The Cardy-like limit of the $\mathcal{N}=1$ superconformal index

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Based on 2012.15208 & 2103.15853
with Antonio Amariti (INFN Milano) and Alessia Segati (Milano Uni)

[see also Cassani-Komargodski 2104.01464
& Arabi Ardehali-Murthy 2104.13932]

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Here focus on d=4 (field theory dimension).
A lot of work in d=3, some results in d=5,6

This talk:

Black hole entropy & superconformal index

New general formula for the Cardy-like limit

Examples (including & beyond $\mathcal{N}$=4 super-YM)

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“1/16-BPS”: 2 out of 32 supercharges

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3 electric charges q_a
2 angular momenta j_i
(constrained by susy)

[Gutowski-Reall,…]

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Ensemble of states in dual $\mathcal{N}=4$ on $\mathbb{R} \times S^3$?

Susy & charges suggest BH entropy should be given by

1/16-BPS states of $\mathcal{N}$=4 super-YM w/ **spins j_i** & U(1) **charges q_a**

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of d=4 $\mathcal{N}$=4 SYM at **large N** ($\sim 1/\sqrt{G_{\text{Newton}}}$) & for **complex chemical potentials**

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associated w/ charge operators Q_a, a=1,2,3

real order-N^2 quantity ~ **$a_{\mathcal{N}=4}$ central charge**

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counts 1/16-BPS states w/ **sign**: different from counting ALL 1/16-BPS states (entropy: hard problem)

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large-N limit from exact evaluation of superconformal index via “Bethe Ansätz” formula $I_{sc} = \sum Z\, H^{-1}$

[Closset-Kim-Willett, Benini-Milan, Benini-Colombo-Soltani-Zaffaroni-Zhang]

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“Cardy-like limit”: small-$|\omega_i|$ limit from matrix model representation of superconformal index

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[Amariti-MF-Segati '21]

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Finite **log Γ_Z correction**: **minimal charge** of matter **under center *Z(G)***
(order of character lattice modulo Weyl symmetry)

explains & expands previous results

$\mathcal{N}$**=4 super-YM** (adj matter): Γ_Z = dim $Z(G)$

[SU: GonzalezLezcano-Hong-Liu-PandoZayas,
USp, SO: Amariti-MF-Segati '20]

toric SU(*N*) quivers (bifundamental matter): $\Gamma_Z = N$

[GonzalezLezcano-Hong-Liu-PandoZayas]

One-slide proof: index as matrix model

$$\mathcal{I}_{\rm sc}(\tau,\Delta) = \frac{(q;q)_\infty^{2\,{\rm rk}_G}}{|{\rm Weyl}(G)|}\int\prod_{i=1}^{{\rm rk}_G}du_i\,\frac{\prod_{I=1}^{n_\chi}\prod_{\rho_I}\tilde\Gamma(\rho_I(\vec u)+\Delta_I)}{\prod_\alpha\tilde\Gamma(\alpha(\vec u))} \equiv \frac{1}{|{\rm Weyl}(G)|}\int\prod_{i=1}^{{\rm rk}_G}du_i\,e^{S_{\rm eff}(\vec u;\tau,\Delta)}$$

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effective action

matter chemical potentials

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u_i gauge holonomies

ρ_I gauge weights
α gauge roots

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ν_I flavor weights;
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v_R R-sym chem potential
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B_n Bernoulli polynomials

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$$\mathcal{I}_{\rm sc}(\tau,\Delta) = \frac{(q;q)_\infty^{2\,{\rm rk}_G}}{|{\rm Weyl}(G)|}\int\prod_{i=1}^{{\rm rk}_G} du_i \frac{\prod_{I=1}^{n_\chi}\prod_{\rho_I}\tilde\Gamma(\rho_I(\vec u)+\Delta_I)}{\prod_\alpha\tilde\Gamma(\alpha(\vec u))} \equiv \frac{1}{|{\rm Weyl}(G)|}\int\prod_{i=1}^{{\rm rk}_G} du_i\, e^{S_{\rm eff}(\vec u;\tau,\Delta)}$$

effective action

$e^{2\pi i u_i}\in S^1$

u_i gauge holonomies

ρ_I gauge weights
α gauge roots

matter chemical potentials

$$\Delta_I = \nu_I(\vec\xi) + R_I v_R$$

ν_I flavor weights;
ξ flavor holonomies

v_R R-sym chem potential
R_I R-charges

EOM of matrix model $\int[du]\, e^{S_{\rm eff}}$ **at leading order**:

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B_n Bernoulli polynomials

$$\log\tilde\Gamma(u) \underset{|\tau|\to 0}{\sim} \frac{B_3(\{u\}_\tau)}{\tau^2} + \frac{B_2(\{u\}_\tau)}{\tau} + B_1(\{u\}_\tau) + \tau$$

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inequivalent ways of selecting
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plug Ansatz back into $S_{\rm eff}$ and impose **physical constraints** on matter chem. potentials Δ_I:
superconformal index computed by ***3d* pure Chern-Simons partition function** &
dependence on SCFT **central charges $a(\Delta_I)$, $c(\Delta_I)$**

$\mathcal{N}$=4 SYM: USp(2N) and SO(N) gauge groups

[Amariti-MF-Segati '20]

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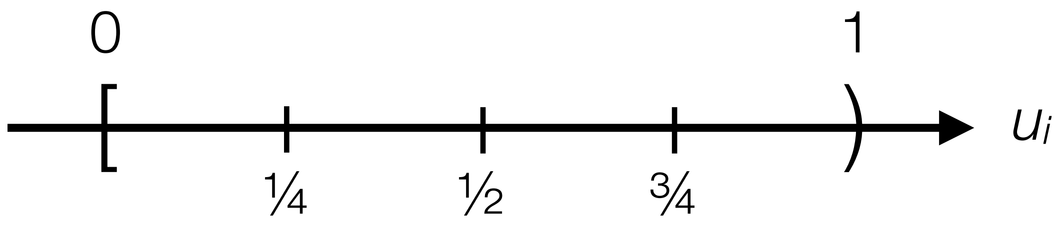
Γ_Z = dim $Z(G)$ = 2 or 4; careful analysis of saddles:

(focus on USp)



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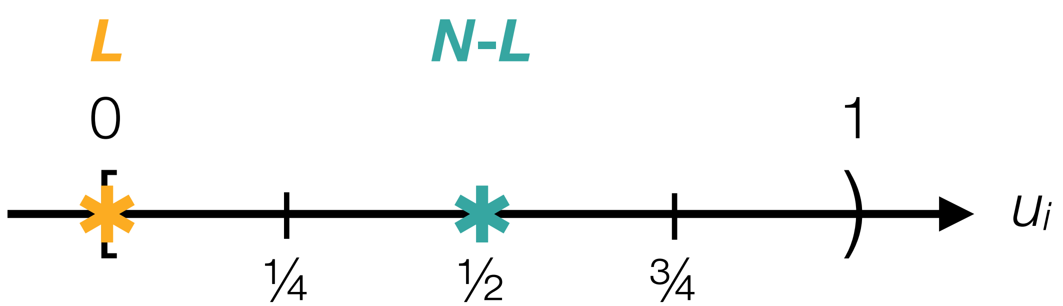

Dominant saddle in BH 'region' (constraints on Δ_I): $\boldsymbol{u_i = m/2 + v_i\,\tau}$; $\boldsymbol{m = \{0,1\}}$ $\Rightarrow$ $\Gamma_Z = 2$

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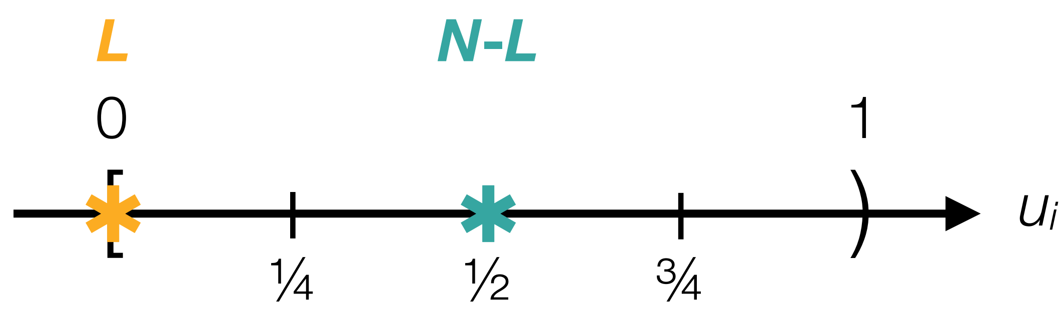

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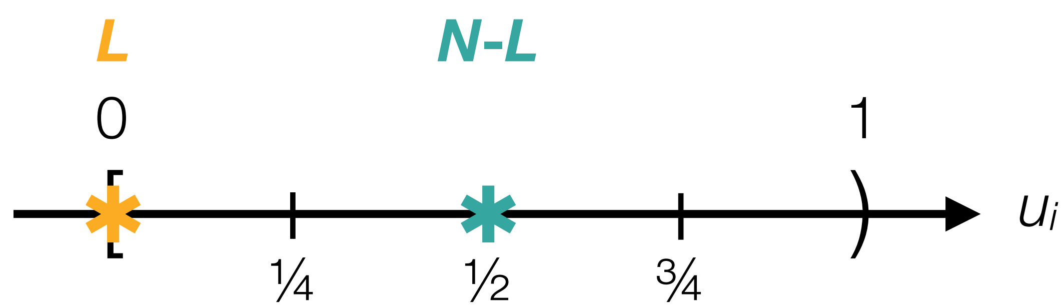

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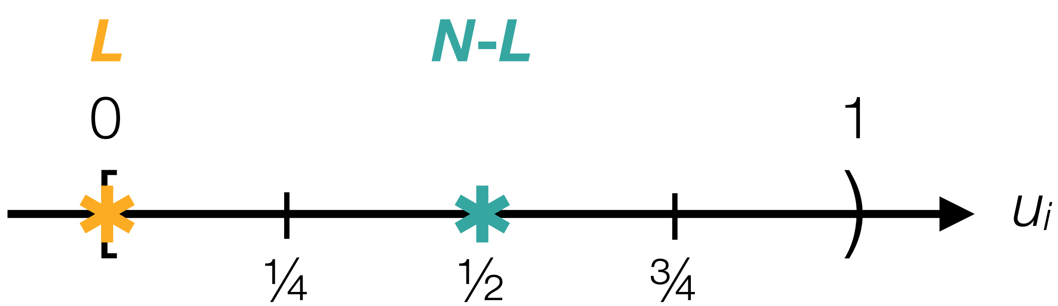

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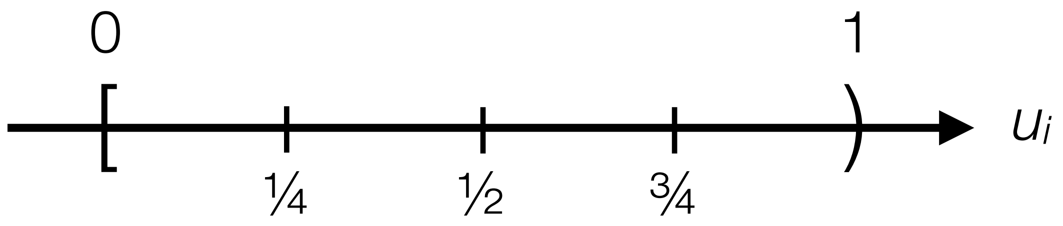

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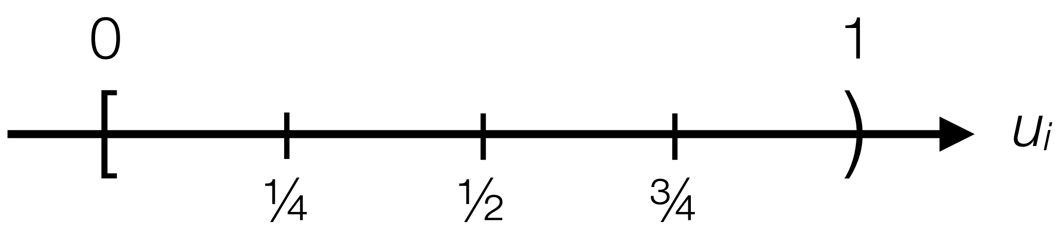

subdominant saddles:

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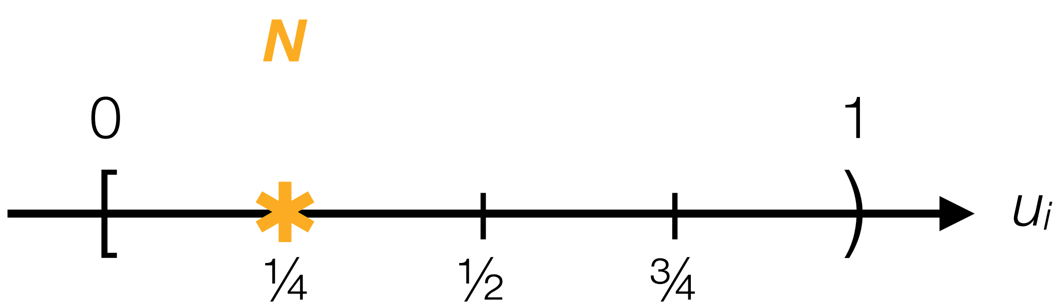

subdominant saddles: ***N* at ¼**

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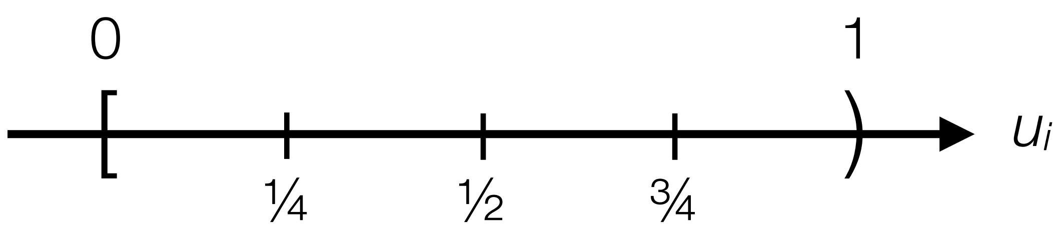

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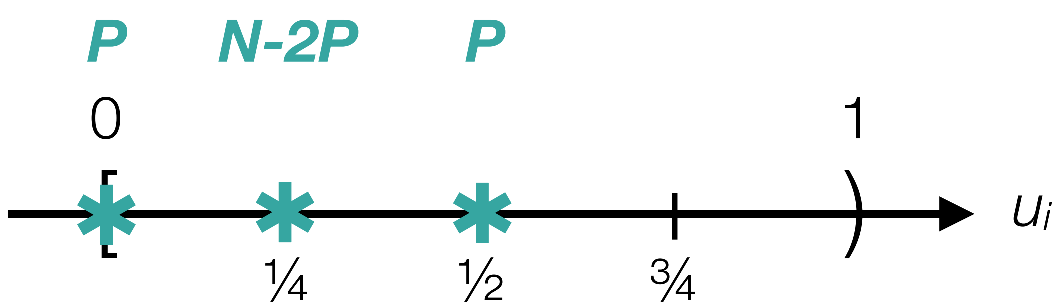

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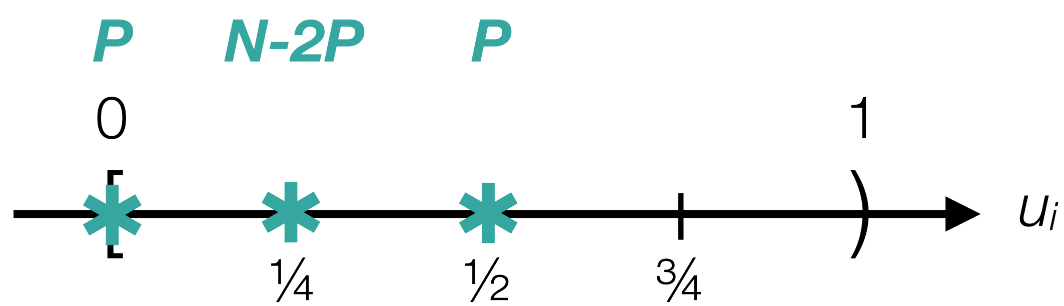

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In both cases, $\boldsymbol{S_{\mathrm{eff}}}$ (ie log index) evaluated at subdominant saddle is **subleading at large *N***:
hierarchy of saddles very important

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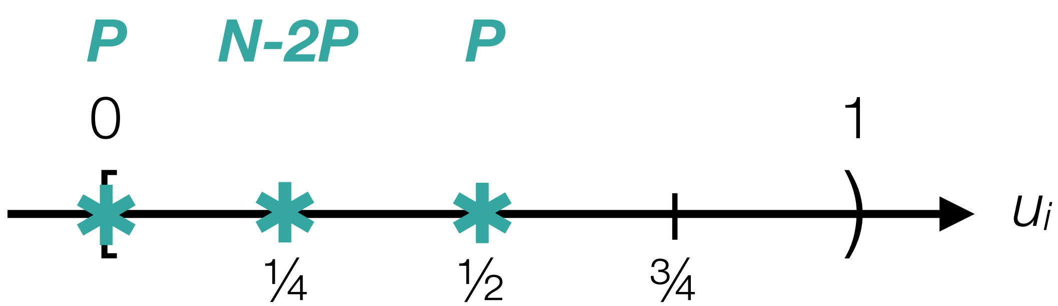

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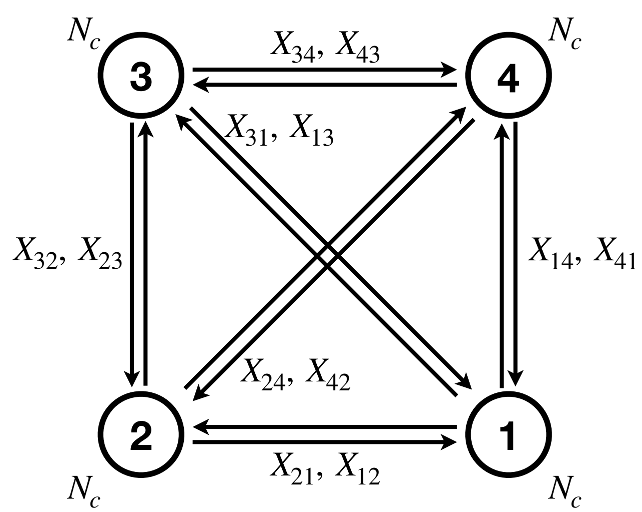
S-duality between USp and SO (identity of superconformal indices) nontrivially realized on saddles

We've looked at more complicated $\mathcal{N}$=1 models:

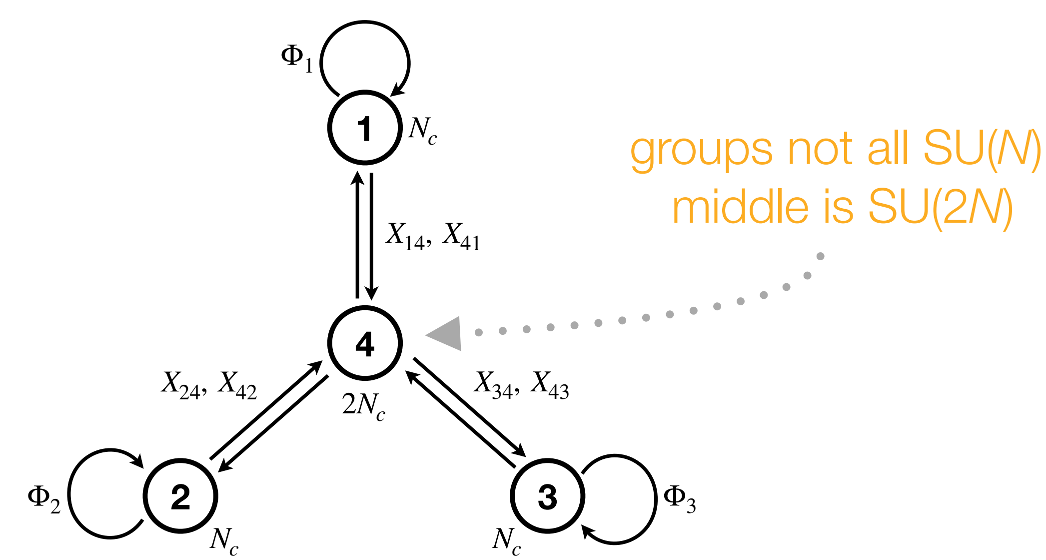
non-toric (toric: $U(1)^3$ global symmetry), not all ranks equal, non-holographic (including subleading corrections and finite terms)

[Amariti-MF-Segati '21]

N D3's probing $\mathbb{C}^3 / \mathbb{Z}_2 \times \mathbb{Z}_2$
(toric model: $U(1)^3$)



Seiberg-dual nontoric phase



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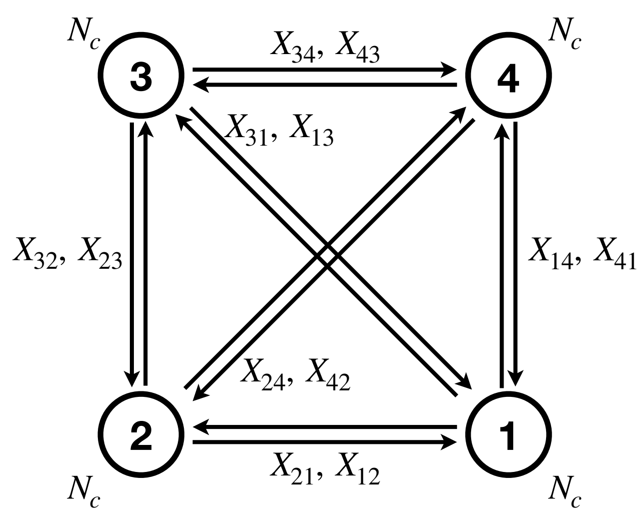
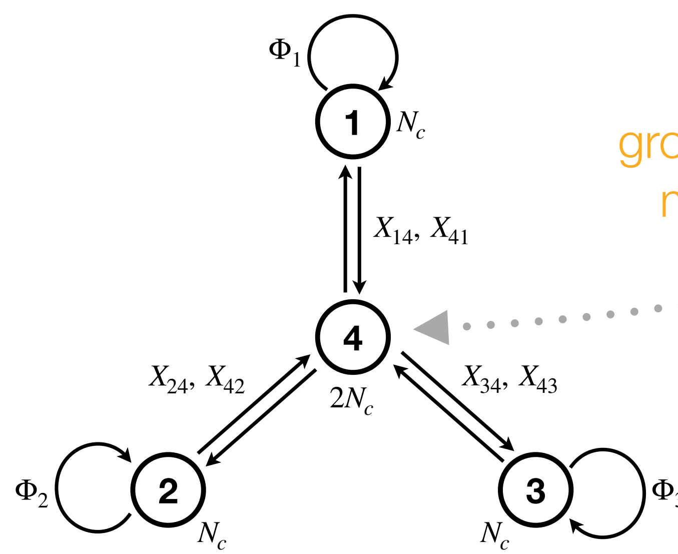


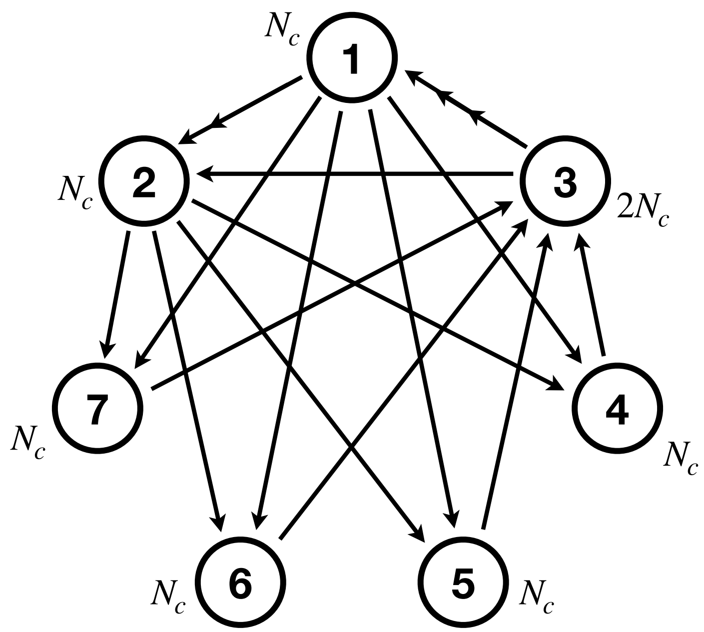


Cardy-like limit of superconformal indices computed independently in two phases from S_{eff} match precisely.
Both given by our new formula with $\Gamma_Z = N$

Nontrivial **check of validity** of our formula

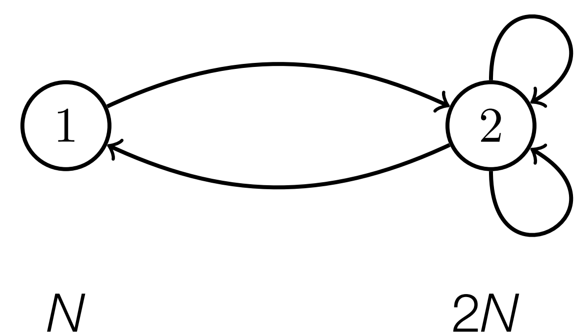
N D3's probing non-toric threefolds (& different ranks):

Cone over dP_4



4 flavor U(1)s

Laufer's theory



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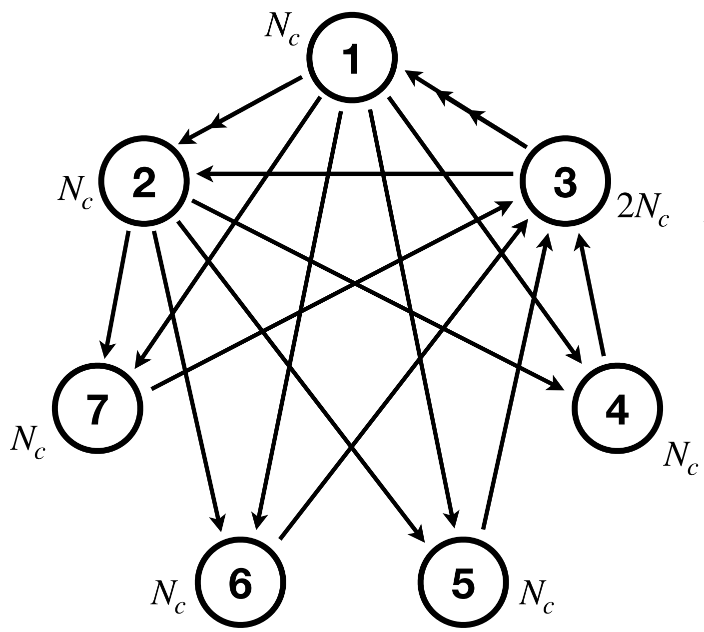

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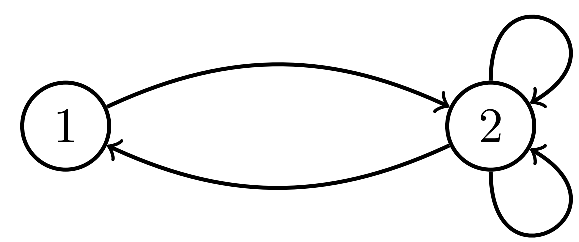

N $2N$

1 flavor U(1)

groups not all SU(N); some SU($2N$):
nontrivial 'complication'

Non-holographic $\mathcal{N}$=1 theories ($a \neq c$ at large N): SQCD

$\mathcal{N}$=2 theories

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SU(N) in conformal window

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family of $\mathcal{N}$=1 SU(n) Lagrangians

enhancing to (A_1, A_{2n-1}) Argyres-Douglas $\mathcal{N}$=2 SCFT

[Maruysohi-Song,…]

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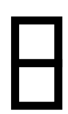
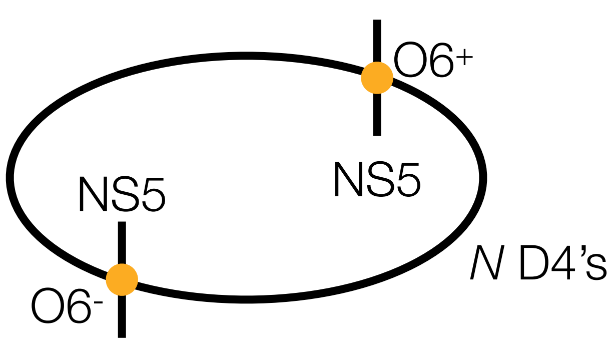
$\mathcal{N}$=2 SCFT: SU(N) w/ hypers  & 

[Ennes-Lozano-Naculich-Schnitzer]



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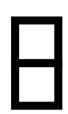
$\mathcal{N}$=2 SCFT: SU(N) w/ hypers  & 

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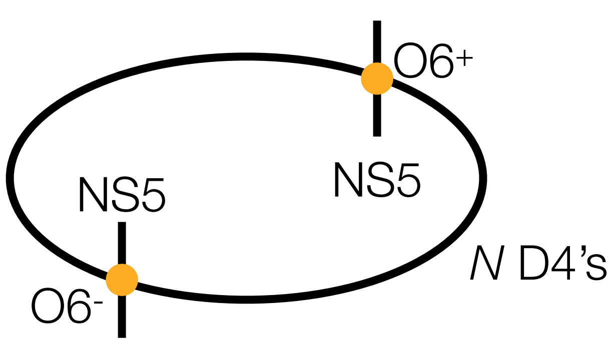

Cardy-like limit of log of SCI matches S_{BH} at large N

$$S_{\mathrm{BH}} = 2\pi \sqrt{Q_2^2 - Q_\ell^2 - Q_{\tilde{\ell}}^2 + 2Q_1(Q_2 - Q_\ell - Q_{\tilde{\ell}}) - \frac{a}{4}(J_1 + J_2)}$$

[Hosseini-Zaffaroni]

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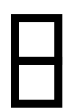
$\mathcal{N}$=2 SCFT: SU(N) w/ hypers  & 

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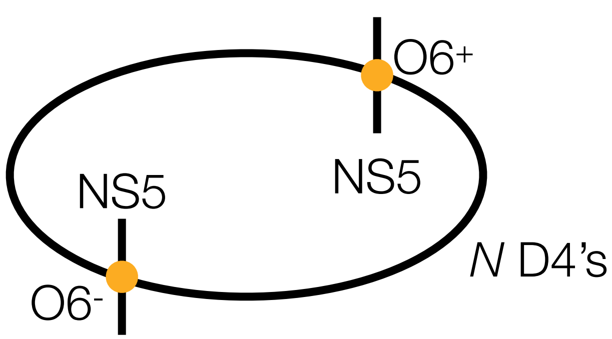

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[Hosseini-Zaffaroni]

peculiarity:
$\Gamma_Z = (3+(-1)^N)/2$ depends on parity of N
(reflected in degeneracy of saddles)

Conclusions

Formula for Cardy-like limit of superconformal index for generic $\mathcal{N}=1$ ABCD SCFTs:
extends previous results valid at lowest order and/or for non-generic theories (super-YM, toric)

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[3*d* EFT interpretation of result given by Cassani-Komargodski]

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[3*d* EFT interpretation of result given by Cassani-Komargodski]

No 'rigorous' proof but can explicitly determine leading & subleading contributions
with very general Ansatz $u_i = u_{*i} + v_i\,\tau$ for matrix model saddle point

Outlook

AdS/CFT derivation/interpretation of finite log correction
(i.e. quantum gravity corrections to asymptotically-AdS BH entropy)

[Bobev-Charles-Gang-Hristov-Reys, Bobev-Charles-Hristov-Reys for AdS_4 BHs]

‘Derivation’ of new formula from 3*d* EFT for SU case: applicable to generic $\mathscr{N}=1$ SCFTs too?

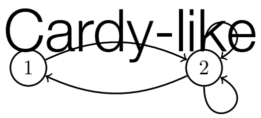
[Cassani-Komargodski for SU case]

Extend formula to 2 different angular momenta: $\omega_1 \neq \omega_2 = \tau$

(Structure of) other subleading saddles?

[ArabiArdehali-Hong-Liu, CaboBizet-Cassani-Martelli-Murthy]

Bethe Ansätz approach in generic case is *terra incognita*; match large-*N* limit to Cardy-like limit.
Very nontrivial: eg ‘basic solutions’ don’t work for Laufer SU(*N*) x SU(2*N*)



[Benini-Colombo-Soltani-Zaffaroni-Zhang for SU(*N*) holographic quivers dual to $AdS_5 \times S^5$]

Beyond τ^0, exponentially suppressed orders in τ^{-1} vs in N^{-1} from Bethe Ansätz. Match? Meaning?

[Aharony-Benini-Mamroud-Milan]

Thanks

Index as matrix model

$q = e^{i\tau}$

$$\mathcal{I}_{\rm sc}(\tau,\Delta) = \frac{(q;q)_\infty^{2\,{\rm rk}_G}}{|{\rm Weyl}(G)|}\int \prod_{i=1}^{{\rm rk}_G} du_i \frac{\prod_{I=1}^{n_\chi}\prod_{\rho_I}\tilde{\Gamma}(\rho_I(\vec{u})+\Delta_I)}{\prod_\alpha \tilde{\Gamma}(\alpha(\vec{u}))} \equiv \frac{1}{|{\rm Weyl}(G)|}\int \prod_{i=1}^{{\rm rk}_G} du_i\, e^{S_{\rm eff}(\vec{u};\tau,\Delta)}$$

$u_i \in (0,1]$

$u_i \sim u_i + 1$

u_i gauge holonomies

$(z_i = e^{2\pi i u_i} \in S^1)$

ρ_I gauge weight

$$\Delta_I = \nu_I(\vec{\xi}) + R_I v_R$$

ν_I flavor weight;
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v_R R-sym chem pot;
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For a=1,…,n_G gauge groups and I=1,…,n_χ matter fields, effective action:

$$S_{\rm eff}(\vec u;\tau,\Delta) = \sum_{I=1}^{n_\chi}\sum_{\rho_I}\log\tilde\Gamma\big(\rho_I(\vec u)+\Delta_I\big) + \sum_{a=1}^{n_G}\sum_{\alpha_a}\log\theta_0\big(\alpha_a(\vec u);\tau\big) + \sum_{a=1}^{n_G} 2\,{\rm rk}_{G_a}\log(q;q)_\infty$$

matter contribution

gauge
[$-\log\Gamma(u) = \log\theta_0(u)$]

q-Pochhammer

Saddles:

Expand all functions in S_{eff} for small τ; eg matter fields contribute as

$$\log \tilde{\Gamma}(u) \underset{|\tau|\to 0}{\sim} \frac{B_3(\{u\}_\tau)}{\tau^2} + \frac{B_2(\{u\}_\tau)}{\tau} + \frac{B_1(\{u\}_\tau)}{\tau^0} + \tau$$

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EOM of matrix model **at leading order**:

$$0 = \frac{\partial S_{\text{eff}}(\vec{u};\tau,\Delta)}{\partial u_{i_a}} = -\frac{i\pi}{\tau^2} \sum_{I=1}^{n_\chi} \sum_{\rho_I} \frac{\partial \rho_I(\vec{u})}{\partial u_{i_a}} B_2(\{\rho_I(\vec{u}) + \Delta_I\}_\tau)$$

Ansatz for saddle points of the form:

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It **captures** up to **finite terms in $\boldsymbol{\tau}$**: goes beyond preexisting results up to τ^{-2}, τ^{-1}.

Number of **inequivalent** ways of selecting **constants** u_{*i} given by $\boldsymbol{\Gamma_Z}$

Plug Ansatz back into S_{eff} and impose physical constraints on matter charges Δ_I

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Superpotential constraint:

Matter fields in each **superpotential term**

$$\sum_{I \in \mathcal{W}} \hat{\Delta}_I = 2 \quad \Rightarrow \quad \sum_{I \in \mathcal{W}} \{\Delta_I\}_\tau = 2\tau - \eta$$

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CONSEQUENCE #1: linear term in holonomies u_i in S_{eff} vanishes for all ABCD algebras

CONSEQUENCE #2: quadratic term reconstructs 3*d* G=**ABCD** pure **CS** partition function at **level -*ηT*(*G*)**

[See also Cassani-Komargodski for 3*d* EFT interpretation.
3*d* CS term previously observed in GonzalezLezcano-Hong-Liu-PandoZayas & Amariti-MF-Segati '20]

GR calculation of BH ⊂ AdS_5 x S^5 entropy:

$$S_{\rm BH}(q_a, j_i) = 2\pi\sqrt{q_1 q_2 + q_1 q_3 + q_2 q_3 - \frac{\pi}{4G_{\rm N}^{(5)} g_{\rm AdS}^3}(j_1 + j_2)}$$

$$N^2 = \frac{\pi}{2G_{\rm N}^{(5)} g_{\rm AdS}^3}$$

Asymptotically AdS$_5$ BH:

Near the horizon:

$$ds^2_{r\sim r_c} \sim -(r-r_c)^2 dt^2 + \frac{dr^2}{(r-r_c)^2} + \mathrm{const}\ ds^2_{\mathcal{M}_{d-1}}$$

Asymptotically AdS$_5$ (with $\mathbb{R} \times M_{d-1}$ conformal boundary):

$$ds^2_{r\to\infty} \sim \frac{dr^2}{r^2} + r^2(-dt^2 + ds^2_{\mathcal{M}_{d-1}}) + \dots$$