The information paradox as a scattering problem

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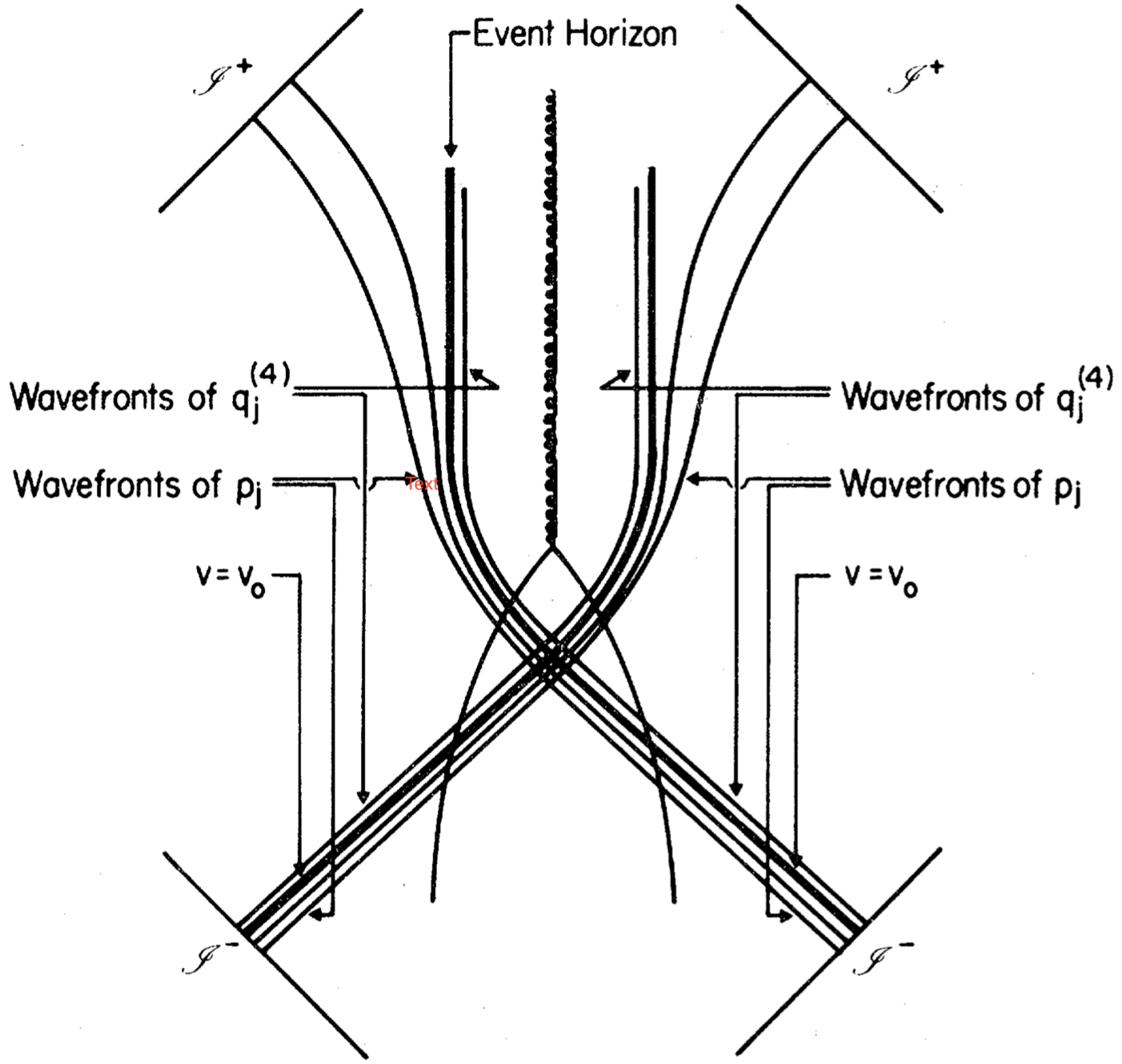
Utrecht University

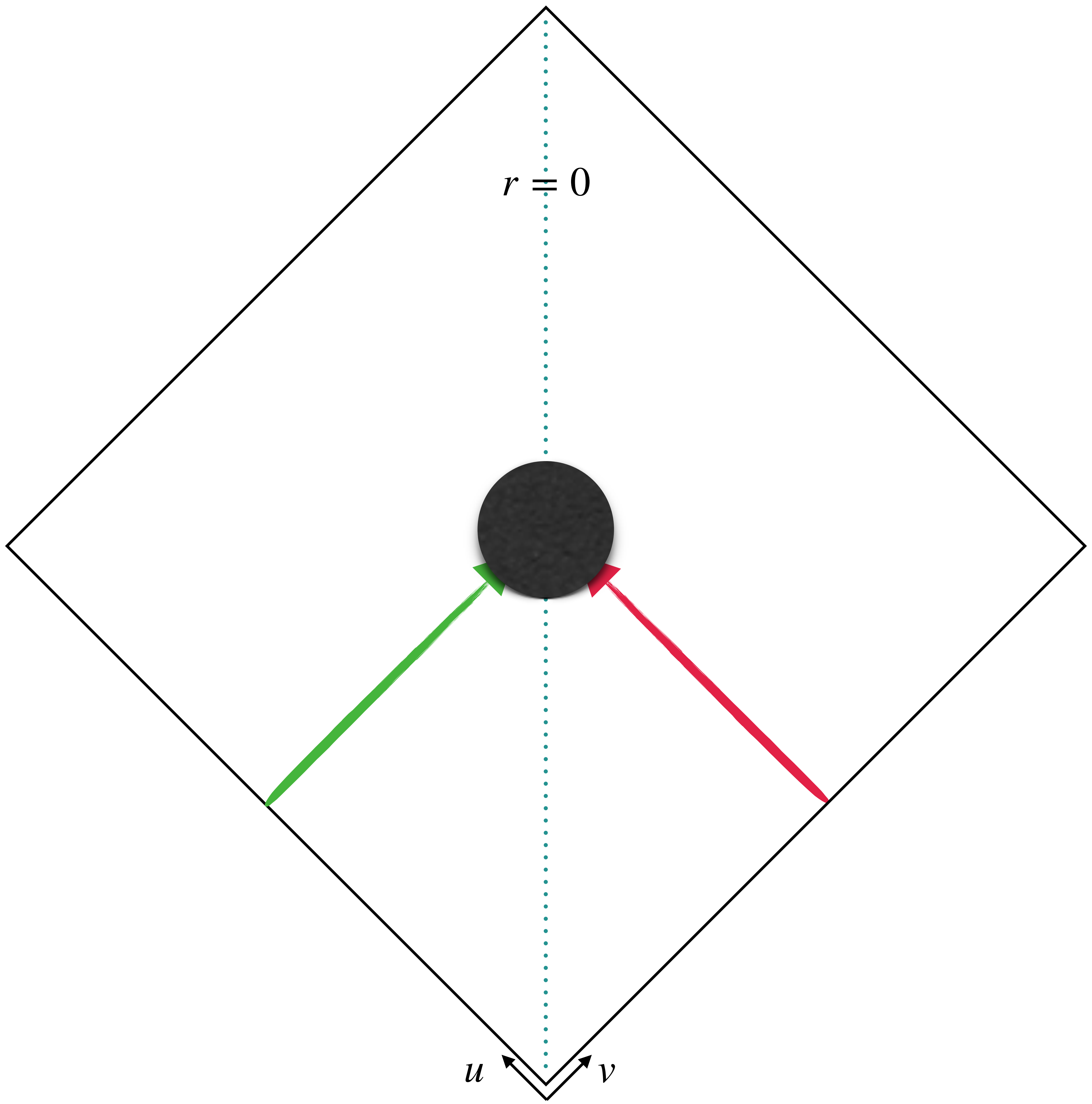
21st May, 2021 — GGI Workshop on Gravitational scattering, inspiral, and radiation

Based on

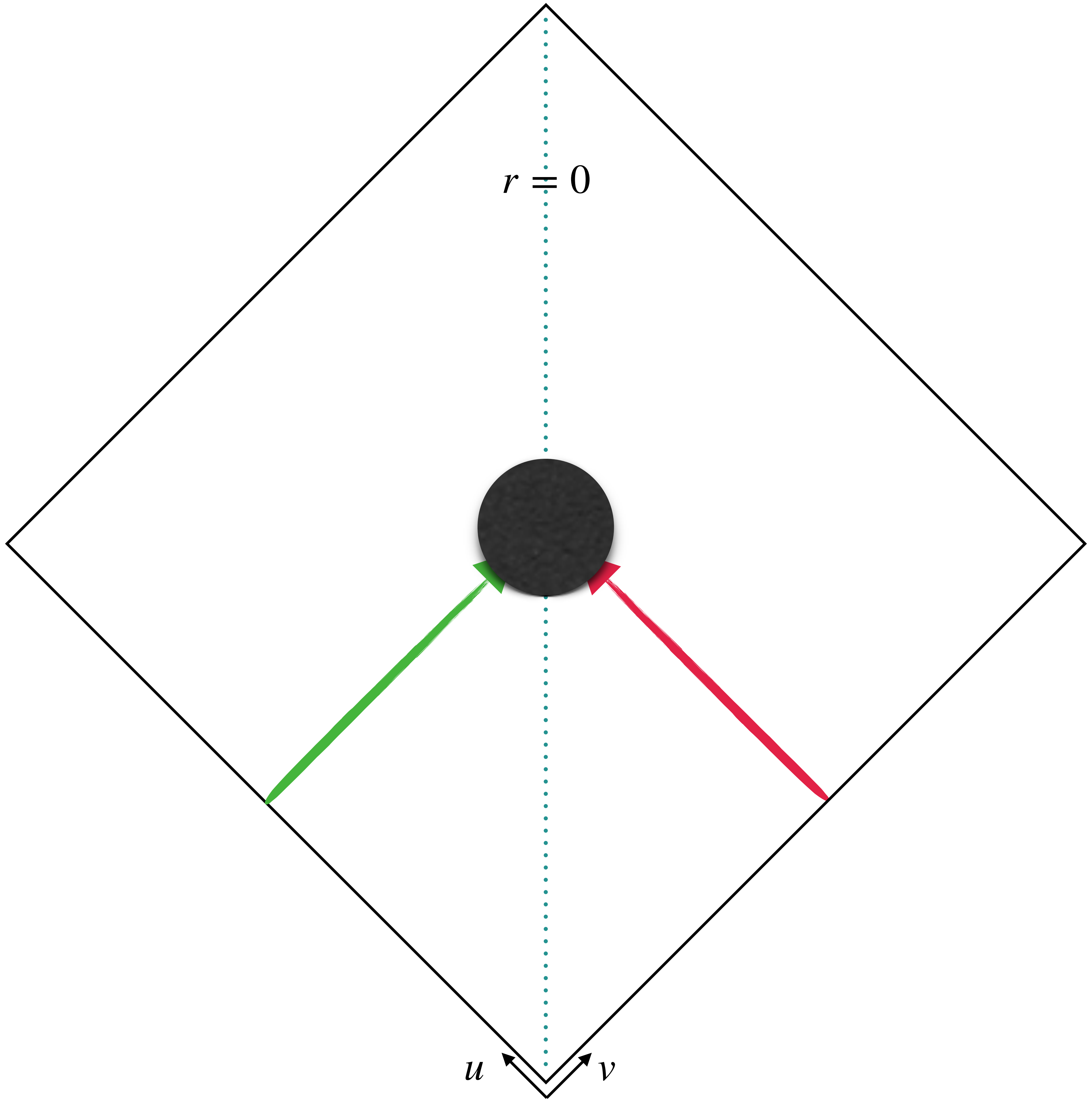
- **2012.02355** (with Nico Groenenboom)
- **2012.02357** (with Nico Groenenboom and Gerard 't Hooft)
- Some work in progress...
- See also 1607.07885 , 2004.09523 and 2012.09834 (with Panos Betzios and Olga Papadoulaki)

Free field theory — Information Loss





- 'Small enough' impact parameters ~ large black hole formation
- Cannot ignore gravitational interactions; what are the set of all out-states?
- Is there a regime of phase space where amplitudes are calculable?



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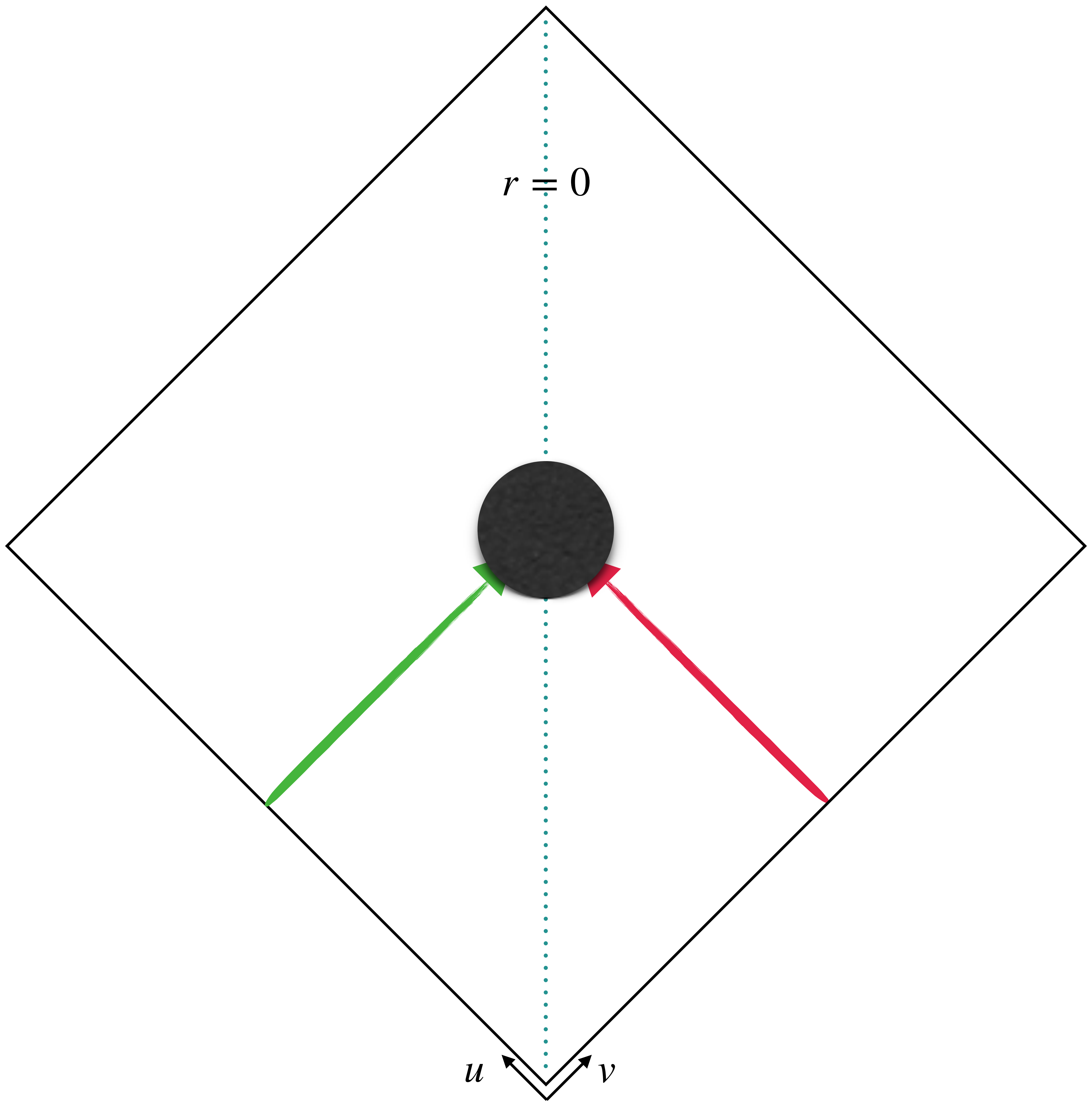
Why would such a regime exist?

- Indications that information problem can be resolved in effective field theory
- Gravity is special: quantum effects arise from large momentum transfer (like QFT) and from emergent scales (Schwarzschild radius): 'classicalisation'



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In this talk:

- Define this regime in effective field theory
- Calculate some amplitudes, promise more
- Ask several questions: What does it say about the information paradox? Post-Newtonian $\longrightarrow$ Post BH? Scrambling time and Page time? IR divergences? Antipodal identification, BMS?

Plan for the talk

- Large impact parameters and the flat space eikonal phase of QG
- Smaller impact parameters and the ***black hole eikonal*** phase of QG
- Comments: 2—N, IR divergences, Antipodal identification, BMS, on-shell vs off-shell bh eikonal,

  PM $\longrightarrow$ PBH, etc.

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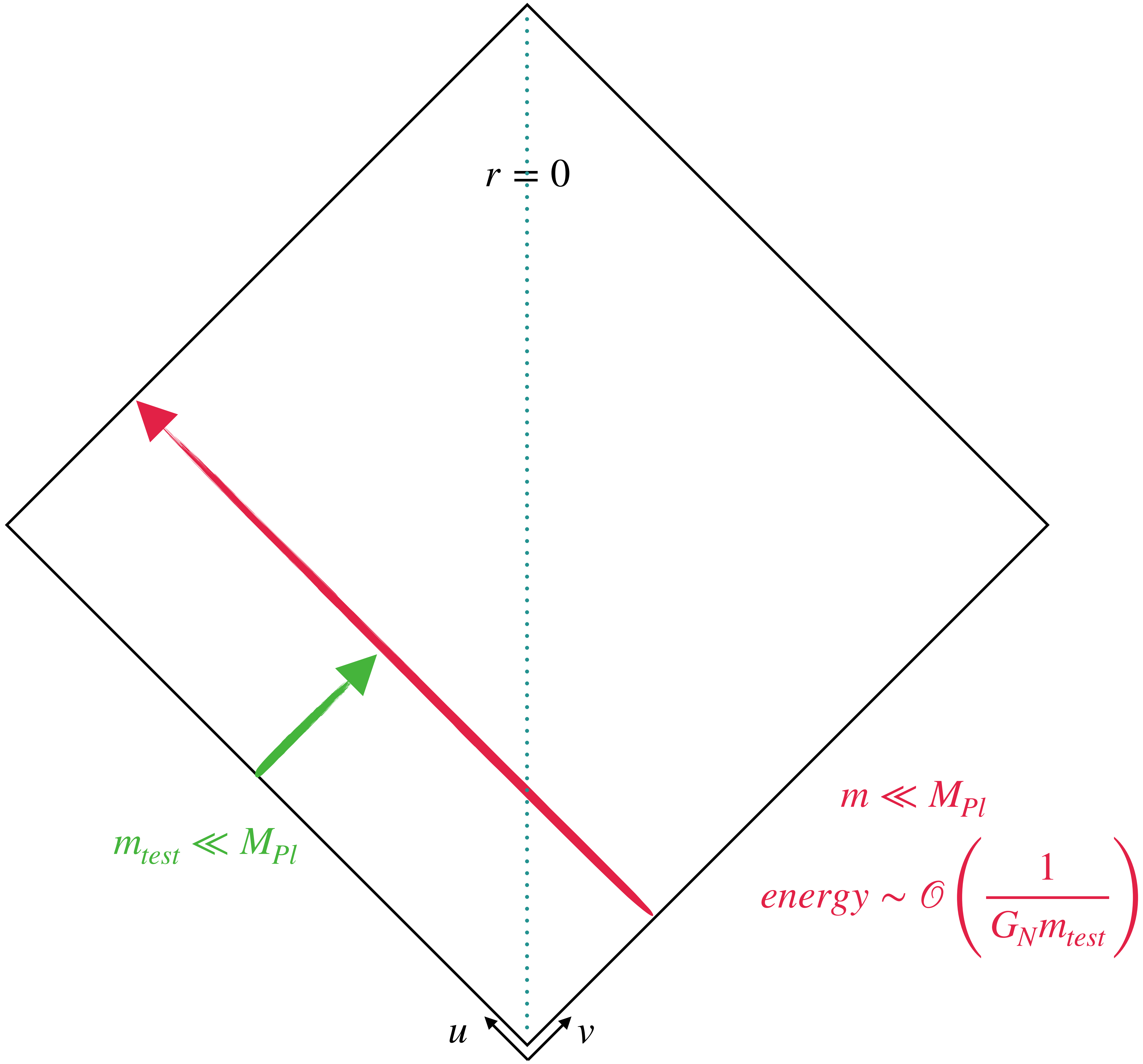

(t, r, Ω)

$(t - r, t + r, \Omega)$

(u, v, Ω)

$R_{-}^{1,3}$ *for* $v < v_0$

$R_{+}^{1,3}$ *for* $v > v_0$

- Large impact parameter: $b \gg G_N \sqrt{s} \gg L_{Pl}$
- Centre of mass energy ~ can be very high (trans-Planckian)!
- Cannot ignore gravitational backreaction

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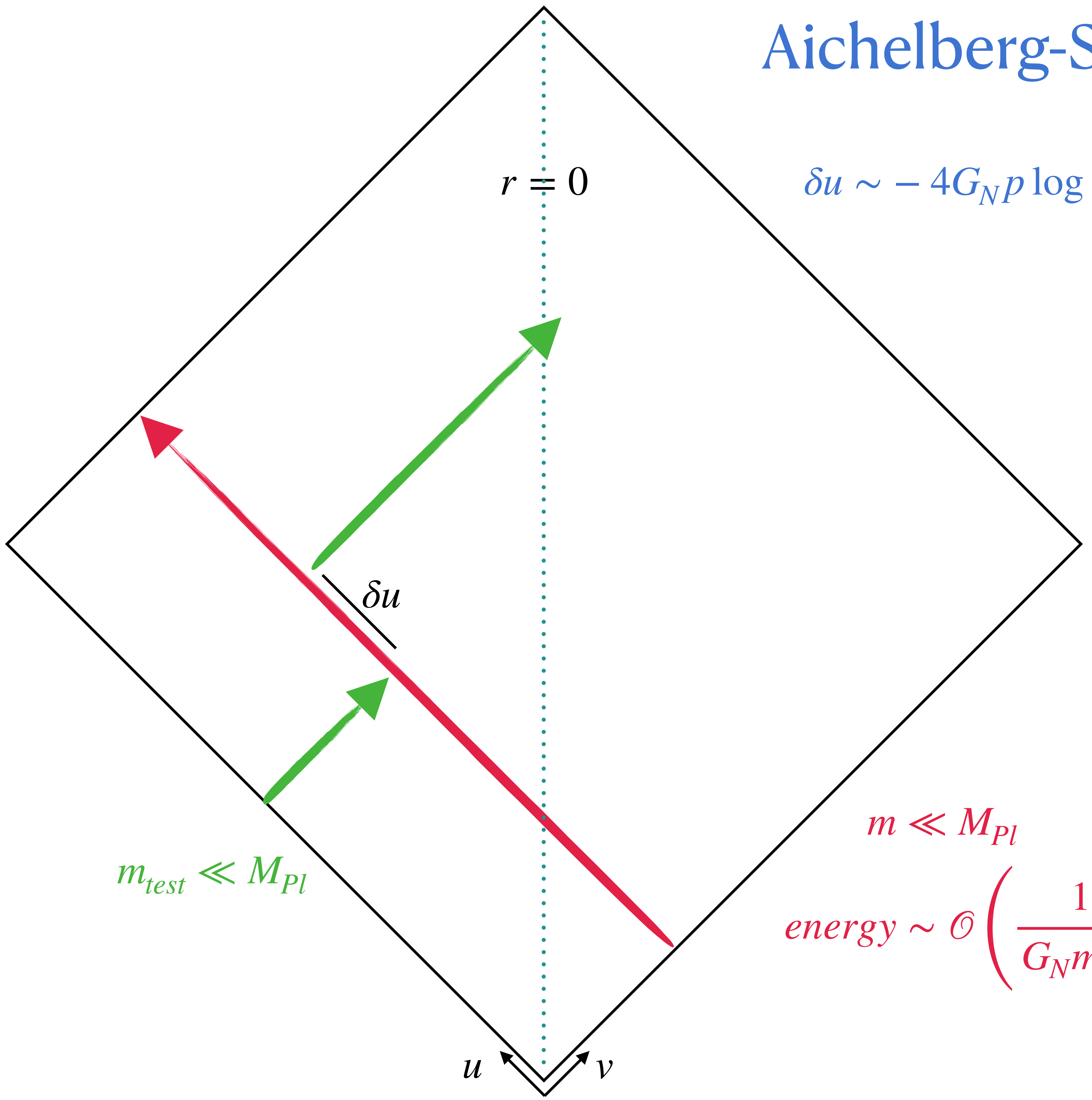
Aichelberg-Sexl Shockwave

$\delta u \sim -4 G_N p \log\left(\Omega - \Omega_{test}\right)$



$m \ll M_{Pl}$

$energy \sim \mathcal{O}\left(\frac{1}{G_N m_{test}}\right)$

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[Connor 1969]
[Penrose 1971]
[Aichelburg - Sexl 1971]
[Dray - 't Hooft 1985]
['t Hooft 1987]
[Damour 2016]
[ACV 1987—1993]...

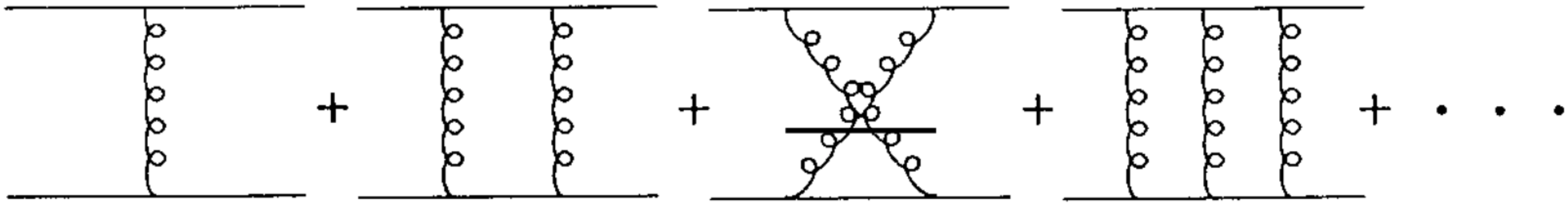
The flat space eikonal regime

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Generated by $\sim \; h^{\mu\nu} T_{\mu\nu}$

Focus on $s \gg t$ and diagrams leading in $\sqrt{s} = E \gg M_{Pl}$

$s = -(p_1 + p_2)^2$ and $t = -(p_1 - p_3)^2$



[De Witt 1967]

[Levy - Sucher 1971]

[Amati - Ciafaloni - Veneziano 1987 — 1993]

[Diebel - Schuecker 1991]

[Kabat - Ortiz 1992]

[Several speakers in this workshop, etc.]

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$$i\mathcal{M} = \frac{2\pi s}{\mu^2} \frac{\Gamma\left(1 - iG_N s\right)}{\Gamma\left(iG_N s\right)} \left(\frac{4\mu^2}{-t}\right)^{1-iG_N s}$$

Determined by

$\chi \approx G_N s \log\left(\mu b\right)$

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μ: graviton mass — IR regulator

Where are the black holes?

[Amati, Ciafaloni, Veneziano, Lipatov, Colferai, Dvali, Verlinde^2, …]

- When impact parameters reach $b \sim G_N\sqrt{s}$, the eikonal amplitudes diverge. This suggests an intermediate state that is produced.
- Gravity is special: quantum effects arise not only due to momentum transfer, but also due to emergent scales (Schwarzschild radius).

Stringy corrections…

[Amati, Ciafaloni, Veneziano, Lipatov, Colferai, Dvali, Verlinde^2, …]

- $R_S \sim G_N\sqrt{s} = G_N E \gg \ell_P$
- $\sqrt{\alpha'} \gg \ell_P \quad \text{when} \quad g_s \ll 1$

- We have three scales: $b, R_S, \sqrt{\alpha'}$

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- $b \gg R_S$ & $b \gg \sqrt{\alpha'}$:
  Flat space eikonal; corrected by R_S/b & $\sqrt{\alpha'}/b$

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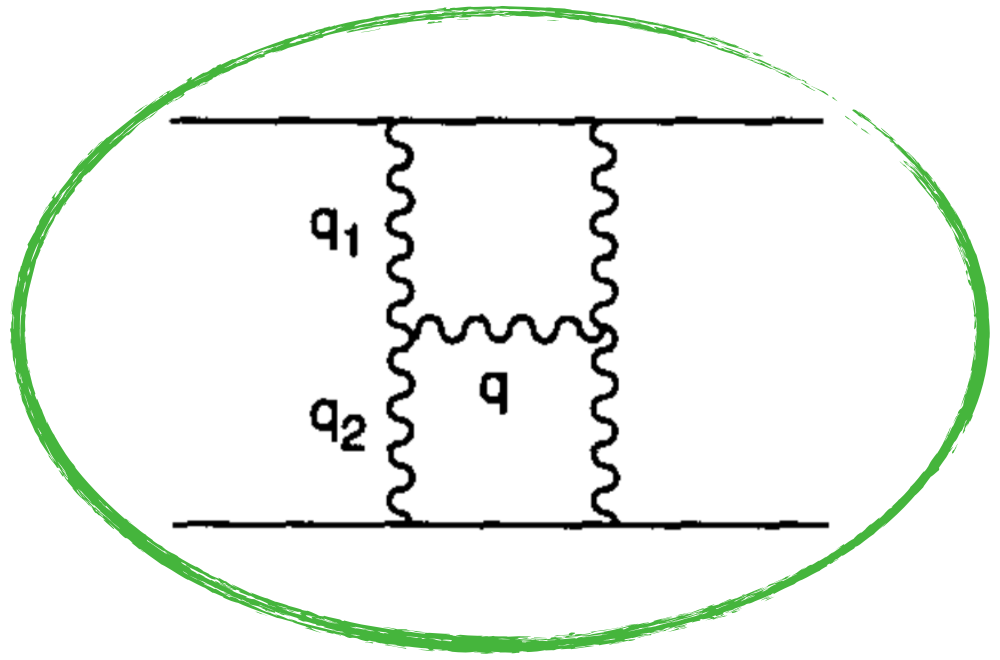

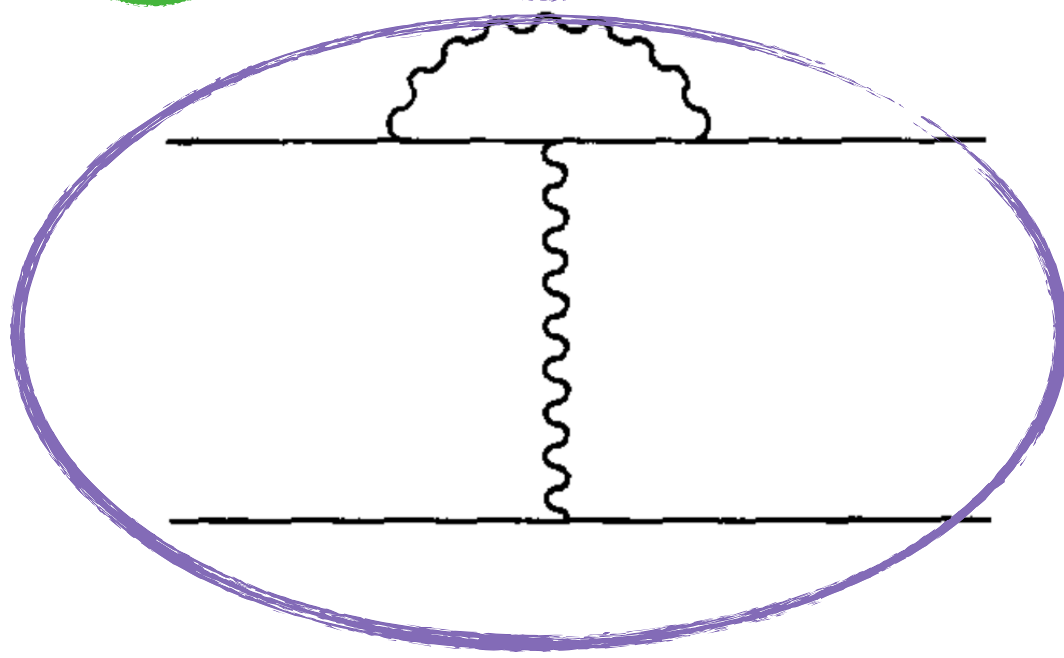

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  $\sqrt{\alpha'} \to R_S > b$ gives BH-like behaviour!
  Final momenta $\sim 1/R_S$

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BH/String transition

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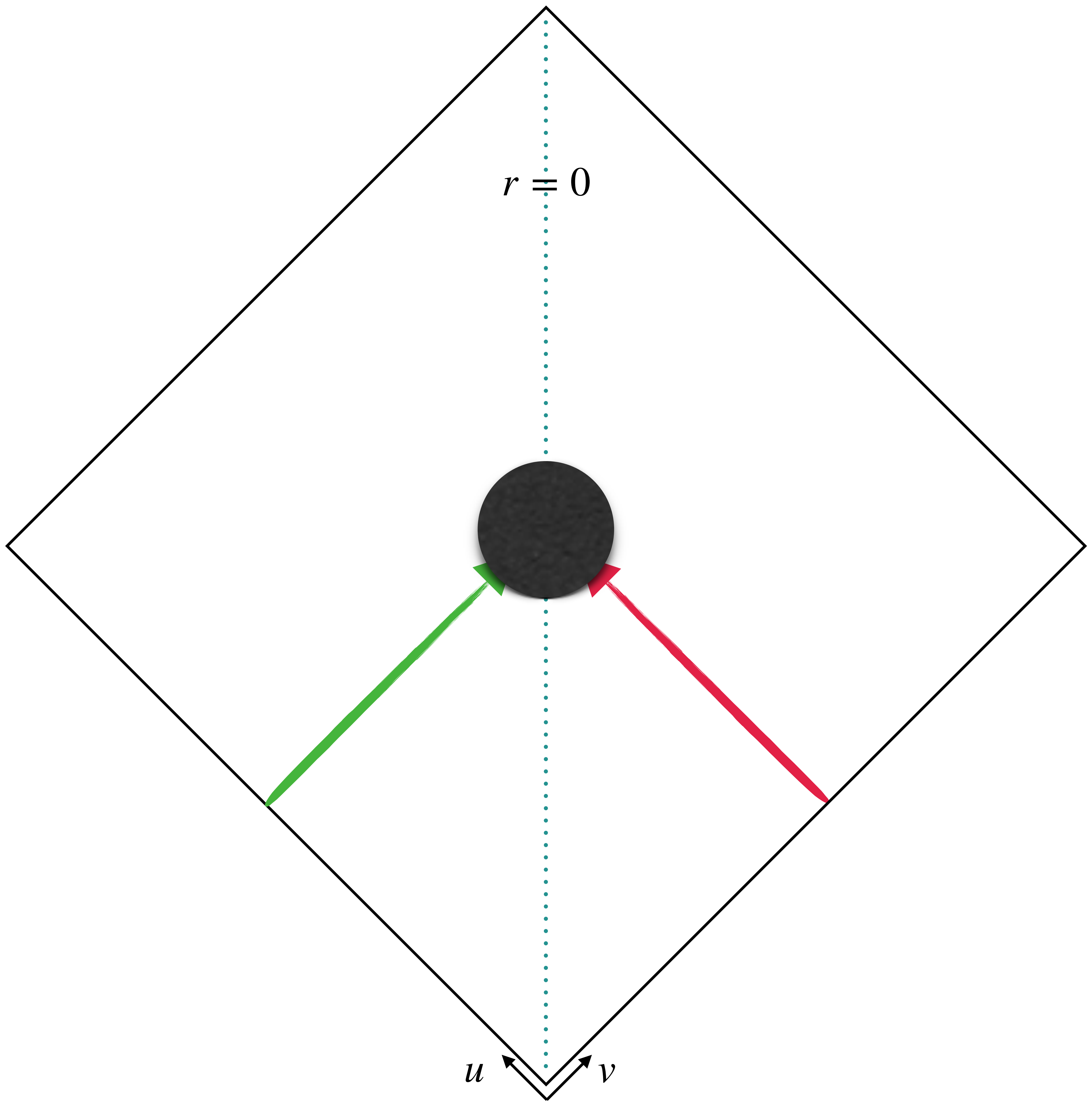
- Focus for today
- Can capture the BH/String transition by tuning g_s



$$b \gg L_{Pl}$$
$$b \lesssim G_N \sqrt{s}$$
$$s \sim M_{BH}^2$$

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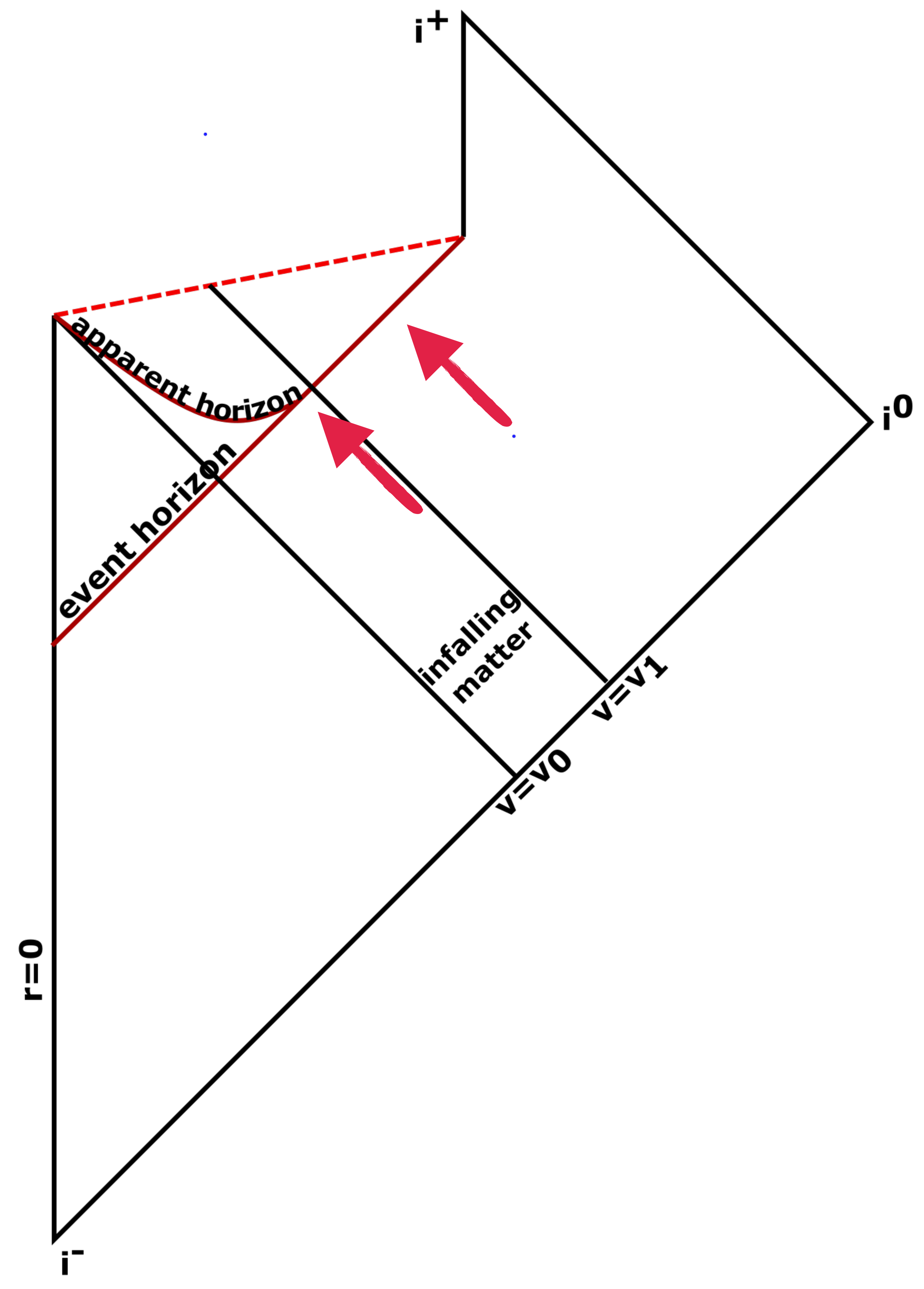


- Apparent horizon forms before the entire collapse.
- Collapse time << BH lifetime. For most of the BH lifetime, apparent horizon ~ event horizon
- Save for the initial part of the (classical) collapse, particles are essentially propagating on an existing black hole background, near the horizon.

The black hole eikonal phase of QG

$$g_{\mu\nu} = g_{\mu\nu}^{Schwarzschild} + h_{\mu\nu}$$

Generated by $\sim \; h^{\mu\nu} T_{\mu\nu}$

[Regge - Wheeler 1957]

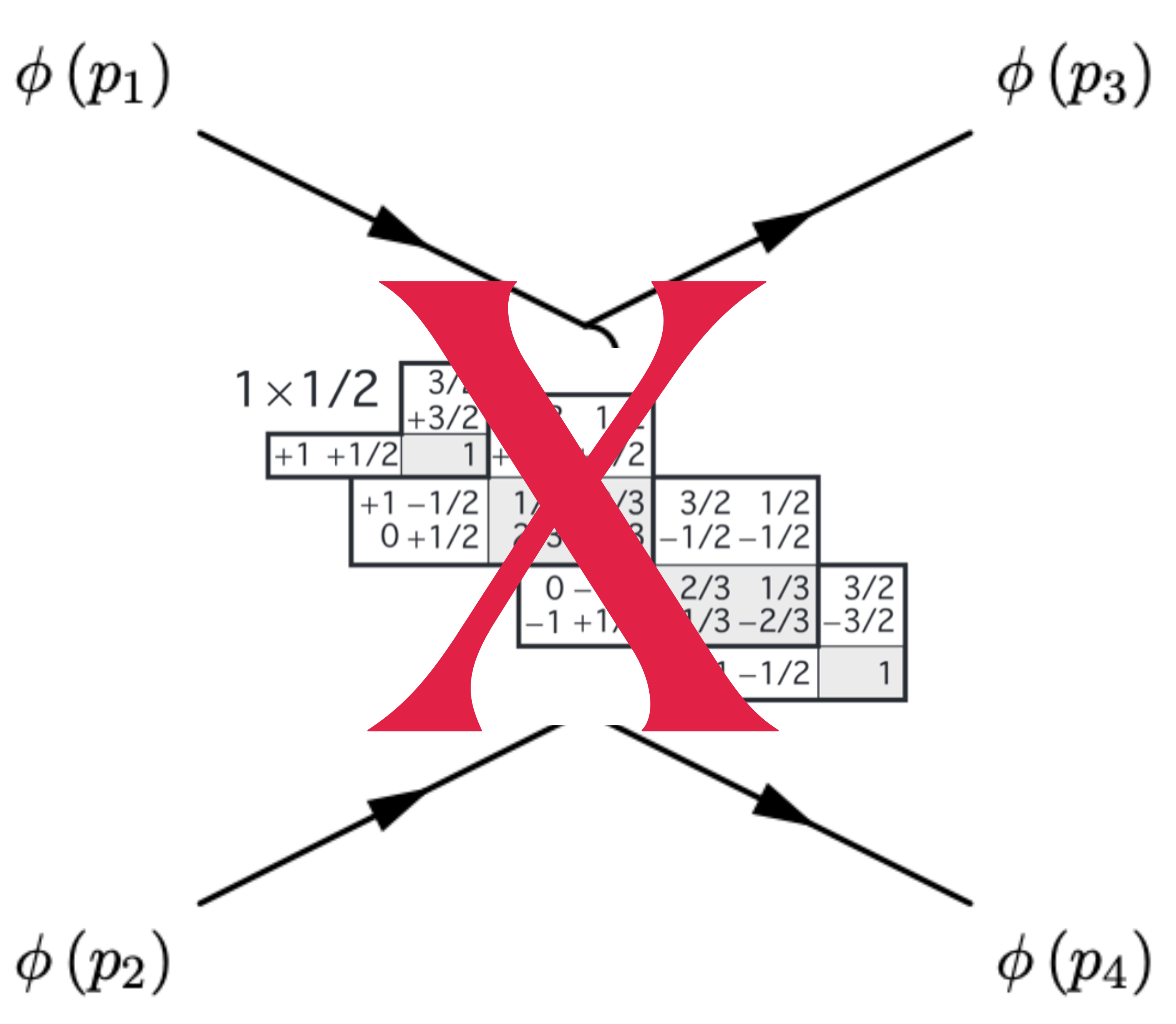
- De-Donder gauge couples different harmonics
- But, Regge-Wheeler gauge fits the bill

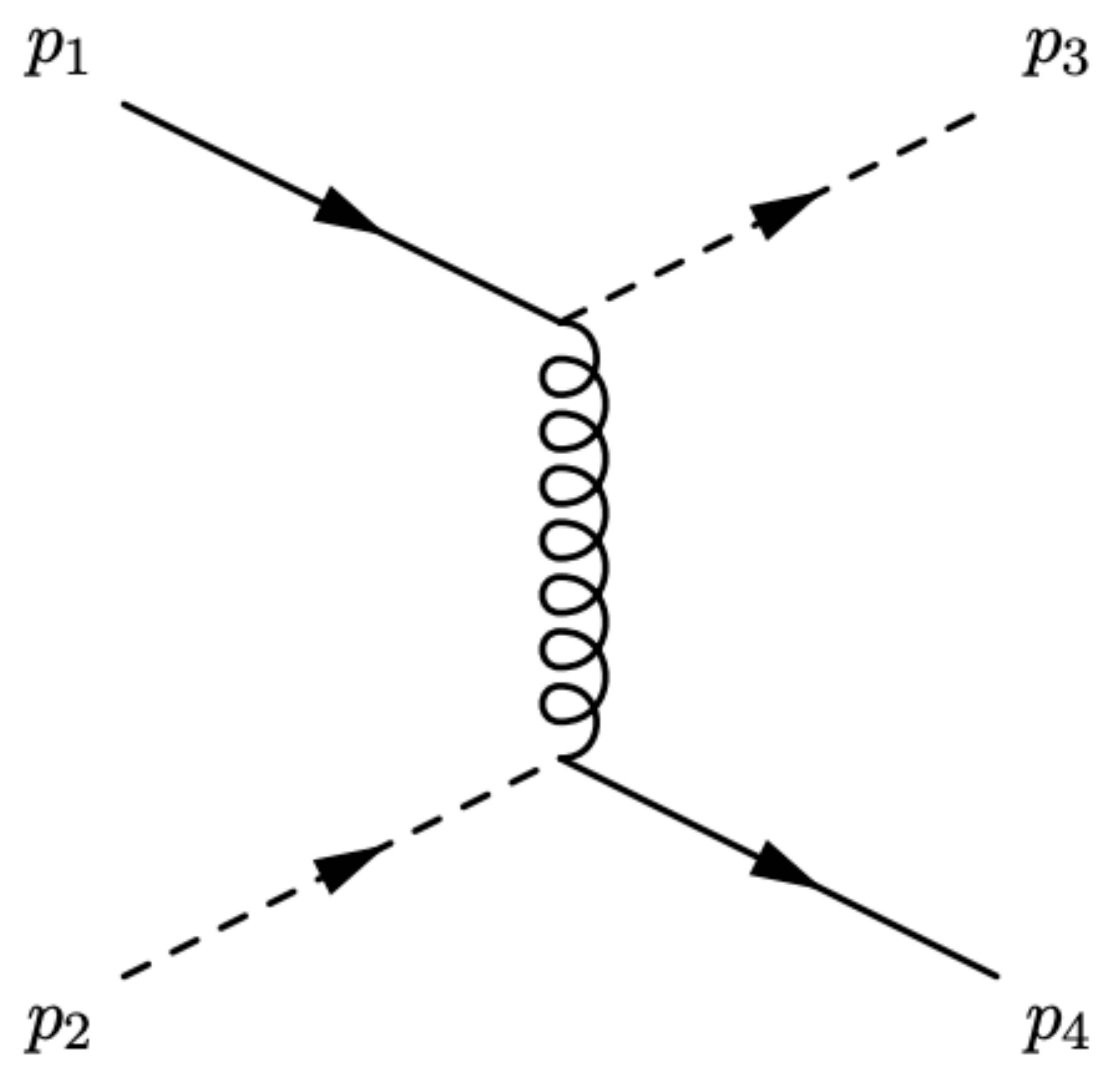
$$h_{\mu\nu} = \begin{pmatrix} h_{ab} & 0 \\ 0 & K \end{pmatrix} + \begin{pmatrix} 0 & X \\ X^T & 0 \end{pmatrix}$$

- Tensorial harmonics reduce to regular spherical harmonics. Also decouples odd and even parity gravitons Tedious, but can be done ...

- Low transverse momenta imply odd-parity graviton X does not contribute! [NG - Groenenboom 2020]

Approximate spherical symmetry



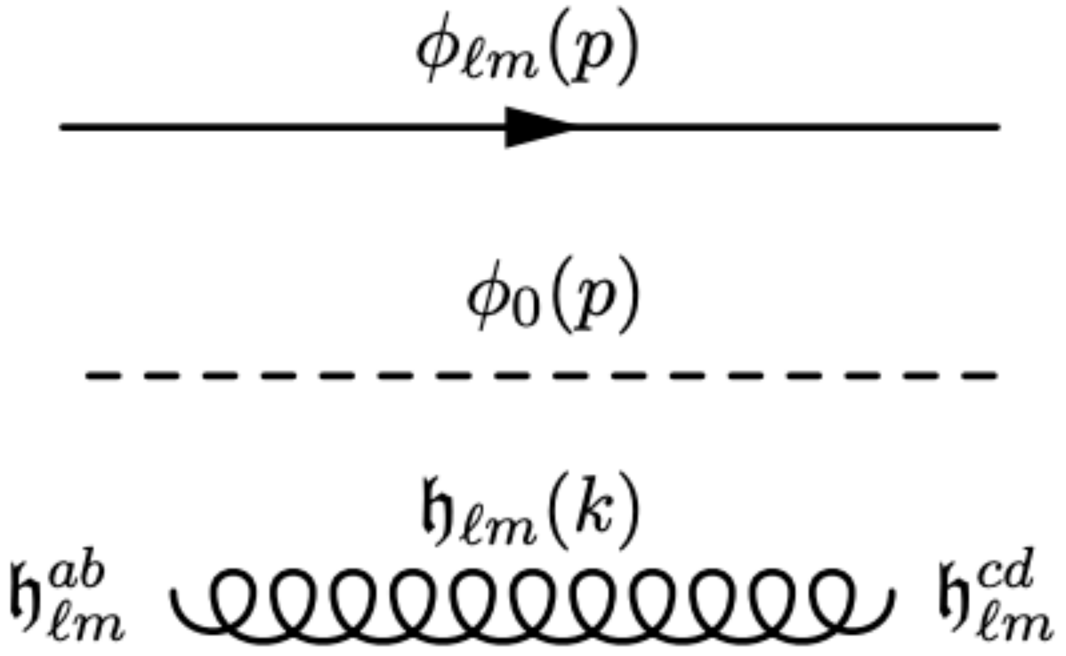


Defining the black hole eikonal phase

[NG - Groenenboom 2020]
[NG - Groenenboom - 't Hooft 2020]

- Separation of longitudinal and transverse effects: partial waves do not mix, transverse momenta can be ignored [Verlinde - Verlinde 1991]
- Impact parameter: $R_S \gtrsim b \gg \ell_{Pl}$. Scatter near the horizon...
- Emergent scale allows for very low collision energies (not necessarily trans-Planckian)
  $E = \sqrt{s} \gg \gamma M_{Pl}$ with $\gamma \sim M_{Pl}/M_{BH}$

Feynman rules on the horizon



$$= \frac{-i}{p^2 + \frac{\lambda}{R_S^2} - i\epsilon}$$

$$= \frac{-i}{p^2 + \frac{1}{R_S^2} - i\epsilon}$$

$$= 2i\mathscr{P}^{abcd}(k) \ ,$$

$$\mathscr{P}^{abcd}(k) = \frac{1}{4} f_l \left(\eta^{ac}\eta^{bd} + \eta^{ad}\eta^{bc} - f_l \left(\frac{\lambda+1}{\lambda-3} \right) \frac{k^a k^b k^c k^d}{4\left(k^2 + \lambda R_S^{-2} - i\epsilon\right)} \right)$$

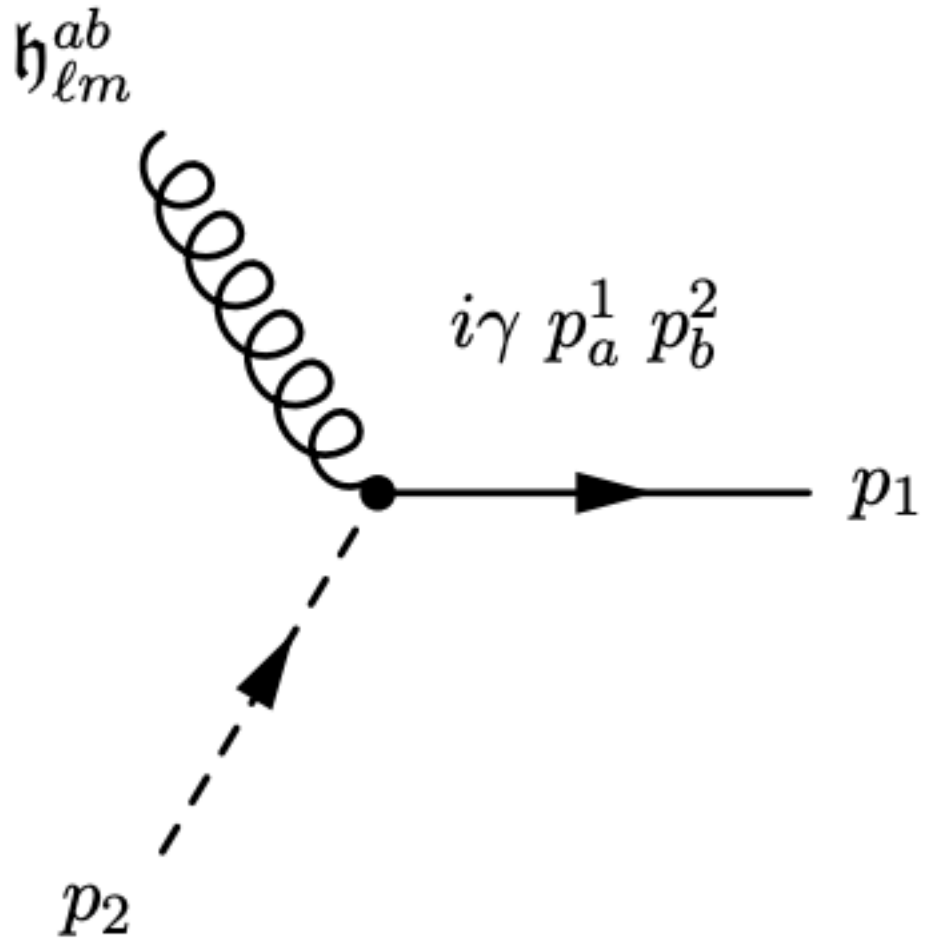
$$f_l = -\frac{4R_S^2}{\lambda+1}$$

$$\lambda = l^2 + l + 1$$

Expected UV divergences: $\mathscr{P}^{abcd}(\infty)$

BUT IR regulated! Graviton gets an effective 2d mass near the horizon: $\mathscr{P}^{abcd}(0)$

Vertex specified by a coupling constant: $\gamma \sim \dfrac{M_{Pl}}{M_{BH}}$



2—2 amplitudes in the black hole eikonal

[NG - Groenenboom 2020]
[NG - Groenenboom - 't Hooft 2020]

Three scales in the problem: M_{Pl}, M_{BH}, $\sqrt{s} = E$ (centre of mass energy of collision)

Perturbative in:

$$E \gg \gamma M_{Pl}$$

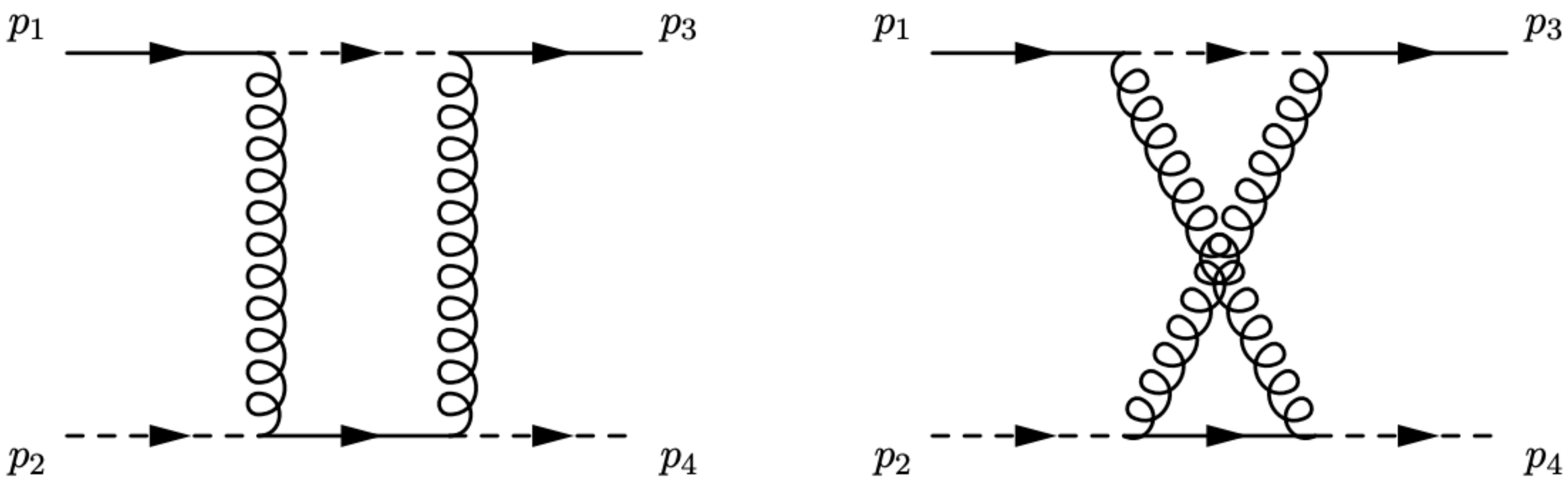
Only soft diagrams contribute
No momentum exchange

Non-perturbative in:

$$\gamma \sim \frac{M_{Pl}}{M_{BH}}$$

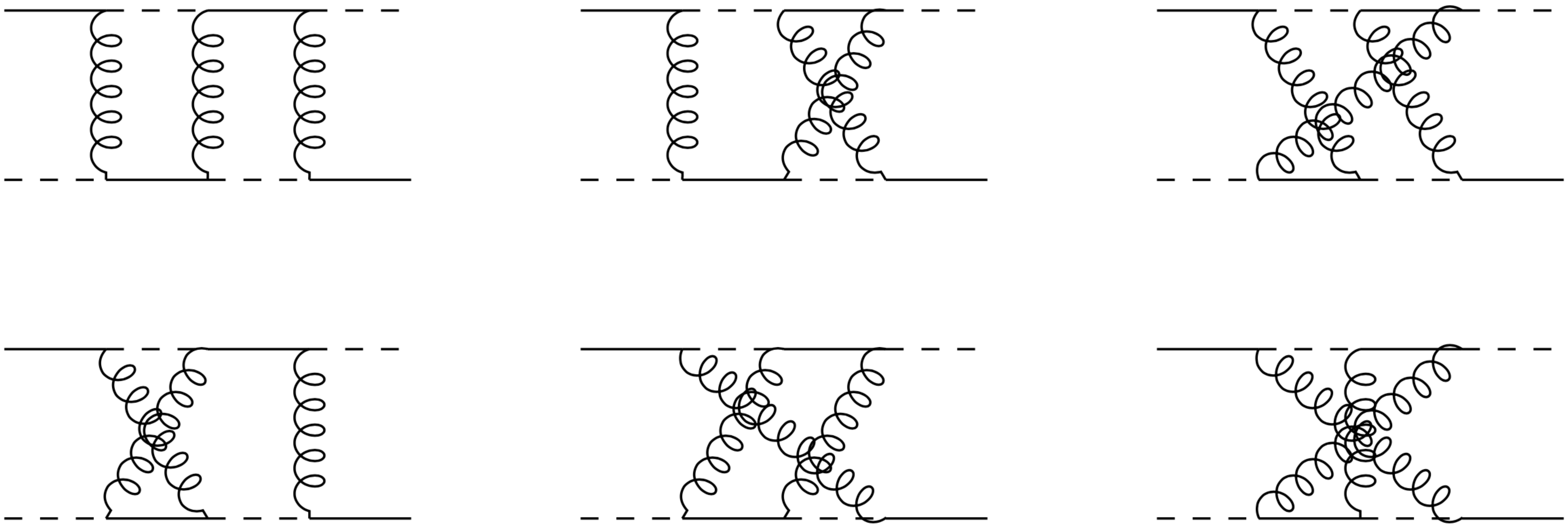
Sum over infinitely many Feynman diagrams

2—2 diagrams: one—loop



All one-loop diagrams: only arise from the conserved channel

2—2 diagrams: two—loop



All two-loop diagrams: only arise from the transfer channel

2—2 ladder diagrams

General $(n-1)$-loop amplitude can be computed:

$$i\mathcal{M}_n = -\frac{\gamma^2 s^2}{n!}\int \frac{\mathrm{d}^2 k}{\left(2\pi^2\right)} 2\, i\, \mathcal{P}^{xxyy}\left(k\right)\int \mathrm{d}^2 x\; e^{-ik\cdot x}(i\chi)^{n-1}$$

$$\chi = -\gamma^2 s^2 \int \mathrm{d}^2 k \left(2\mathcal{P}^{xxyy}\left(k\right)\right) e^{-ik\cdot x}\delta\left(2p_1\cdot k\right)\delta\left(2p_2\cdot k\right) = -\frac{1}{2}\gamma^2\, s\, \mathcal{P}^{xxyy}(0)$$

$$= \frac{R_S^2\,\gamma^2\, s}{\ell^2+\ell+2} = \frac{8\pi G_N\, s}{\ell^2+\ell+2}$$

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Soft limit is enforced upon us

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Flat space eikonal:

$$\chi \sim G_N\, s \log\left(\ell^2+\ell\right)$$

BH leading eikonal exponentiation

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Diagrams can all be neatly re-summed:

$$i\mathcal{M} = i\sum_n \mathcal{M}_n = 4s\left[\exp\left(i\frac{\gamma^2 R_S^2}{l^2+l+2}s\right)-1\right] = 4E^2\left[\exp\left(i\frac{8\pi G_N}{l^2+l+2}E^2\right)-1\right]$$

BH leading eikonal exponentiation

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Classical on-shell counterpart: Dray - 't Hooft shockwave

['t Hooft 2015, 2016, 2018]
[Shenker-Stanford 2014]
[Betzios - NG - Papadoulaki 2016, 2020]

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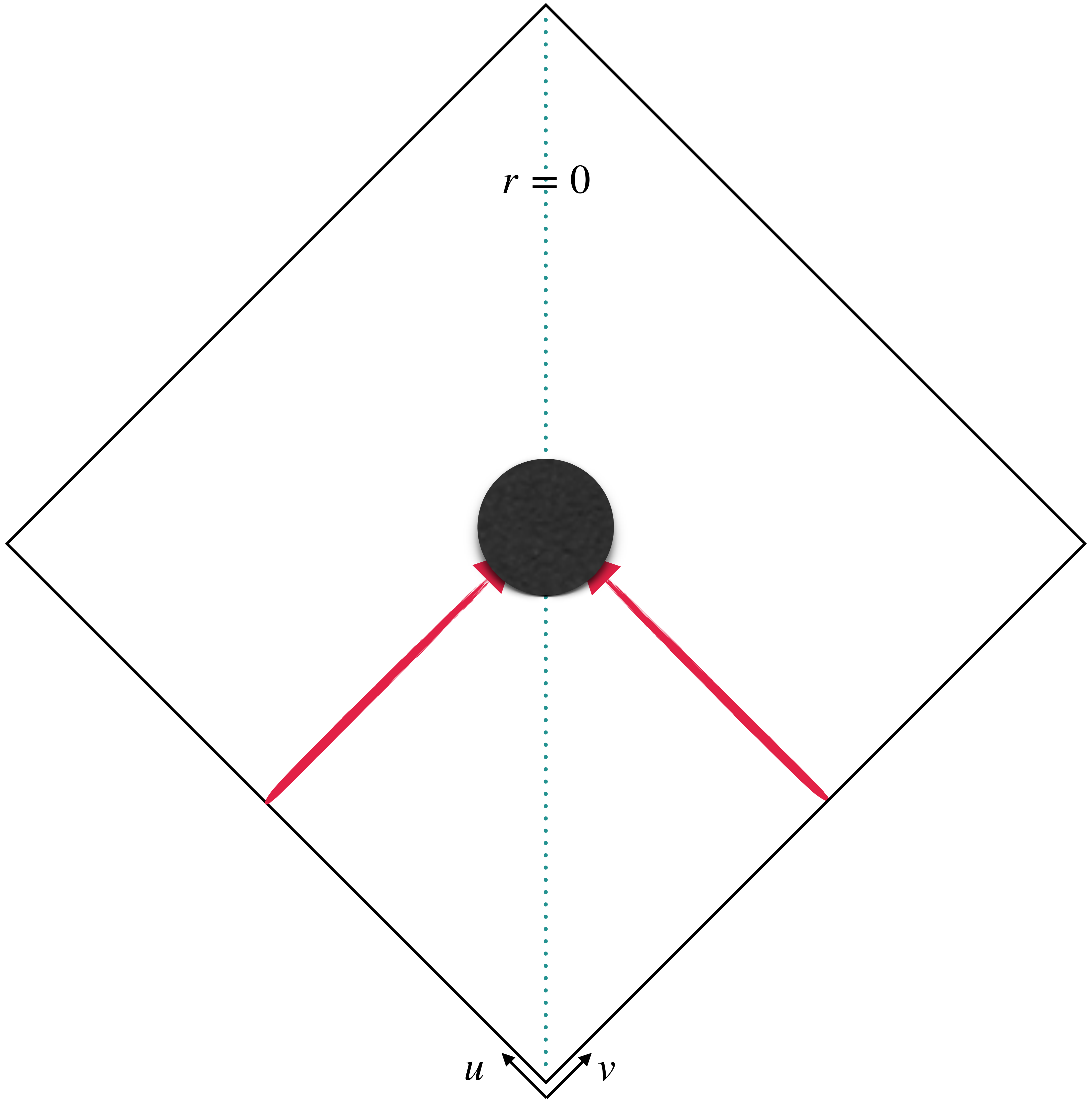
2—N amplitudes



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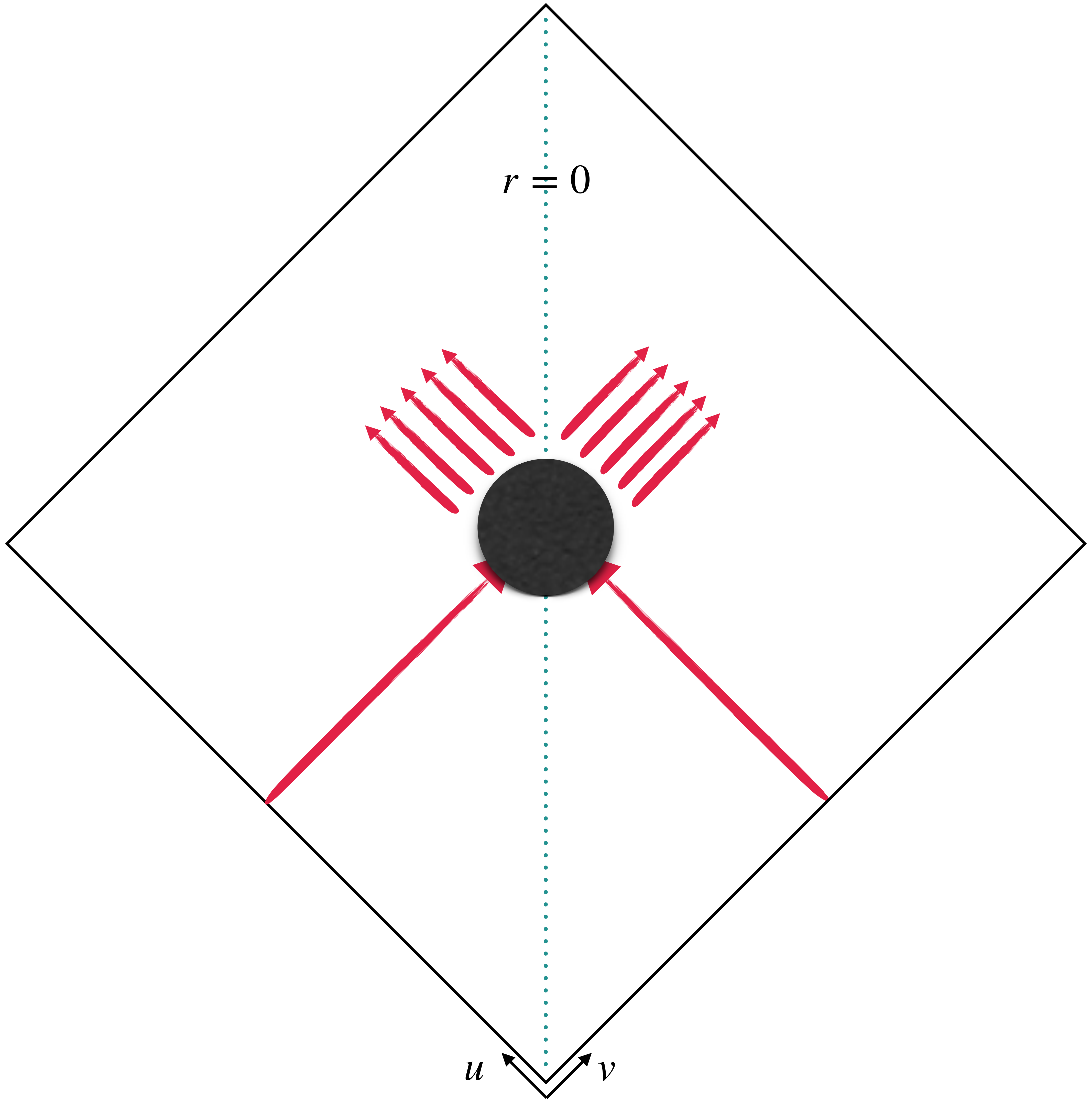
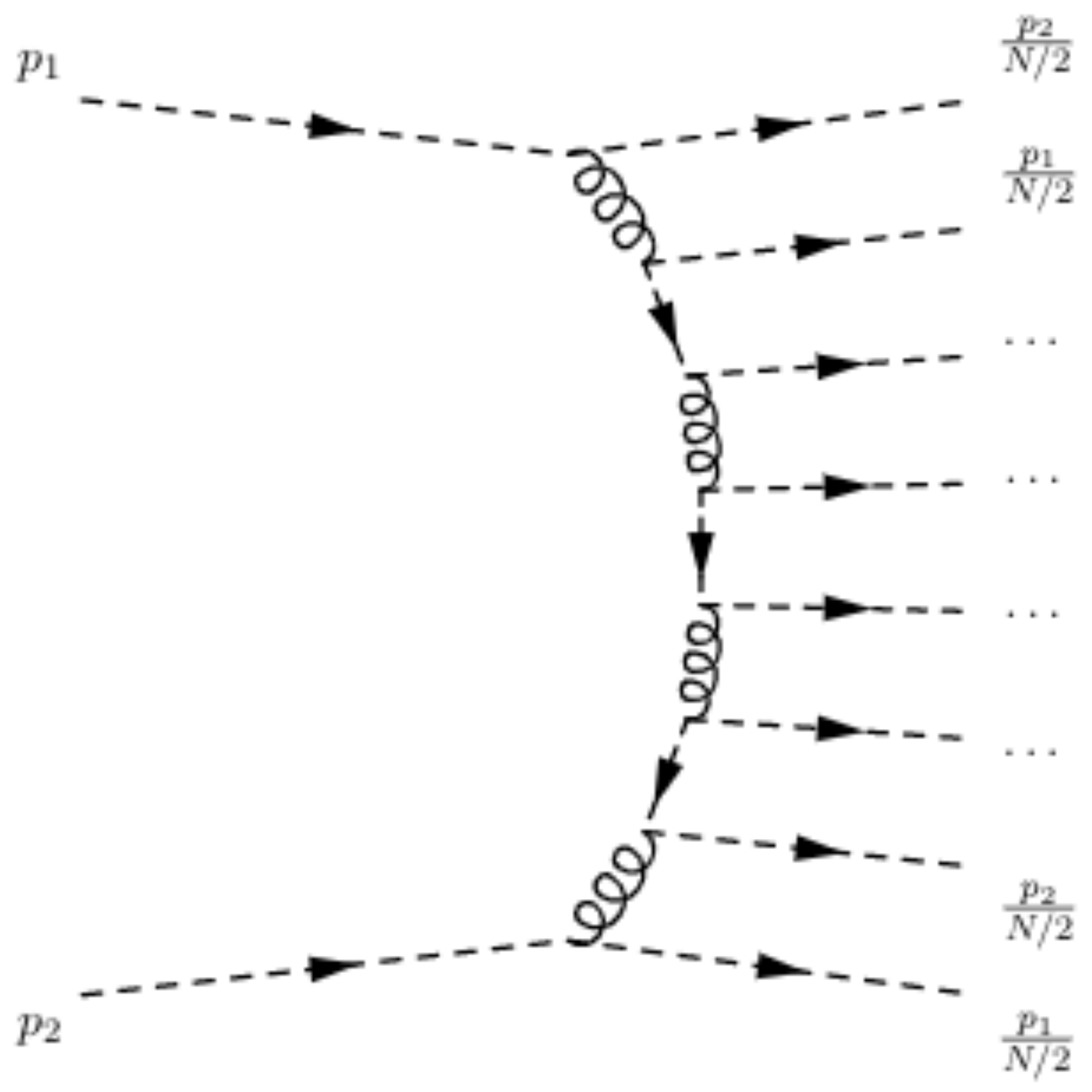
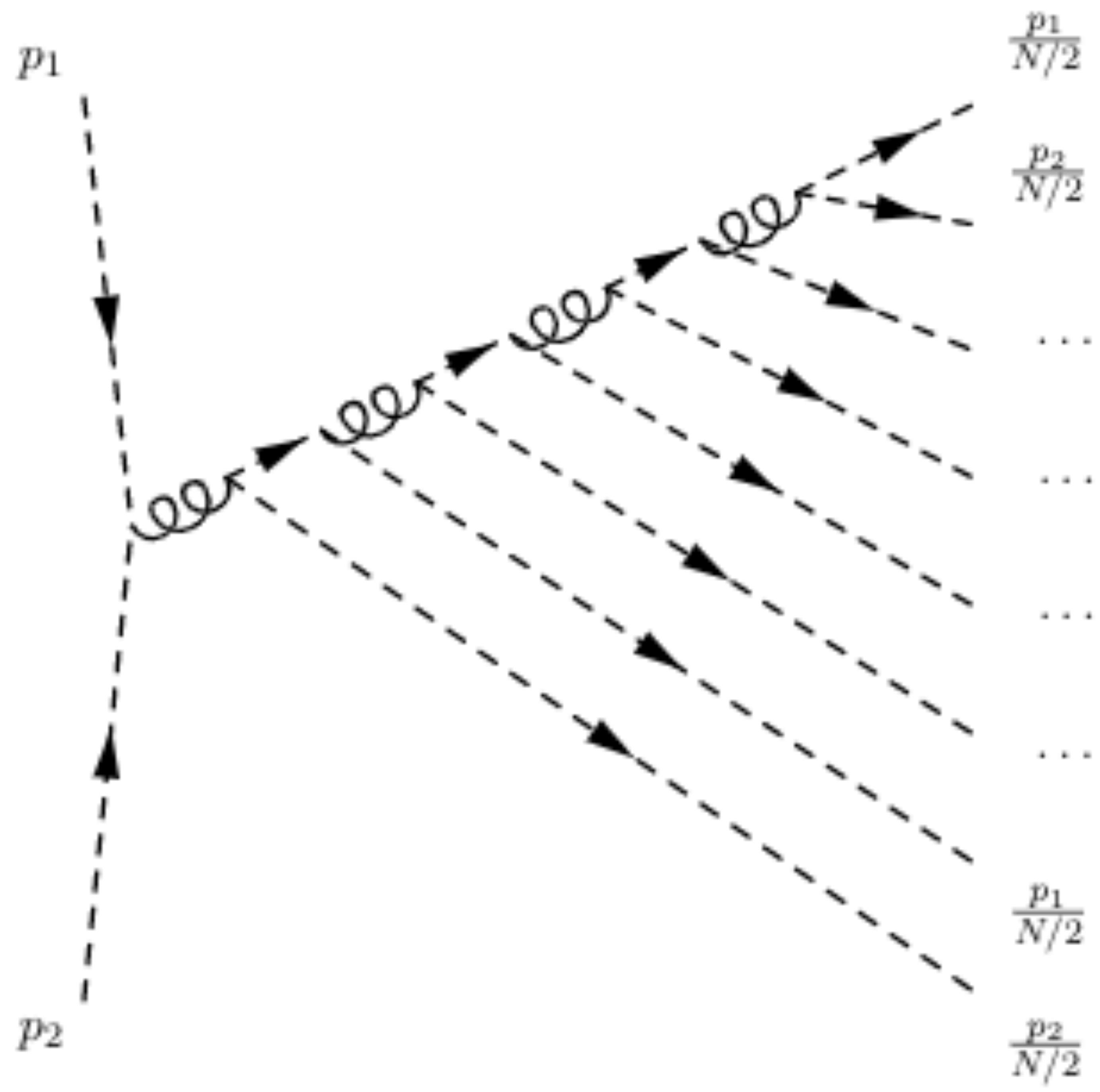
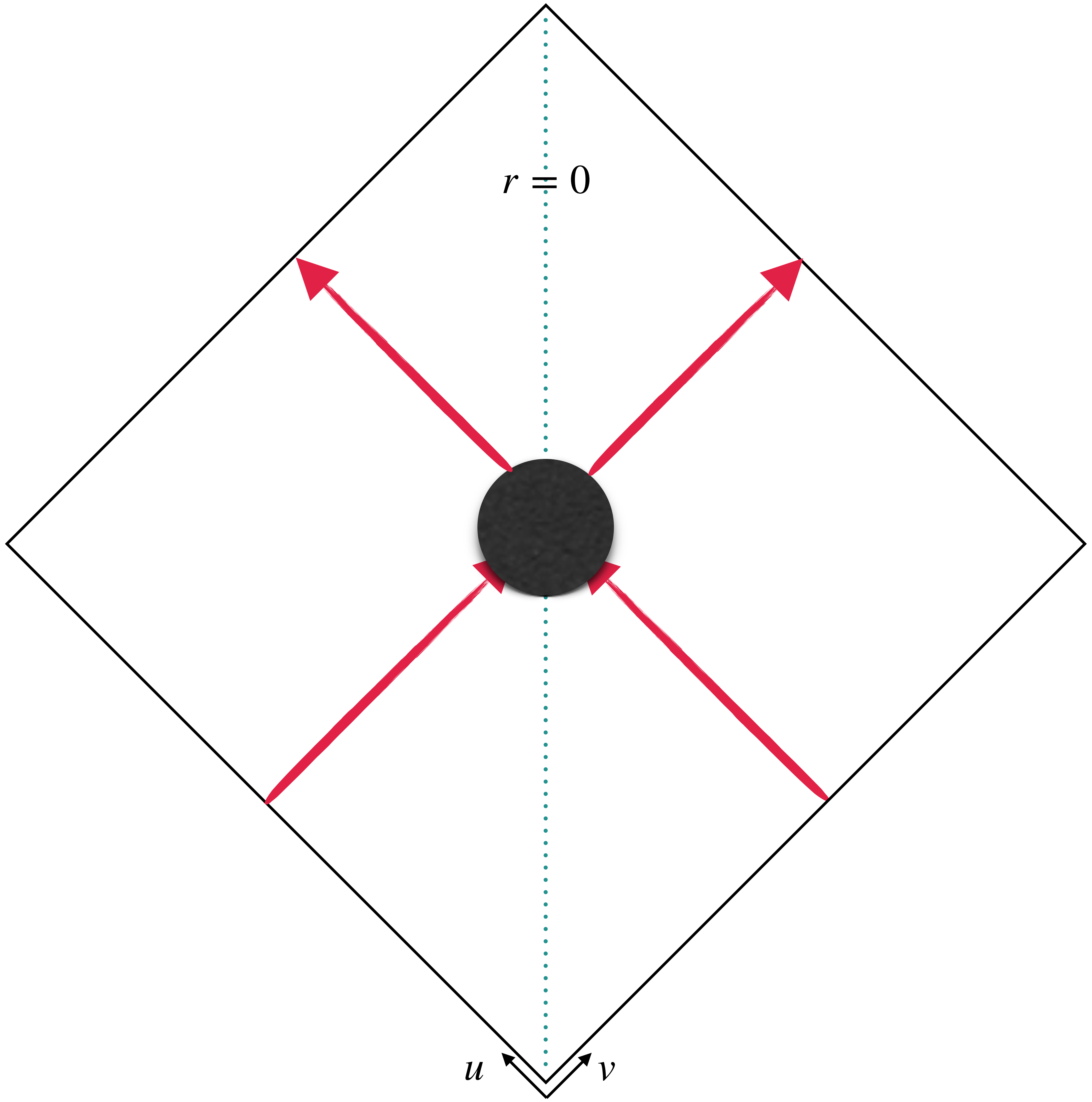


2—N amplitudes : work in progress





- Momentum transfer is important
- Many out-states allowed
- All diagrams calculable
- Can extend to loops
- Adding ladders/eikonalisation; N—point chaos bound!
- Wigner's time delay: Scrambling time & Page time



Antipodal identification

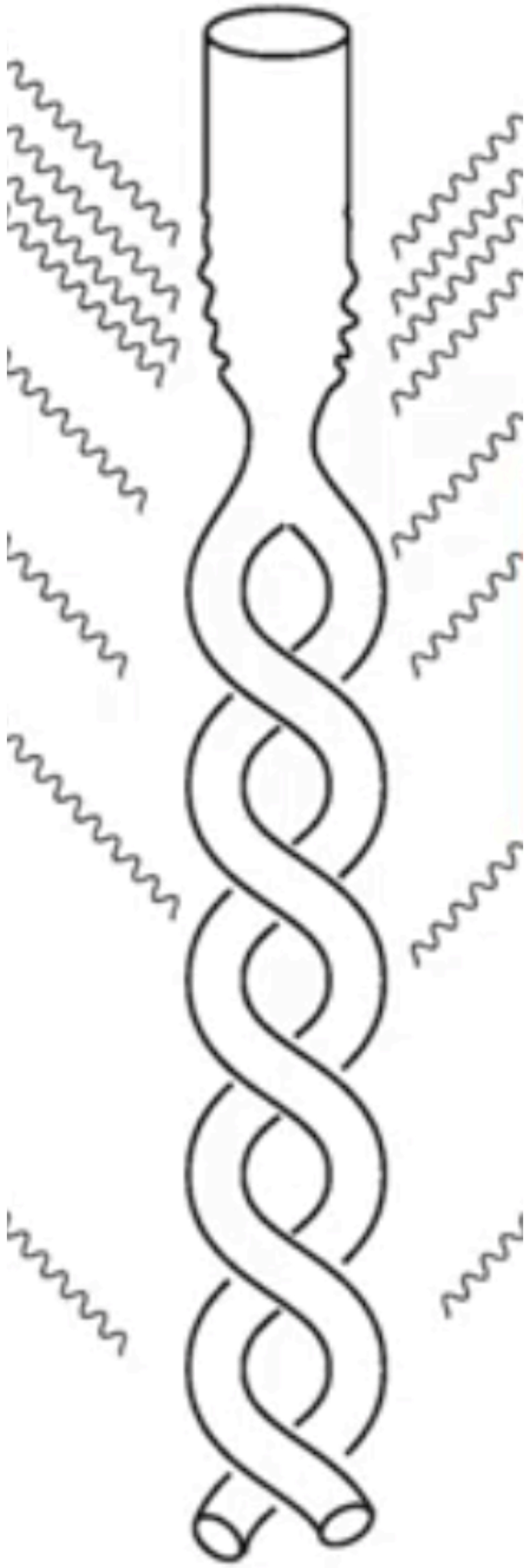
- Does not arise from a condition due to $\mathscr{I}^+_-$ and $\mathscr{I}^-_+$
- A condition on the horizon

BMS

- No IR divergences due to the emergent scale. Therefore, no violation of conservation laws.
- But what are the symmetries? Near-horizon analogs of BMS
- Shockwaves imprint supercharges on the horizon (some work in progress)

Post-Minkowskian ⟶ Post-BH

- Natural $\mathcal{O}\left(G_N^{\#}\right)$ expansion from the BH eikonal ...
- When impact parameters are smaller, use the BH graviton propagator. EOB formulation?
- Unlike in the flat space eikonal, the bh eikonal is dominated by low angular momentum modes!



What should you take away?

- Remarkable new phase of quantum gravity capturing black hole evolution
- Information paradox can be addressed in this phase; no UV completion needed, exponentially suppressed corrections natural.
- Can easily extend to include stringy corrections and treat BH/String transition
- Several aspects (analogous to the flat space eikonal) waiting to be discovered
- Post—Minkowskian $\longrightarrow$ Post—BH; exactly what physics is captured here?
- ~~Gravitational echoes [2012.09834 with P. Betzios and O. Papadoulaki]~~

Further ideas ...

- Hints for ultra-violet physics by perturbatively including corrections away from the black hole eikonal phase
- All extensions of the flat space eikonal regime can be repeated with these tools.
- Bekenstein-Hawking entropy — sum over all out-states and look for entropic scaling
- Include standard model fields. Higher order interactions
- Ideal for small black holes in AdS/CFT; find a CFT interpretation
- Cosmological eikonal
- Infalling observer? (in-in formalism)

Thank you