Integrable open quantum systems

Chiara Paletta

Trinity College Dublin

palettac@maths.tcd.ie

arXiv:2101.08279 with Marius de Leeuw & Balázs Pozsgay

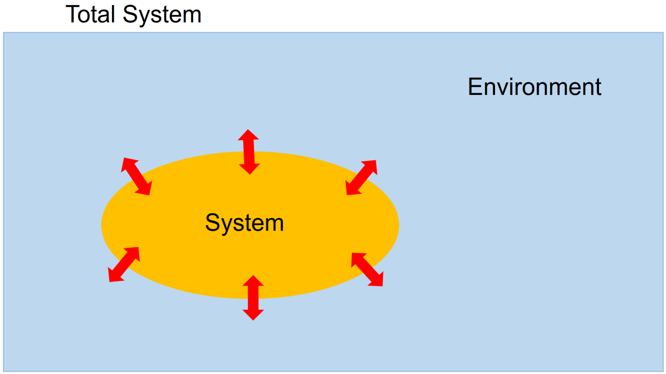
Cortona Young 2021

June 11, 2021

General picture

In nature, the interaction of a system with the environment cannot be avoided.

Closed quantum systems are an idealization of the real ones.



Main goal: Understand the dynamics of a system in touch with an environment, from the EoM of the total system.

[Manzano, 2020; Manzano, Hurtado, 2018; Petruccione, Breuer, 2002, Medvedyeva, Essler, Prosen, 2016]

Idea:

We are looking for a simple way to remove the environment from the EoM

$\rightarrow$ Approximations!

E.g. Weak coupling to the environment can have interesting effects to the dynamics of an open system: it can result in Non-equilibrium steady states.

Context:

Quantum optics, Condensed matter, atomic physics, quantum information, quantum biology, quantum circuits, ...

Mathematical tool: density matrix

Isolated systems: States are unit vector of $\mathcal{H}$. Pure states $|\psi\rangle$

Open system: Ensemble of pure states: $\{|\psi_i\rangle, p_i\}$. Mixed states ρ

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i| \qquad \text{Density matrix}$$

How do quantum states evolve?

Pure states:

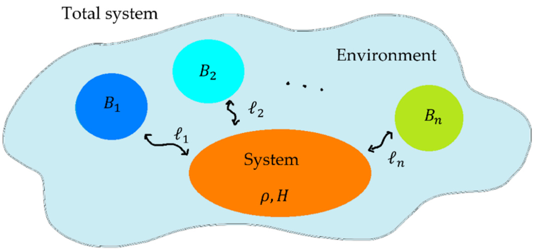
$$\frac{d}{dt}|\psi(t)\rangle = -iH|\psi(t)\rangle \qquad \text{Schrödinger equation}$$

Mixed states:

$$\dot{\rho} = i[\rho, H] \qquad \text{von-Neumann equation}$$

How does ρ for an open quantum system evolve?

Lindblad Master equation



$$\mathcal{H}_T = \mathcal{H} \otimes \mathcal{H}_E$$

$$H_T = H \otimes 1_E + 1 \otimes H_E + \alpha H_I$$

Taking the partial trace of $\dot{\rho}_T = i[\rho_T, H_T]$, remove the environment d.o.f.

Approximation

- weak coupling regime $\alpha \ll 1$,
- Markovian approximation $\tau_{environment} \ll \tau_{system}$

Lindblad equation

$$\underbrace{\dot{\rho} = i\,[\rho, H]}_{\text{Liouville equation}} + \underbrace{\sum_{a=1}^{n} \left[\ell_a \rho \ell_a^\dagger - \frac{1}{2}\{\ell_a^\dagger \ell_a, \rho\} \right]}_{\text{Dissipator}}$$

Approaches to solve open system

Open systems are in general **hard to solve**.

Progress made by employing approximative methods:

- Numerical methods
- Perturbative methods

Wish: Find exactly *solvable* cases: *Yang Baxter Integrable Lindblad systems*

Reasons: The out of equilibrium dynamics can be studied:

- the Non-Equilibrium steady states can be constructed with exact method,
- the generator of the dynamics can be diagonalized.

Spin Ladders construction

Spin 1/2 chain of length L, $H = \sum_{i=1}^{L} h_{i,i+1}$ and $h_{L,L+1} = h_{L,1}$.

Lindblad equation

$$\dot{\rho} = i\,[\rho, H] + \sum_{a=1}^{n} \left[\ell_a \rho \ell_a^\dagger - \frac{1}{2}\{\ell_a^\dagger \ell_a, \rho\} \right],$$

$\ell_a = \ell_{i,i+1}$, ρ, H and $\ell \in \ \mathcal{H}$

Write the Lindblad equation in the form $\dot{\rho} \equiv \mathcal{L}\rho$

Superoperator formalism

Linear map: $\underbrace{\mathcal{H}}_{|\phi_i\rangle\langle\phi_k|} \mapsto \underbrace{\text{Ket} \otimes \text{Bra}}_{|\phi_i,\phi_k\rangle\rangle}$

Lindblad equation: $\dot{\rho} \equiv \mathcal{L}\rho, \quad \mathcal{L} \in \text{Ket} \otimes \text{Bra} \equiv \mathcal{H} \otimes \mathcal{H}^* = \mathcal{H}^{(1)}\mathcal{H}^{(2)}$

Lindblad superoperator, $\mathcal{L} = \sum_j \mathcal{L}_{j,j+1}$

$$\mathcal{L}_{i,j} = -i\, h_{i,j}^{(1)} + i h_{i,j}^{(2)*} + \ell_{i,j}^{(1)} \ell_{i,j}^{(2)*} - \frac{1}{2} \ell_{i,j}^{(1)\dagger} \ell_{i,j}^{(1)} - \frac{1}{2} \ell_{i,j}^{(2)\,T} \ell_{i,j}^{(2)*}$$

Yang-Baxter integrable superoperator

Idea: Identify $\mathcal{L}$ as a (non-Hermitian) Hamiltonian of a spin ladder $\mathbb{H}$
Goal: Find models with *Yang-Baxter integrable* $\mathbb{H} = \mathcal{L}$.

Definition

$\mathcal{L}$ is one of the conserved charges of an Integrable model.

A bit more on Integrable models:

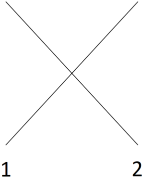
Models with a high amount of symmetry and a tower of conserved charges

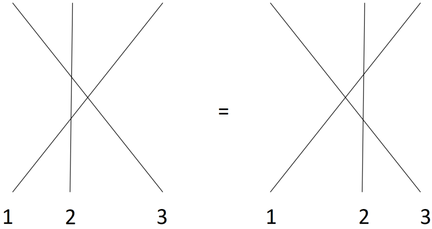
$$[\mathbb{H}, \mathbb{Q}_r] = [\mathbb{Q}_r, \mathbb{Q}_s] = 0, \quad r, s = 1, 2, \ldots, \infty$$

Integrable models are characterized by an R-matrix, solution of the *Yang-Baxter equation*.

Yang-Baxter equation

$R(u_1, u_2) =$





Yang-Baxter equation

$$R_{12}(u_1, u_2)R_{13}(u_1, u_3)R_{23}(u_2, u_3) = R_{23}(u_2, u_3)R_{13}(u_1, u_3)R_{12}(u_1, u_2)$$

$R \in V \otimes V$ solution of the YBE defines an integrable spin chain $\in \bigotimes^L V$

Construct the conserved charges

R-matrix $\rightarrow$ transfer matrix $t(u, \theta) \rightarrow$ conserved charges $[\mathbb{Q}_r, \mathbb{Q}_s] = 0$

$\mathbb{Q}_2 = \mathbb{H} = \sum_j \mathcal{H}_{j,j+1}(\theta)$, range 2

Identification: $\mathbb{Q}_2 = \mathcal{L}$

Integrable models $\rightarrow$ Eigenvalues and eigenvectors of the conserved charges can be found with exact methods (*Bethe ansatz* technique).

Typically, Integrability is broken due to the environment!
$\rightarrow$ There exist *Integrable Lindblad models*!

For these models, the non-equilibrium steady states and the relaxation toward them can be computed with exact methods.

Examples through known spin ladder system [Essler,Katsura, Medvedyeva, Prosen, Shibata, Ziolkowska]

Systematic *construction* of new integrable cases. [de Leeuw, CP, Pozsgay]

Construction of YB Integrable models

R solution of YBE

⇩

Conserved charges: momentum, Hamiltonian, ...

⇩

dynamics

We follow:

What do we do with this?

⇧

R-matrix

⇧

??? Our method

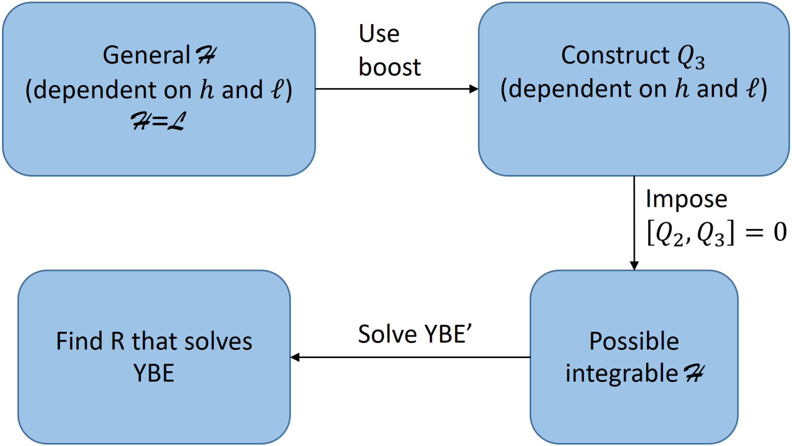
⇧

Hamiltonian

Bottom up approach: Boost automorphism mechanism

[de Leeuw, CP, Pribytok, Retore, Ryan]

Steps:



We found some known models: for example the Hubbard model with imaginary coupling and 4 new models.

New model

$$h = \frac{1}{2}\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & e^{i\phi} & 0 \\ 0 & e^{-i\phi} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \ell = \sqrt{\frac{\gamma}{2}}\begin{pmatrix} \gamma & 0 & 0 & 0 \\ 0 & 1 & i(\gamma-1)e^{i\phi} & 0 \\ 0 & -i(\gamma+1)e^{-i\phi} & -1 & 0 \\ 0 & 0 & 0 & \gamma \end{pmatrix}$$

$\phi \in \mathbb{R}, \quad \gamma \in \mathbb{R}^+$ coupling constant

$h_{j,j+1} = \frac{1}{2}\left[e^{i\phi}\sigma_j^+\sigma_{j+1}^- + e^{-i\phi}\sigma_j^-\sigma_{j+1}^+\right]$: *free-fermion* hopping model

The R-matrix is new!

Why is this model physically interesting?

Classical flow

The flow equation

$$\dot{\rho} = \mathcal{L}\rho \qquad \underbrace{\rightarrow}_{\text{projection on } \rho_{ii}} \qquad \partial_t |P\rangle = W|P\rangle$$

ρ_{ii} classical probabilities of finding the system in a given state,
$W = \sum_{j=1}^{L} w_{j,j+1}$: generator of the classical flow

[Henkel, Schütz, Vanicat]

$\gamma = 1$, diagonal preserving model: the operator space spanned by the diagonal elements is left invariant by the Lindblad superoperator.

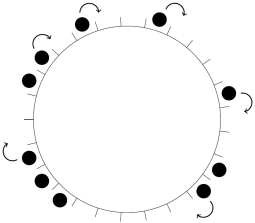
$$w_{i,i+1} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} = \sigma_i^- \sigma_{i+1}^+ - n_i(1 - n_{i+1})$$

Totally Asymmetric Simple Exclusion Process (TASEP)!

Particle current

$$J_k = (1-\gamma^2) J_k^0 + \frac{\gamma(1+\gamma)^2}{2} n_k(1-n_{k+1}) - \frac{\gamma(1-\gamma)^2}{2}(1-n_k)n_{k+1},$$

$n_k = \frac{1+\sigma_j^z}{2}$ local occupation number



Realization of the TASEP model using an integrable Lindbladian.
Embedding known in the literature. [Eisler, Essler, Piroli, Temme, Wolf, Verstraete]

New result:

Existence of YB integrable superoperator behind the classical flow.

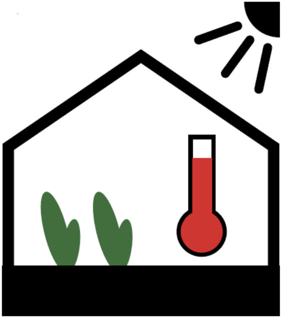
Non-equilibrium steady states $\dot{\rho}_{NESS} = \mathcal{L}\rho_{NESS} = 0,\ \rho_{NESS} = |\Psi\rangle\langle\Psi|$

Particle current in the NESS of given spin

$$\lim_{L\to\infty} \langle J_k \rangle = (1+\gamma^2)\,\langle n \rangle \left(1 - \langle n \rangle\right)$$

Even if $\gamma \to 0$, there is a finite particle current: pumping effect

This phenomenon was discussed in Nature Communication. [Lange, Lenarčič, Rosch]



We found a solvable example for this.

Summary

We give a structured approach to construct Yang-Baxter integrable Lindblad systems.

We gave a new model with a coupling constant.
Physically interesting: support classical flows on the diagonal of the density matrix, solvable example of the *pumping effect*.

Future work

- Find the Bethe ansatz solution for the new model
- Apply the method to $\mathcal{L}$ acting on more than 2 sites of the spin chain
- Construct *all* integrable diagonal preserving models
- Explore space of solutions with more than one family of jump operators
- Study open integrable quantum chains

Thank you