QUANTUM GENERALIZED HYDRODYNAMICS

PAOLA RUGGIERO

With: P. Calabrese, B. Doyon, J. Dubail

Based on: *Phys. Rev. Lett. 124, 140603 (2020)*

Cortona Young 2021

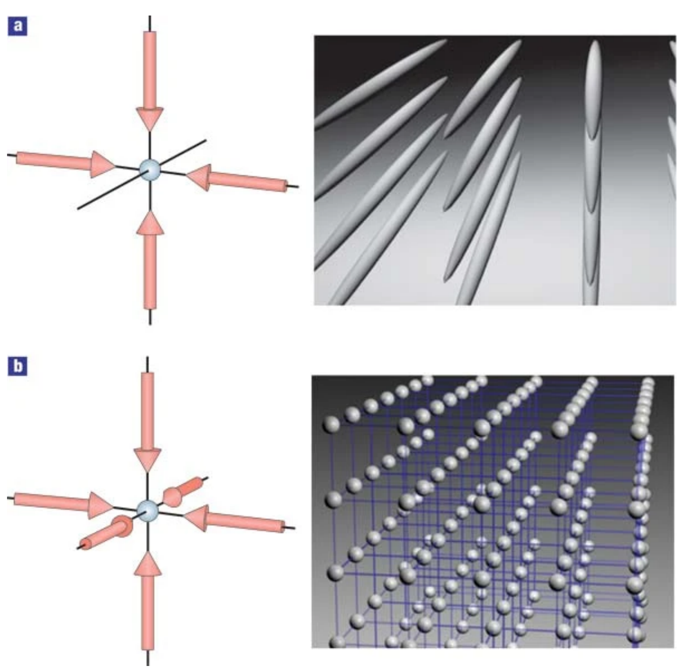
UNIVERSITÉ DE GENÈVE

SOME CONTEXT...

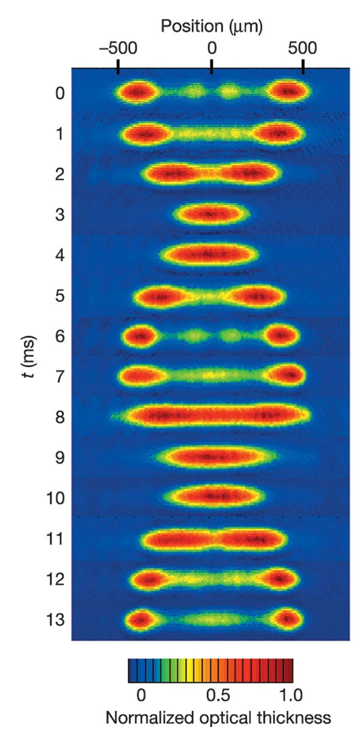
Interacting 1D Quantum Many-Body systems : WHY INTERESTING?

- Effects of correlations and interaction enhanced and lead to dramatic effects (ex: Fermi liquid replaced by Luttinger Liquid theory)
- Many available tools, both analytical (ex: CFT 1+1D, lattice/field theory integrability) and numerical (ex: tensor-networks based algorithms, as DMRG)

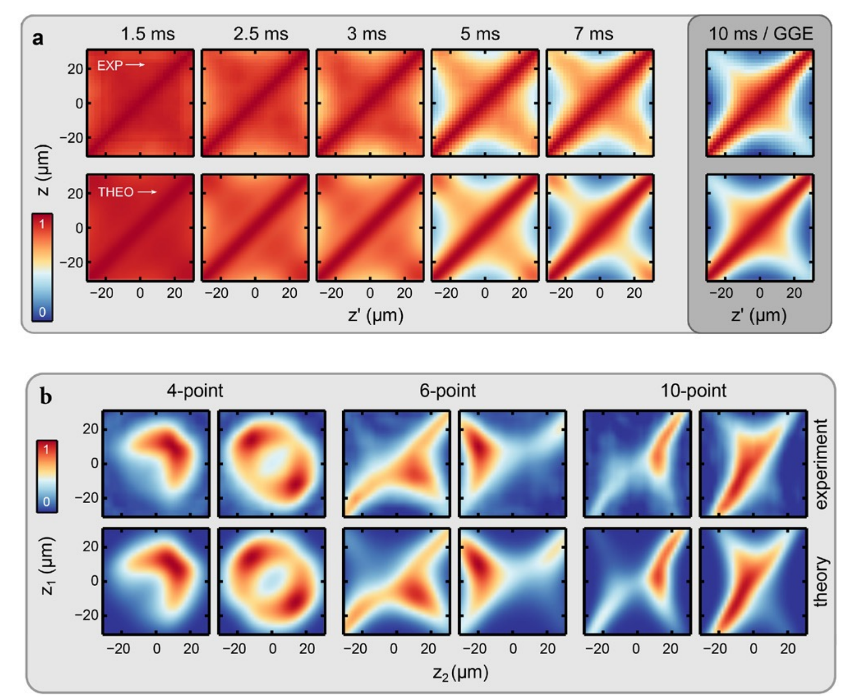
ULTRACOLD ATOMS EXPERIMENTS [Weiss, Bloch, Schmiedmayer, Dalibard, Bouchoule ...]



Rev. Mod. Phys. **83**, 1405



Nature **440** 900



Science **348**, 6231

THERMALIZATION OF 1-D SYSTEMS?

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EQUILIBRIUM : $\lim_{t\to\infty} \langle \psi_t | O_{loc} | \psi_t \rangle = \mathrm{Tr}\left(O_{loc}\rho_{stat}\right), \qquad \rho_{stat} = ?$

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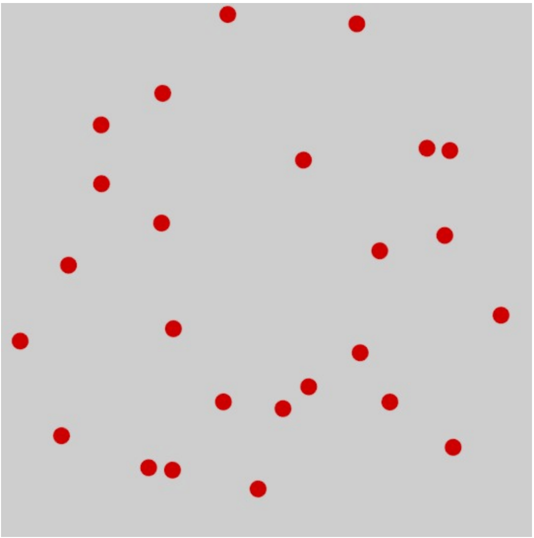

Gibbs ensemble

$$\rho_{Gibbs} = e^{-\beta H}$$

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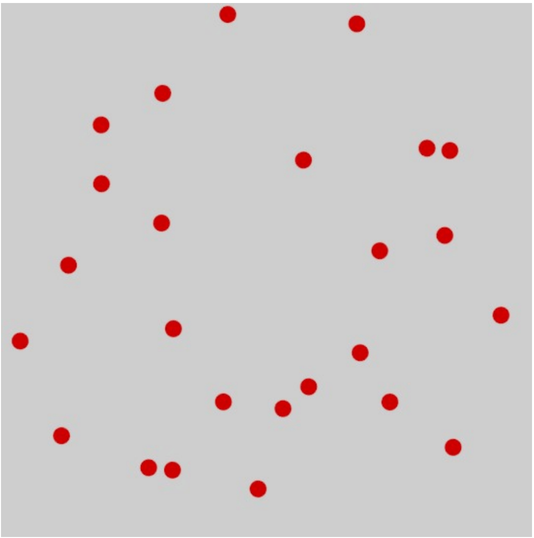

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GENERALIZED

Generalized Gibbs ensemble (GGE)

$$\rho_{GGE} = e^{-\sum_i \beta_i Q_i}$$

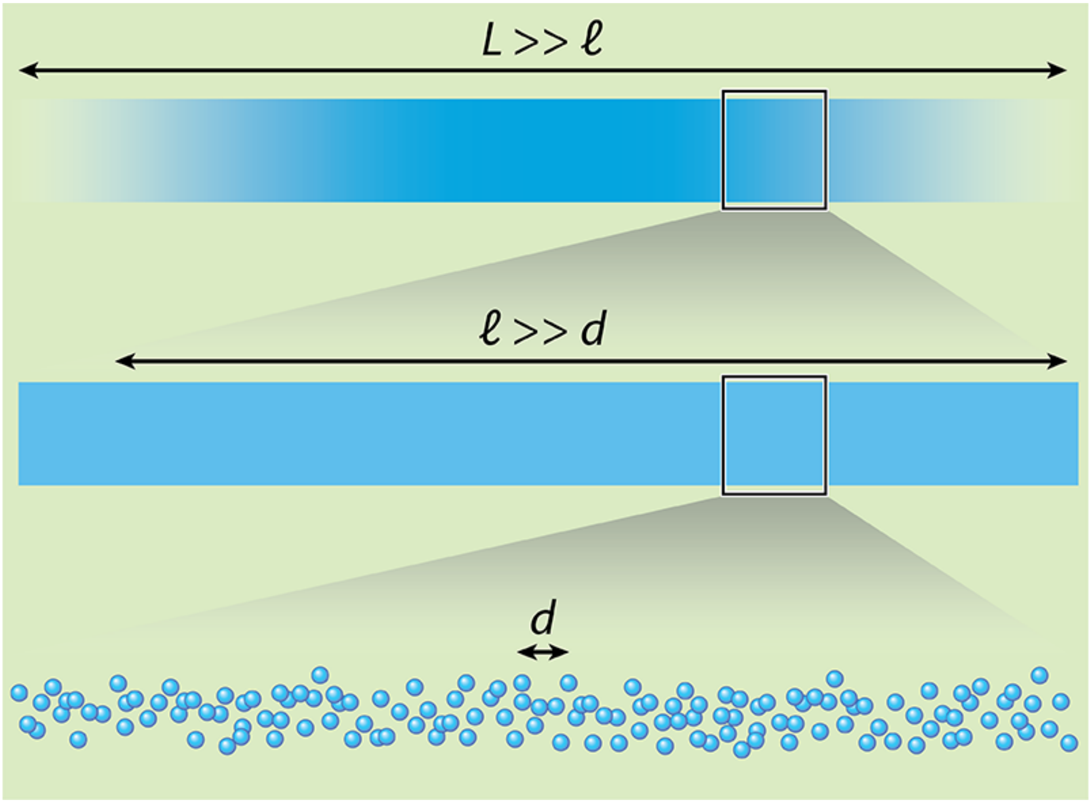
HYDRODYNAMICS: THE EULER SCALE

SEPARATION OF *LENGTH* SCALES

- Macroscopic scale : $L = \rho / \partial_x \rho$
- Microscopic scale : d
- MESOSCOPIC scale: $d \ll \ell \ll L$

SEPARATION OF *TIME* SCALES

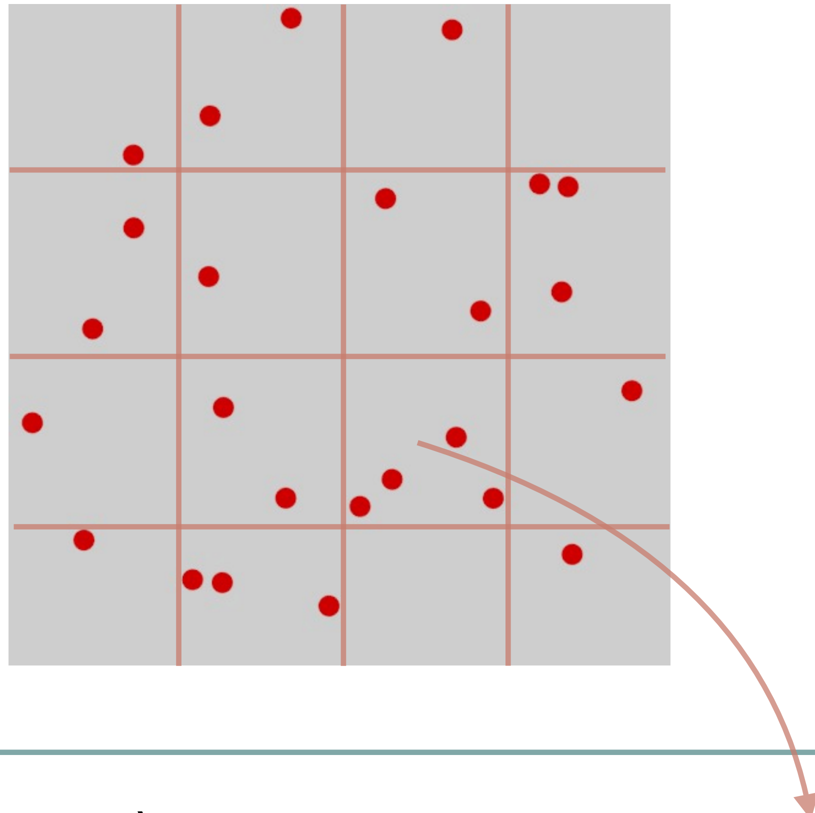
- Macroscopic time : $T = \rho / \partial_t \rho$
- Microscopic time : τ_{relax}
- MESOSCOPIC time: $\tau_{relax} \ll t \ll T$



[PICTURE FROM: DUBAIL, PHYSICS 9, 153]

H. SPOHN, *Large Scale Dynamics of Interacting Particles* (1991)

(CONVENTIONAL) HYDRODYNAMICS



$$\langle O(x) \rangle_t = \langle O(x) \rangle_{\rho_{Gibbs}}, \quad \rho_{Gibbs} = \rho_{Gibbs}(x,t)$$

CONSERVATION LAWS:

- Mass
- Momentum
- Energy

CONTINUITY EQUATIONS:

$$\left\langle \begin{aligned} \partial_t \rho + \partial_x j &= 0 \\ \partial_t \rho_P + \partial_x j_P &= 0 \\ \partial_t \rho_E + \partial_x j_E &= 0. \end{aligned} \right\rangle$$

GENERALIZED HYDRODYNAMICS (*GHD*)

CONSERVATION LAWS:

$$Q_i = \int q_i(x)\, dx \quad i = 1,2,\cdots$$

→

CONTINUITY EQUATIONS:

$$\partial_t q_i + \partial_x j_i = 0 \quad i = 1,2,\cdots$$

Castro-Alvaredo, Doyon, Yoshimura, PRX 6, 041065 (2016)
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EIGENSTATES: $|\theta_1, \cdots, \theta_N\rangle$ "Rapidities" ---TDL---> $n(\theta)$ "Occupation function"

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"Rapidities" "Occupation function"

$$\partial_t n(\theta) + v^{\mathrm{eff}}(\theta)\partial_x n(\theta) = 0 \quad + \quad v^{\mathrm{eff}}(\theta) = \frac{(E')^{\mathrm{dr}}(\theta)}{(p')^{\mathrm{dr}}(\theta)}$$

$$f^{\mathrm{dr}}(\theta) = f(\theta) + \int \frac{d\theta'}{2\pi}\frac{d\phi(\theta-\theta')}{d\theta} n(\theta') f^{\mathrm{dr}}(\theta')$$

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"FREE" GHD

$$\theta \to p, \qquad v^{\text{eff}} \to p,$$

$$n(x,\theta;t) \to n(x,p;t)$$

GHD $\equiv$ *Evolution Wigner function*

EIGENSTATES: $|\theta_1, \cdots, \theta_N\rangle$ TDL

"Rapidities" "Occupat

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CASTRO-ALVAREDO, DOYON, YOSHIMURA, PRX 6, 041065 (2016)
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ZERO-ENTROPY STATES

DOYON, DUBAIL, KONIK, YOSHIMURA, PRL119, 195301 (2017)

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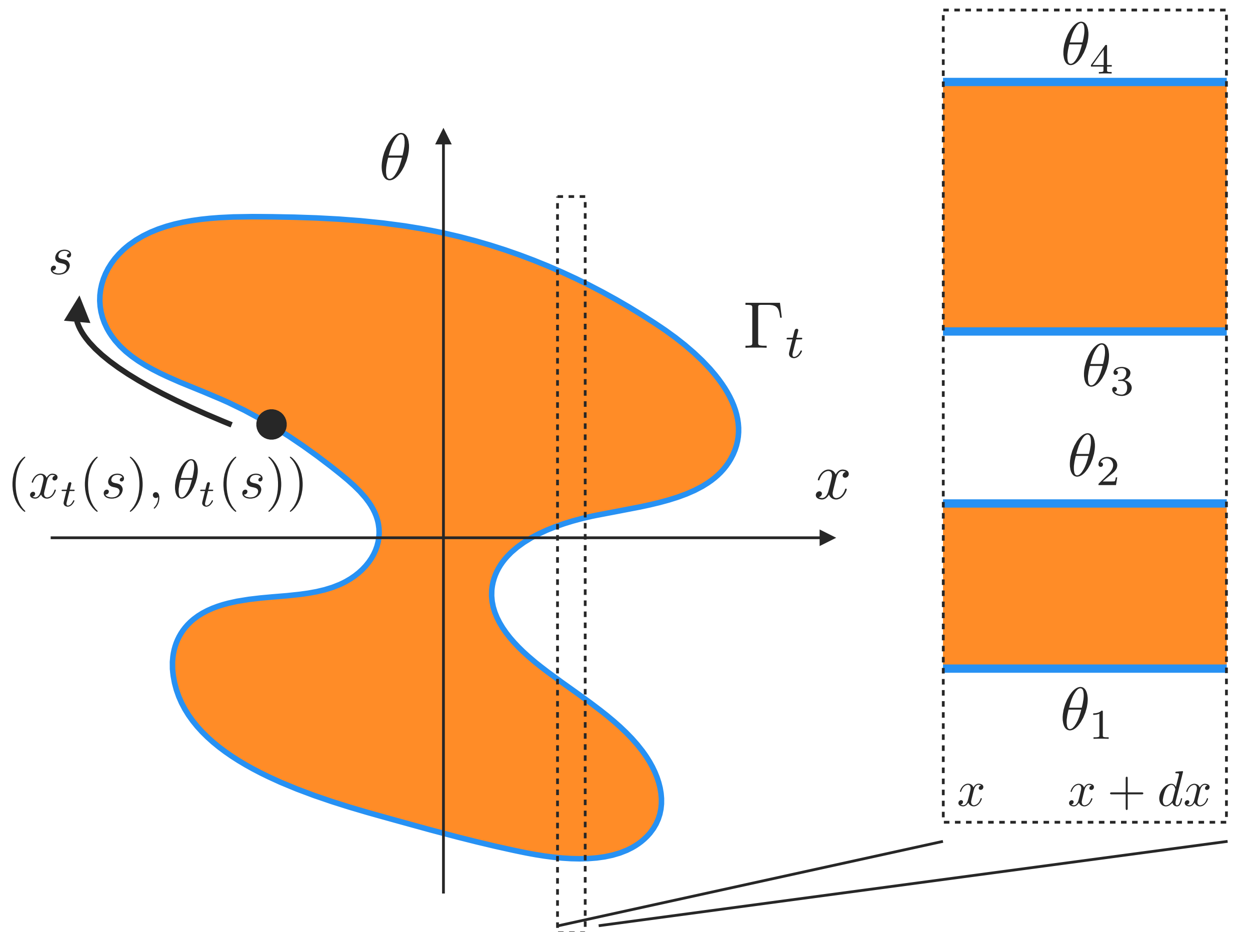
$$n(x,\theta) = \begin{cases} 1 & \text{if} \quad (x,\theta) \text{ in } \Gamma_t \\ 0 & \text{otherwise} . \end{cases}$$



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EXAMPLES:
Ground states within
inhomogeneous potentials

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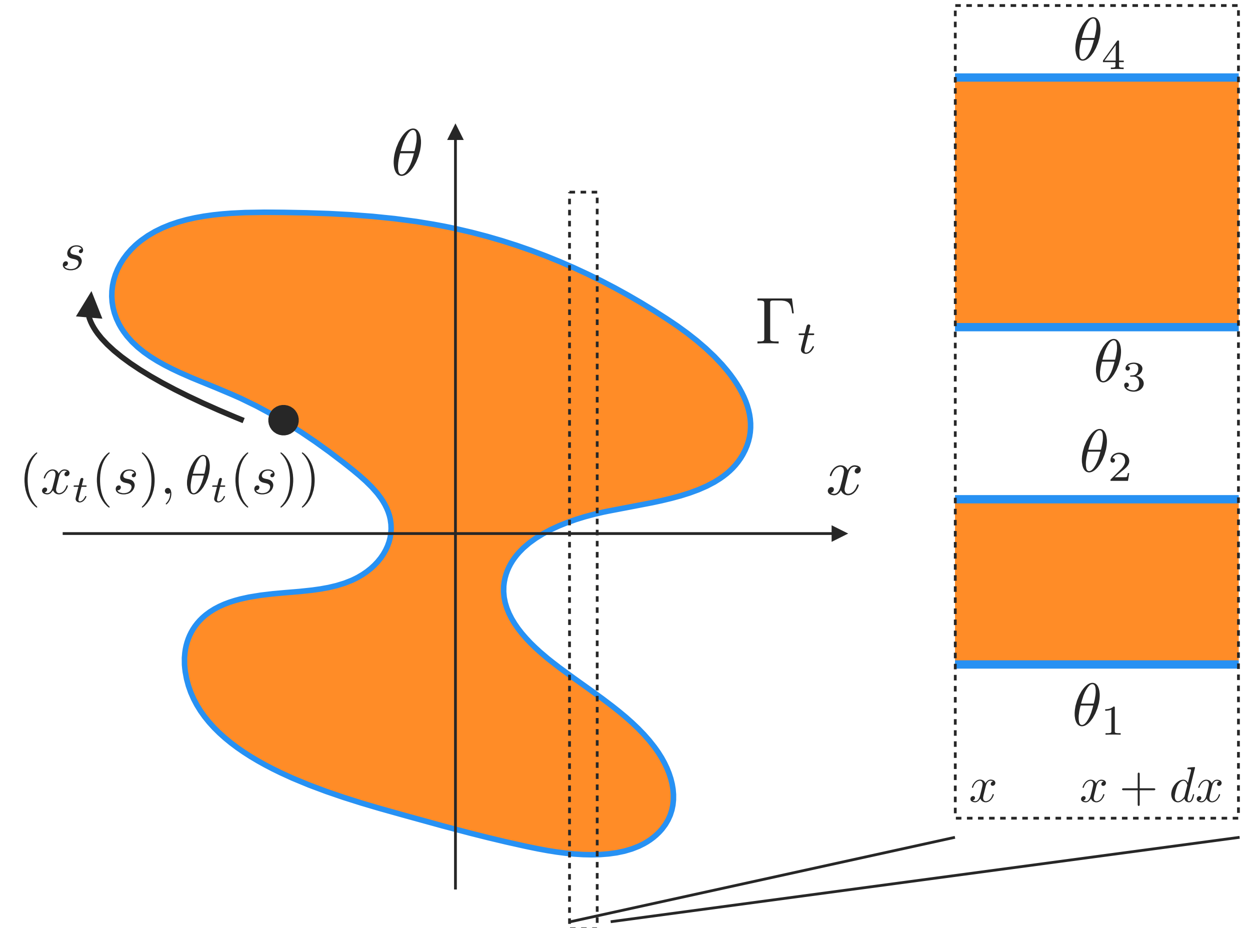

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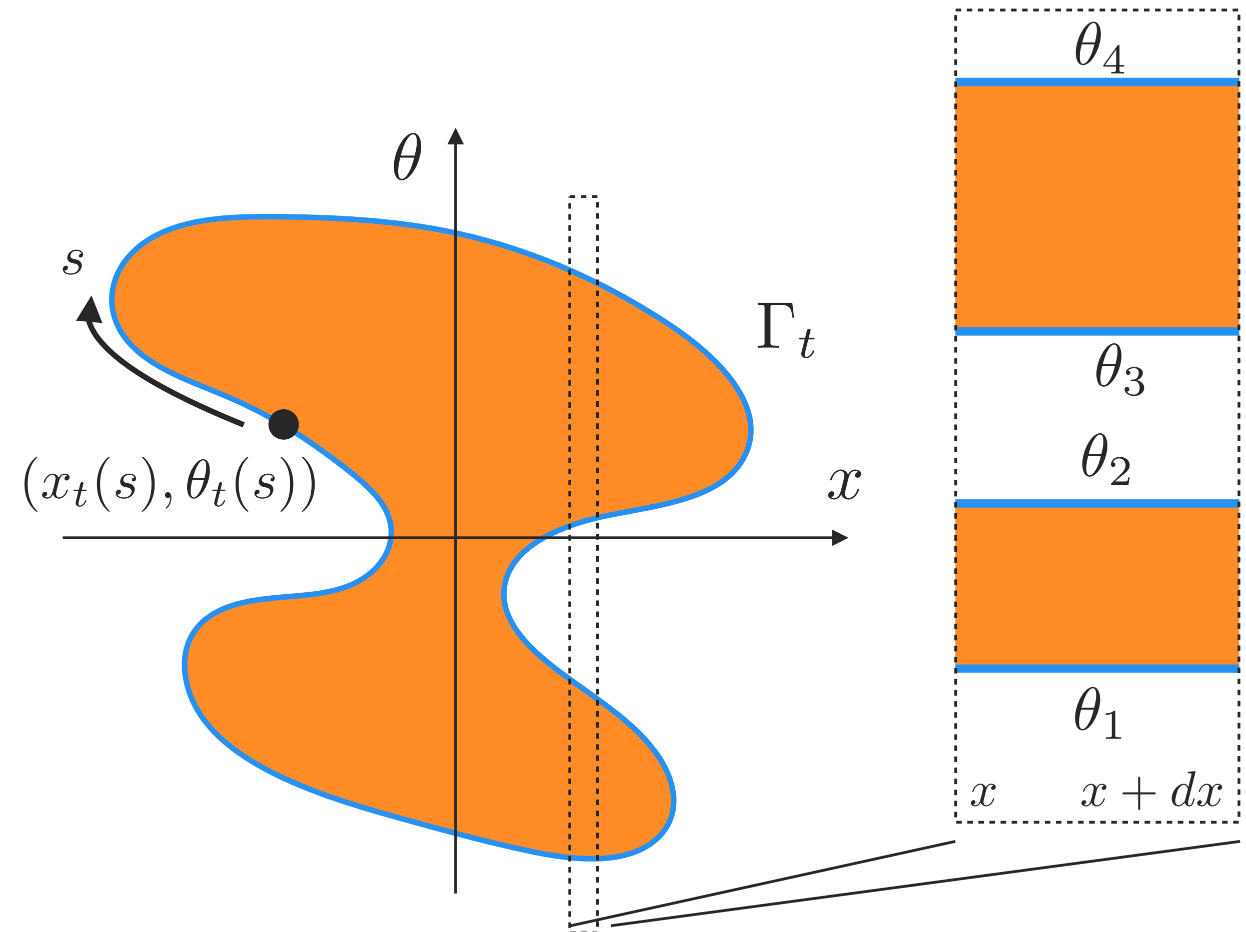

"Split Fermi seas"

Fokkema, Eliens, Caux, PRA 89, 033637 (2014)
Eliens, Caux, J.Phys.A 49, 495203 (2016)
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$$\theta \in [\theta_1, \theta_2] \cup \ldots \cup [\theta_{2q-1}, \theta_{2q}]$$

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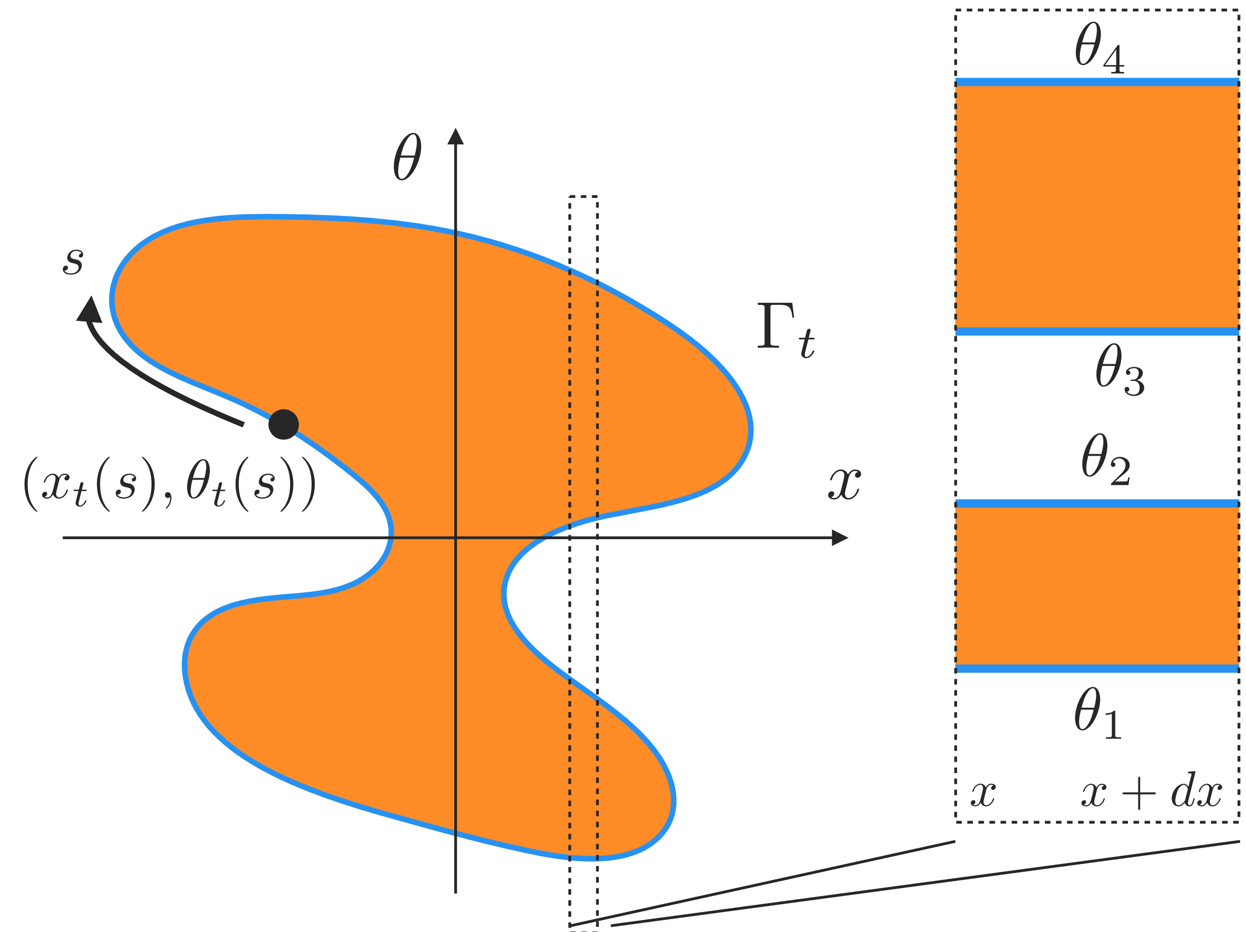

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FINITE-DIMENSIONAL **GHD**

$$\partial_t \theta_j + v^{eff}_{\{\theta\}}(\theta_j)\, \partial_x \theta_j = 0$$

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GHD IN EXPERIMENTS…

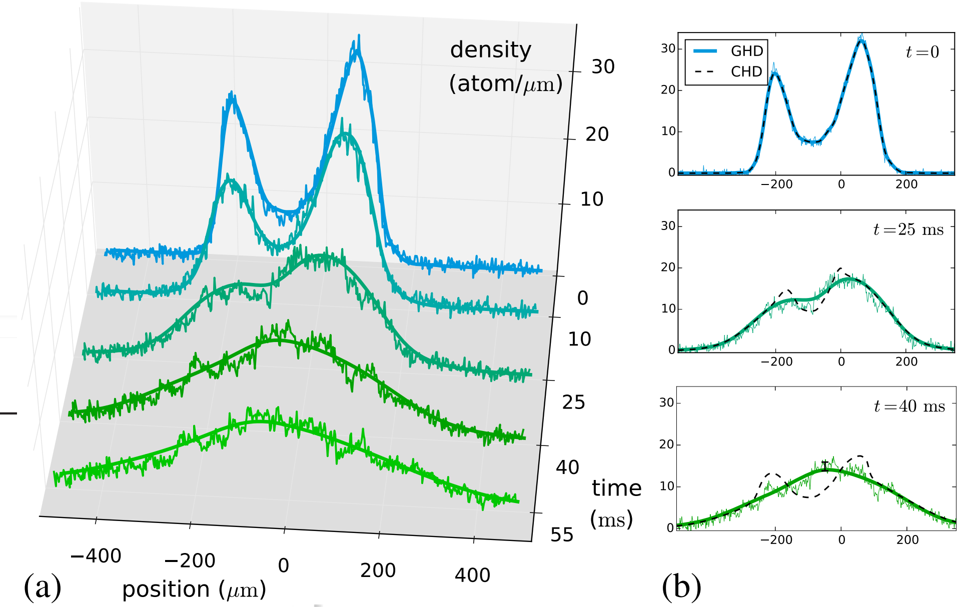
PHYSICAL REVIEW LETTERS **122,** 090601 (2019)

Editors' Suggestion   Featured in Physics

Generalized Hydrodynamics on an Atom Chip

M. Schemmer,[1] I. Bouchoule,[1] B. Doyon,[2] and J. Dubail[3]

[1]*Laboratoire Charles Fabry, Institut d'Optique, CNRS, Université Paris-Saclay, 91127 Palaiseau cedex, France*
[2]*Department of Mathematics, King's College London, Strand, London WC2R 2LS, United Kingdom*
[3]*Laboratoire de Physique et Chimie Théoriques, CNRS, Université de Lorraine, UMR 7019, F-54506 Vandoeuvre-les-Nancy, France*

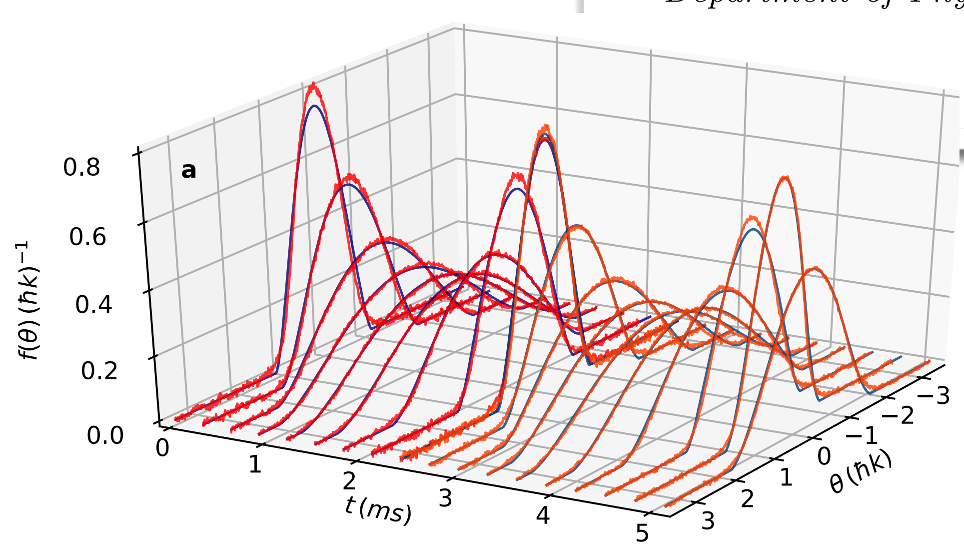


Generalized hydrodynamics in strongly interacting 1D Bose gases

Neel Malvania*,[1] Yicheng Zhang*,[1] Yuan Le,[1] Jerome Dubail,[2] Marcos Rigol,[1] and David S. Weiss[1]

[1]*Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802, USA*
[2]*Université de Lorraine, CNRS, LPCT, F-54000 Nancy, France*
These authors contributed equally to this work

arXiv:2009.0665



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CLASSICAL EQUATIONS → QUANTIZE THEM

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- Superconductivity BARDEEN, COOPER, SCHRIEFFER, PHYS REV.. 104 (4)
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Different problem:
Corrections
to GHD equations:

FAGOTTI,
SCIPOST PHYS. 8, 048 (2020)

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EPL, 126 20006 (2019)

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$\neq$

HERE:
Look at propagation of linear sound waves on top of GHD and quantise them:

! We talk about "***quantum fluids***" in the same sense in which a Luttinger Liquid is a quantum fluid **!**

QUANTUM GENERALIZED HYDRODYNAMICS

$$H = \sum_{i=1}^{N} [-\frac{\hbar^2}{2}\partial_{x_i}^2 + V(x_i)] + g\sum_{i<j}\delta(x_i - x_j)$$

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PR, Calabrese, Doyon, Dubail, PRL 124, 140603 (2020)

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PLAN:

1. Look at *fluctuations* around a (zero-entropy) GHD background
2. *Quantize* them in a semiclassical fashion.

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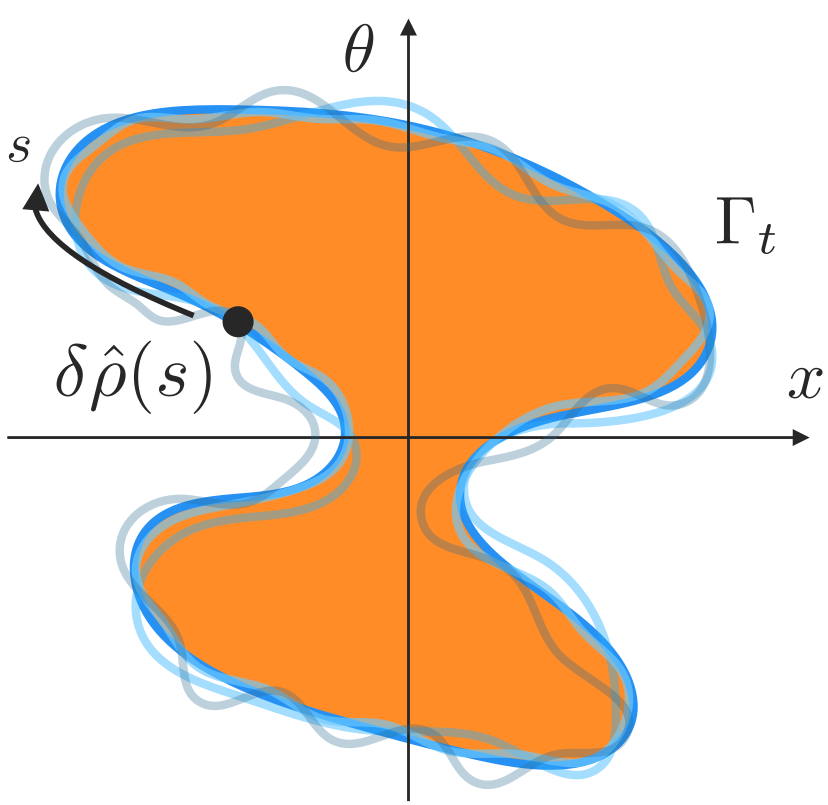

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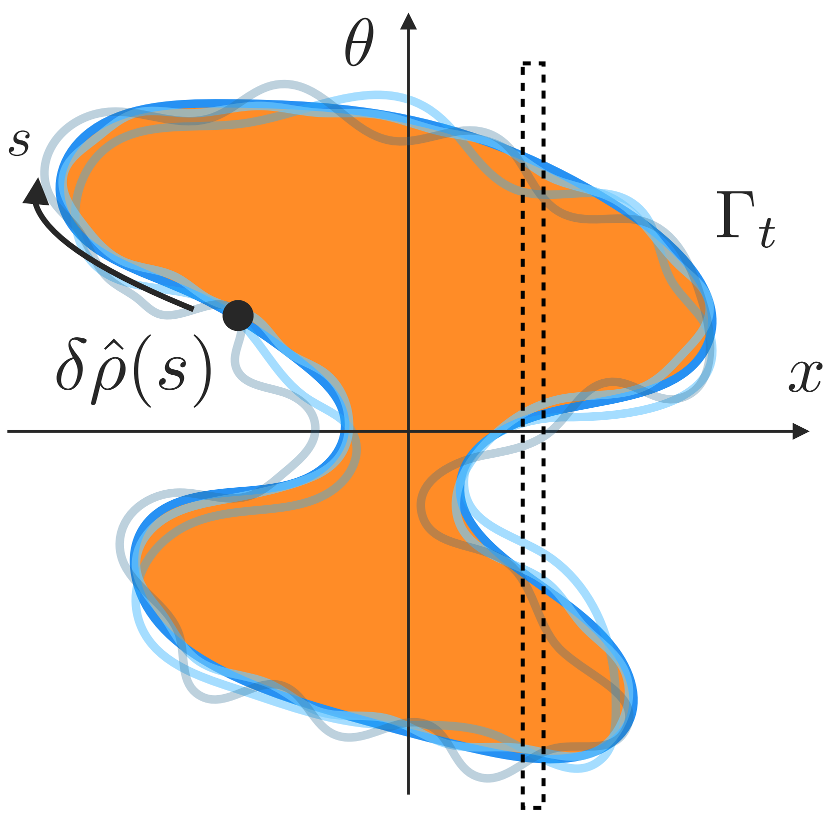

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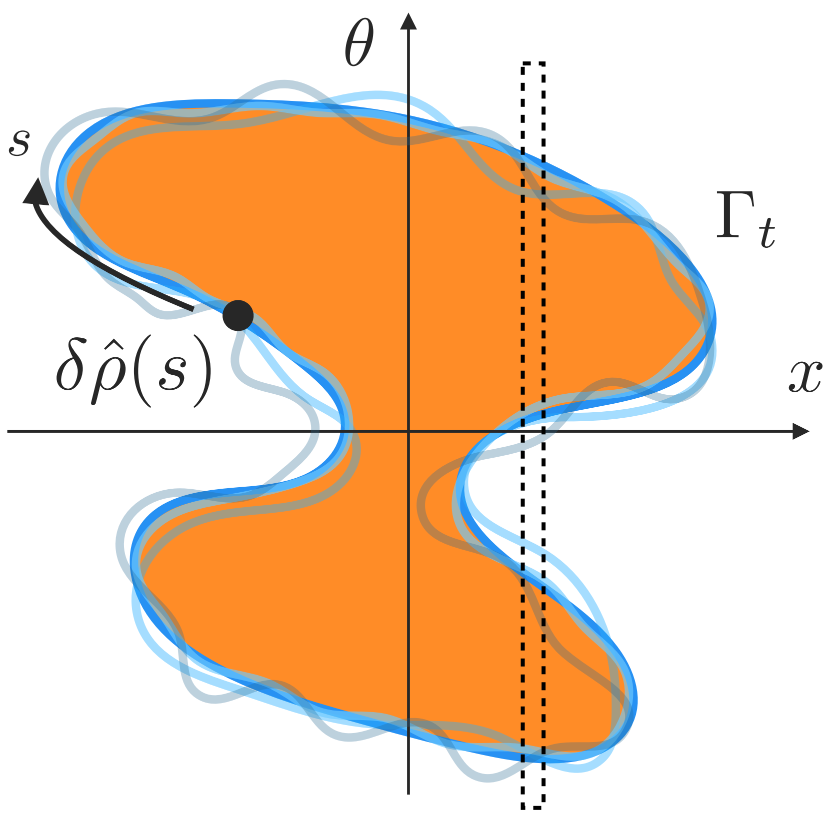

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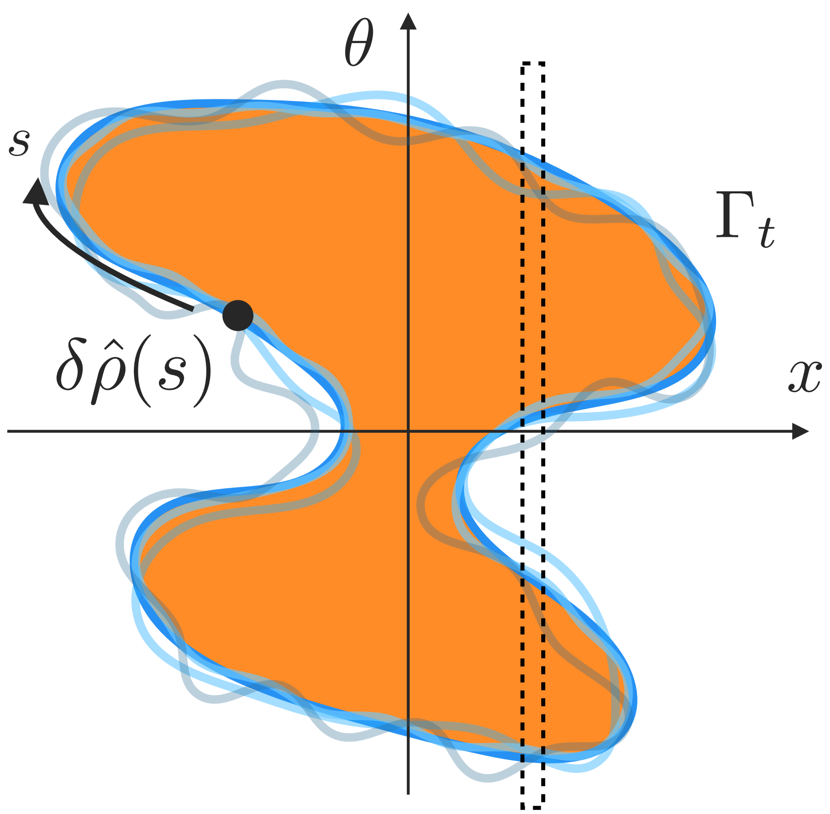

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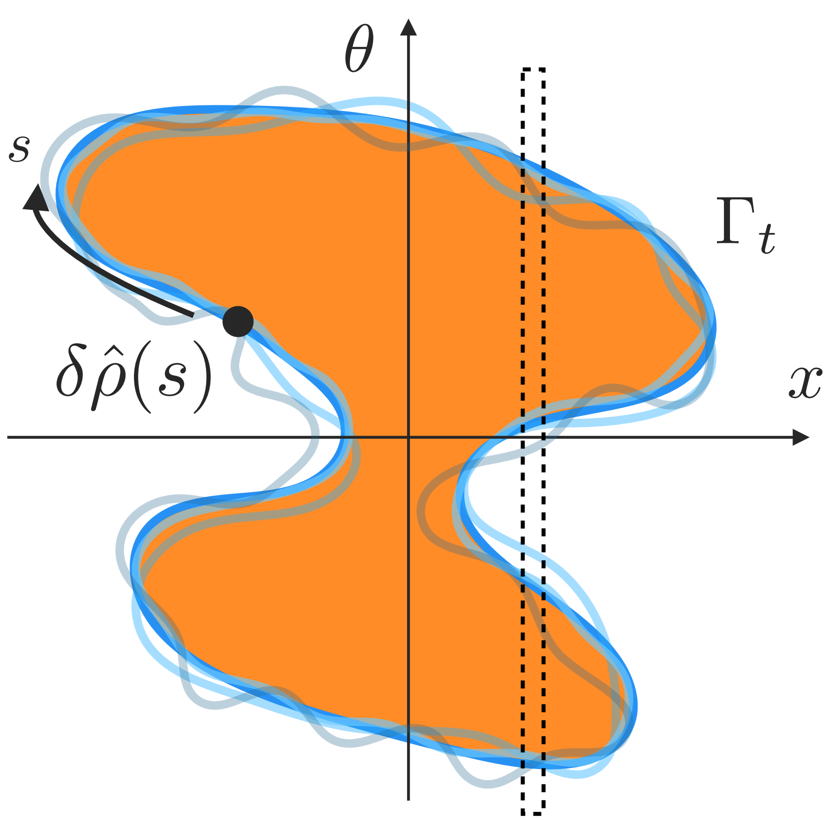

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2. $\delta p_a \to \delta\hat{p}_a : \quad e^{iS} \approx e^{iS_{\text{classical}} + i\sum_{ab} S^{(2)}_{ab} \delta p_a \delta p_b}$

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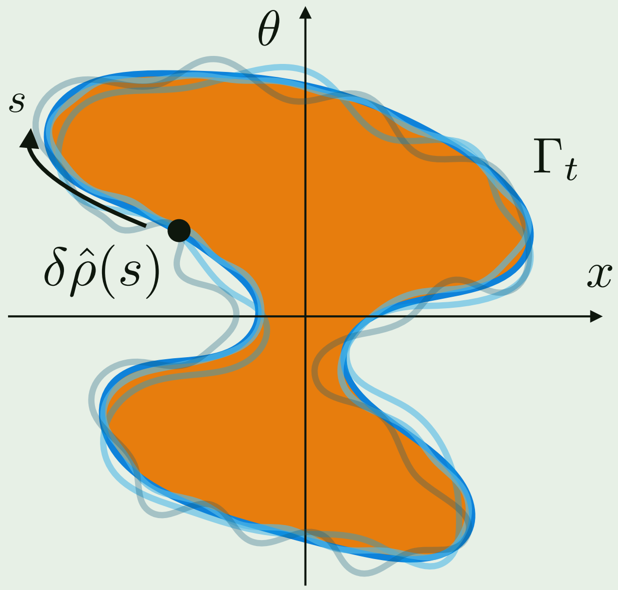
QUANTUM GENERALIZED HYDRODYNAMICS

"Quantum GHD HAMILTONIAN"

$$\hat{H}[\Gamma_t] = \frac{1}{4\pi\hbar}\int dx \sum_{a,b} \delta\hat{p}_a(x)\, \mathsf{A}^{ab}(x,t)\, \delta\hat{p}_b(x)$$

$$\left[\delta\hat{p}_a(x), \delta\hat{p}_b(y)\right] = -\, i\sigma_a 2\pi\hbar^2 \delta_{ab}\, \delta'(x-y)$$



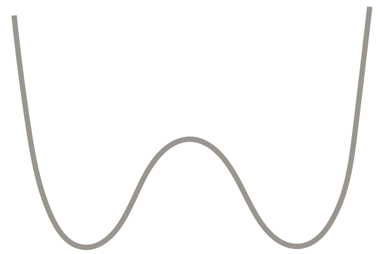
Multicomponent time-dependent and spatially inhomogeneous *CHIRAL LUTTINGER LIQUID* or, (eq.) *FREE CHIRAL BOSON*:

$$\delta\hat{p}_a(x) = \partial\phi_a(x)$$

PR, Calabrese, Doyon, Dubail, PRL 124, 140603 (2020)

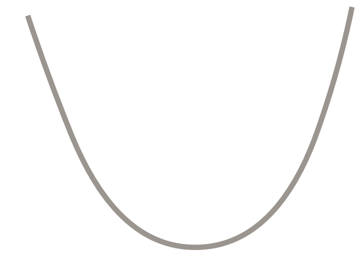
QGHD: Numerical check in the Lieb-Liniger model

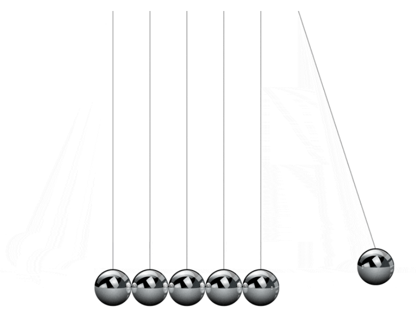
$$V_0(x) = a_4 x^4 - a_2 x^2$$

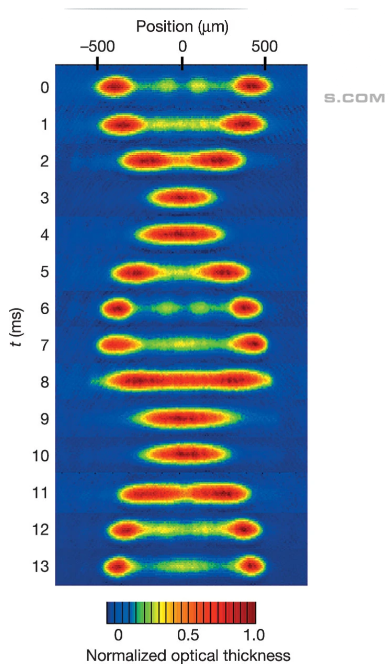


$$\longrightarrow$$

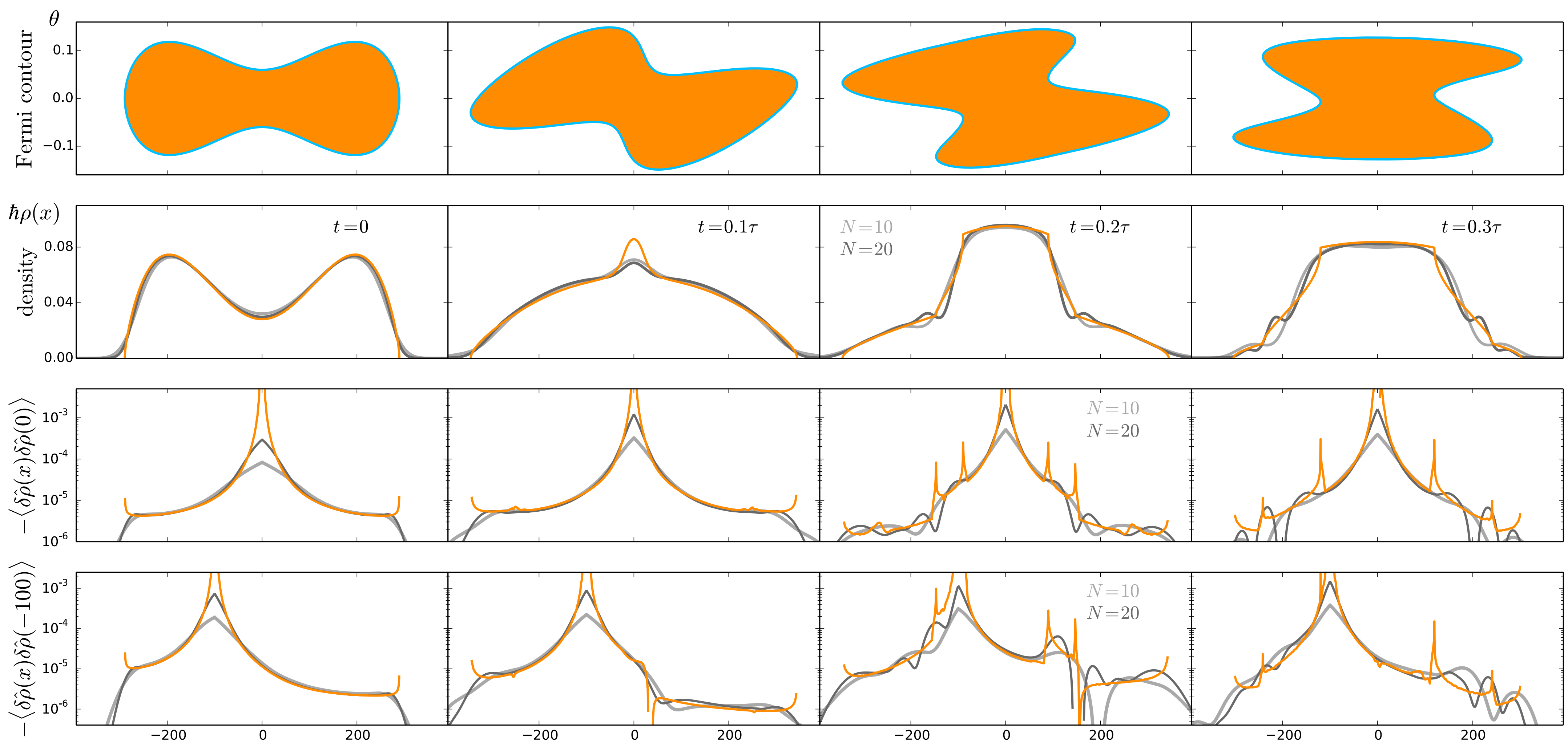
$$V(x) = \omega^2 x^2 / 2$$





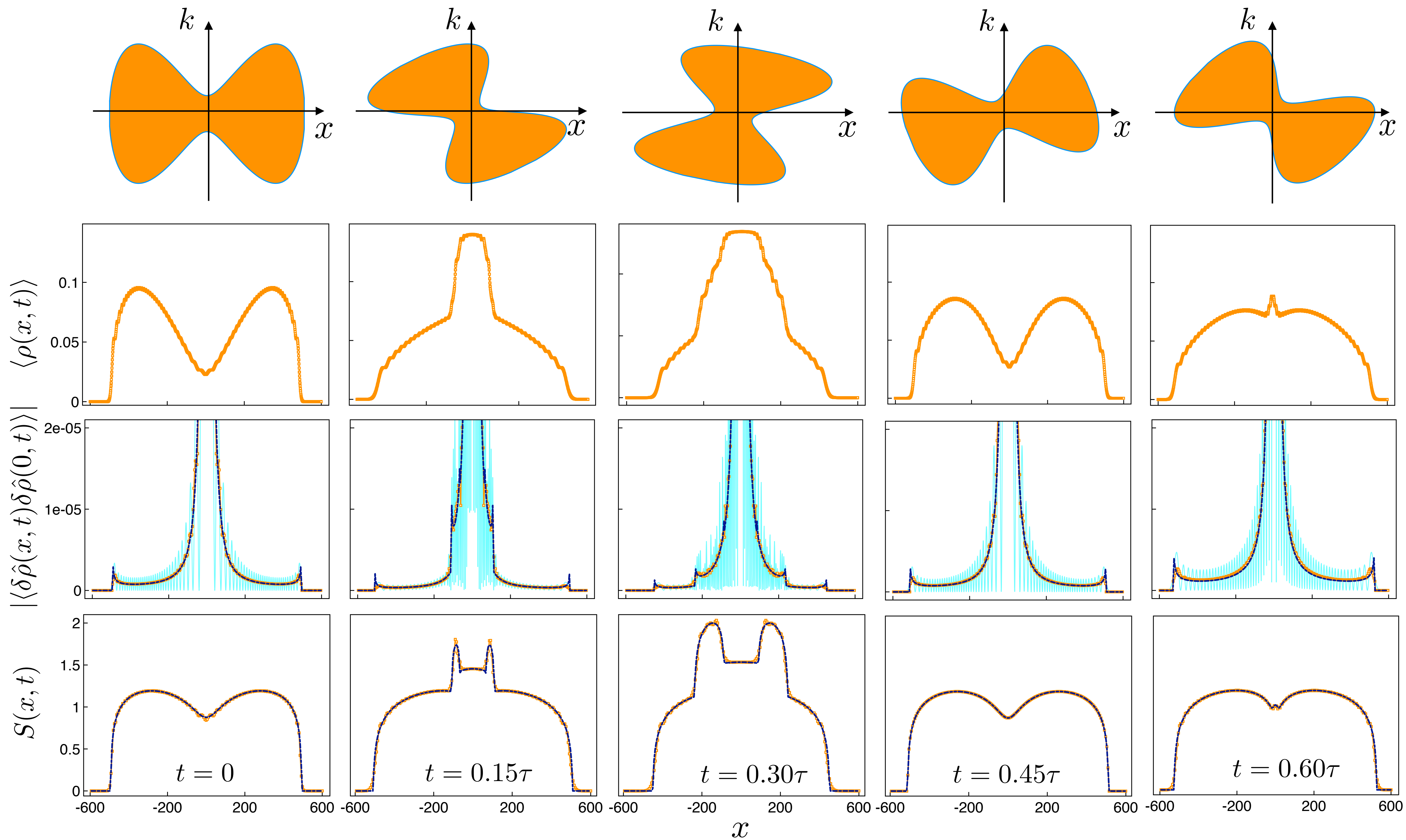


Nature 440 900 (2006)



PR, Calabrese, Doyon, Dubail, PRL 124, 140603 (2020)

FREE QGHD: checks in the Tonks-Girardeau limit



PR, Calabrese, Doyon, Dubail, In Preparation

SUMMARY

1. GHD provides an efficient way to describe interacting quantum particles in 1D
2. Still, it misses important quantum effects
3. Quantum GHD gives a way to construct quantum fluctuations around GHD
4. This is not the end of the story...

THANK YOU.