Lepton Masses and Mixing from Residual Modular Symmetries

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Takeaways/Outline

1. Flavor structure may be a hint for new physics
2. Modular symmetry is a predictive framework for describing flavor
3. Hierarchical mass patterns arise naturally within this framework [Penedo, Petcov, PN, 2102.07488]

Flavor structure:
hint for new physics

Flavor structure of the Standard Model

flavors ∼ families, generations: $q_i \quad u^c_i \quad d^c_i \quad l_i \quad e^c_i$

The family problem occurs when three generations
have to live together

[A. Zee, "Group theory in a nutshell for physicists"]

charged leptons

$e^c_i M^e_{ij} l_j$

$\downarrow V_e$

$$\begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

$V_e^\dagger V_\nu$

$\theta_{ij}, e^{i\delta}$

neutrinos

$l_i M^\nu_{ij} l_j$

$\downarrow V_\nu$

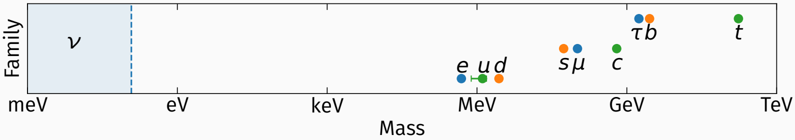
$$\begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}$$

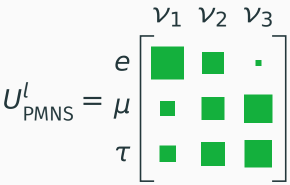
Three generations living together

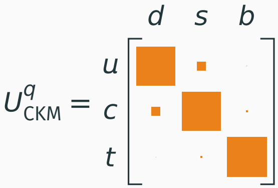
quarks: m_d m_u m_s m_c m_b m_t θ^q_{12} θ^q_{23} θ^q_{13} δ^q

leptons: m_e m_1 m_μ m_2 m_τ m_3 θ^l_{12} θ^l_{23} θ^l_{13} δ^l $\left(\alpha^l_{21}\, \alpha^l_{31}\right)$

bosons: g_1 g_2 g_3 v_H λ_H







Flavor symmetry approach

Flavor symmetry group G mixing different generations:

$$\phi_i \to U_{ij}\,\phi_j$$

Froggatt-Nielsen:

$$M \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^1 \\ \epsilon^2 & \epsilon^1 & 1 \end{pmatrix}$$

☺ hierarchies
☹ predictivity

Discrete symmetries:

$$U \simeq \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

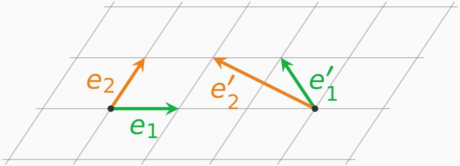
☺ mixing
☹ symmetry breaking

Modular symmetry as a flavor symmetry

Modular group

Modulus: $\tau = \frac{e_2}{e_1} \in \mathbb{C}$

$$\tau' = \frac{e'_2}{e'_1} = \frac{a\tau + b}{c\tau + d}$$



Modular group: $\Gamma = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad \begin{matrix} a, b, c, d \in \mathbb{Z}, \\ ad - bc = 1 \end{matrix} \right\}$

Flavors mix and rescale: $\phi'_i = \boxed{(c\tau + d)^{-k}} \; \boxed{\rho\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]_{ij}} \; \phi_j$

weight $k \in \mathbb{Z}$

unitary representation of Γ

Modular invariance

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}: \qquad \tau \to \frac{a\tau + b}{c\tau + d} \qquad \phi_i \to (c\tau + d)^{-k} \rho\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]_{ij} \phi_j$$

$$Y(\tau) \qquad \phi_1 \phi_2 \ldots \phi_n$$

$$k = k_1 + k_2 + .. + k_n$$

$$\rho \otimes \rho_1 \otimes \rho_2 \otimes \ldots \otimes \rho_n \supset \mathbf{1}$$

$$(c\tau + d)^k \rho \qquad (c\tau + d)^{-(k_1 + k_2 + \ldots + k_n)} \rho_1 \otimes \rho_2 \otimes \ldots \otimes \rho_n$$

× itself

$$Y_i\left(\frac{a\tau + b}{c\tau + d}\right) = (c\tau + d)^k \rho\left[\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right]_{ij} Y_j(\tau)$$

couplings are modular forms

Modular invariance: example

- Two neutrinos $\nu_i\,,\, i=1,2:\quad k=-1,\quad \rho=\mathbf{2}$
- Mass term $\nu_i\nu_j:\quad \boxed{k=-2},\quad \rho=\mathbf{2}\otimes\mathbf{2}=\mathbf{1}\oplus\mathbf{1'}\oplus\boxed{\mathbf{2}}$

$$(\nu\nu)_{\mathbf{1}}=\nu_1\nu_1+\nu_2\nu_2 \qquad (\nu\nu)_{\mathbf{1'}}=\nu_1\nu_2-\nu_2\nu_1=0 \qquad (\nu\nu)_{\mathbf{2}}=\begin{pmatrix}\nu_2\nu_2-\nu_1\nu_1\\ \nu_1\nu_2+\nu_2\nu_1\end{pmatrix}$$

- Modular forms $Y(\tau):\quad \boxed{k=2},\quad \rho=\boxed{\mathbf{2}}\oplus\mathbf{3'}$

$$\left[\begin{pmatrix}Y_1(\tau)\\ Y_2(\tau)\end{pmatrix}_{\mathbf{2}}\otimes\begin{pmatrix}\nu_2\nu_2-\nu_1\nu_1\\ \nu_1\nu_2+\nu_2\nu_1\end{pmatrix}_{\mathbf{2}}\right]_{\mathbf{1}}$$

$$=Y_1(\tau)\left(\nu_2\nu_2-\nu_1\nu_1\right)+Y_2(\tau)\left(\nu_1\nu_2+\nu_2\nu_1\right)$$

$$=(\nu_1\,\nu_2)\begin{pmatrix}-Y_1(\tau) & Y_2(\tau)\\ Y_2(\tau) & Y_1(\tau)\end{pmatrix}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

Modular invariant model of lepton flavor

Choose field representations and weights → mass matrices:

$$M_\nu = g_1 \begin{pmatrix} 2\,Y_1(\tau) & 0 & 0 \\ 0 & \sqrt{3}\,Y_2(\tau) & -Y_1(\tau) \\ 0 & -Y_1(\tau) & \sqrt{3}\,Y_2(\tau) \end{pmatrix}$$

$$+\, g_2 \begin{pmatrix} 0 & -Y_4(\tau) & Y_5(\tau) \\ -Y_4(\tau) & -Y_3(\tau) & 0 \\ Y_5(\tau) & 0 & Y_3(\tau) \end{pmatrix}$$

$$\chi^2\,(\tau, g_1, g_2, \alpha) \to \min$$

$$M_e = \alpha \begin{pmatrix} 0 & Y_5(\tau) & -Y_4(\tau) \\ -Y_5(\tau) & 0 & Y_3(\tau) \\ Y_4(\tau) & -Y_3(\tau) & 0 \end{pmatrix}$$

Flavor patterns are unexplained

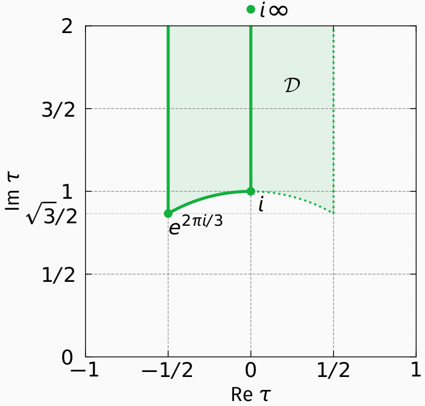
Flavor patterns from residual modular symmetries

Residual symmetries

$$\tau \sim \frac{a\tau + b}{c\tau + d} \quad \Rightarrow \quad \tau \in \text{fundamental domain } \mathcal{D}$$

Symmetric points:

τ_{sym}	inv. under	group
i	$\tau \to -\frac{1}{\tau}$	$\mathbb{Z}_2$
$e^{2\pi i/3}$	$\tau \to -\frac{1}{\tau+1}$	$\mathbb{Z}_3$
$i\infty$	$\tau \to \tau + 1$	$\mathbb{Z}$



Key idea: some flavor observables may vanish as $\tau \to \tau_{\text{sym}}$

Hierarchical mass matrices

$\epsilon \equiv |\tau - \tau_{\text{sym}}|$

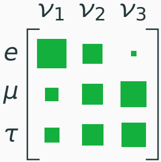
$\phi_i^c \, M_{ij}(\tau) \, \phi_j$

$$\left.\begin{matrix} \phi_i \sim \mathbf{3} & \rightsquigarrow \mathbf{1}_0 \oplus \mathbf{1}_2 \oplus \mathbf{1}_3 \\ \phi_i^c \sim \mathbf{3}' & \rightsquigarrow \mathbf{1}_0 \oplus \mathbf{1}_1 \oplus \mathbf{1}_4 \end{matrix}\right\} \Rightarrow M(\tau) \sim \begin{pmatrix} 1 & \epsilon^4 & \epsilon \\ \epsilon^3 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & \epsilon & \epsilon^3 \end{pmatrix} \Rightarrow \begin{matrix} m_1 \sim \epsilon^4 \\ m_2 \sim \epsilon \\ m_3 \sim 1 \end{matrix}$$

Key results:

- Hierarchical masses are possible
- List of field representations which yield them

Large mixing in the symmetric limit



$$\xrightarrow{\tau \to \tau_{\text{sym}}} \begin{bmatrix} * & * & 0 \\ * & * & * \\ * & * & * \end{bmatrix} \quad \text{or} \quad \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

[Reyimuaji, Romanino, 1801.10530]

1. $\begin{cases} l \rightsquigarrow 1 \oplus 1 \oplus 1 \\ e^c \rightsquigarrow 1 \oplus \dots \end{cases}$
2. $\begin{cases} l \rightsquigarrow \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}^* \\ e^c \rightsquigarrow \mathbf{1}^* \oplus \dots \end{cases}$

Other options:

3. $m_e, m_\mu, m_\tau \to 0$
4. $m_1^\nu, m_2^\nu, m_3^\nu \to 0$

Example model

- hierarchical masses
- large mixing
- predictivity

→

$$e^c \sim (\hat{\mathbf{3}}, 4) \qquad \epsilon \simeq \left|\tau - e^{2\pi i/3}\right|$$
$$\nu^c \sim (\mathbf{3}', 1)$$
$$l \sim (\hat{\mathbf{1}} \oplus \hat{\mathbf{1}} \oplus \hat{\mathbf{1}}', 2)$$

$$M_e \propto \begin{pmatrix} 1 & \alpha - 2\beta & 2\sqrt{3}i\gamma \\ \sqrt{3}\epsilon & \sqrt{3}(\alpha + 2\beta)\epsilon & 2i\gamma\epsilon \\ \frac{5}{2}\epsilon^2 & \left(\frac{5}{2}\alpha - \beta\right)\epsilon^2 & -\frac{5}{\sqrt{3}}i\gamma\epsilon^2 \end{pmatrix}$$

$$m_e = \mathcal{O}(\epsilon^2)$$
$$m_\mu = \mathcal{O}(\epsilon)$$
$$m_\tau = \mathcal{O}(1)$$

$$M_\nu \propto \epsilon \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & a \\ 1 & a & 2i\sqrt{\frac{2}{3}}b \end{pmatrix}$$

$\epsilon \simeq 0.02$ $\quad$ $\alpha = 2.45 \pm 0.44$
$a = 1.5 \pm 0.15$ $\quad$ $\beta = 2.14 \pm 0.32$
$b = 2.22 \pm 0.17$ $\quad$ $\gamma = 0.91 \pm 0.05$

$\Sigma m^\nu \approx 0.059$ eV $\quad \delta = \pi + \mathcal{O}(10^{-6}) \quad |\langle m \rangle| = (1.44 \pm 0.33)$ meV

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Thank you!