UNIVERSITÀ DI PISA

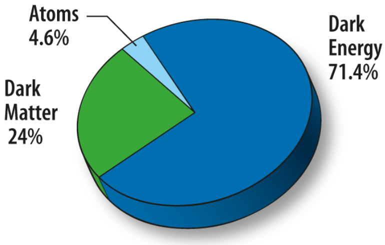
Accidental Dark matter in gauge theories

Giacomo Landini
Cortona Young 2021

Introduction

Dark Matter paradigm

Astrophysical and cosmological evidence for DM : $\boxed{\Omega_{\rm DM} \approx 0.26}$



DM properties

- neutral
- stable (or $\tau_{\rm DM} \gg t_{\rm U}$)
- cold (non-relativistic at structure formation time)
- weakly interacting with the Standard model particles

None of the Standard Model particles is a good DM candidate: **physics Beyond the Standard Model is required!**

Accidental symmetries and DM stability

- The proton is stable or very long-lived: $\tau_p > 10^{34}$ years
- Consequence of the *accidental* baryon number symmetry of the renormalizable SM Lagrangian $\mathrm{U}(1)_B : q \to e^{i\theta} q$
- $\mathrm{U}(1)_B$ broken by dimension-6 operators: $\tau_p \sim \Lambda_{\mathrm{UV}}^4 / m_p^5$

Accidental symmetries and DM stability

- The proton is stable or very long-lived: $\tau_p > 10^{34}$ years
- Consequence of the *accidental* baryon number symmetry of the renormalizable SM Lagrangian $\mathrm{U}(1)_B \,:\, q \to e^{i\theta} q$
- $\mathrm{U}(1)_B$ broken by dimension-6 operators: $\tau_p \sim \Lambda^4_{\mathrm{UV}}/m^5_p$

Accidental DM models

Can DM be accidentally stable as the proton?

1. dark sector: confining and/or spontaneously broken dark *gauge theory*

Accidental symmetries and DM stability

- The proton is stable or very long-lived: $\tau_p > 10^{34}$ years
- Consequence of the *accidental* baryon number symmetry of the renormalizable SM Lagrangian $\mathrm{U}(1)_B : q \to e^{i\theta} q$
- $\mathrm{U}(1)_B$ broken by dimension-6 operators: $\tau_p \sim \Lambda^4_{\mathrm{UV}}/m_p^5$

Accidental DM models

Can DM be accidentally stable as the proton?

1. dark sector: confining and/or spontaneously broken dark *gauge theory*

$$\Downarrow$$

2. set of accidental global symmetries of the action up to dimension $d \geq 4$
   $\Rightarrow$ DM is the lightest state with non-trivial transformation properties

Accidental symmetries and DM stability

- The proton is stable or very long-lived: $\tau_p > 10^{34}$ years
- Consequence of the *accidental* baryon number symmetry of the renormalizable SM Lagrangian $\mathrm{U}(1)_B : q \to e^{i\theta} q$
- $\mathrm{U}(1)_B$ broken by dimension-6 operators: $\tau_p \sim \Lambda^4_{\mathrm{UV}}/m_p^5$

Accidental DM models

Can DM be accidentally stable as the proton?

1. dark sector: confining and/or spontaneously broken dark *gauge theory*

$$\Downarrow$$

2. set of accidental global symmetries of the action up to dimension $d \geq 4$
$\Rightarrow$ DM is the lightest state with non-trivial transformation properties

$$\Downarrow$$

3. Higher dimensional operators can break the accidental symmetries but are suppressed by powers of $\Lambda_{\mathrm{UV}} \Rightarrow \tau_{\mathrm{DM}} \gg t_U$.

Accidental symmetries in gauge theories

Gauge group $G = \{\,\mathrm{SU}(N),\, \mathrm{SO}(N),\, \mathrm{Sp}(N)\}$

$$\mathcal{L} = -\frac{1}{4}\,\mathsf{Tr}\big[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\big] + \bar{\psi} i \gamma^{\mu}\mathcal{D}_{\mu}\psi + (\mathcal{D}_{\mu}\phi)^{\dagger}\mathcal{D}_{\mu}\phi + \ldots$$

Accidental symmetries in gauge theories

Gauge group $G = \{\,\mathrm{SU}(N),\, \mathrm{SO}(N),\, \mathrm{Sp}(N)\}$

$$\mathcal{L} = -\frac{1}{4}\,\mathrm{Tr}\big[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\big] + \bar{\psi} i \gamma^{\mu} \mathcal{D}_{\mu}\psi + (\mathcal{D}_{\mu}\phi)^{\dagger}\mathcal{D}_{\mu}\phi + \ldots$$

- **Global $\mathbf{U(1)}$:** $\phi \to e^{i\theta}\phi, \qquad \psi \to e^{i\alpha}\psi, \qquad \ldots$
  Same of baryon number within the dark sector. DM is the lightest state charged under $\mathrm{U}(1)$. DM must be complex.

Accidental symmetries in gauge theories

Gauge group $G = \{\,\mathrm{SU}(N),\,\mathrm{SO}(N),\,\mathrm{Sp}(N)\}$

$$\mathcal{L} = -\frac{1}{4}\,\mathrm{Tr}\big[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\big] + \bar{\psi} i \gamma^{\mu}\mathcal{D}_{\mu}\psi + (\mathcal{D}_{\mu}\phi)^{\dagger}\mathcal{D}_{\mu}\phi + \ldots$$

- **Global** $\mathbf{U(1)}$**:** $\phi \to e^{i\theta}\phi, \qquad \psi \to e^{i\alpha}\psi, \qquad \ldots$
  Same of baryon number within the dark sector. DM is the lightest state charged under $\mathrm{U}(1)$. DM must be complex.
- **Group parity:** $\mathcal{G}_{ij} \to (-1)^{\delta_{1i}+\delta_{1j}}\mathcal{G}_{ij}, \qquad \phi_i \to (-1)^{\delta_{1i}}\phi_i, \qquad \ldots$
  $\mathcal{Z}_2$ symmetry acting as a reflection of any of the N equivalent directions in group space. DM is the lightest $\mathcal{Z}_2$-odd state.

Accidental symmetries in gauge theories

Gauge group $G = \{\,\mathrm{SU}(N),\, \mathrm{SO}(N),\, \mathrm{Sp}(N)\}$

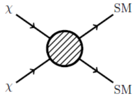
$$\mathcal{L} = -\frac{1}{4}\,\mathrm{Tr}\big[\mathcal{G}_{\mu\nu}\mathcal{G}^{\mu\nu}\big] + \bar{\psi} i\gamma^{\mu}\mathcal{D}_{\mu}\psi + (\mathcal{D}_{\mu}\phi)^{\dagger}\mathcal{D}_{\mu}\phi + \ldots$$

- **Global $\mathrm{U}(1)$:** $\phi \to e^{i\theta}\phi, \qquad \psi \to e^{i\alpha}\psi, \qquad \ldots$
  Same of baryon number within the dark sector. DM is the lightest state charged under $\mathrm{U}(1)$. DM must be complex.
- **Group parity:** $\mathcal{G}_{ij} \to (-1)^{\delta_{1i}+\delta_{1j}}\mathcal{G}_{ij}, \qquad \phi_i \to (-1)^{\delta_{1i}}\phi_i, \qquad \ldots$
  $\mathcal{Z}_2$ symmetry acting as a reflection of any of the N equivalent directions in group space. DM is the lightest $\mathcal{Z}_2$-odd state.
- **Group charge conjugation:** $\mathcal{G} \to -\mathcal{G}^*, \qquad \phi \to \phi^*, \qquad \ldots$
  $\mathcal{Z}_2$ symmetry of the $\mathrm{SU}(N)$ Lagrangian. It can be extended to SO and Sp groups. DM is the lightest $\mathcal{Z}_2$-odd state.

DM thermal production: freeze out

DM is kept in thermal equilibrium with the SM thermal bath by annihilation processes with interaction rate

$$\Gamma_{\rm int} = n_{\rm DM} \langle \sigma v \rangle$$



.

The Universe is expanding with rate $H \sim T^2/M_{\rm Pl}$.

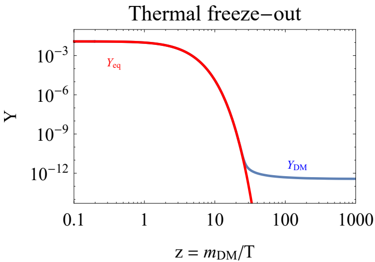
When $\Gamma_{\rm int} \lesssim H$ (at $T_{\rm dec} \lesssim m_{\rm DM}$) DM decouples.

- DM (number/entropy) density

  $Y_{\rm DM} \equiv n_{\rm DM}/s \sim 1/T_{\rm dec} M_{\rm Pl} \langle \sigma v \rangle.$

- DM relic abundance

  $\Omega_{\rm DM} h^2 = 0.12 \times (m_{\rm DM}/0.4\ {\rm eV}) Y_{\rm DM}.$



Scalar gauge dynamics and Dark Matter

D. Buttazzo, L. Di Luzio, G. L., A. Strumia, and D. Teresi [JHEP 1910, 1907.11228 (2019)]

D. Buttazzo, L. Di Luzio, P. Ghorbani, C. Gross, G. L., A. Strumia, *et al.* [JHEP 2001, 1911.04502 (2019)]

G. L. and J.-W. Wang [JHEP 2006, 2004.03299 (2020)]

The model(s)

Dark sector

- a new dark-gauge group $G = \{\mathrm{SU}(\mathcal{N}), \mathrm{SO}(\mathcal{N}), \mathrm{Sp}(\mathcal{N})\}$
- a scalar field $\mathcal{S}$ in the *fundamental* representation of G and singlet of the SM gauge group

The most general renormalizable Lagrangian is

$$\mathcal{L} = \mathcal{L}_{\rm SM} - \frac{1}{4}\mathcal{G}^A_{\mu\nu}\mathcal{G}^{A\,\mu\nu} + |\mathcal{D}_\mu \mathcal{S}|^2 - V_{\mathcal{S}}$$

$$V_{\mathcal{S}} = -M_{\mathcal{S}}^2|\mathcal{S}|^2 + \lambda_{\mathcal{S}}|\mathcal{S}|^4 - \lambda_{H\mathcal{S}}|H|^2|\mathcal{S}|^2$$

The Higgs portal connects the Dark sector within the SM
We work under the additional assumption of scale invariance which leads to a more predictive model ($M_{\mathcal{S}}^2 = 0$).

Phases of the theory

The model admits two apparently different **phases**

- **Higgs phase:** the scalar field gets a vev $\langle \mathcal{S} \rangle = w$.
  SSB of the gauge group $G \to H$ which (if Non-Abelian) confines at Λ_H.
  (Scale invariance $\Rightarrow$ symmetry breaking *à la* Coleman-Weinberg.)
- **Confined phase:** we have $\langle \mathcal{S} \rangle = 0$ but scalar condensates $\langle \mathcal{S}^\dagger \mathcal{S} \rangle$ form.
  The gauge group G confines at some Λ_G (analogous to QCD).

Fradkin-Shenker theorem: from $\mathrm{SU}(2)$ duality...

Fradkin, Shenker et al. proved that for $\mathrm{SU}(2)$ gauge group the two phases are smoothly connected and share the same spectrum of asymptotic particles and same accidental global symmetries.
Can we generalize this statement?

...to general dualities?

We claim a *duality* among the two phases based on the one-to-one mapping of

- the asymptotic states
- the accidental symmetries

Generalization of Fradkin-Shenker theorem to $\{SU(N), SO(N), \mathrm{Sp}(N)\}$ gauge groups (it works also for G_2.)

Group	Higgs phase	Condensed phase
$\mathrm{SU}(\mathcal{N}) \to \mathrm{SU}(\mathcal{N}-1)$	s $\mathcal{Z}_\mu$ $\epsilon_{\mathcal{N}-1}\mathcal{W}^{\mathcal{N}-1}$ $\mathcal{A}\mathcal{A}$ $d\mathcal{A}\mathcal{A}\mathcal{A}$	$\mathcal{S}^\dagger\mathcal{S}$ $\mathcal{S}^\dagger\mathcal{D}_\mu\mathcal{S}$ $\epsilon_{\mathcal{N}}\mathcal{S}^{\mathcal{N}}$ $\mathcal{G}\mathcal{G}$ $d\mathcal{G}\mathcal{G}\mathcal{G}$
$\mathrm{SO}(\mathcal{N}) \to \mathrm{SO}(\mathcal{N}-1)$	s $\epsilon_{\mathcal{N}-1}\mathcal{A}\ldots\mathcal{A}$ (for odd $\mathcal{N}$) $\epsilon_{\mathcal{N}-1}\mathcal{W}\mathcal{A}\ldots\mathcal{A}$ (for even $\mathcal{N}$) $\mathcal{A}\mathcal{A}$	$\mathcal{S}^T\mathcal{S}$ $\epsilon_{\mathcal{N}}\mathcal{S}\mathcal{G}\ldots\mathcal{G}$ (for odd $\mathcal{N}$) $\epsilon_{\mathcal{N}}\mathcal{G}\ldots\mathcal{G}$ (for even $\mathcal{N}$) $\mathcal{G}\mathcal{G}$
$\mathrm{Sp}(\mathcal{N}) \to \mathrm{Sp}(\mathcal{N}-2)$	$s, \mathcal{X}^\dagger\mathcal{X}$ $\mathcal{Z}_\mu, \mathcal{X}^\dagger\mathcal{D}_\mu\mathcal{X}$ $\mathcal{W}_\mu, \mathcal{X}^T\gamma_{\mathcal{N}-2}\mathcal{D}_\mu\mathcal{X}$ $\mathcal{A}\mathcal{A}$	$\mathcal{S}^\dagger\mathcal{S}$ $\mathcal{S}^\dagger\mathcal{D}_\mu\mathcal{S}$ $\mathcal{S}^T\gamma_{\mathcal{N}}\mathcal{D}_\mu\mathcal{S}$ $\mathcal{G}\mathcal{G}$
$G_2 \to \mathrm{SU}(3)$	s $\mathrm{Re}\,\mathcal{W}\mathcal{W}\mathcal{W}$ $\mathcal{A}\mathcal{A}$	$\mathcal{S}^T\mathcal{S}$ $\mathcal{S}\mathcal{S}\mathcal{S}, \mathcal{S}\mathcal{G}\mathcal{G}\mathcal{G}$ $\mathcal{G}\mathcal{G}$

Further study is required to confirm the claim (ex: lattice computations...)

Accidental composite DM in scalar gauge theories

$$G \overset{\langle\mathcal{S}\rangle=w}{\to} H\text{: spectrum} \begin{cases} \text{massless vectors } \mathcal{A} \\ \text{massive vectors } m_{\mathcal{W}} \sim \mathfrak{g}_{\rm DC} w \\ \text{a light scalar } \mathfrak{s} \end{cases}$$

H confines at $\Lambda_{\rm DC}$: the acceptable asymptotic states are singlet of H. Most of them are composite particles (baryons, mesons, glue-balls,...)

One of more of them is stable because of accidental symmetries
$\to$ Dark Matter.

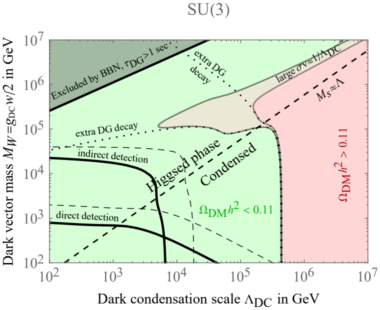
DM is a composite particle with mass $m_{\rm DM} \sim \max\{\Lambda_{\rm DC}, m_{\mathcal{W}}\}$

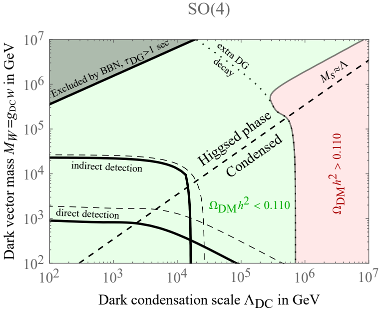
Group	Global symmetry	DM candidate	DM Annihilation
$\mathrm{SU}(N)$	U(1)	Baryon $\epsilon\mathcal{S}^{N} \cong \mathcal{W}^n$	Bohr-like – $1/\Lambda^2_{\rm DC}$
	Charge conjugation	Glue-balls $d\mathcal{GGG} \cong d\mathcal{AAA}$	$1/\Lambda^2_{\rm DC}$
$\mathrm{SO}(N_{\rm even})$	group parity	1-ball $\epsilon\mathcal{G}^{N/2} \cong \mathcal{W}\mathcal{A}^{(n-1)/2}$	$1/\Lambda^2_{\rm DC}$
$\mathrm{SO}(N_{\rm odd})$	group parity	0-ball $\epsilon\mathcal{S}\mathcal{G}^{(N-1)/2} \cong \mathcal{A}^{n/2}$	$1/\Lambda^2_{\rm DC}$
$\mathrm{Sp}(N)$	U(1)	Meson $\mathcal{S}\gamma\mathcal{S} \cong \mathcal{W}, \mathcal{XX}$	Perturbative

DM production

DM is produced thermally with

$$\langle \sigma v \rangle_{\rm ann} \approx \begin{cases} 1/\Lambda_{\rm DC}^2 & \text{if } \Lambda_{\rm DC} \gg M_{\mathcal{W}} \\ 1/\alpha_{\rm DC} M_{\mathcal{W}}^2 & \text{if } \Lambda_{\rm DC} \ll M_{\mathcal{W}} \end{cases}$$

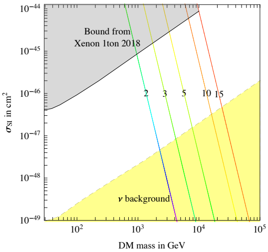


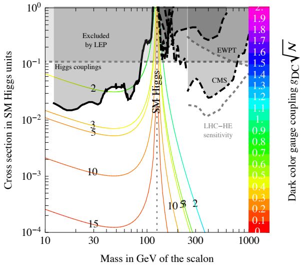


DM phenomenology

Elastic scattering of DM on target nuclei. Tree level exchange of $\mathfrak{s}$ and SM Higgs h.

$$\sigma_{\rm SI} \propto \frac{N^2 m_N^4 g_{\rm DC}^2 v^2 \lambda_{H\mathcal{S}}}{M_h^4 M_{\mathfrak{s}}^4}$$





The light scalar $\mathfrak{s}$ can be produced at colliders through the Higgs portal.

$$\frac{\sigma_{\mathfrak{s}}}{\sigma_h} \propto \frac{(4\pi)^4 \lambda_{H\mathcal{S}}^3}{\mathfrak{g}_{\rm DC}^8}$$

An high-quality axion model

M. Ardu, L. Di Luzio, G. L., A. Strumia, D. Teresi and J.-W. Wang [2007.12663 (2020)]

Axion overview

- CP-violation in QCD $\mathcal{L}_{\text{QCD}} \supset \theta G\tilde{G} \to$ neutron EDM $d_n \propto \theta$ but $d_n^{\text{exp}} < 10^{-26} e\cdot$ cm $\Rightarrow |\theta| \lesssim 10^{-10} \Rightarrow$ **Strong CP-problem**
- **Peccei-Quinn** solution : a new chiral $\mathrm{U}(1)_{\mathrm{PQ}}$ global symmetry spontaneously broken (at f_a) and anomalous.
  - new light pseudoscalar particle, the *axion*, Goldstone of $\mathrm{U}(1)_{\mathrm{PQ}}$
  - the axion potential $V_{\mathrm{QCD}}(a) \sim \Lambda^4_{\mathrm{QCD}} \cos(a/f_a + \theta)$ is minimized for

$$\langle a/f_a\rangle_{\mathrm{QCD}} + \theta = 0 \Rightarrow \text{no CP violation!}$$

- New Physics (e.g gravity) breaks $\mathrm{U}(1)_{\mathrm{PQ}}$ in the UV
  - extra contribution $\Delta V_{\mathrm{UV}}(a) \Rightarrow$ new minimum at $\langle a/f_a\rangle_{\mathrm{UV}}$
  - We must assure $\langle a/f_a\rangle_{\mathrm{UV}} + \theta \lesssim 10^{-10} \Rightarrow$ **PQ-quality problem**

We parametrize NP as non-renormalizable operators suppressed by $\Lambda_{\mathrm{UV}} = M_{\mathrm{Pl}}$

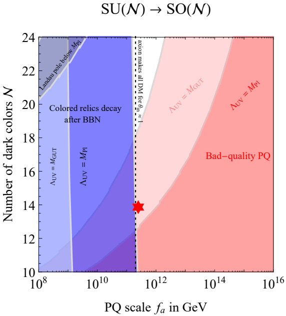
The model

- $\mathrm{SU}(\mathcal{N})$ gauge group with a scalar field $\mathcal{S}$ in the symmetric representation (singlet under SM)
- new fermions provide an **accidental** $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry at renormalizable level
- $\langle \mathcal{S} \rangle$ breaks $\mathrm{SU}(\mathcal{N}) \times \mathrm{U}(1)_{\mathrm{PQ}} \to \mathrm{SO}(\mathcal{N})$
- axion field $\mathcal{S} \sim (w + \mathfrak{s}) e^{ia/f_a}$ with $f_a = w/\sqrt{2\mathcal{N}}$
- an *accidental* SU group parity is preserved by SO dynamics

Field name	Lorentz spin	Gauge symmetries				Global accidental symmetries		
		$\mathrm{U}(1)_Y$	$\mathrm{SU}(2)_L$	$\mathrm{SU}(3)_c$	$\mathrm{SU}(\mathcal{N})$	$\mathrm{U}(1)_{\mathrm{PQ}}$	$\mathrm{U}(1)_{\mathcal{Q}}$	$\mathrm{U}(1)_{\mathscr{L}}$
$\mathcal{S}$	0	0	1	1	$\mathcal{N}\mathcal{N}$	$+1$	0	0
$\mathcal{Q}_L$	1/2	$+Y_{\mathcal{Q}}$	1	3	$\mathcal{N}$	$+1/2$	$+1$	0
$\mathcal{Q}_R$	1/2	$-Y_{\mathcal{Q}}$	1	$\bar{3}$	$\mathcal{N}$	$+1/2$	-1	0
$\mathscr{L}_L^{1,2,3}$	1/2	$+Y_{\mathscr{L}}$	1	1	$\bar{\mathcal{N}}$	$-1/2$	0	$+1$
$\mathscr{L}_R^{1,2,3}$	1/2	$-Y_{\mathscr{L}}$	1	1	$\bar{\mathcal{N}}$	$-1/2$	0	-1

Constraints

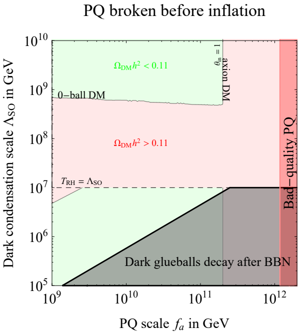
- The first $\mathrm{U}(1)_{\mathrm{PQ}}$-breaking operator arises at dimension $\mathcal{N}$: $\det \mathcal{S}$.
  $$\Delta V_{\mathrm{UV}}(a) \sim \Lambda_{\mathrm{UV}}^4 \left(\frac{f_a}{\Lambda_{\mathrm{UV}}}\right)^{\mathcal{N}} \cos\left(\frac{a}{2f_a}\right)$$
  PQ-quality problem solved if $f_a \lesssim \frac{\Lambda_{\mathrm{UV}}}{\sqrt{\mathcal{N}}} \left(\frac{\Lambda_{\mathrm{QCD}}}{\Lambda_{\mathrm{UV}}}\right)^{4/\mathcal{N}} \times 10^{-10/\mathcal{N}}$
- Colored relics ($\mathcal{Q}$ fermions) must decay before BBN
- No Landau Poles below M_{Pl}

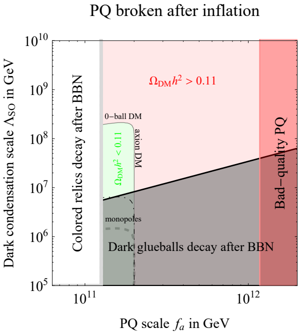


DM phenomenology

The model predicts multicomponent DM

- axions produced though the vacuum realignment mechanism
- composite bound states (glue-balls) odd under the accidental group parity are produced with thermal abundance set by $\langle \sigma v \rangle_{\rm ann} \sim 1/\Lambda^2_{\rm SO}$



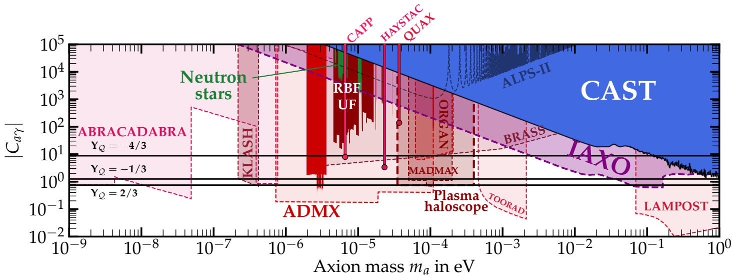


Axion-photons coupling

$$\mathcal{L}_{\rm eff} = \frac{\alpha_{\rm em} C_{a\gamma}}{8\pi f_a} a F^{\mu\nu} \tilde{F}_{\mu\nu}$$

$$C_{a\gamma} = 6(Y_{\mathcal{Q}}^2 - Y_{\mathscr{L}}^2) - 1.92(4)$$

- $Y_{\mathscr{L}} = 0$ is needed to avoid charged relics;
- $Y_{\mathcal{Q}} = \frac{\{2,-1,-4\}}{3}$ is required to allow $\mathcal{Q}$ decays (avoid colored relics).



Conclusions

Concluding remarks

- A new dark gauge sector provides a set of accidental global symmetries which lead to automatically stable DM candidates;
- we claimed a duality among the Higgs/confined phases of scalar gauge theories with a fundamental scalar. These models also provide accidental composite DM.
- a $\mathrm{SU}(N)$ gauge model with one scalar field and new fermions provides accidental PQ symmetry automatically preserved by higher dimensional operators as well as multicomponent DM.

Backup slide: Higher-order representations

The claimed duality holds for scalars in the fundamental representation.

What about higher order representations?

We considered a scalar field in a two-index representation (symmetric, antisymmetric, adjoint) of G.

Stable DM candidates in the Higgsed phase: $\mathrm{SU}(\mathcal{N})$

Unbroken phase		Broken phase: perturbative				Broken condensed
scalar rep $\mathcal{S}$	accidental global	unbroken gauge $\mathcal{H}$	accidental global $\mathcal{H}$	massive vectors	massive scalars	Dark Matter
symmetric	$\mathrm{U}(1)_{\mathcal{S}}$	$\mathrm{SU}(\mathcal{N}-1)$	U(1)	$\mathcal{W}$, $\mathcal{Z}$	$\mathfrak{s}$, $\tilde{\mathcal{S}}$	$\mathcal{B} \sim \mathcal{W}^{\mathcal{N}-1}$, $\tilde{\mathcal{S}}$
		$\mathrm{SO}(\mathcal{N})$	$\mathcal{P}_{\mathrm{U}}, \mathcal{C}$	$\mathcal{W}$	$\mathfrak{a}$, $\mathfrak{s}$, $\tilde{\mathcal{S}}$	$\mathfrak{a}$ + 0-ball if $\mathcal{N}$ even
antisymm	$\mathrm{U}(1)_{\mathcal{S}}$	$\mathrm{SU}(\mathcal{N}-2) \otimes \mathrm{SU}(2)$	U(1)	$\mathcal{W}$, $\mathcal{Z}$	$\mathfrak{s}$, $\tilde{\mathcal{S}}$	$\mathcal{B} \sim \mathcal{W}^{\mathcal{N}-2}$; $\mathcal{B}\mathcal{B} \sim \mathcal{W}^{2(\mathcal{N}-2)}$
		$\mathrm{Sp}(\mathcal{N})$	$\mathcal{C}$	$\mathcal{W}$	$\mathfrak{a}$, $\mathfrak{s}$, $\tilde{\mathcal{S}}$	$\mathfrak{a}$
		$\mathrm{Sp}(\mathcal{N}-1)$	$\mathcal{C}$, U(1)	$\mathcal{W}$, $\mathcal{Z}$, $\mathcal{X}$	$\mathfrak{s}$, $\tilde{\mathcal{S}}$	$\mathcal{Z}$, $\mathcal{M} \sim \mathcal{X}\gamma_{\mathcal{N}-1}\mathcal{X}$
adjoint	–	$\mathrm{SU}(\mathcal{N}-1) \otimes \mathrm{U}(1)$	–	$\mathcal{W}^{\pm}$	$\mathfrak{s}$, $\tilde{\mathcal{S}}$	$\mathcal{B} \sim \mathcal{W}^{\mathcal{N}-1}$

Stable DM candidates in the confined phase: $\mathrm{SU}(\mathcal{N})$

Representation	$\mathrm{SU}(\mathcal{N})$ even	$\mathrm{SU}(\mathcal{N})$ odd even
symmetric	$\epsilon(\mathcal{G}\mathcal{S})^{\mathcal{N}/2}$, $\epsilon\epsilon\mathcal{S}^{\mathcal{N}}$	$\epsilon\epsilon\mathcal{S}^{\mathcal{N}}$
anti-symmetric	$\epsilon\mathcal{S}^{\mathcal{N}/2}$, $\epsilon\epsilon\mathcal{S}^{\mathcal{N}}$	$\epsilon\epsilon\mathcal{S}^{\mathcal{N}}$
adjoint	—	—

We found *no general dualities*!

Backup slide: axion cosmology

$$\ddot{a} + 3H\dot{a} + m_a^2 a = 0$$

- early times $T \gg m_a$: the axion field is frozen at constant value $a_{\rm in} = f_a \theta_{\rm in}$
- late times $T \lesssim m_a$: the axion oscillates toward the minimum of its potential with frequency $\omega_a = m_a \to$ **cold dark matter** equation of state

$$\frac{\Omega_a h^2}{0.12} \approx \theta_{\rm in}^2 \left(\frac{f_a}{2.0 \times 10^{11}\ \text{GeV}} \right)^{7/6}$$