Galilelo Galilei Institute (GGI)

The Fate of long modes in Cosmology

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Cortona Young –June 11th, 2021

Outline

1) Primordial Non-Gaussianity

2) The controversy:

observability of PNG in Single-field Inflation

- The source of the problem

- Solution and consequences

[a]= Matarrese, Pilo, Rollo, 2020,

Resilience of long modes in cosmological observables

Recall of Cosmology

- **Cosmological principle:** Our Universe results to be homogeneous and Isotropic at sufficiently large scales.
- **Idea:** Homogeneous and isotropic background+ small perturbations

-Background $$ds^2 = \bar{g}_{\mu\nu}\, dx^\mu dx^\nu = -dt^2 + a(t)^2 \frac{\delta_{ij}}{(1 - x^2\chi)^2} dx^i dx^j$$

-Perturbations $$g_{\mu\nu} = \bar{g}_{\mu\nu}(t) + g^{(1)}_{\mu\nu}(t,x) + \frac{1}{2} g^{(2)}_{\mu\nu}(t,x)\ldots$$

$$g_{00} = -e^{2\Psi}, \qquad g_{0i} = a\,(\partial_i F + G_i), \qquad g_{ij} = a^2\left[e^{2\Phi}\delta_{ij} + \partial_{ij}E + \partial_j C_i + \partial_i C_j + h_{ij}\right];$$

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-Curvature perturbations: $$\boxed{R = -\Phi + H\, v,} \qquad \zeta = -\Phi + H\frac{\delta\rho}{\partial_t \rho}, \ldots$$

Statistical properties of Cosmological Perturbations

- At the linear level we have no phase-correlation among different modes…

  Gaussianity

$$< R(x_1)..R(x_{2n}) >= \sum_{Perm.} \prod_{Pairs} < R(x_i)\, R(x_j) >,$$

$$< R(x_1)..R(x_{2n+1}) > = 0.$$

- At the non-linear level we get phase-correlation…

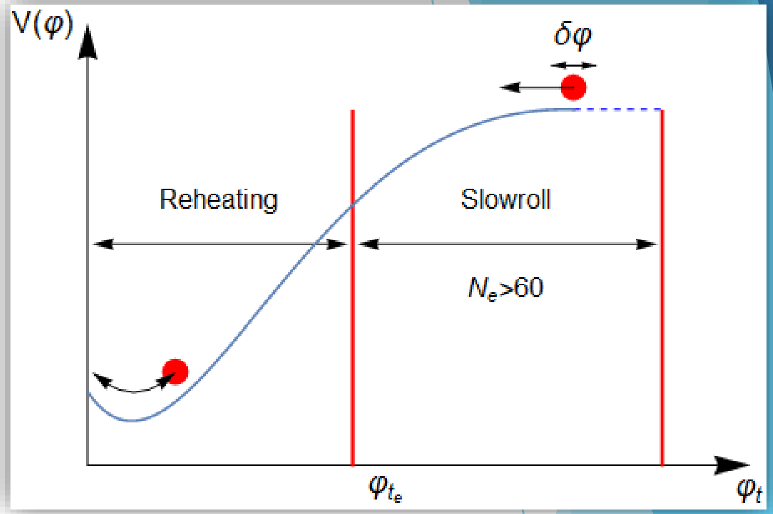
  non-Gaussianity

$$< R(x_1)..R(x_{2n+1}) > \neq 0.$$

$$< R(x_1)..R(x_{2n}) > \; - < R(x_1)..R(x_{2n}) > \Big|_G \neq 0.$$

Single-field Inflation

- Action: $S = M_{pl}^2 \int dx^4 \sqrt{-g}\left[R + \frac{1}{2}\, g^{\mu\nu}\partial_\mu\varphi\, \partial_\nu\varphi + V(\phi)\right]$
- SR limit: $\epsilon = -\frac{\dot{H}}{H^2} \ll 1 \rightarrow p \approx -\rho, \qquad \eta = \frac{\dot{\epsilon}}{\epsilon H} \ll 1,$



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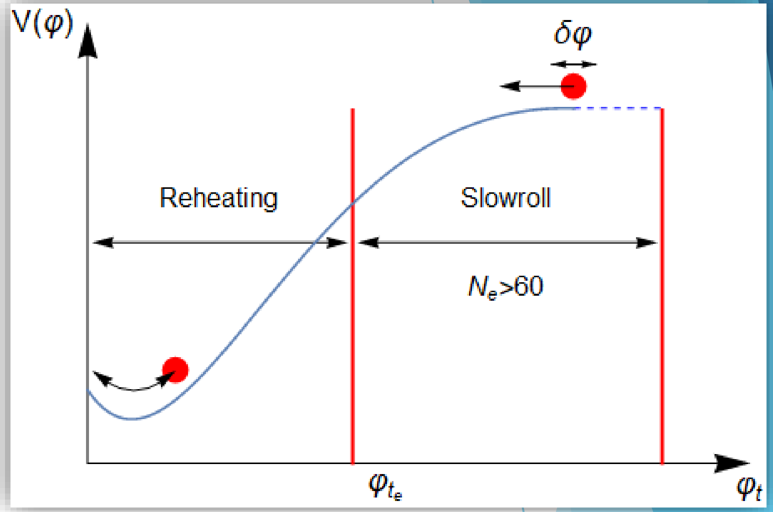

- Scalar PS $\left(\frac{q}{H} \ll 1\right)$: $< \mathrm{R}_p \mathrm{R}_q > = 4\,\pi\, \boxed{q^3 \boldsymbol{\mathcal{P}}_{\mathrm{R}}(q)}\,\delta^{(3)}(q+p),$

$$\boldsymbol{\mathcal{P}}_R(q) \equiv \boldsymbol{\mathcal{P}}_\zeta(q) = \cdots; \quad \boldsymbol{\mathcal{P}}(\mathrm{q}) = \boldsymbol{\mathcal{P}}_{SF}\, q^{n_s-1} \begin{cases} \boldsymbol{\mathcal{P}}_{SF} = \dfrac{H^2}{8\pi^2\, M_{pl}^2 \epsilon} \; (2.4\ 10^{-9}), \\ n_s - 1 = -2\epsilon - \eta \;\; (0.9652). \end{cases}$$

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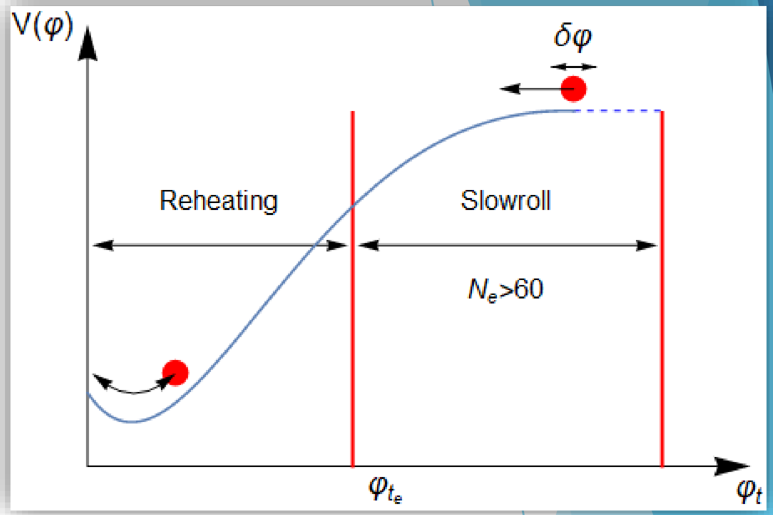

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- Tensor PS $\left(\frac{q}{H} \ll 1\right)$: $h_{ij} = \sum_{s=-2}^{2} \varepsilon_{ij}^{(s)}\, h_k^{(s)}, \qquad < h_p^{(s)}\, h_q^{(r)} >= 4\, \delta_s^r\, \pi \boxed{q^3 \boldsymbol{\mathcal{P}}_{\mathrm{h}}(q)}\, \delta^{(3)}(q+p),$

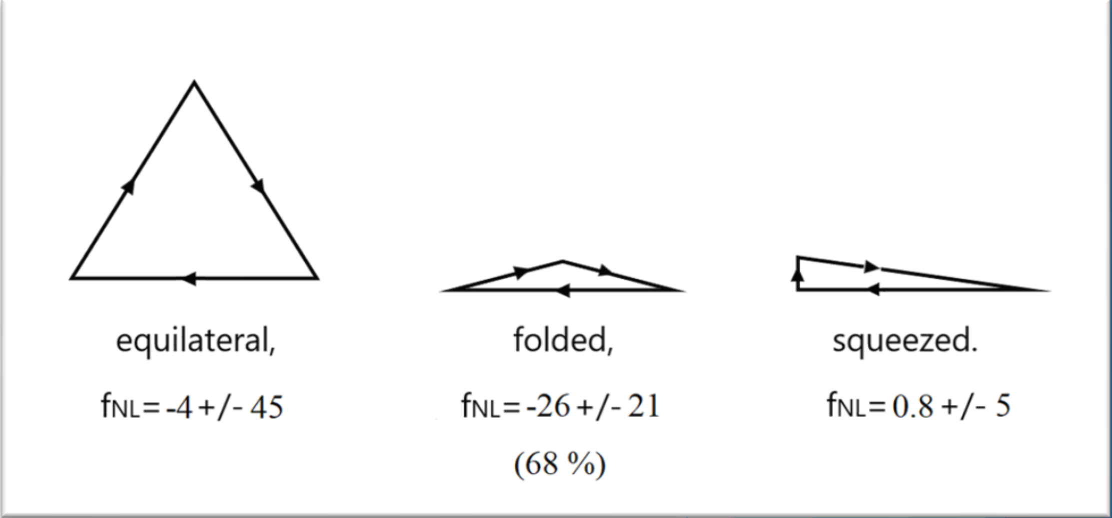
$$\boldsymbol{\mathcal{P}}_h(\mathrm{q}) = r\, \boldsymbol{\mathcal{P}}_{SF}\, q^{n_T - 1} \begin{cases} r = 8\,\epsilon \ \ (\leq 0.06), \\ n_T - 1 = -2\epsilon \ \ (\pm 0.6). \end{cases}$$

Single-field Inflation NG

- Scalar 3-point functions:

$$< \mathrm{R}_{k_1}\mathrm{R}_{k_2}\mathrm{R}_{k_3} >= (2\pi)^3 B_{\mathrm{R}}(k_1, k_2, k_3)\ \delta^{(3)}(k_1 + k_2 + k_3),$$

$$B_{\mathrm{R}} \equiv f_{NL}\frac{6}{5}\left(\sum_{ij-Perm} P_{\mathrm{R}}(k_i)P_{\mathrm{R}}(k_j)\right)$$



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- **Validity of the consistency relation**[b,c]

$$B_R(k_1 = \mathrm{k_L} \ll k_2 \sim k_3) = -\frac{1}{2}(n_s - 1)P_R(k_L)P_R(k_S), \qquad f_{NL}^{sq} = -\frac{5}{12}(n_s - 1)$$

[b] Maldacena, 2003. Non-Gaussian features of primordial fluctuations in single-field inflationary models.

[c] Acquaviva et al. 2003. Second order cosmological perturbations from inflation.

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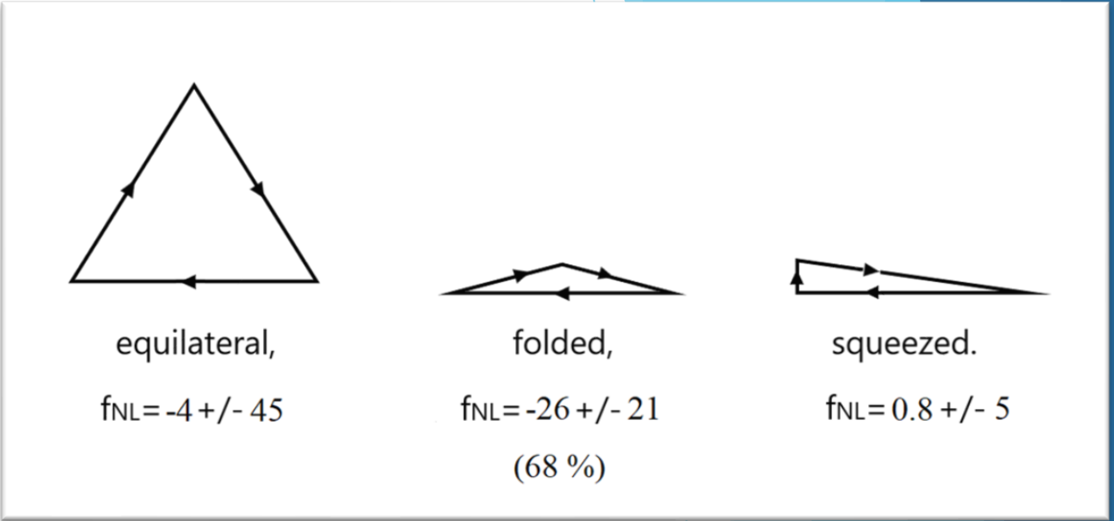

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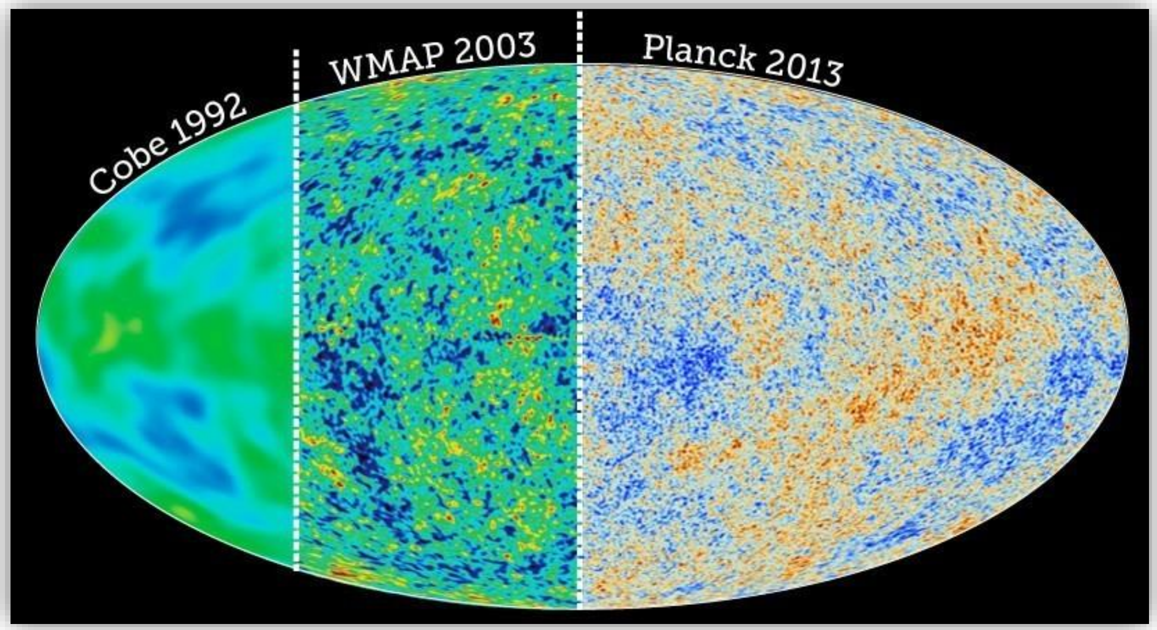
- Gravitons, Graviton-Scalars correlators:

TTT	$< h_p^{(s)}\ h_q^{(r)}\ h_k^{(o)} >$	$f_{TTT} = \frac{B_{TTT}}{P_h(p)P_h(k)}$	SQ: 300 ± 200	$\sim O(\epsilon)\ \varepsilon_{ij}^{s}(k_S)\ \varepsilon_{lj}^{p}(k_S)\ \varepsilon_{li}^{r}(k_L)$
TTS	$< h_p^{(s)}\ h_q^{(r)}\mathrm{R}_k >$	$f_{TTS} = \frac{B_{TTS}}{P_h(p)P_{\mathrm{R}}(k)}$	??	$\sim O(\epsilon)\ \varepsilon_{ij}^{s}(k_S)\ \varepsilon_{ij}^{p}(k_S)$
TSS	$< h_p^{(s)}\mathrm{R}_q\ \mathrm{R}_k >$	$f_{TSS} = \frac{B_{TSS}}{P_h(p)P_{\mathrm{R}}(k)}$	SQ: 90 ± 40	$\sim O(\epsilon)\ \varepsilon_{ij}^{s}(k_L)\ k_S^i k_S^j$

f_{NL} detection…

- $< \frac{\delta T}{T}\frac{\delta T}{T}\frac{\delta T}{T} >$ Improving CMB (LiteBIRD) …



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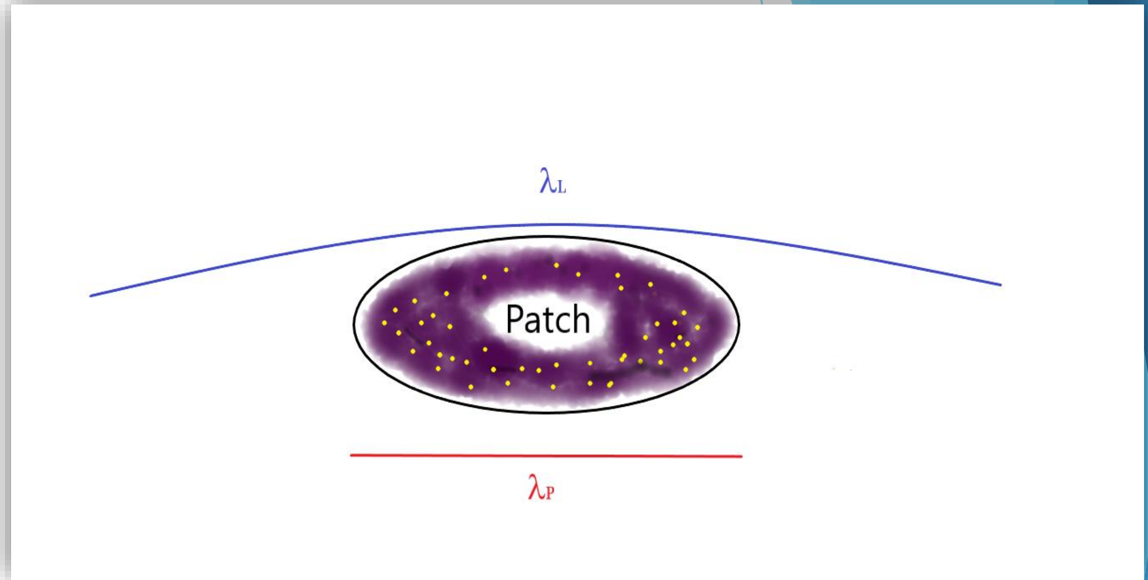

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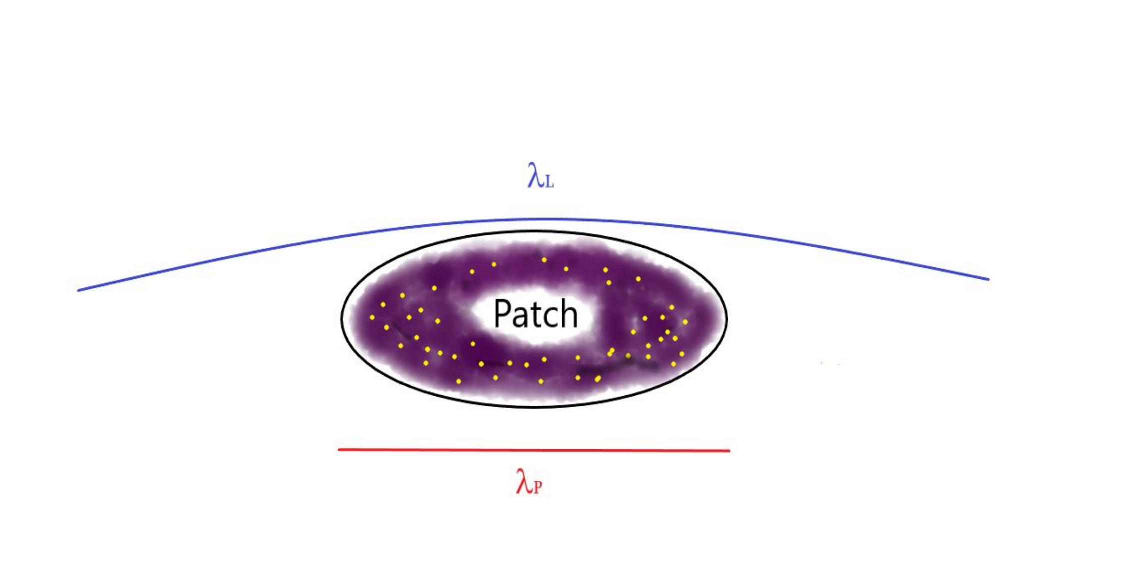

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$$\delta \equiv \frac{\delta\rho}{\rho} \sim (\dots)\left(\nabla^2\Phi - 2\,\Phi\,\nabla^2\Phi + \frac{1}{2}(\nabla\Phi)^2\right)$$

$$\delta g \sim \left(\mathrm{b}_0 + (\dots)\frac{f_\delta^{sq}}{k^2}[d]\right)\delta$$

$$f_\delta^{sq} = -\frac{5}{3} + f_{NL}^{sq}$$

[d] Verde, Matarrese, 2009. Detectability of the effect of Inflationary non-Gaussianity on halo bias

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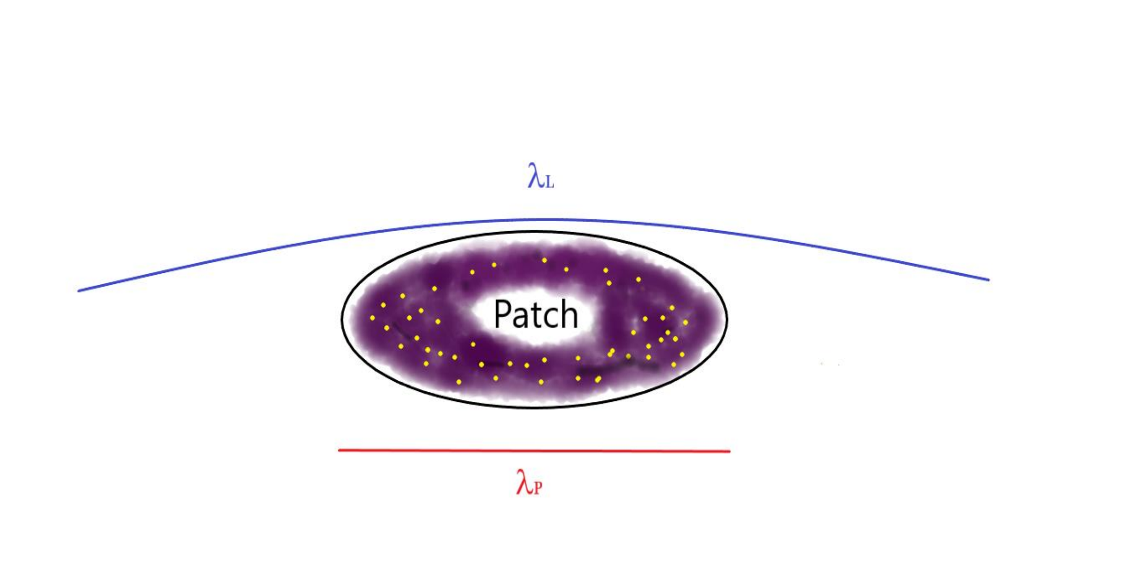

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$$\delta g \sim \left(\mathrm{b}_0 + (\dots)\frac{f_\delta^{sq}}{k^2}[d] + (\dots)f_{TSS}^{sq}\,\varepsilon_{ij}^{(s)}\frac{k^i k^j}{k^2}[e,f]\right)\delta \qquad f_\delta^{sq} = -\frac{5}{3} + f_{NL}^{sq}$$

[d] Verde, Matarrese, 2009. Detectability of the effect of Inflationary non-Gaussianity on halo bias
[e] Jeong, Kamionkowski 2012. Clustering Fossils frome the Early-Universe.
[f] Akhshik, 2015. Clustering fossils in Solid Inflation

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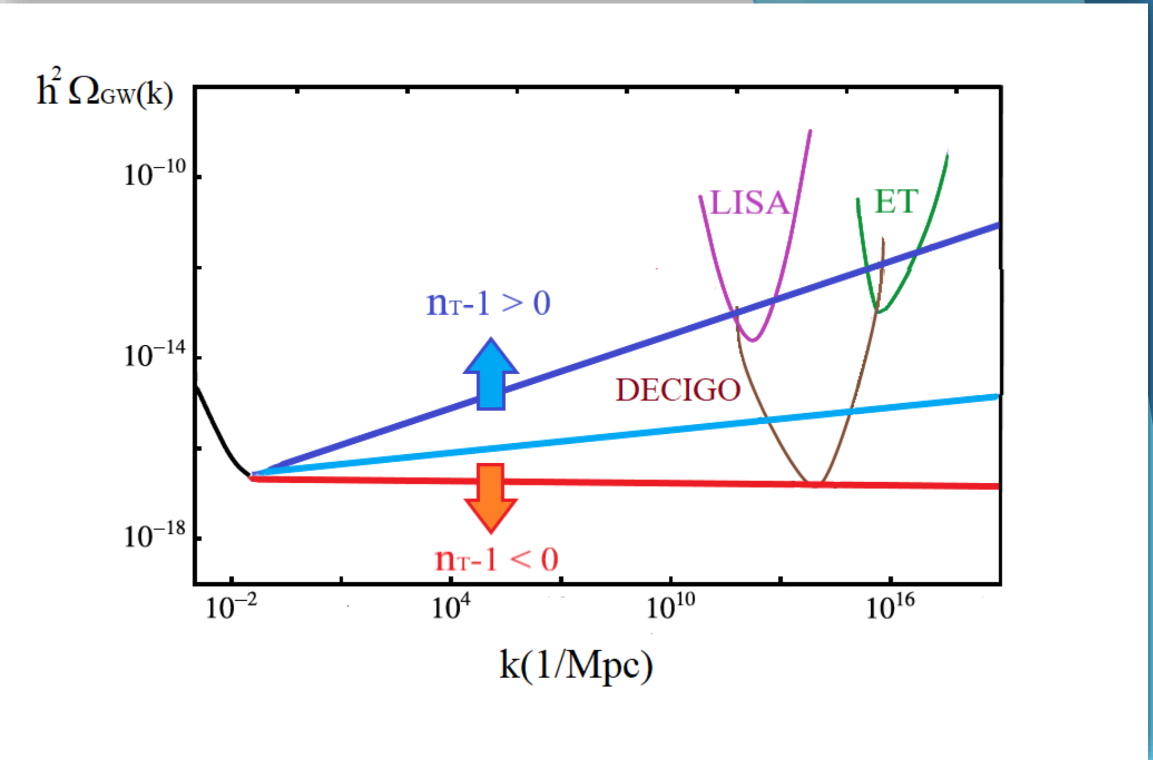
- Detection of GWs background [g] and local non-linear corrections of tensor PS [h][i]

$$\Omega_{GW} \sim k^{n_T - 1} \qquad < h^2 > \Big|_{\mathrm{Non-}Lin.} \sim [1 + (\dots)\, f_{TTS}^{sq}] < \mathrm{h}^2 >$$



[g]: Bartolo et al., 2016. Probing inflation with gravitational waves.
[h]: Malhotra, Dimastrogiovani, Fasiello, Shiraishi, 2020. Cross-correlations as a Diagnostic Tool For PGWs.
[i]: Adshead et al., 2020. Multimessanger Cosmology: Correlating CMB and SGWB measurements.

Question: is the consistency relation trivial?

In literature: The consistency relation can be cancelled with a spatial diff. [l, m, n…]

[l]= Pajer, Schmidt, Zaldarriaga, 2013, The Observed Squeezed Limit of Cosmological Three-Point Functions;

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$$f_{NL}^{sq} = \cancel{-\frac{5}{12}(n_s - 1)}, \qquad f_{\delta}^{sq} = \cancel{-\frac{5}{3} + f_{NL}^{sq}}, \qquad f_{TTT}^{sq}, \qquad \cancel{f_{TTS}^{sq}}, \qquad f_{TSS}^{sq} \cdots$$

Question: is the consistency relation trivial?

In literature: The consistency relation can be cancelled with a spatial diff. [l, m, n…]

The action is a scalar under a spatial diffeomorphism: $x_i' = x_i + \varepsilon_i(x)$

If R is a scalar $\{R'(x') = R(x)\}$ $\Longrightarrow$ $\Delta B_R = B_R'(x_1, x_2, x_3) - B_R(x_1, x_2, x_3) \equiv 0$

The aim of the discussion: to show that R is a good scalar

k=0 world

- Validity of the Cosmological principle: Universe homogeneous and isotropic at k=0;

- so(4,1) algebra: Spatial translations; Spatial rotations;

  Dilatations: $\boxed{x^i \to e^{\lambda} x^i}$

  Special conformal transformations

$$x^i \to x^i + 2b^j x_j x^i - b^i x^2$$

k=0 world

- ***FUNDAMENTAL*** : Let us apply a dilatation, we can impose a gauge redundancy!
- For instance:

Initial gauge: comoving E=v=0

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Final gauge: comoving

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- Intuitive argument:

$$ds^{(3)} = 2\, a^2\, \mathrm{e}^{2\,\mathrm{R}} \mathrm{dx^i dx^j} \to 2 \boxed{\mathrm{e}^{2\lambda} a^2} \mathrm{e}^{-2\,(\mathrm{R}+\lambda)} \mathrm{dx^i dx^j},$$

Local scale factor a’

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- Formally: Sym. Breaking pattern so(4,1) → rotations + translations.

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- Applications:

  -The Weinberg Theorem;

  -The Consistency Relation.

The Weinberg Theorem [o]

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- **Statment** : At sufficiently large scales $k \to 0$, if we have only one scalar DoF (no entropy) and a vanishing anisotropic stress tensor. ζ and *R* are equivalent and conserved in time.
- Matching the k=0 world with the Adiabatic mode (physics)

$$k = 0:\ \Delta g\Big|_{redundant} = \text{Solution}$$

$$k \neq 0:\ \mathrm{k}^{\mathrm{i}}\mathrm{k}^{\mathrm{j}}(\Psi_{\mathrm{k}} - \Phi_{\mathrm{k}}) = 0$$

$$\Psi_{\mathrm{k}} \equiv \Phi_{\mathrm{k}}$$

$$\lambda = \lim_{k \to 0} R_k = \int d^3k\ \delta^{(3)}(k)\ R_k$$

- We can extract physics from a gauge redundancy!

The Consistency Relation

[p]: Creminelli, Norena, Simonovich. Conformal consistency relation for single-field inflation. 2012 .
[q]: Hinterbichler et al. An Infinite Set of Ward Identities for Adiabatic Modes in Cosmology. 2014 .
[r]: Hinterbichler et al. Conformal Symmetries Adiabatic Modes in Cosmology. 2012 .

- Single-field: Spontaneous breaking of so(4,1) global symmetries [p][q][r]

de Sitter: so(4,1) $\rightarrow$ rotations + translations.

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- Dilatation is a symmetry non-linearly realized. This implies a Norther current and charge:

$$Q = \int dx^3 \{P_R, \delta R_\lambda\};$$

- Using Ward identities, one can extract the consistency relation[q]:

$$\lim_{k\to 0}\langle R_k R_{k_1} R_{k_2}\rangle = \frac{1}{2} P(k)\left[3 + \sum_{a=1}^{2} k_a\, \partial_{k_a}\right]\langle R_{k_1} R_{k_2}\rangle$$

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de Sitter: so(4,1) → rotations + translations.

- Dilatation is a symmetry non-linearly realized. This implies a Norther current and charge:

$$Q = \int dx^3 \{P_R, \delta R_\lambda\};$$

- Using Ward identities, one can extract the consistency relation[q]:

$$\lim_{k\to 0}\langle R_k R_{k_1} R_{k_2}\rangle = \frac{1}{2}\boxed{P(k)}\left[3 + \sum_{a=1}^{2} k_a\,\partial_{k_a}\right]\langle R_{k_1} R_{k_2}\rangle$$

$$\boxed{0???}$$

- Applying a second dilatation:

$$\boxed{R_k \to R_k - \lambda = 0}$$

Outside k=0 world: a deformed dilatation..

- Are the CFC-like transformations purely constant dilatations?

- Take a deformed dilatation $\lambda_k = W_{k_L}(k)\, R_k, \qquad x^i = (1+\lambda)x^i,$

- λ_{CFC} structure:

$$\lambda_{CFC} = \sum_n \alpha^{(n)} \left[k^{i_1} \dots k^{i_n} \partial_{k_{i_1}} .. \partial_{k_{i_n}}\right] R_k$$

Deformed dilatation: the first order

- A standard gauge transformation: $\Delta g_{ij} = -a^2\left[2\lambda\delta_{ij} + x^j\partial_i\lambda + x^i\partial_j\lambda\right]$
- Basic element $$x^i\partial_j\lambda = \frac{-1}{(2\pi)^{3/2}}\int dk^3\, e^{ikx}\partial_{k^i}\left(k^j\lambda_k\right) + BT.$$
- Final result $$\Delta g_{ij} = 2a^2\frac{k^ik^j}{k}\partial_k\lambda_k$$

$$\Delta R_k = 0, \qquad \Delta E_k = \frac{-2}{k}\partial_k\lambda_k$$

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- A standard gauge transformation: $\Delta g_{ij} = -a^2\left[2\lambda\delta_{ij} + \boxed{x^j\partial_i\lambda + x^i\partial_j\lambda}\right] \longrightarrow 0??$

  Instead of $\Delta g_{ij} = -a^2[2\lambda\,\delta_{ij}]$ ...

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- Basic element $x^i\partial_j\lambda = \frac{-1}{(2\pi)^{3/2}}\int dk^3\, e^{ikx}\boxed{\partial_{k^i}(k^j\lambda_k)} + BT.$ $\longrightarrow \sim \lambda_k$

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- Final result

$$\boxed{\Delta g_{ij} = 2a^2\frac{k^ik^j}{k}\partial_k\lambda_k \qquad \Delta R_k = 0, \qquad \boxed{\Delta E_k = \frac{-2}{k}\partial_k\lambda_k}}$$

A gauge change!

A DISCONTINUITY IN THE GRADIENT EXPANSION!

Non-linear deformed dilatation: $\Delta B_R = 0??$

- In [a] we give two independent demonstrations:

  -1) Using the in-in formalism;

  -2) Using field redefinitions.

$$\Rightarrow \Delta B =< R'(x)^3 >-< R(x)^3 >\equiv \mathrm{BT} = 0.$$

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- We solved the halo bias scale dependence [s][t]

$$\Delta b(k)\; k^2 \propto f_\delta^{sq} = -\frac{5}{3} + \mathrm{f_{NL}} \cancel{\rightarrow \mathbf{0}}$$

- **Such effects are physical and observable in principle by future high-sensitivity experiments!**

[s]: Cabass, Pajer, Schmidt, 2018. Imprints of oscillatory Bispectra on Galaxy Clustering.

[t]: de Putter, Dorè, Green, 2015. Is there scale-dependent bias in single-field inflation?

Future developments

- The ambiguity is quite diffused…

  - In [a] we analyzed the scalar sector only

$$x^{j\prime} = e^{\lambda}\, x^{j} + \omega^{j}_{\ l} . x^{l} \quad \textit{Spin-2} \text{ [o]}$$

k=0: $h'_{ij} = h_{ij} - \omega_{ij} \rightarrow 0$ [l,h,i]

k $\neq$ 0: $h'_{ij} - h_{ij} = 0,$

[o]: S. Weinberg, Adiabatic modes in Cosmology. 2003.

[l]: Pajer et al., 2013, The Observed Squeezed Limit of Cosmological Three-Point Functions;

[h]: Dimastrogiovanni et al., 2015.Inflationary tensor fossils in LSS.

[i]: Adshead et al., 2020. Multimessanger Cosmology: Correlating CMB and SGWB measurements.

Thank you for your attention!