Topology vs. interaction –

the strong, the weak and the fragile

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Outline:

1. The context:

    A system of non-interacting electrons, where topology forces a gapless spectrum.

    The question:

    Can interaction – in principle - gap the spectrum?

2. Two warm-up examples – chiral edge state of the IQHE (No way), and a Dirac cone on the surface of a 3D topological insulator (yes, but it will cost you).

3. The main subject – can interactions gap Dirac cones protected by fragile topology?
    - Motivation – what is fragile topology? Relevance to twisted bi-layer graphene.
    - Gapping the Dirac cones by adding another trivial band.
    - Gapping by interactions.

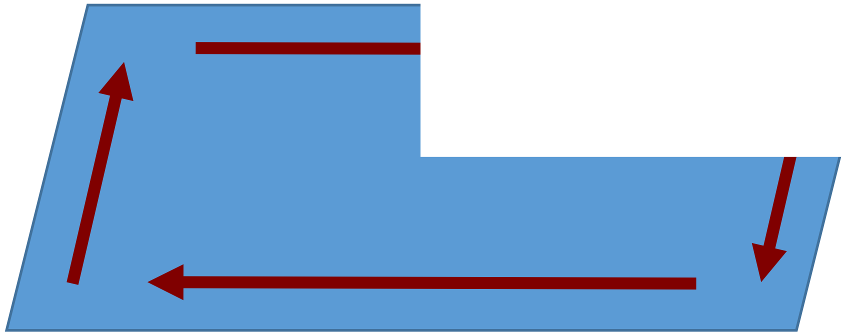
Can interactions gap a spectrum that topology wants to keep gapless?

Motivation:

1. As a matter of principle
2. Magic angle twisted bi-layer graphene at the charge neutrality point

1st example:

- Integer quantum Hall effect (or Chern bands)
- Hall conductivity is quantized to integer units
- Gapless chiral modes propagate on the edge



Weak interactions – no effect

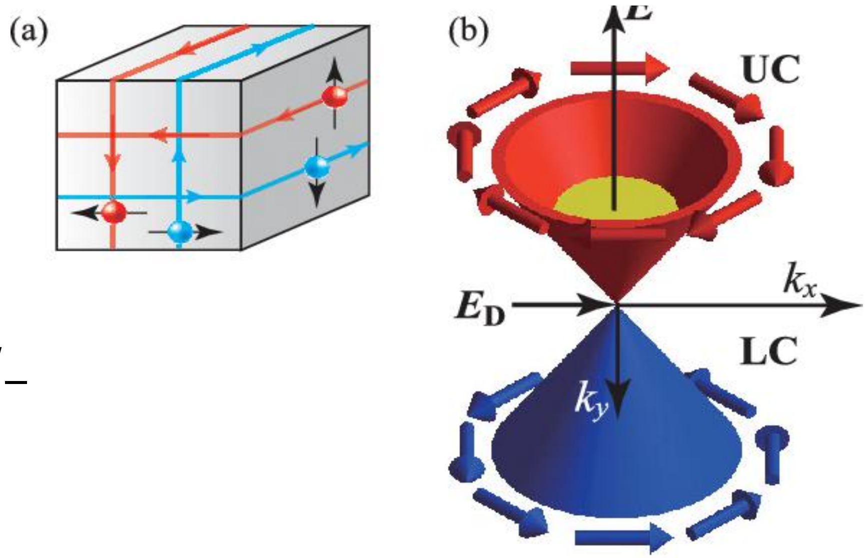
Strong interactions – transition to the world of fractionalized states

2^{nd} example

- Three dimensional topological insulator

- A three dimensional band with time reversal symmetry $\mathcal{T}_-$

  anti-unitary, $\mathcal{T}_-^2 = -1.$

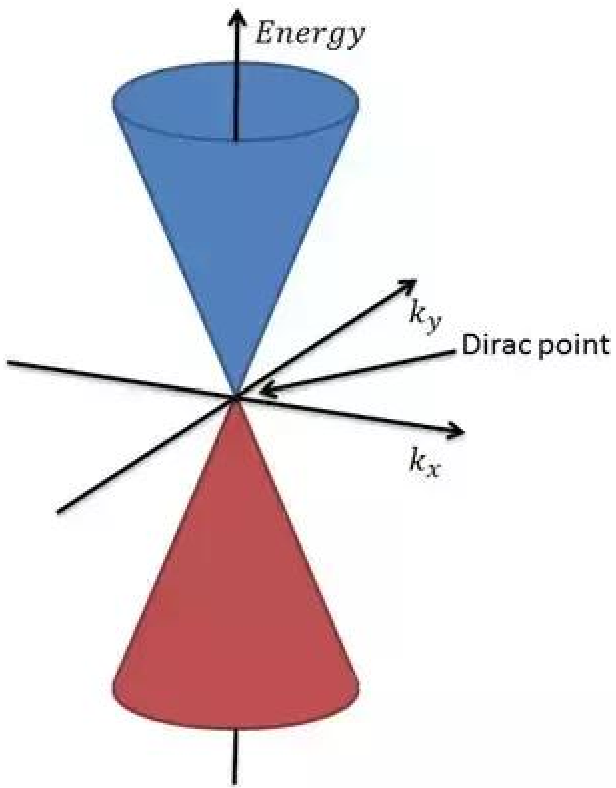


A gapless Dirac cone on each surface

Can interactions gap the surface Dirac cone, without breaking the protecting symmetries (neither explicitly nor spontaneously)?

The main player – the Dirac cone

$$H = \pm \underbrace{k_x \sigma_x + k_y \sigma_y}_{\text{Dirac point}} + \underbrace{M(k)\,\sigma_z}_{\text{Mass term}}$$

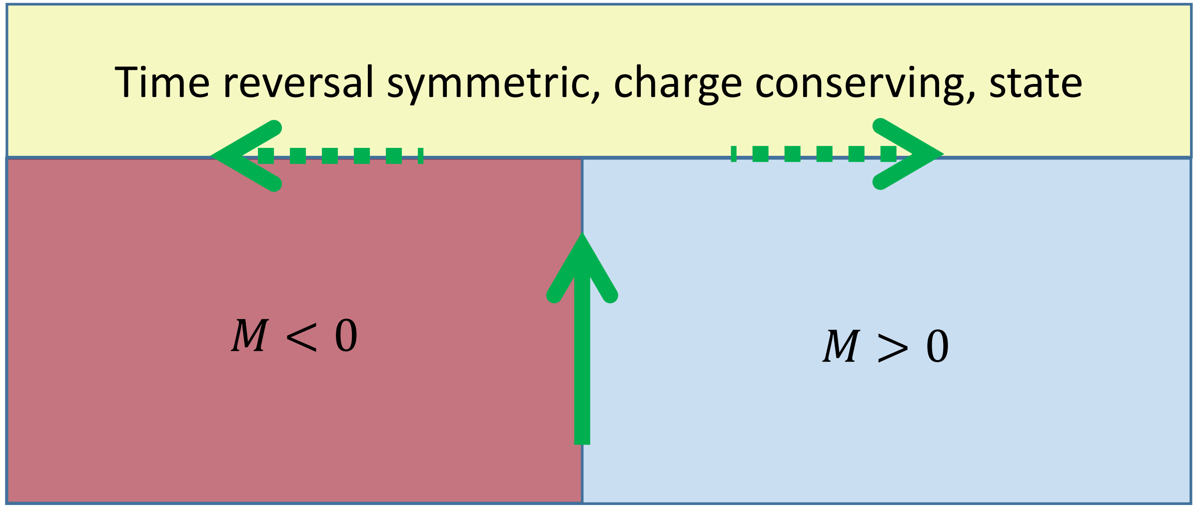


$$M(k) = m_0 + \alpha k^2$$

Berry curvature near $k = 0$ carries half of e^2/h.

Can interactions gap the surface Dirac cone, without breaking the protecting symmetries (neither explicitly nor spontaneously)?

Yes, but the resulting state must have topological order (Levin et al, 2011, Mross et al. 2015)



A gapped symmetric topologically ordered state may be constructed, in principle.

(Bonderson, Wang, Chen, Metlitsky et al. 2013)

3Rd example – the focus of this talk

Can interactions gap the topologically protected gapless spectrum of topologically fragile bands?

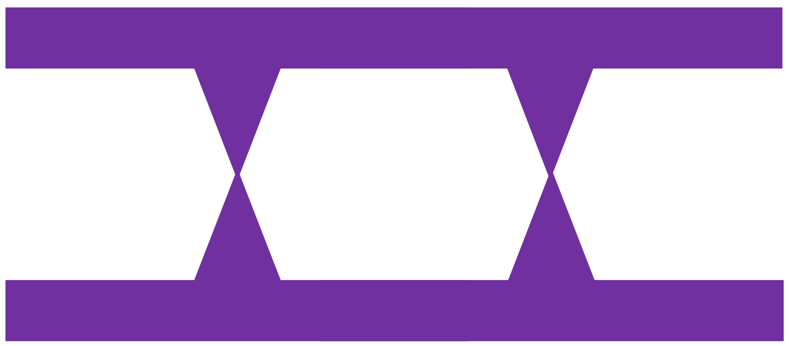
If so, is topological order essential?

What are fragile bands? (Watanabe, Bernevig, Dai, Slager…et al)

A pair of bands that are attached to one another by two Dirac cones, protected by a crystalline symmetry.

- A stand alone 2D band structure.
- Two Dirac cones in graphene – may be merged to gap.

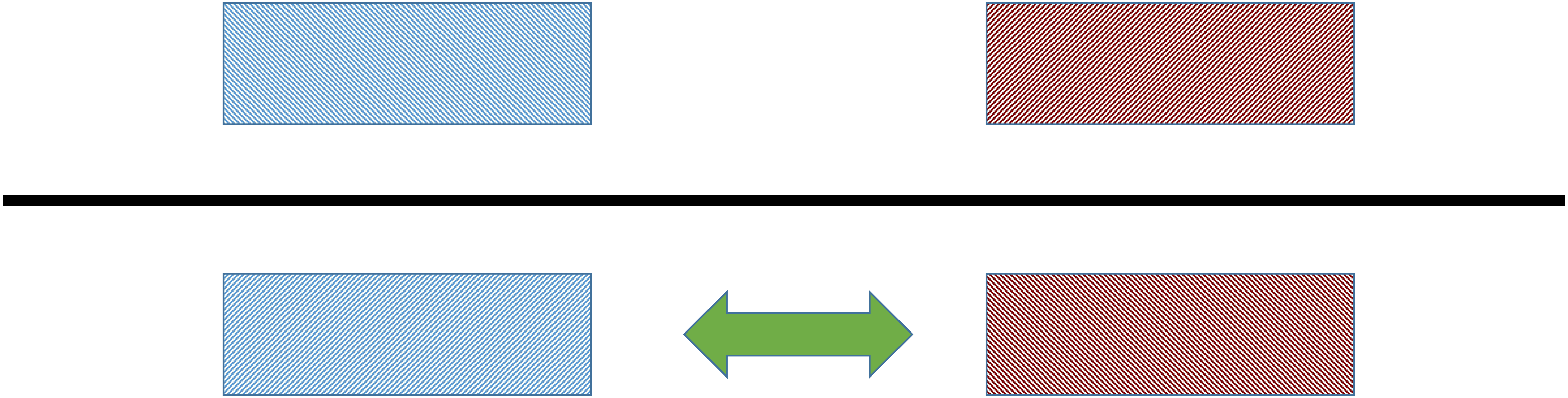
Here – Cannot be gapped by being merged, since they carry the same chirality.



What are fragile bands?

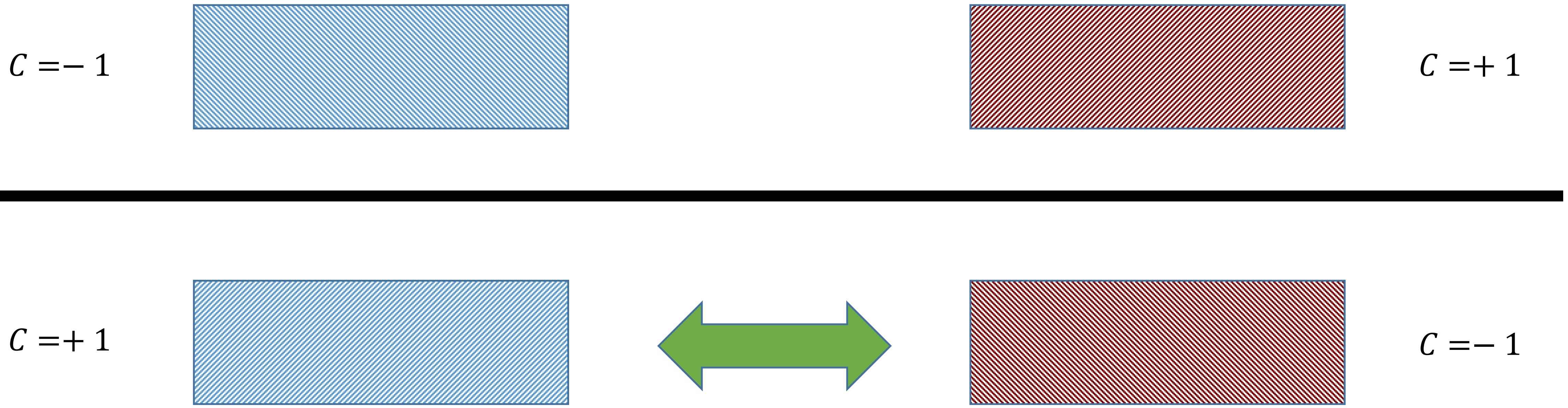
The “Stern conjecture”:
There is no end to the richness and beauty of physics originating from the quantum Hall effect.

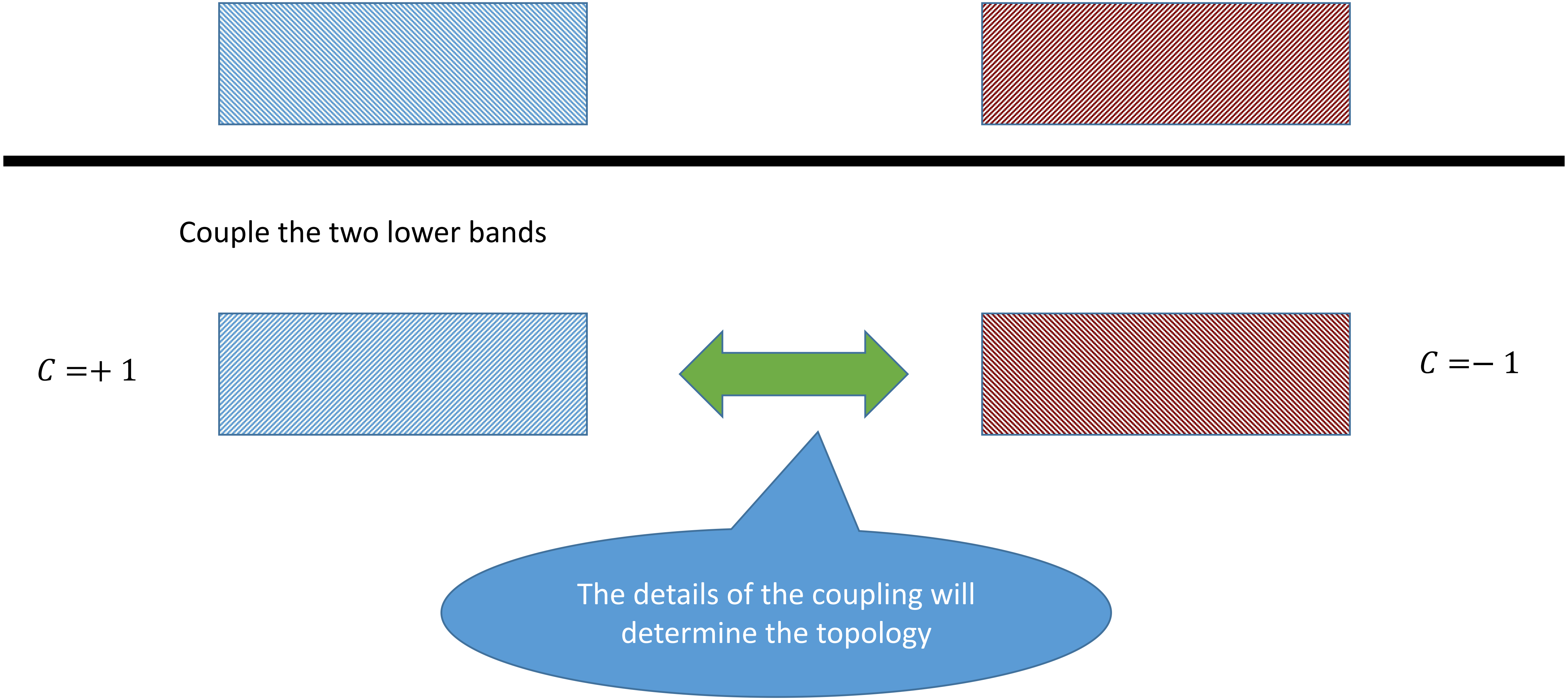
Take two pairs of Chern bands:



Constructing a pair of fragile bands

Take two pairs of Chern bands:




Couple the two lower bands
$C = +1$
$C = -1$
The details of the coupling will determine the topology

The properties of one pair of Chern bands –

- Each band carries a quantized Hall conductivity $\pm C \frac{e^2}{h}$

- The wave function is a two-component spinor, covers the entire Bloch sphere.

- No gauge choice in which the wave function is smooth.

$$H = (m - \cos k_x - \cos k_y)\sigma_z \ \pm \ \sin k_x \sigma_x \ + \ \sin k_y \sigma_y$$

The essence may be seen close to the Γ-point.

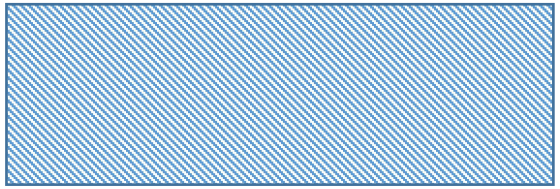
$$H = \underbrace{(m - 2 + \frac{k^2}{2})}\sigma_z \ \pm \ \underbrace{k_x \sigma_x \ + \ k_y \sigma_y}$$

Mass term
Relative sign at $k = 0, \infty$
determines $C = 0, \pm 1$

Dirac point
The relative sign determines
the sign of the Chern
number

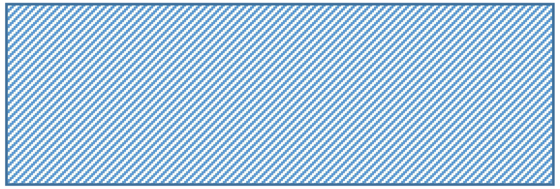
$$\psi(k) = \begin{pmatrix} \cos \frac{\theta(k)}{2} \\ \sin \frac{\theta(k)}{2} \ e^{i\phi(k)} \end{pmatrix}$$

The properties of one pair of Chern bands –



- Each band carries a quantized Hall conductivity $\pm C \, \frac{e^2}{h}$

- The wave function is a two-component spinor, covers the entire Bloch sphere.

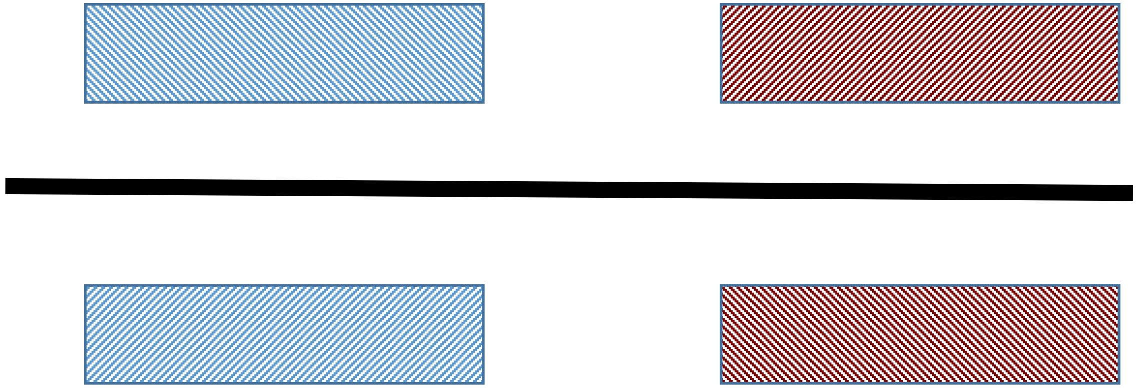


- No gauge choice in which the wave function is smooth.

$$\psi(k) = \begin{pmatrix} \cos\theta(k)/2 \\ \sin\theta(k)/2 \, e^{i\phi(k)} \end{pmatrix}$$

The two lower bands:

$$H = (m - \cos k_x - \cos k_y)\sigma_z \quad + \quad \sin k_x \sigma_x \tau_z \quad + \quad \sin k_y \sigma_y$$



The properties of two coupled Chern bands of opposite Chern number

- No quantized Hall conductivity.
- No total "vorticity" of the wave function
- The outcome depends on the symmetries obeyed by the coupling

If the coupling satisfies time reversal symmetry $\mathcal{T}_-$, we get a 2D topological insulator

The case here:

A system that satisfies a $C_2\mathcal{T}_+$ symmetry.

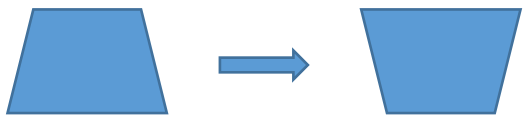
What is $C_2\mathcal{T}_+$ symmetry?

$\mathcal{T}_+$: In real space, it is complex conjugation × a local operation

In momentum space also maps $k \leftrightarrow -k$

C_2 : In real space maps $r \leftrightarrow -r$

In momentum space maps $k \leftrightarrow -k$



The product $C_2\mathcal{T}_+$ is local in momentum space, is anti-unitary, and $(C_2\mathcal{T}_+)^2 = +1$

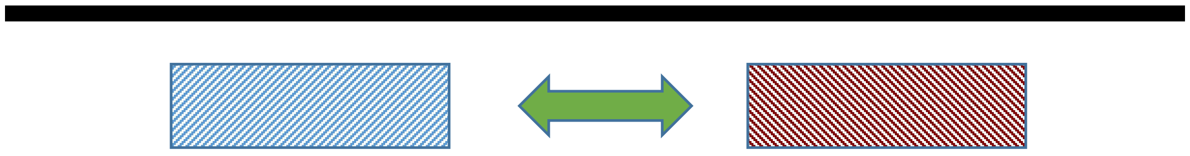
It is capable of keeping a Dirac cone from gapping.

- Being anti-unitary, $C_2\mathcal{T}_+$ symmetry forbids one Pauli matrix in the subspace of the two lower bands.

- Span the subspace by $\eta_0, \eta_x, \eta_y, \eta_z$.   Forbid η_y.

  The Hamiltonian is real

$$h_0(k) + \begin{pmatrix} h_2(k) & h_1(k) \\ h_1(k) & -h_2(k) \end{pmatrix}.$$



- If a Dirac point exists, no mass term can be introduced to gap it.

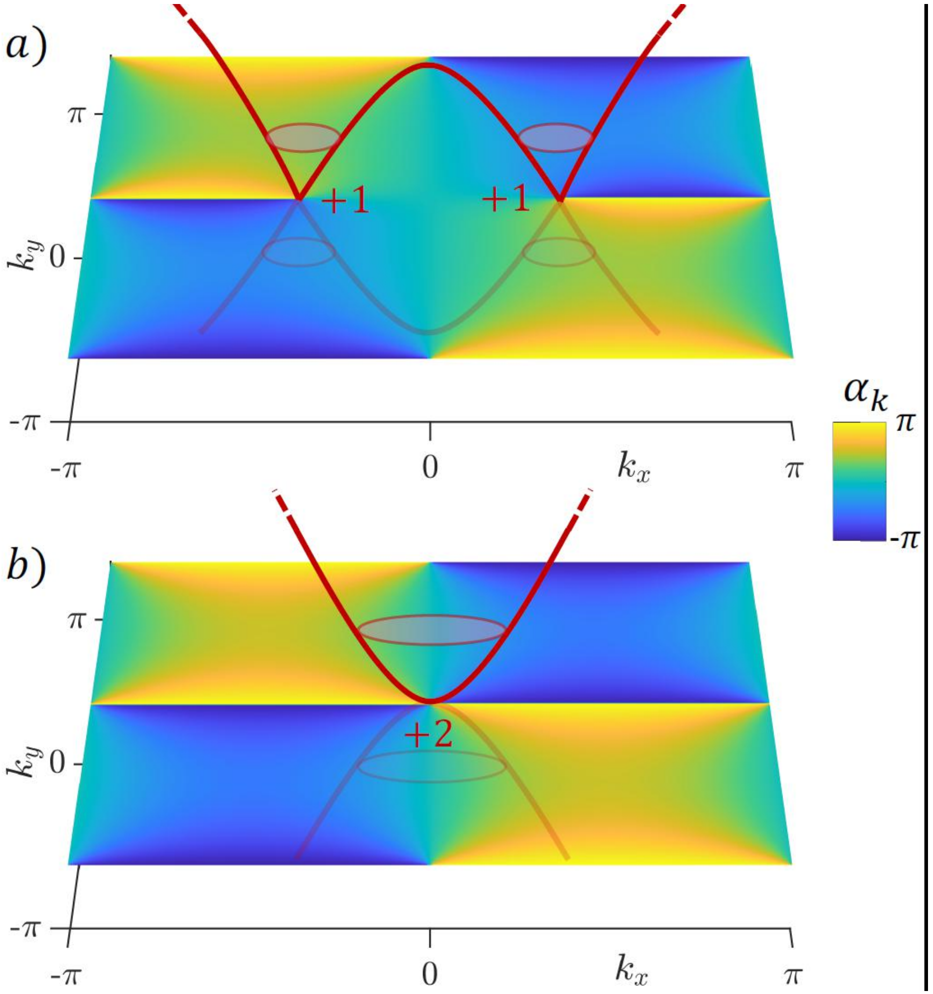
- Winding points of the $(h_1(k), h_2(k))$ vector around a point k_0 - may originate from two sources:

  - $h(k_0) = 0$ – Dirac point
  - Singularity in the basis vectors of the two lower bands.

$\Longrightarrow$ When the pair of bands carries opposite Chern numbers, Dirac points are unavoidable. May be moved around, may be merged to a quadratic band touching.

The two Dirac cones may be merged, but not gapped.

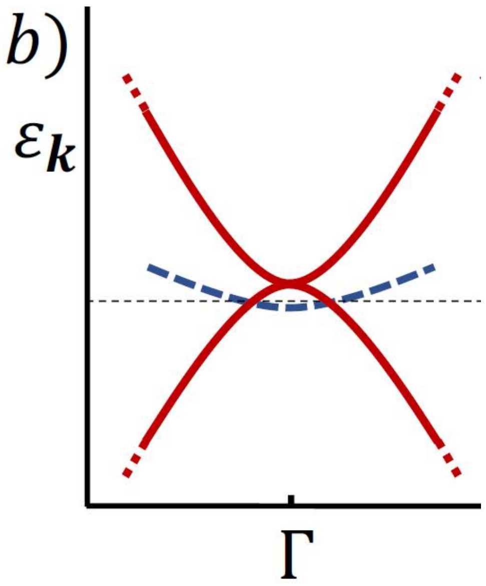
We consider them merged to a quadratic band touching.

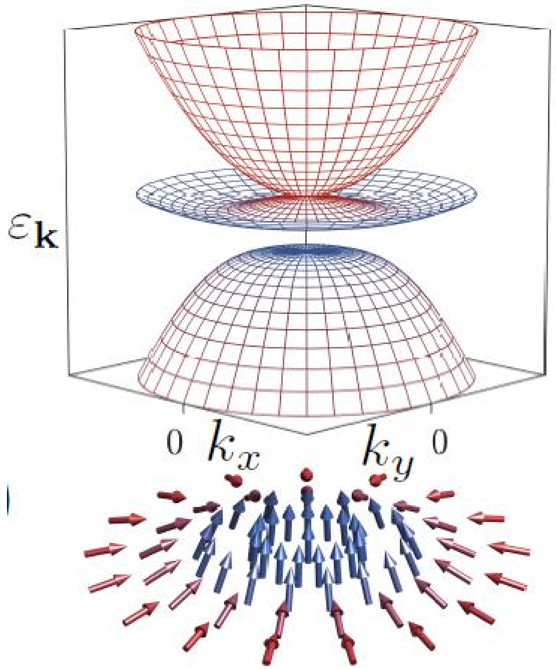


How can the Dirac points be gapped:

- Breaking of $C_2\mathcal{T}_+$ - allows for a mass term
- Adding a third band (even if trivial !)

$$H = \begin{pmatrix} \frac{k^2}{2m_1}\cos 2\varphi & \frac{k^2}{2m_1}\sin 2\varphi & k\cos\varphi \\ \frac{k^2}{2m_1}\sin 2\varphi & -\frac{k^2}{2m_1}\cos 2\varphi & k\sin\varphi \\ k\cos\varphi & k\sin\varphi & \epsilon_0 + k^2/m \end{pmatrix}$$





And now turn interactions on

The rules of the game:

We are allowed to:

- Change the band structure at the non-interacting level, keeping all symmetries.
- Introduce all interactions we need, avoiding spontaneous symmetry breaking.

We need interactions to

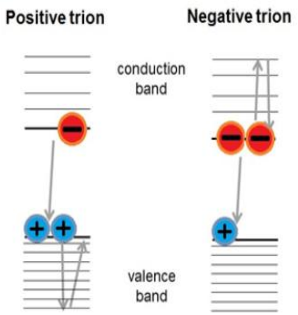
- form a new band of low-momentum fermions all constructed from our available two bands
- To make these new fermions interact with the electrons close to the Γ point, and gap them.

A source of inspiration – bound states in semi-conductors

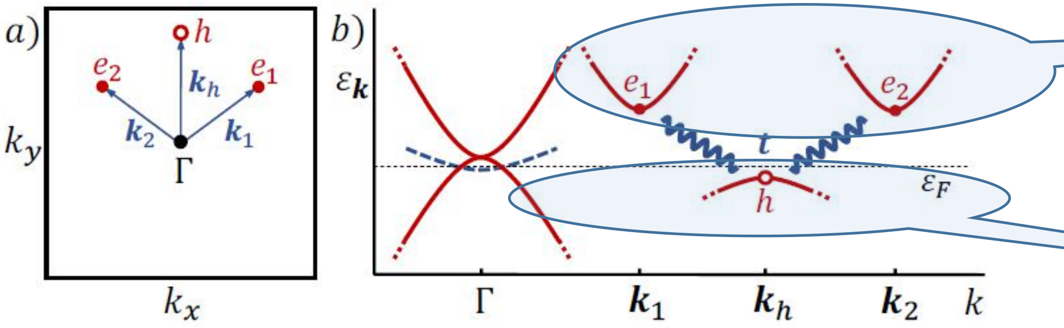
1. Excitons – one electron + one hole, bosonic, not what we want

$$\epsilon_{ex} = \epsilon_e + \epsilon_h - \epsilon_b$$

2. Trions – 2 electrons + one hole



Change the band structure to:

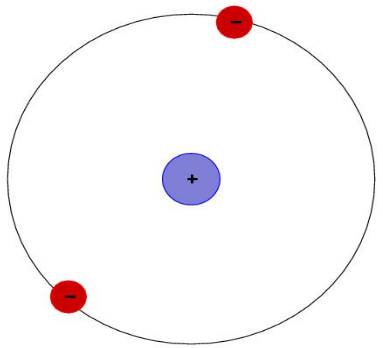


Two electron pockets

One hole pocket

The trion forming interaction

$$-u \sum_{i=1,2} \int dr\, \psi_h^+(r)\psi_h(r)\psi_{e,i}^+(r)\psi_{e,i}(r)$$



Trion energy $2\epsilon_e + \epsilon_h - 2\epsilon_b$

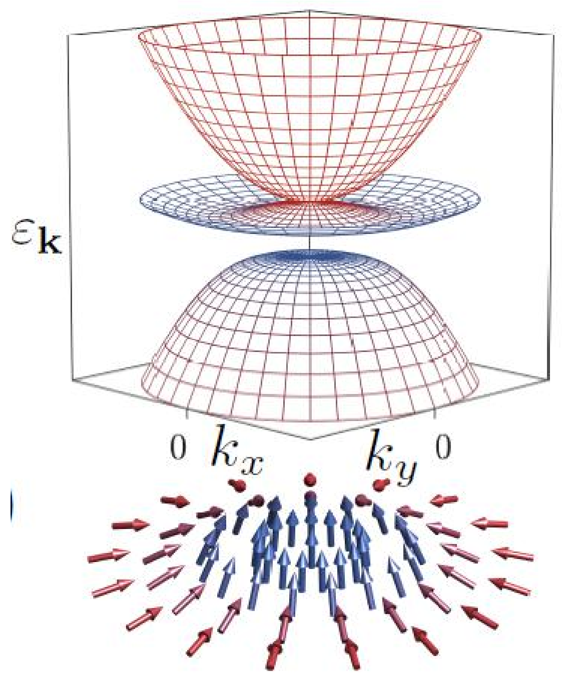
We need the trion energy to be negative, and the exciton energy to be positive, so

$$\epsilon_e + \frac{1}{2}\epsilon_h < \epsilon_b < \epsilon_e + \epsilon_h$$

Need a threshold level of interaction

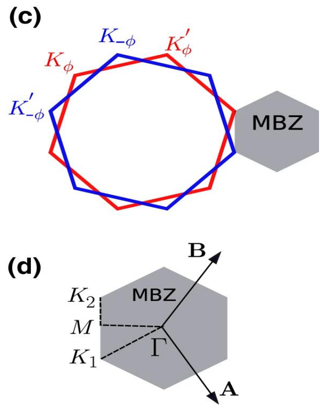
Coupling the trions to the electrons

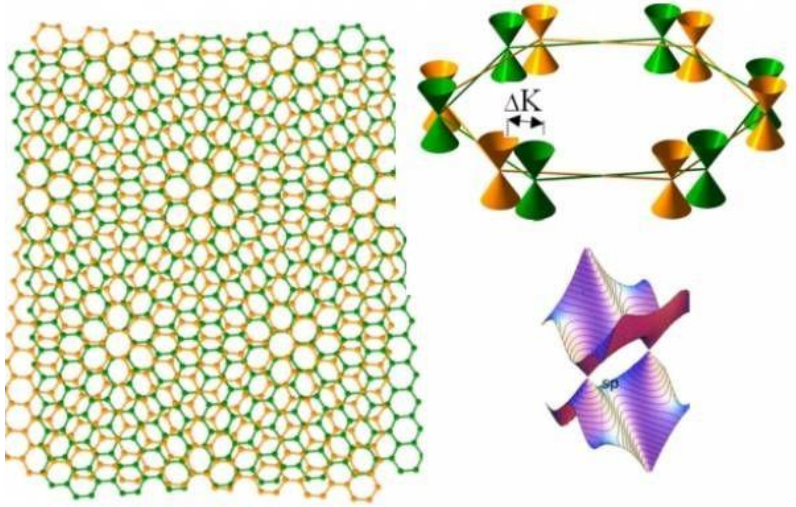
$$t^\dagger(\boldsymbol{r}_\mathrm{t}) = \int d^2r_{\mathrm{e},1} d^2r_{\mathrm{e},2}\, F^\star(\boldsymbol{r}_{\mathrm{e},1} - \boldsymbol{r}_\mathrm{t}, \boldsymbol{r}_{\mathrm{e},2} - \boldsymbol{r}_\mathrm{t}) \times \psi^\dagger_\mathrm{h}(\boldsymbol{r}_\mathrm{t}) \psi^\dagger_{\mathrm{e},1}(\boldsymbol{r}_{\mathrm{e},1} - \boldsymbol{r}_\mathrm{t}) \psi^\dagger_{\mathrm{e},2}(\boldsymbol{r}_{\mathrm{e},2} - \boldsymbol{r}_\mathrm{t}),$$



$$H = \begin{pmatrix} \frac{\mathrm{k}^2}{2\mathrm{m}_1} \cos 2\varphi & \frac{\mathrm{k}^2}{2\mathrm{m}_1} \sin 2\varphi & \cos\varphi \\ \frac{\mathrm{k}^2}{2\mathrm{m}_1} \sin 2\varphi & -\frac{\mathrm{k}^2}{2\mathrm{m}_1} \cos 2\varphi & \sin\varphi \\ \cos\varphi & \sin\varphi & \epsilon_0 + k^2/m \end{pmatrix}$$

Twisted bi-layer graphene





Our gapping procedure

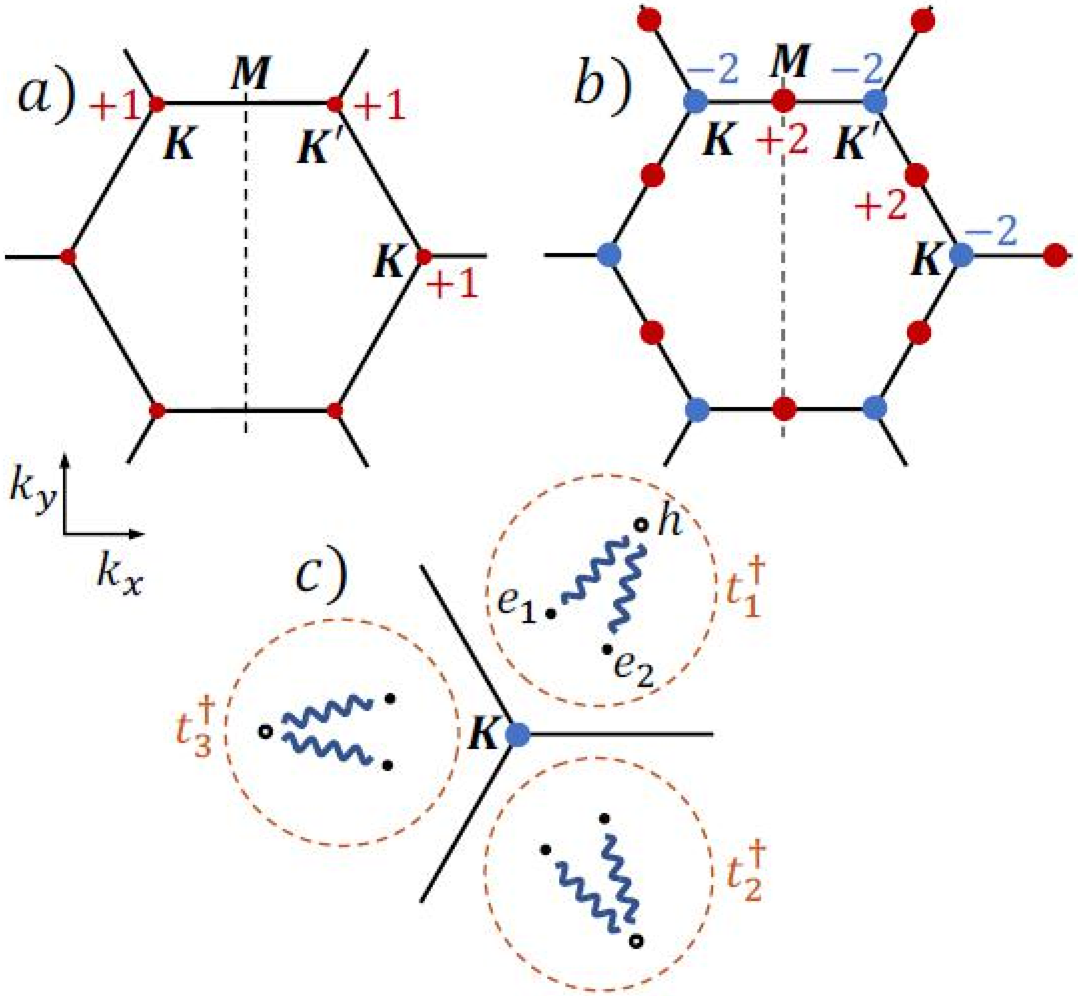
At the single particle level:

- Merge the two Dirac cones to a quadratic band touching
- Deform the band to have two electrons and one hole valleys

At the interaction level:

- Form a trion band
- Couple the trion band to the electrons

For a twisted bi-layer graphene, the Dirac points are pinned by C_3 symmetry



Nucleating pairs of Dirac cones of opposite chiralities at the M- points allow for turning the K, K' points into quadratic touching points at the cost of introducing quadratic touching points at M.

Summary:

1. Fragile topology lives up to its name – a pair of fragile bands may be gapped by interactions.
2. Put differently, a featureless insulator may be constructed in a half filled pair of fragile bands.