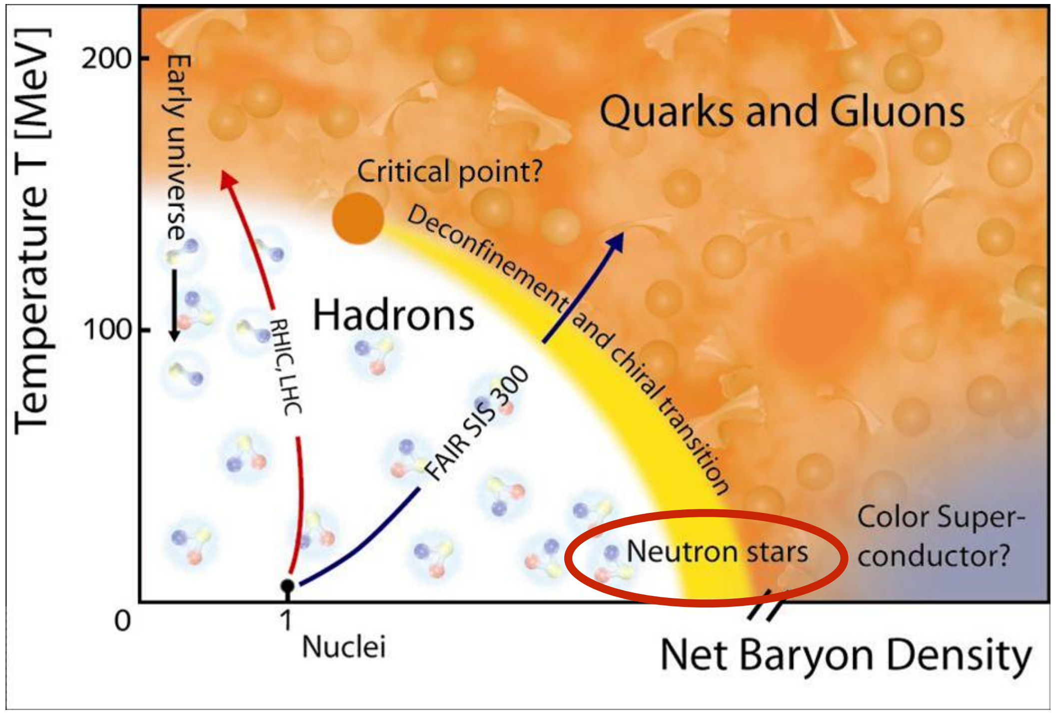
Love numbers
(5PN order onwards)

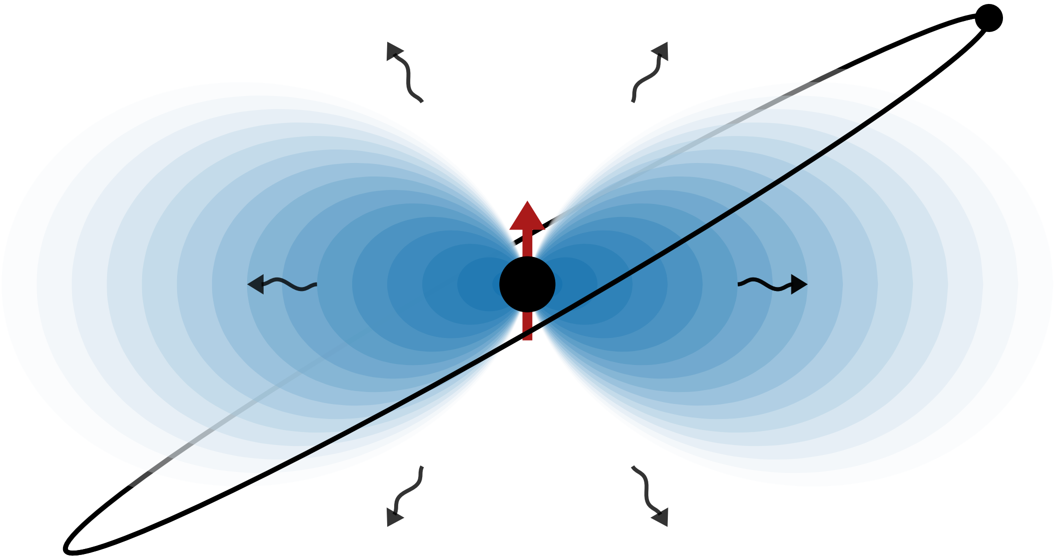
A precise measurement of these phase imprints would allow us to probe the nature of the binary constituents.

Poisson, Sasaki [9412027]
Tagoshi, Mano, Takasugi [9711072]
Flanagan, Hinderer [0709.1915]

Probing Binary Constituents with Tidal Effects

For binary neutron stars, these tidal effects probe the high-density and low-temperature regime of the QCD phase diagram.





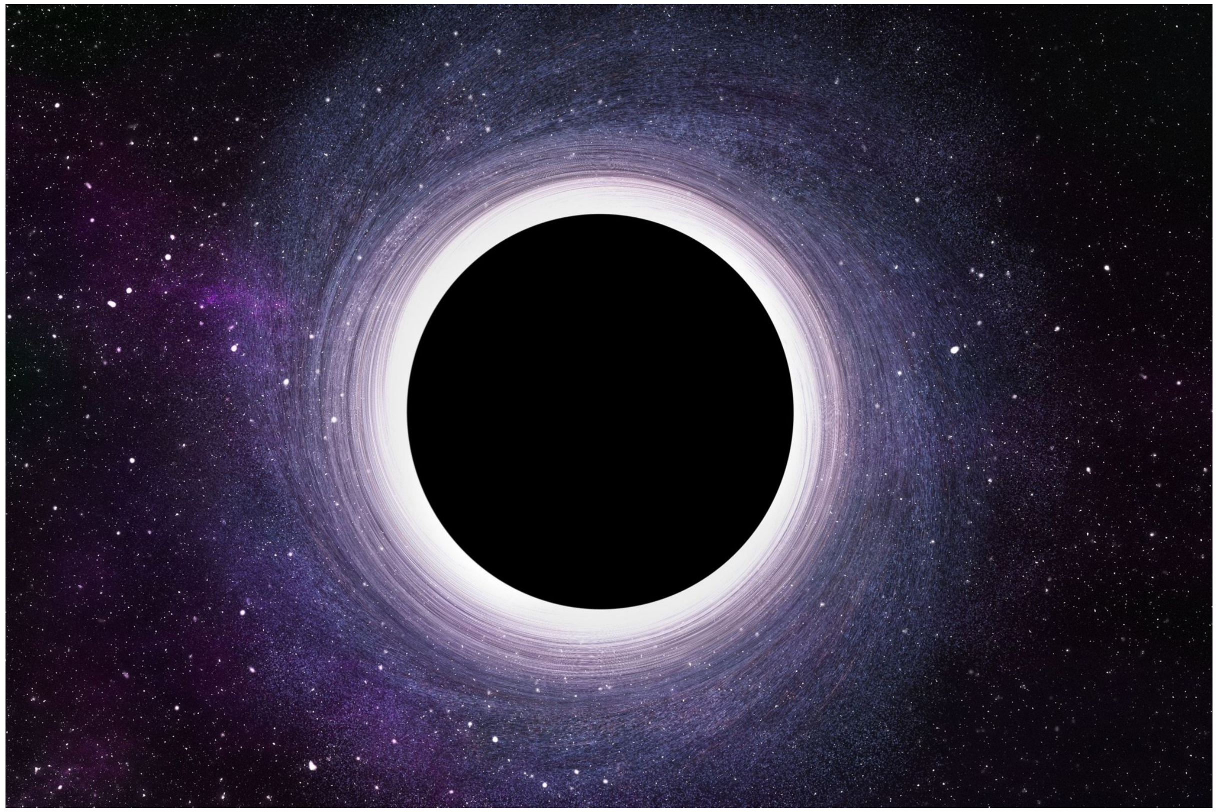
Measurements of these tidal effects could also provide hints for the existence of new types of compact objects, e.g. superradiant clouds, boson stars, etc.

Raithel, Özel, Psaltis [1803.07687], Chatziioannou [2006.03168]

Figures adapted from Aarts [1512.05145] & Baumann, HSC, Porto, Stout [1912.04932]

Tidal Response of Black Holes

Black holes are the simplest and most abundant compact objects that are detected by the LIGO and Virgo observatories.

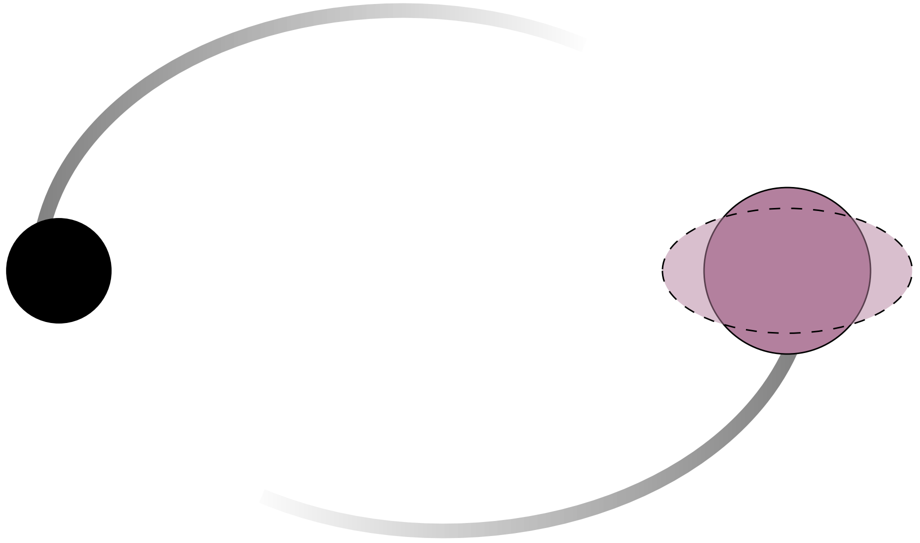


A detailed understanding of their tidal deformation and dissipation is a key goal in gravitational astrophysics and fundamental physics.

Kerr (1963)
Carter (1971)
Robinson (1975)

Black Holes do not Fall in Love

Black holes do not deform when they are perturbed by a static tidal field.



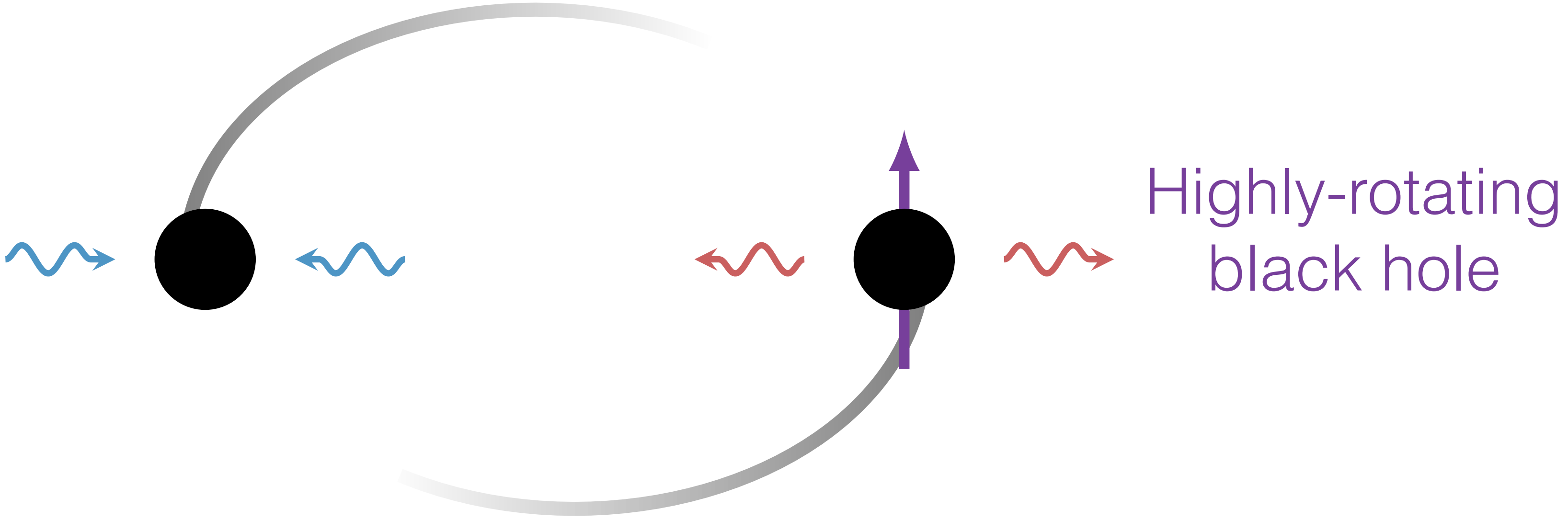
This conclusion holds for all values of black hole spin, ranging from Schwarzschild black holes to extremal Kerr black holes.

HSC [2010.07300]

Charalambous, Dubovsky, Ivanov [2102.08917]

Black Hole Dissipation

In general, energy and angular momentum are absorbed into the black hole.



For highly-rotating black holes, the dissipative response would trigger mode amplification, which is a phenomenon known as superradiance.

HSC [2010.07300]

Charalambous, Dubovsky, Ivanov [2102.08917]

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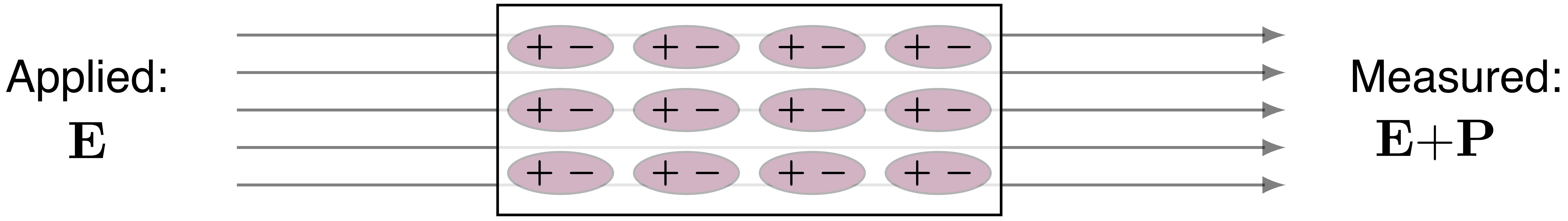
- Comments on "Spinning black holes fall in Love" claim
- Love is unambiguous

Electrostatic Response

In electromagnetism, a dielectric medium would respond to its external **electrostatic** field $\mathbf{E}$ by acquiring an induced polarization:

$$\mathbf{P} = \chi_E \mathbf{E}\,,$$

where χ_E is the electric susceptibility of the medium.



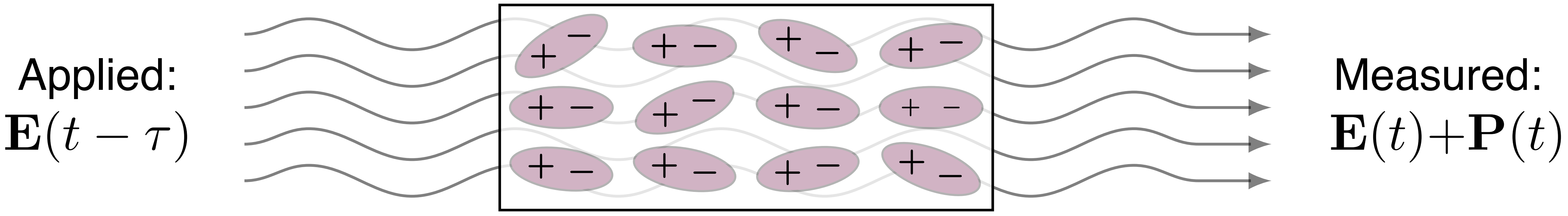
For a static external field, the material **polarizes instantaneously.**

Jackson (Classical Electrodynamics textbook)

Electrodynamic Response

In general, the external electric field **varies with time** and the material **does not polarize instantaneously**. For a slowly-varying external field,

$$\begin{aligned}\mathbf{P}(t) &= \int_0^\infty \mathrm{d}\tau\, \chi_E(\tau)\, \mathbf{E}(t-\tau)\,, \\ &= \chi_E^{(0)}\, \mathbf{E}(t) - \tau_0\, \chi_E^{(1)}\, \dot{\mathbf{E}}(t) + \cdots\,,\end{aligned}$$



where overdot denotes time derivative, and τ_0 is the typical time lag.

Jackson (Classical Electrodynamics textbook)

Conservative and Dissipative Effects

In Fourier space, the electrodynamic response reads

$$\mathbf{P}(\omega) = \chi_E(\omega)\mathbf{E}(\omega)$$

where ω is the frequency of the external electric field, and

$$\boxed{\chi_E(\omega) = \chi_E^{(0)} + i\omega\tau_0\chi_E^{(1)} + \cdots}$$

real part =
conservative effect

imaginary part =
dissipative effect

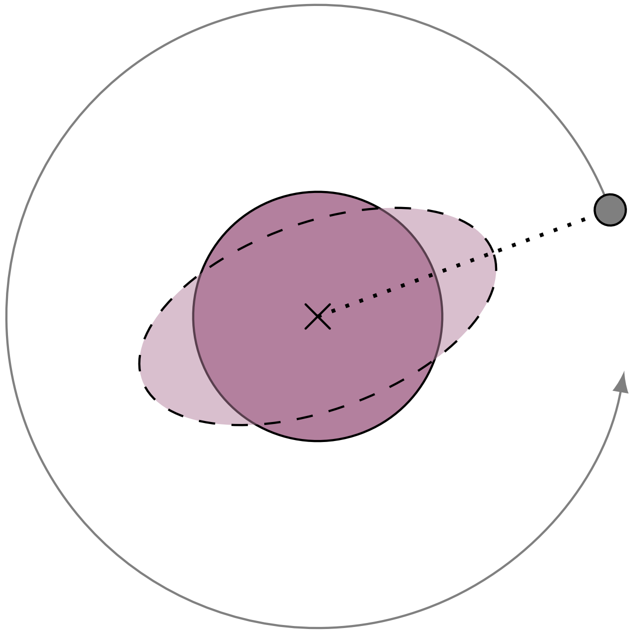
Jackson (Classical Electrodynamics textbook)

Static Tidal Response

In Newtonian gravity, a non-rotating body would respond to a **static** external tidal field by acquiring induced mass-type moments:

$$\delta Q_{\ell m} \propto 2k_{\ell m}\mathcal{E}_{\ell m}$$

where the proportionality constants $k_{\ell m}$ are called the Love numbers.



Love (1912)

Poisson, Will (Gravity textbook)

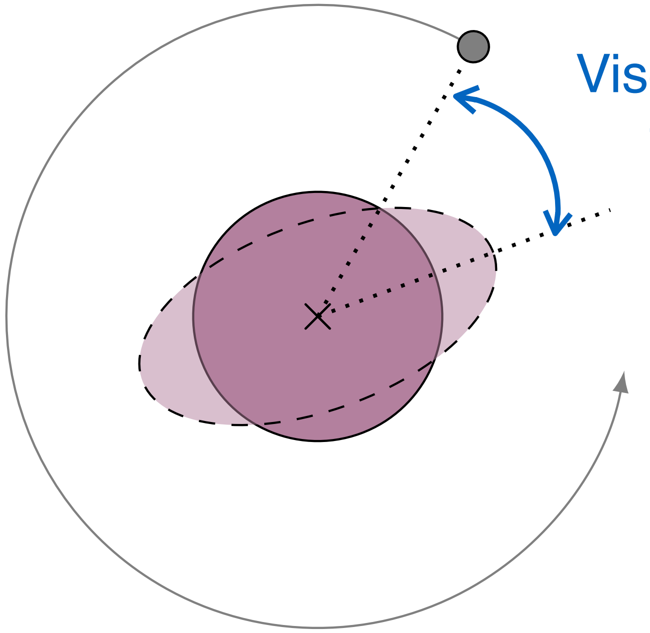
Time-Dependent Tidal Response

For a **slowly-varying** external tidal field, the induced response is

$$\delta Q_{\ell m}(t) \propto 2k_{\ell m}\mathcal{E}_{\ell m}(t) - \tau_0 \nu_{\ell m}\dot{\mathcal{E}}_{\ell m}(t) + \cdots \,,$$

static tides   tidal dissipation   dynamical tides (subleading)

where $\nu_{\ell m}$ are the dissipation numbers associated to the object's viscosity.



Viscosity-induced time lag, τ_0

Poisson, Will (Gravity textbook)

Tidal Response of a Non-rotating Body

In Fourier space, the tidal response of a non-rotating body is

$$\delta Q_{\ell m}(\omega) = F_{\ell m}(\omega)\mathcal{E}_{\ell m}(\omega)\,,$$

where ω is the frequency of the external tidal field, and

$$\boxed{F_{\ell m}(\omega) = 2k_{\ell m} + i\omega\tau_0\nu_{\ell m} + \cdots}$$

real part =
conservative effect

imaginary part =
dissipative effect

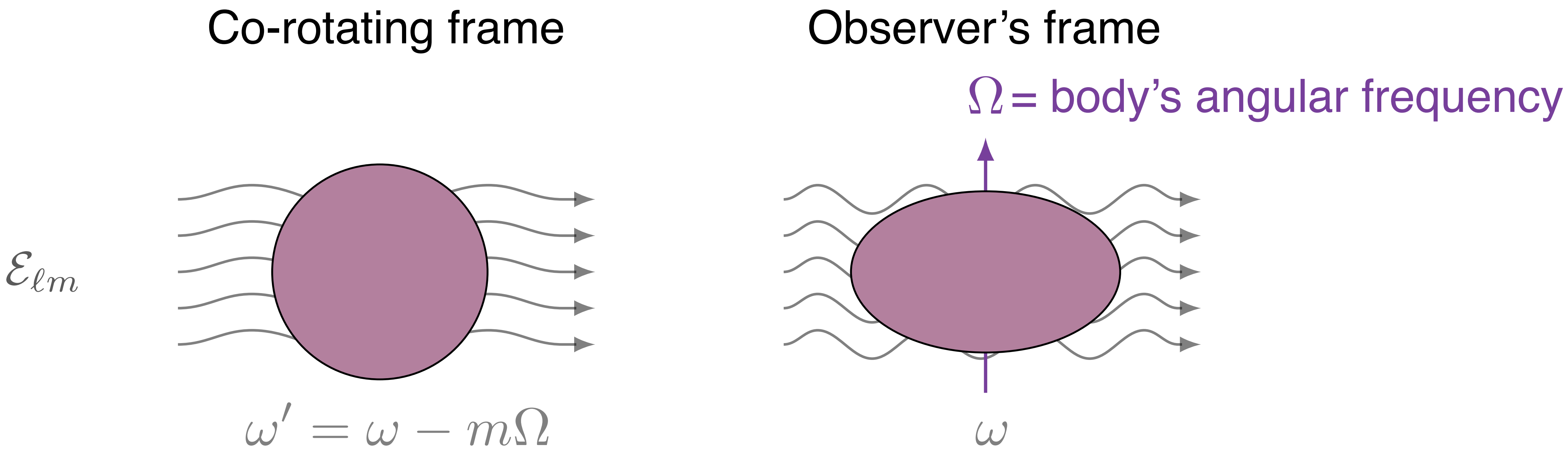
Goldberger, Rothstein [0409156, 0511133]
Chakrabarti, Delsate, Steinhoff [1306.5820]

Tidal Response of a Rotating Body

For a rotating body, the functional form

$$F_{\ell' m'}(\omega') = 2k_{\ell' m'} + i\omega' \tau_0 \nu_{\ell' m'} + \cdots$$

holds in the **co-rotating frame**, in which case ω' and $\{\ell', m'\}$ are tidal field's frequency and angular momentum numbers as perceived in this frame.



For a rotating body, tidal dissipation can still occur in a static tidal field, $\omega = 0$, due to the presence of relative motion between the body and the external field.

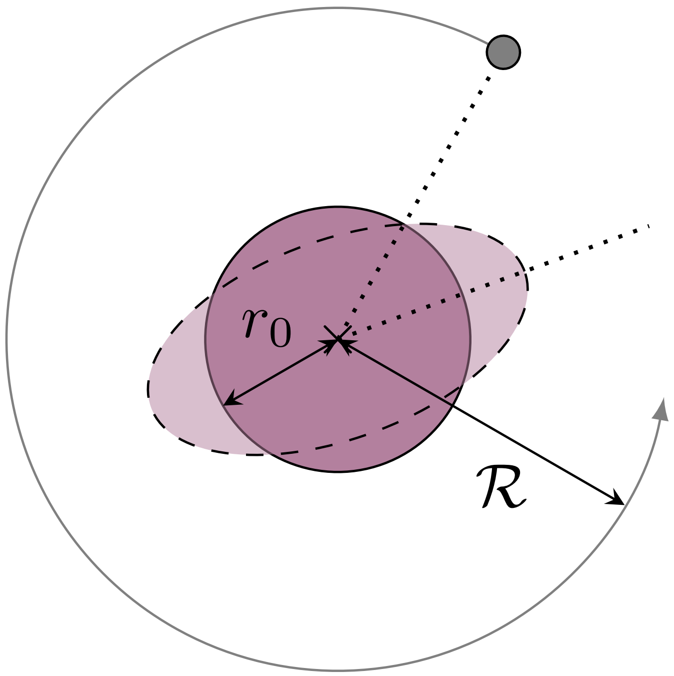
Measuring the Response Function

In Fourier space, the total potential of the perturbed non-rotating body is

$$U = -\frac{M}{r} + \sum_{\ell m} \frac{\mathcal{E}_{\ell m} r^{\ell}}{\ell(\ell-1)} \left[1 + \boxed{F_{\ell m}(\omega)} \left(\frac{r_0}{r}\right)^{2\ell+1}\right] Y_{\ell m}(\theta, \phi)$$

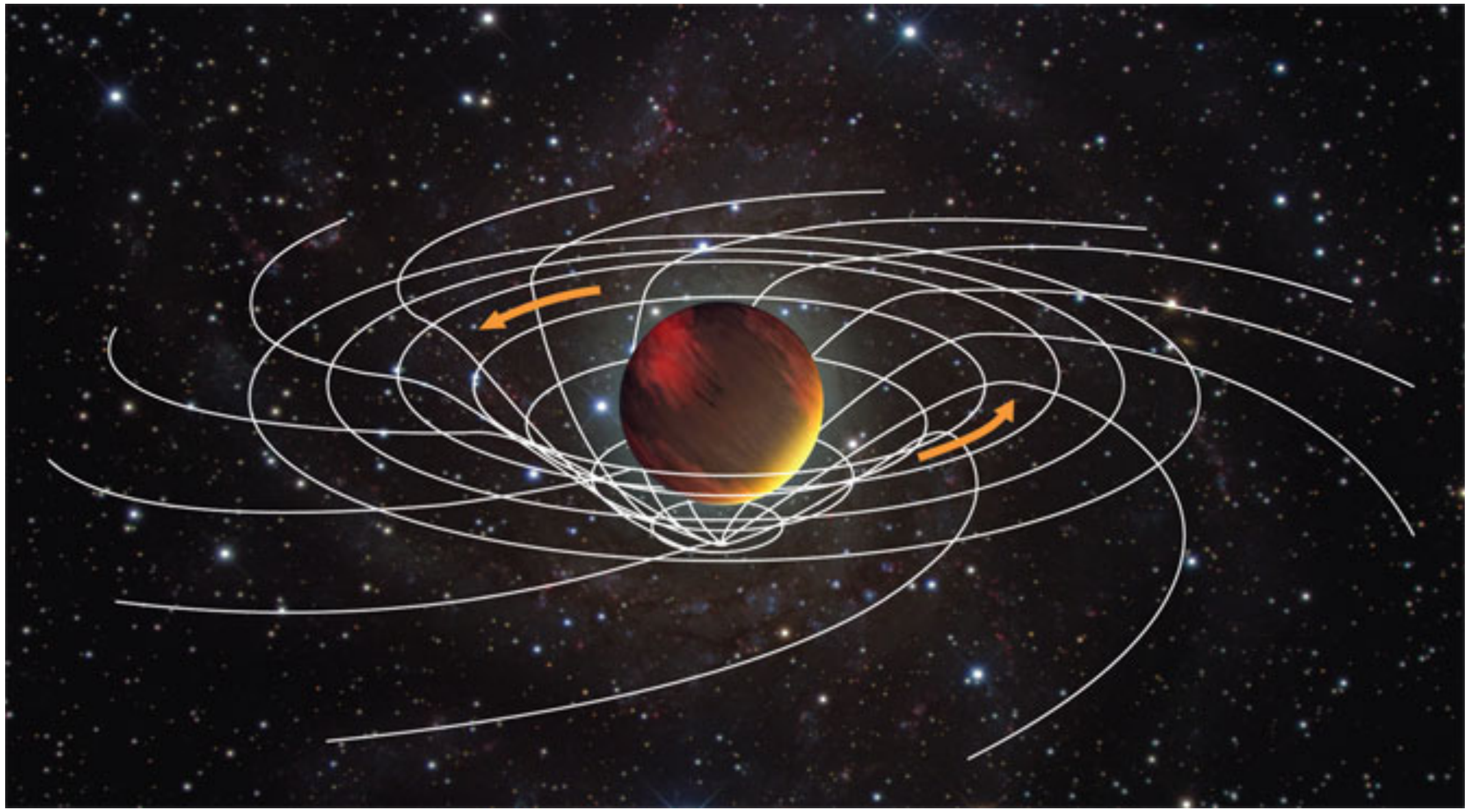
$\sim r^{\ell}$ “growing term”
(applied external tidal field)

$\sim r^{-\ell-1}$ “decaying term”
(object’s response)



Relativistic Tides

In addition to the electric-type tides $\mathcal{E}_{\ell m}$, tidal perturbations in General Relativity are also described by the magnetic-type tides $\mathcal{B}_{\ell m}$



These magnetic-type tides source induced current-type moments, and the tidal response is analogous to magnetic susceptibilities in electromagnetism.

Zhang (1986)

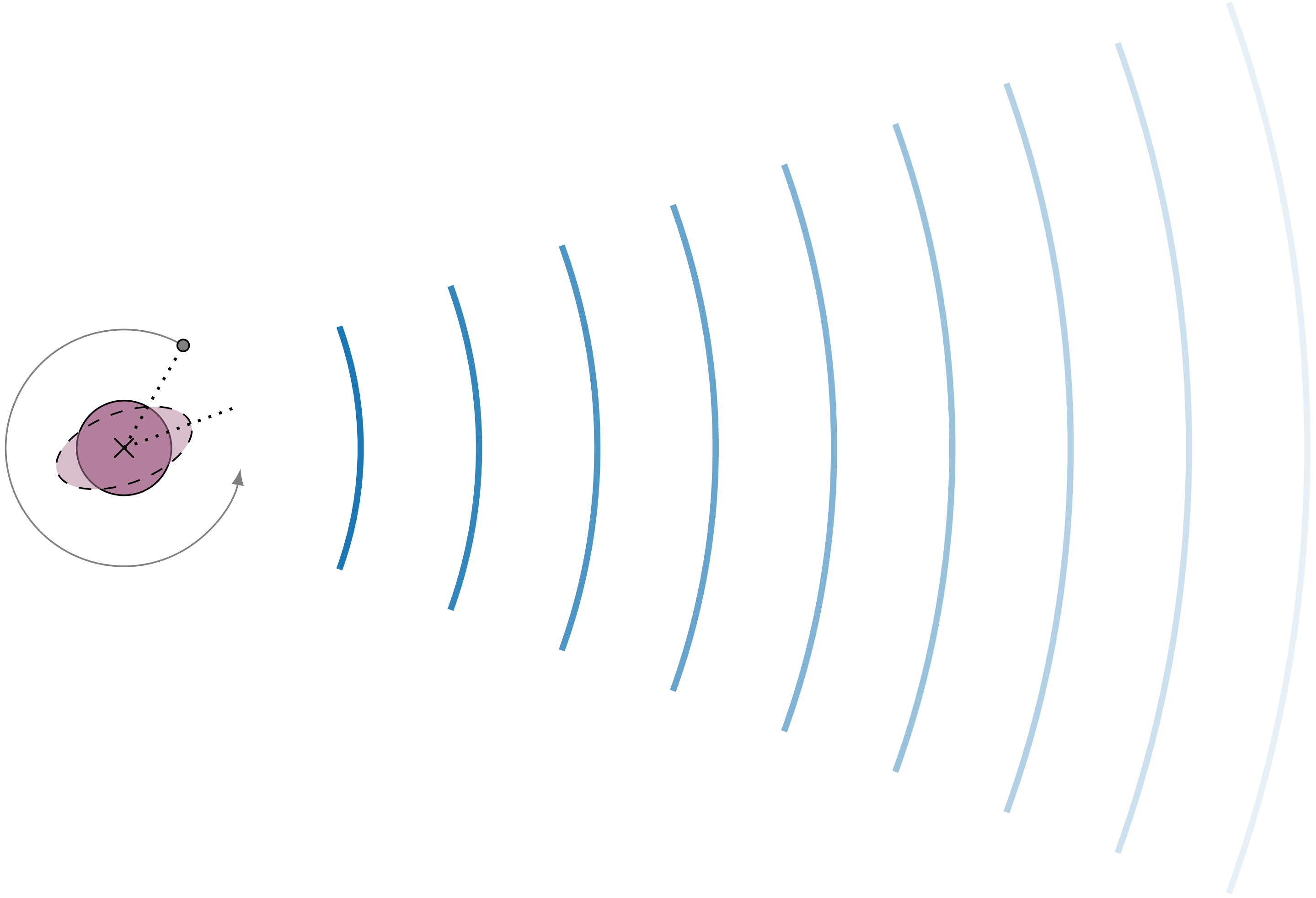
Binnington Poisson [0906.1366], Damour Nagar [0906.0096]

Weyl Scalars

For various practical reasons, it is convenient to decompose the Weyl tensor into five complex Weyl scalars in the Newman-Penrose formalism:

$$\{\psi_0,\ \psi_1,\ \psi_2,\ \psi_3,\ \psi_4\}$$

The "peeling theorem" states that $\psi_n \sim \mathcal{O}(r^{-5+n})$ at $r \to \infty$.



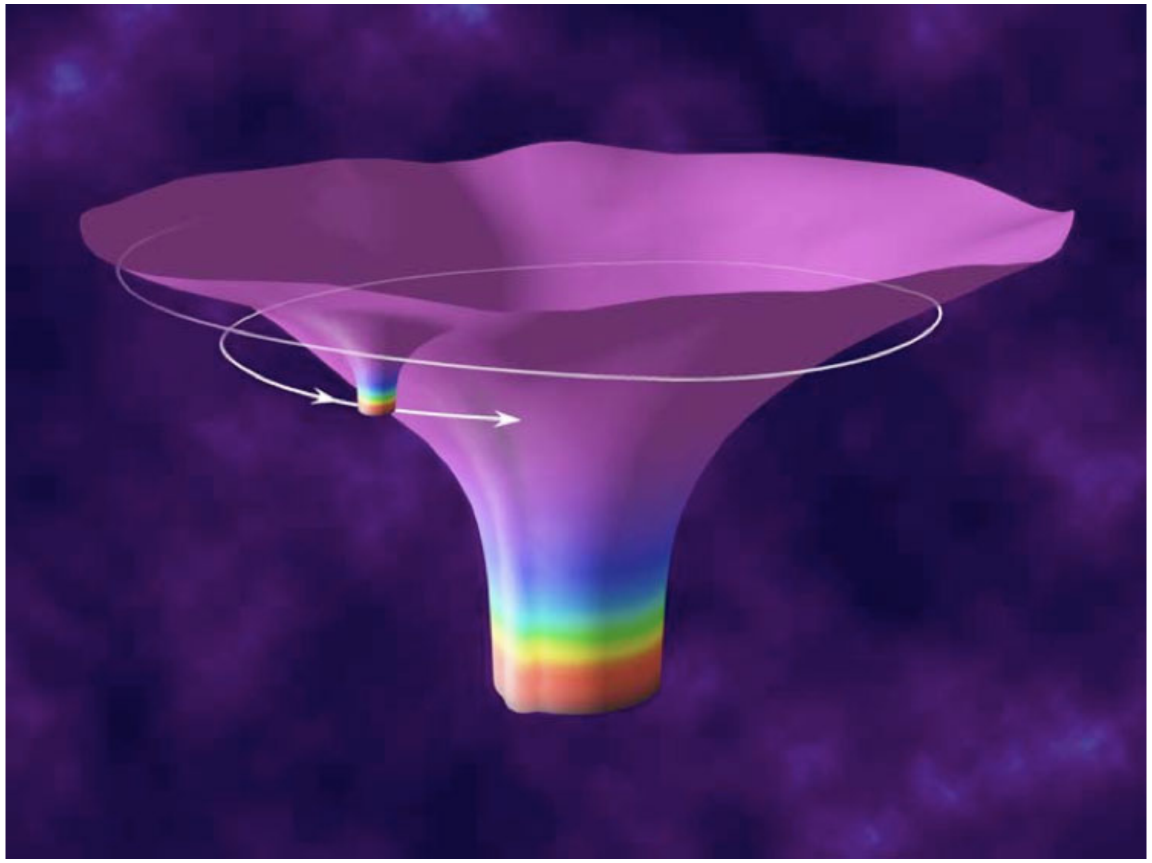
Measured:

$$\psi_4 \propto \ddot{h}_+ - i\ddot{h}_\times$$

Newman and Penrose (1962)

Black Hole Perturbation Theory

For Kerr black holes, the linear perturbations of ψ_4 (and ψ_0) can be solved relatively easily through the **Teukolsky equation**.



Using this method, the gravitational wave fluxes emitted by extreme mass ratio inspirals have been computed to high perturbative orders.

Teukolsky (1972, 1973)
Press and Teukolsky (1973, 1974)
Sasaki, Tagoshi (Living Review)

Tidal Moments as Amplitude Modulations

The Teukolsky equation separates ψ_4 into a set of coupled ordinary differential equations. This is achieved through the separable form

$$\rho^4 \psi_4 = \sum_{\ell m} e^{-i\omega v + im\phi} R_{\ell m}(r) {}_{-2}S_{\ell m}(\theta) \times \mathcal{M}_{\ell m} \,,$$

where $\{v, r, \theta, \phi\}$ are the spacetime coordinates, ${}_sS_{\ell m}$ is the spin-weighted spheroidal harmonic, and $\mathcal{M}_{\ell m}$ are amplitude constants.

One can show that the tidal moments **modulate the amplitudes** via

$$\mathrm{Re}\left(\mathcal{M}_{\ell m}\right) \propto \mathcal{E}_{\ell m} \,, \qquad \mathrm{Im}\left(\mathcal{M}_{\ell m}\right) \propto \mathcal{B}_{\ell m}$$

Teukolsky (1972, 1973)
Chatziioannou, Poisson, Yunes [1211.1686]
HSC [2010.07300]

The Radial Teukolsky Equation

The radial Teukolsky equation is*

$$\frac{\mathrm{d}^2 R}{\mathrm{d}r^2} + \left(\frac{2iP_+ - 1}{r - r_+} - \frac{2iP_- + 1}{r - r_-} - 2i\omega\right)\frac{\mathrm{d}R}{\mathrm{d}r} + \Bigg(\frac{4iP_-}{(r - r_-)^2} \boxed{- \frac{4iP_+}{(r - r_+)^2}} + \frac{A_- + iB_-}{(r - r_-)(r_+ - r_-)} - \frac{A_+ + iB_+}{(r - r_+)(r_+ - r_-)}\Bigg) R = \frac{T}{\Delta}\,,$$

Dominates as $r \to r_+$

where we have organised the radial equation in terms of its poles at $r_\pm, \infty$, and

$$P_+ = \frac{am - 2r_+ M\omega}{r_+ - r_-} \propto m\Omega_H - \omega$$

The P_+ "superradiance factor" captures all of the physics of the event horizon.

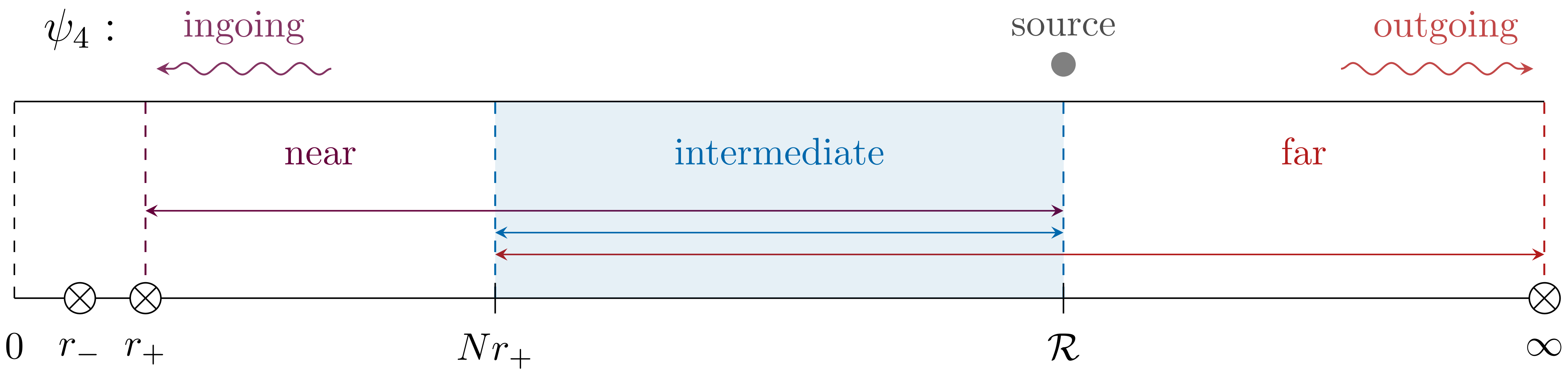
*In the the ingoing-Kerr coordinates and the Kinnersley null tetrad

Press and Teukolsky (1974)

HSC [2010.07300]

Boundary Conditions

The Teukolsky equation allows for the easy incorporation of the boundary conditions at the outer horizon, $r = r_+$, and at asymptotic infinity, $r \to \infty$.



The asymptotic radial behaviours at these locations are

$$\psi_4 \sim \boxed{Y_{\text{in}}(r - r_+)^2} + Y_{\text{out}}(r - r_+)^{-2iP_+}\,, \qquad r \to r_+\,,$$

$$\psi_4 \sim Z_{\text{in}}/r^5 + \boxed{Z_{\text{out}}e^{2i\omega(r+2M\log r)}/r\,,} \qquad r \to \infty\,,$$

Press and Teukolsky (1974)

Figure from HSC [2010.07300]

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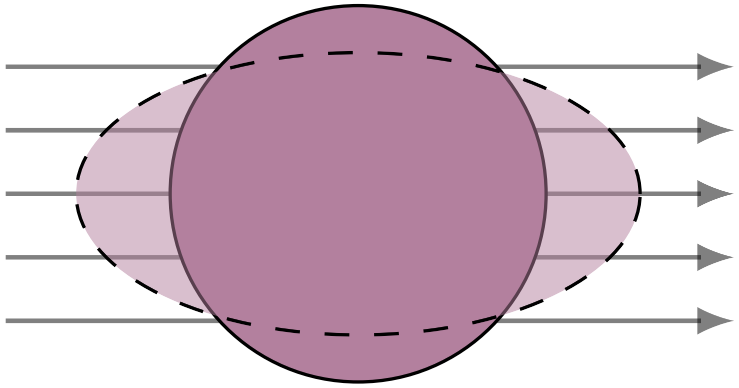
Perturbed Spherical Star: Metric

The metric of a perturbed relativistic spherical star has been computed in the literature for the static limit, $\omega = 0$ (no information about dissipation). E.g.

$$g_{vv} = -\left(1 - \frac{2M}{r}\right) - \frac{2\mathcal{E}_{\ell m}}{\ell(\ell-1)}\left[r^\ell A(r) + \mathbf{2k^E_{\ell m}} r^\ell \left(\frac{r_0}{r}\right)^{2\ell+1} B(r)\right] Y_{\ell m}(\theta,\phi)\,,$$

where $k^E_{\ell m}$ are the electric-type Love numbers. Other metric components depend on the magnetic-type Love numbers, $k^B_{\ell m}$, as well.

$\mathcal{E}_{\ell m}, \mathcal{B}_{\ell m}$
$(\omega = 0)$



$k^I_{\ell m} \neq 0$

Binnington, Poisson [0906.1366]
Damour, Nagar [0906.0096]

Perturbed Spherical Star: Weyl Scalar

We can condense all information of the perturbed metric into the Weyl scalar:*

$$\psi_4^{\mathrm{Sph}}(\omega = 0) = \sum_I \sum_{\ell m} \mathcal{M}^I_{\ell m}\, {}_{-2}Y_{\ell m}(\theta, \phi)$$
$$\times \left[r^{\ell-2} G_\ell(r) + 2\, k^I_{\ell m} r^{\ell-2} \left(\frac{r_0}{r}\right)^{2\ell+1} D_\ell(r) \right] ,$$

$\sim r^{\ell-2}$ "growing term"
(applied external tidal field)

$\sim r^{-\ell-3}$ "decaying term"
(object's response)

where $I = \{E, B\}$ label quantities of electric- and magnetic-type characters:

$$\mathcal{M}^E_{\ell m} \propto \mathcal{E}_{\ell m} , \qquad \mathcal{M}^B_{\ell m} \propto i\mathcal{B}_{\ell m} ,$$

HSC [2010.07300]

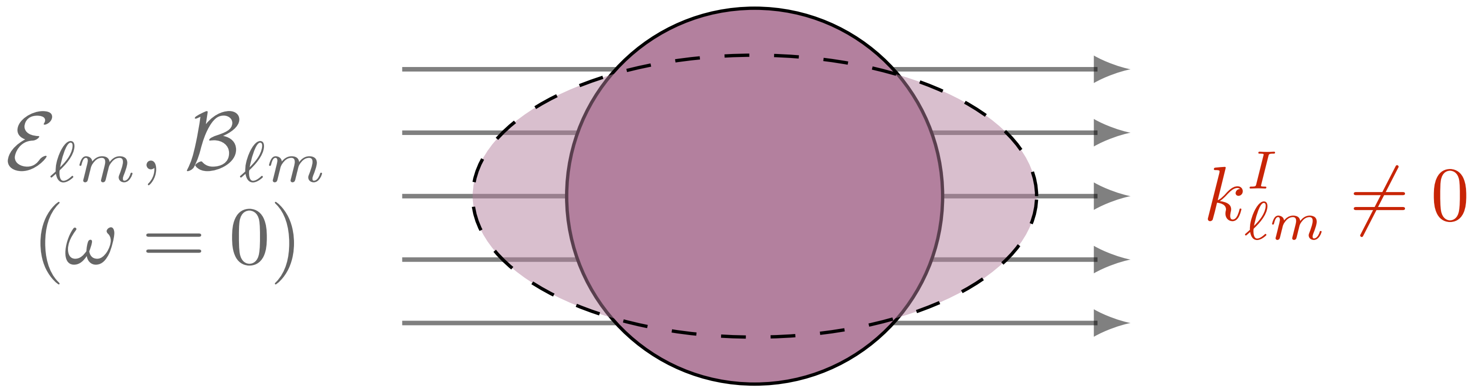
*Recall the analogous $\sim r^\ell$ and $\sim r^{-\ell-1}$ scalings in the Newtonian potential

Spherical Star: Love Numbers

The Love numbers are determined by matching the exterior metric with the boundary condition at the star's surface, $r = r_0$.

$$\psi_4^{\mathrm{Sph}}(\omega = 0) \propto r^{\ell-2} G_\ell(r) + 2\, k^I_{\ell m} r^{\ell-2} \left(\frac{r_0}{r}\right)^{2\ell+1} D_\ell(r)\,.$$

For a general star, $r_0 > 2M$. In addition, one would need to specify an equation of state for the star's interior region to obtain boundary condition.



Binnington, Poisson [0906.1366]

Damour, Nagar [0906.0096]

Schwarzschild Black Hole: Love Numbers

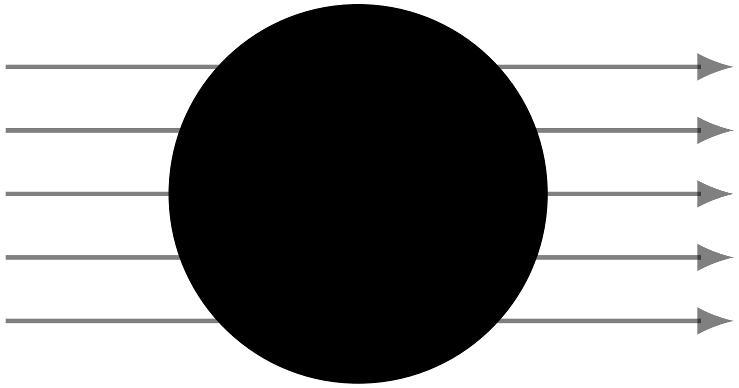
For a Schwarzschild black hole, $r_0 = 2M$.

$$\psi_4^{\text{Sph}}(\omega = 0) \propto r^{\ell-2} G_\ell(r) + 2\, k_{\ell m}^I r^{\ell-2} \left(\frac{r_0}{r}\right)^{2\ell+1} D_\ell(r)\,.$$

Furthermore, as we approach the event horizon,

$$r^{\ell-2} G_\ell(r) \sim (r - 2M)^2\,, \qquad r \to 2M\,,$$

while D_ℓ diverges logarithmically. The boundary condition at the event horizon* forces the decaying terms to vanish identically, which is only possible if $k_{\ell m}^I = 0$.



$\mathcal{E}_{\ell m}, \mathcal{B}_{\ell m}$
$(\omega = 0)$

$k_{\ell m}^I = 0$

No deformation

Binnington, Poisson [0906.1366], Damour, Nagar [0906.0096], Kol, Smolkin [1110.3764]

*Recall that the purely ingoing wave scales as $\psi_4 \sim (r - r_+)^2$

Schwarzschild Black Hole: Solution

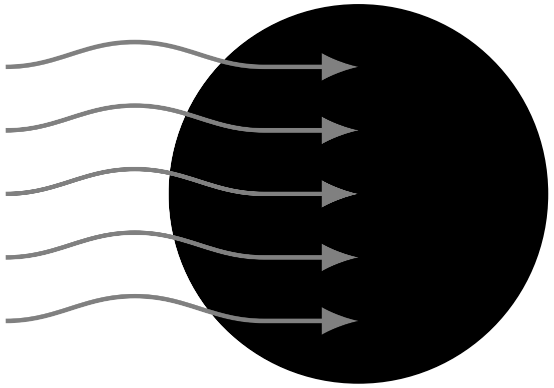
The discussion above was restricted to the static limit.

$$\psi_4^{\rm Schw}(\omega = 0) \propto {}_2\mathsf{F}_1(2-\ell, \ell+3; 3; 1 - r/2M)\,.$$

To derive the dissipative response of the Schwarzschild black hole, we solve the Teukolsky equation perturbatively in $M\omega \ll 1$. At leading order, we find

$$\psi_4^{\rm Schw}(M\omega \ll 1) \propto {}_2\mathsf{F}_1(2-\ell, \ell+3; 3 + 2i\tilde{P}_+; 1 - r/2M)\,.$$

where $i\tilde{P}_+ = -i\omega(2M)$ is the zero-spin limit of the superradiance factor.



$\mathcal{E}_{\ell m}, \mathcal{B}_{\ell m}$
$(\omega \neq 0)$

$k_{\ell m}^I = 0$

$\nu_{\ell m}^I \neq 0$

Absorption of energy and angular momentum fluxes

HSC [2010.07300]

Schwarzschild Black Hole: Total Response

The tidal response is obtained by expanding the Weyl scalar at large distances:

$$\psi_4^{\rm Schw} \propto r^{\ell-2}\left[(1+\cdots)+\boxed{F_{\ell m}^{I,{\rm Schw}}}\left(\frac{2M}{r}\right)^{2\ell+1}(1+\cdots)\right], r \gg 2M$$

$$\boxed{F_{\ell m}^{I,{\rm Schw}}(\omega) = 0 + i\omega(2M)\nu_{\ell m}^{\rm Schw} + \cdots}$$

Well-known result in the literature

We obtain the **general expression** for the dissipation numbers $\nu_{\ell m}^{\rm Schw}$, which recovers known results for the first few orders of ℓ in the literature.

HSC [2010.07300]

Binnington, Poisson [0906.1366], Damour, Nagar [0906.0096], Kol, Smolkin [1110.3764]

Goldberger, Rothstein [0511133]

Newtonian vs Schwarzschild Response

Non-rotating body in Newtonian gravity:

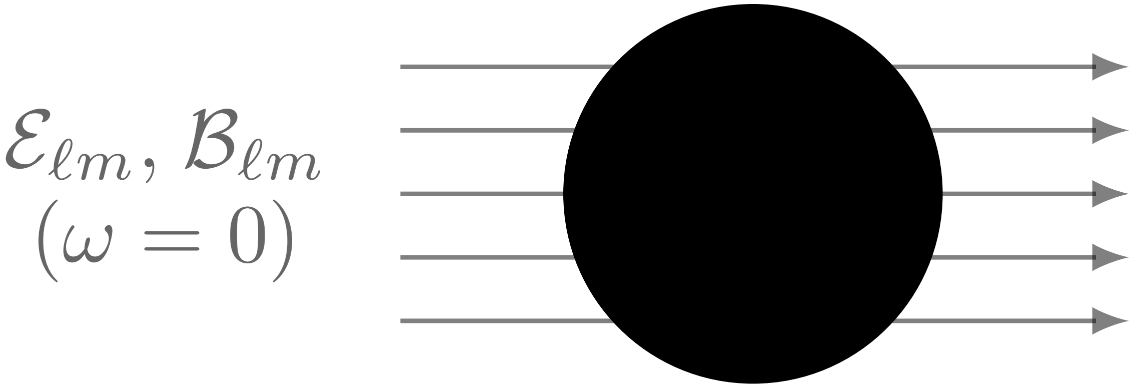
$$F_{\ell m}(\omega) = 2k_{\ell m} + i\omega\tau_0\nu_{\ell m} + \cdots$$

Schwarzschild black hole:

$$F^{I,\mathrm{Schw}}_{\ell m}(\omega) = 0 + i\omega(2M)\nu^{\mathrm{Schw}}_{\ell m} + \cdots$$

where $\tau_0 = 2M$ is the black hole light crossing time.

At $\omega = 0$, tidal dissipation and all higher order terms of the Schwarzschild response function vanish identically.



Schwarzschild black holes *do not* respond to *static* tides

HSC [2010.07300]

Poisson [2012.10184]

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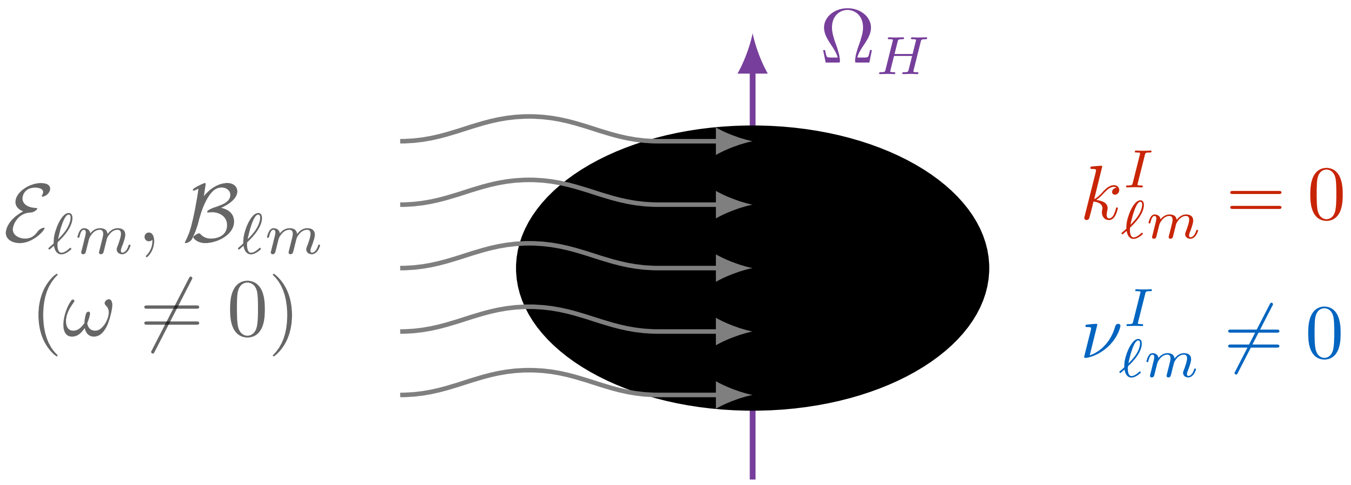
Kerr Black Hole: Solution

Repeating the same exercise as above, we can derive the tidal response of Kerr black hole's tidal response through the Teukolsky equation:

$$\psi_4^{\mathrm{Kerr}}(M\omega \ll 1) \propto {}_2\mathsf{F}_1\big(2-\ell, \ell+3; 3+2iP_+; (r_+-r)/(r_+-r_-)\big)\,.$$

Easy to check that this reduces to the Schwarzschild solution at $a=0$.



Deformation is spin-induced,
not tidally-induced!

HSC [2010.07300]

Kerr Black Hole: Tidal Response

We also solve for the response function of the Kerr black hole:

$$\boxed{F^{I,\mathrm{Kerr}}_{\ell m}(\omega,\Omega_H) = 0 - i(m\Omega_H - \omega)\,[2Mr_+/(r_+ - r_-)]\,\nu^{\mathrm{Kerr}}_{\ell m} + \cdots}$$

- **Rotating black holes *do not* fall in Love**

  - true for all spins, all $\{\ell, m\}$, and both electric-type and magnetic-type tides
  - generalizes partial results known in the literature

- **Tidal dissipation** is proportional to the so-called **superradiance** factor

$$\propto m\Omega_H - \omega$$

which can either be negative (energy loss) or positive (energy extraction)

HSC [2010.07300]

Poisson [1411.4711], Pani, Gualtieri, Maselli, Ferrari [1503.07365]

Goldberger, Li, Rothstein [2012.14869], Charalambous, Dubovsky, Ivanov [2102.08917, 2103.01234]

Schwarzschild vs Kerr Response

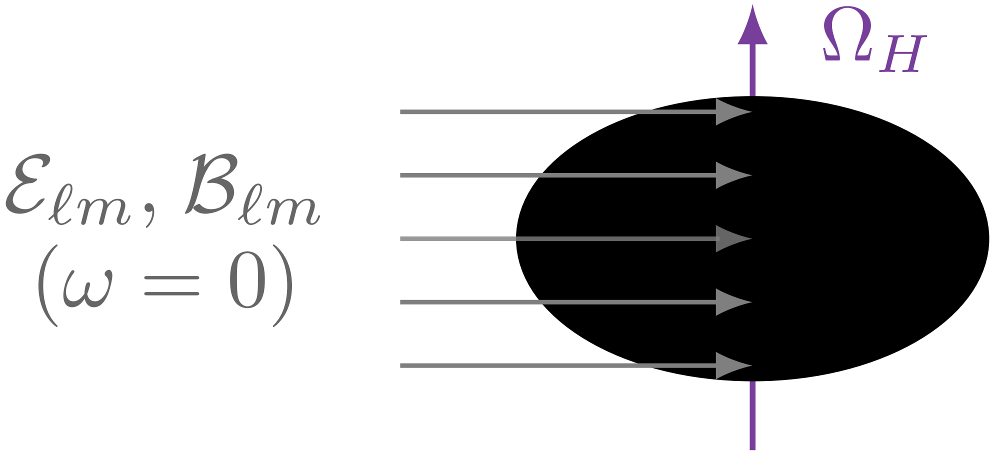
Schwarzschild black hole:

$$F_{\ell m}^{I,\mathrm{Schw}}(\omega) = 0 + i\omega(2M)\nu_{\ell m}^{\mathrm{Schw}} + \cdots$$

Kerr black hole:

$$F_{\ell m}^{I,\mathrm{Kerr}}(\omega, a) = 0 - i(m\Omega_H - \omega)\,[2Mr_+/(r_+ - r_-)]\,\nu_{\ell m}^{\mathrm{Kerr}} + \cdots$$

Unlike the Schwarzschild black hole, the Kerr black hole still experiences dissipation at $\omega = 0$ due to the relative motion between the black hole's rotation and the static tidal field.



The Kerr black hole *does respond* to *static* tides through dissipation

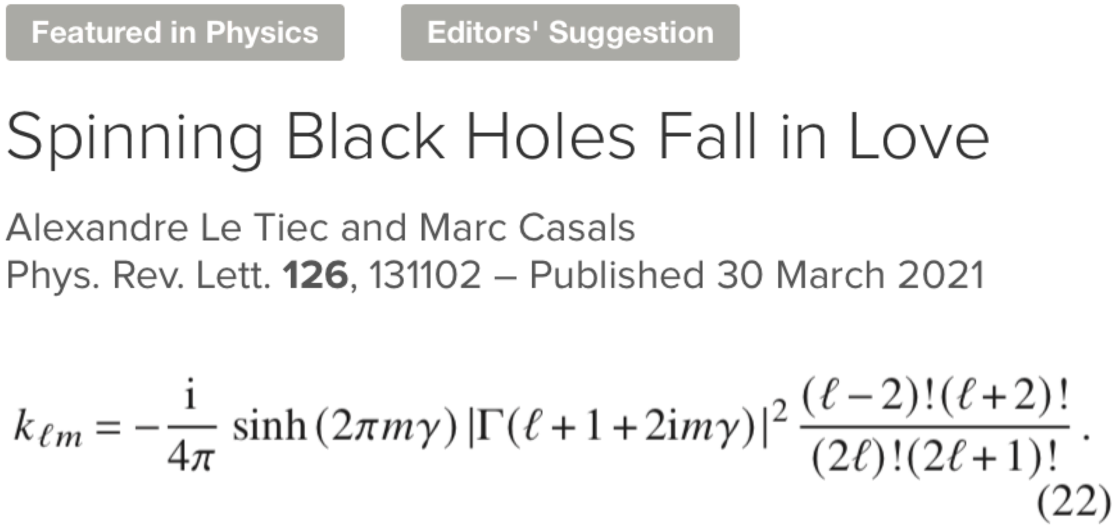
HSC [2010.07300]

Goldberger, Li, Rothstein [2012.14869]

Charalambous, Dubovsky, Ivanov [2102.08917]

Claims of “Spinning Black Holes Fall in Love”

Le Tiec, Casals [2007.00214] (PRL) + Franzin [2010.15795]:



Featured in Physics   Editors' Suggestion

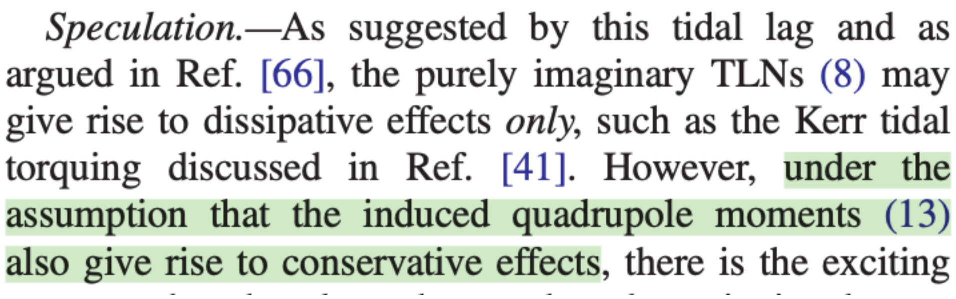
Spinning Black Holes Fall in Love

Alexandre Le Tiec and Marc Casals

Phys. Rev. Lett. **126**, 131102 – Published 30 March 2021

$$k_{\ell m} = -\frac{\mathrm{i}}{4\pi} \sinh(2\pi m\gamma)\,|\Gamma(\ell+1+2\mathrm{i}m\gamma)|^2 \frac{(\ell-2)!(\ell+2)!}{(2\ell)!(2\ell+1)!}. \tag{22}$$

In the revised version of the draft, the authors redefined what they mean by “Love numbers”. They also added:



Speculation.—As suggested by this tidal lag and as argued in Ref. [66], the purely imaginary TLNs (8) may give rise to dissipative effects *only*, such as the Kerr tidal torquing discussed in Ref. [41]. However, under the assumption that the induced quadrupole moments (13) also give rise to conservative effects, there is the exciting

↑ $\omega = 0$

$$F^{I,\mathrm{Kerr}}_{\ell m}(\omega, \Omega_H) = 0 - i(m\Omega_H - \omega)\,[2Mr_+/(r_+ - r_-)]\,\nu^{\mathrm{Kerr}}_{\ell m} + \cdots$$

In the first version of the arXiv paper, the authors claimed that the non-vanishing term is the conservative tidal Love numbers. Instead, those are dissipation numbers at the static limit, which are non-vanishing due to frame dragging.

HSC [2010.07300]

Poisson [1411.4711], Pani, Gualtieri, Maselli, Ferrari [1503.07365]

Goldberger, Li, Rothstein [2012.14869], Charalambous, Dubovsky, Ivanov [2102.08917, 2103.01234]

Black Holes do not Fall in Love

The GW community commonly refers to $k_{\ell m}$, not $F_{\ell m}$, as Love numbers

$$F^{I,\text{Kerr}}_{\ell m}(\omega, \Omega_H) = 0 - i(m\Omega_H - \omega)\,[2Mr_+/(r_+ - r_-)]\,\nu^{\text{Kerr}}_{\ell m} + \cdots$$

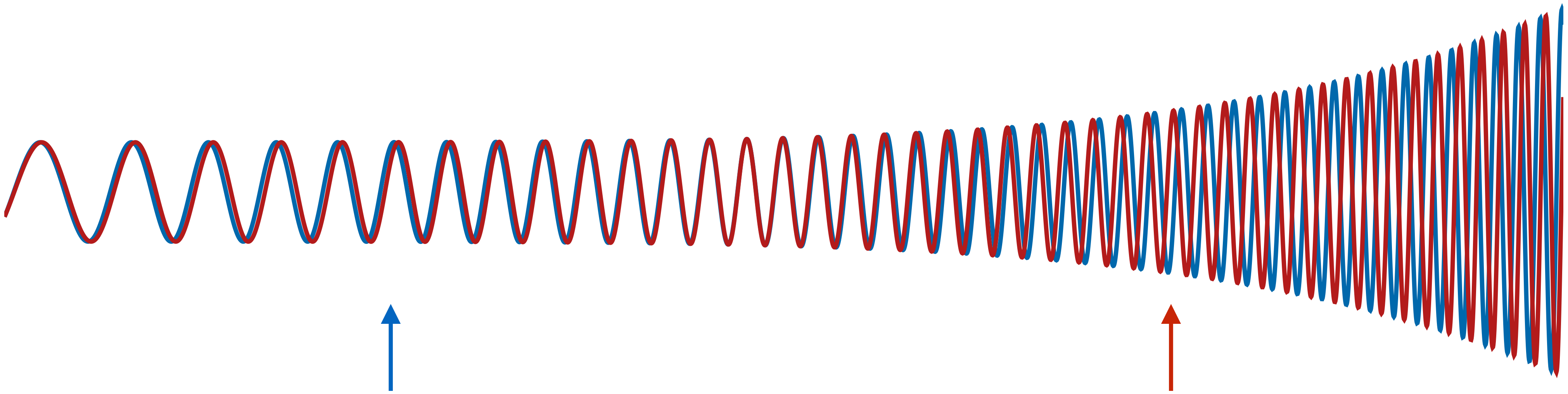
Regardless of difference in definitions/nomenclatures, the **physical imprints** of those tidal effects on waveforms are **unambiguous**.

- **Tidal deformation** (first appears at **5PN** in waveform phase): 0
- **Tidal dissipation** (first appears at **2.5PN** in waveform phase): $\propto m\Omega_H - \omega$

HSC [2010.07300]
Poisson [1411.4711], Pani, Gualtieri, Maselli, Ferrari [1503.07365]
Goldberger, Li, Rothstein [2012.14869], Charalambous, Dubovsky, Ivanov [2102.08917, 2103.01234]

Tidal Dissipation in Schwarzschild vs Kerr BHs

The superradiance factor is responsible for an enhancement of tidal dissipation in rotating black holes, compared to Schwarzschild black holes.



Tidal dissipation
(2.5PN order onwards)

$$P_+ \propto m\Omega_H - \omega$$

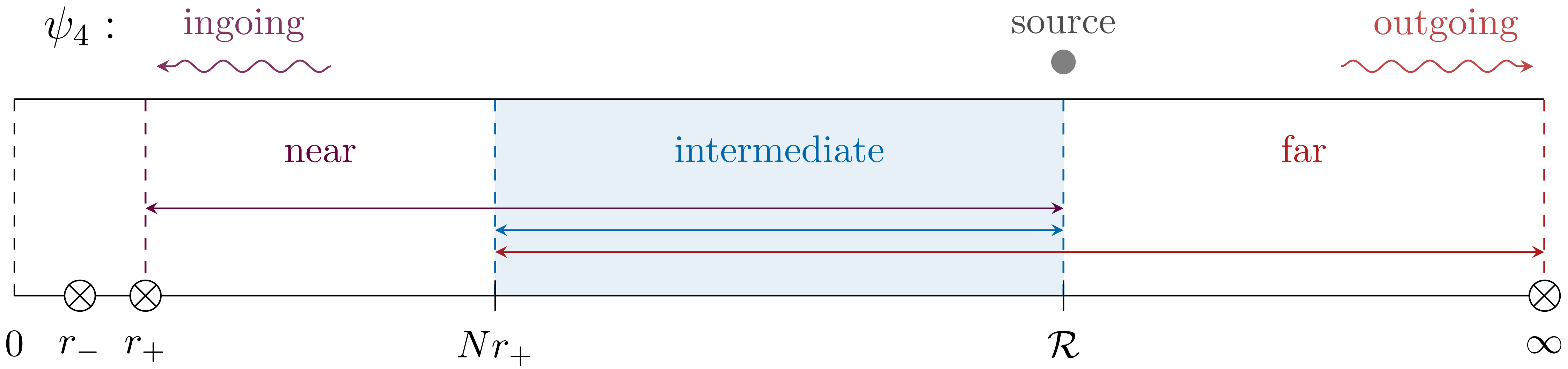
Love numbers
(5PN order onwards)

In the PN expansion, $M\omega \sim v^3$, where v is the typical binary velocity. For a Schwarzschild black hole, where $\Omega_H = 0$, tidal dissipation is suppressed by an additional factor of v^3 and therefore first appears at 4PN order.

Poisson, Sasaki [9412027]
Tagoshi, Mano, Takasugi [9711072]
Flanagan, Hinderer [0709.1915]

Love is Unambiguous

There are also claims in the literature that Love numbers are “ambiguous” in General Relativity (related to coordinate transformation arguments)



However, this is a red-herring and can be resolved by performing a **matched asymptotic expansion** between the solutions in the regions of spacetime.

Gralla [1710.11096]
HSC [2010.07300]
Charalambous, Dubovsky, Ivanov [2102.08917]

Why is Love only Vanishing at D=4?

It is known for some time that Love numbers of (Schwarzschild black holes) are only zero at four spacetime dimensions, but not at higher dimensions.

From a field theory perspective:

Vanishing Love number $\leftrightarrow$ **Symmetry?**

Recent work has found a remarkable **hidden $SL(2,\mathbb{R})$ symmetry** and a **ladder symmetry** associated to the black hole horizon, which is responsible for vanishing Love only at D=4.

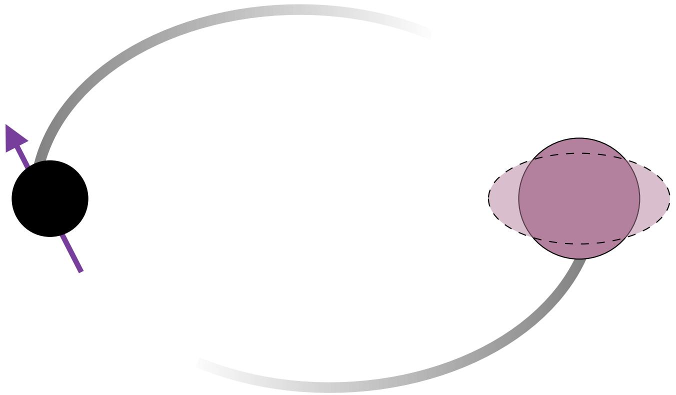
Kol, Smolkin [1110.3764]

Hui, Joyce, Penco, Santoni, Solomon [2010.00593, 2105.01069]

Charalambous, Dubovsky, Ivanov [2103.01234]

Thank you for your attention!

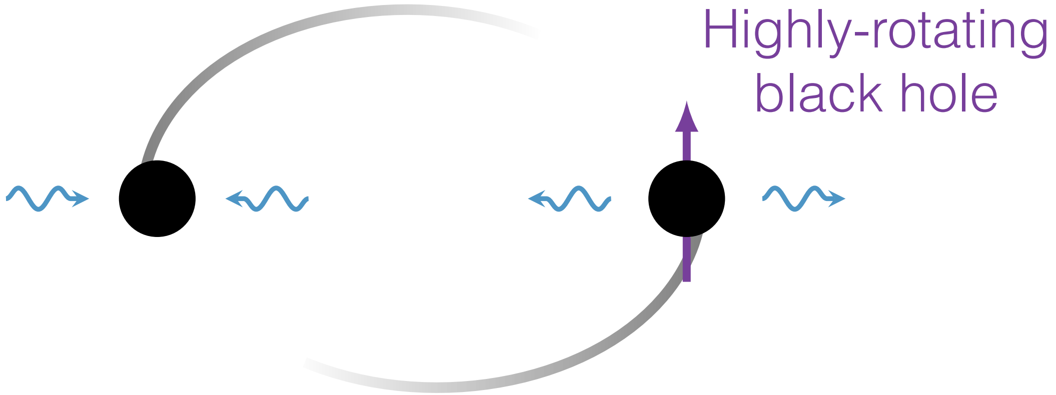
Tidal deformation



Rotating black holes

***do not* fall in Love**

Tidal dissipation



Black hole dissipation can induce
mode absorption and **amplification**

HSC [2010.07300]

Supplementary Slides

Relativistic Tides

In General Relativity, the tidal perturbations are characterised by both the electric-type and magnetic-type tidal moments

$$\mathcal{E}_L \propto \left[u^\alpha u^\beta C_{\alpha\mu_1\beta\mu_2\,;\,\mu_3\cdots\mu_\ell}\right]^{\mathrm{STF}} ,$$

$$\mathcal{B}_L \propto \left[u^\alpha u^\beta \epsilon^{\lambda\sigma}{}_{\beta\mu_2} C_{\alpha\mu_1\lambda\sigma\,;\,\mu_3\cdots\mu_\ell}\right]^{\mathrm{STF}} ,$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor, and u^μ is the time-like unit vector of a Fermi comoving observer.

The tidal moments in the STF representation can be rewritten in terms of the spherical harmonic representation:

$$\mathcal{E}_L x^L \propto r^\ell \sum_{\ell m} \mathcal{E}_{\ell m} Y_{\ell m}(\theta, \phi) ,$$

$$\mathcal{B}_L x^L \propto r^\ell \sum_{\ell m} \mathcal{B}_{\ell m} Y_{\ell m}(\theta, \phi) ,$$

Zhang (1986)

Newman-Penrose Weyl Scalars

In the Newman-Penrose formalism, the Weyl tensor can be decomposed into five complex Weyl scalars through a set of null tetrad:

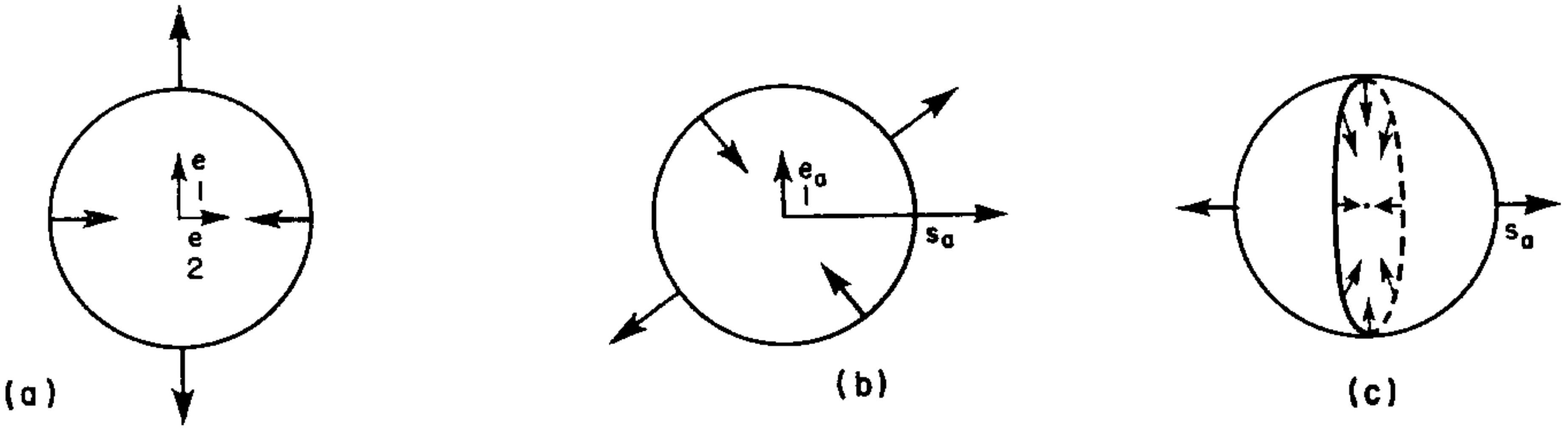
$\psi_0 = C_{\mu\nu\rho\sigma} l^\mu m^\nu l^\rho m^\sigma \,,$	Ingoing transverse wave
$\psi_1 = C_{\mu\nu\rho\sigma} l^\mu n^\nu l^\rho m^\sigma \,,$	Ingoing longitudinal wave
$\psi_2 = C_{\mu\nu\rho\sigma} l^\mu m^\nu \bar{m}^\rho n^\sigma \,,$	“Coulomb” field
$\psi_3 = C_{\mu\nu\rho\sigma} l^\mu n^\nu \bar{m}^\rho n^\sigma \,,$	Outgoing longitudinal wave
$\psi_4 = C_{\mu\nu\rho\sigma} n^\mu \bar{m}^\nu n^\rho \bar{m}^\sigma \,,$	Outgoing transverse wave

where l, n are real, $\bar{m}$ is the complex conjugate of m, and they obey $l^\mu n_\mu = -m^\mu \bar{m}_\mu = -1$, with all other inner products vanish.

Newman and Penrose (1962)
Szekeres (1965)

“Shearing” Forces of the Weyl Scalars

Each of the Weyl scalars represents different “shearing” forces:



ψ_0, ψ_4
transverse

ψ_1, ψ_3
longitudial

ψ_2
“Coulomb”

For a “star”, such as the Kerr black hole, only ψ_2 is non-vanishing.
For gravitational waves, only ψ_0, ψ_4 are non-vanishing.

Newman and Penrose (1962)
Szekeres (1965)

The Peeling Theorem

The “peeling theorem” states that each of these Weyl scalars decays as

$$\psi_n \sim \mathcal{O}\left(r^{-5+n}\right) \, , \qquad r \to \infty$$

For example:

- the $\psi_2 \sim \mathcal{O}\left(r^{-3}\right)$ scaling of the “Coulomb” field represents the dominant tidal force that are sourced by stars;
- the $\psi_4 \sim \mathcal{O}\left(r^{-1}\right)$ scaling implies that the outgoing transverse waves dominate the gravitational field at large distances.

For an outgoing plane wave, $\psi_0 = \psi_1 = \psi_2 = \psi_3 = 0$, while ψ_4 encodes the two independent GW polarizations that we measure at large distances

$$\psi_4 = -\ddot{h}_+ + i\ddot{h}_\times \, , \qquad r \to \infty$$

where dot denotes time derivative.

Newman and Penrose (1962)